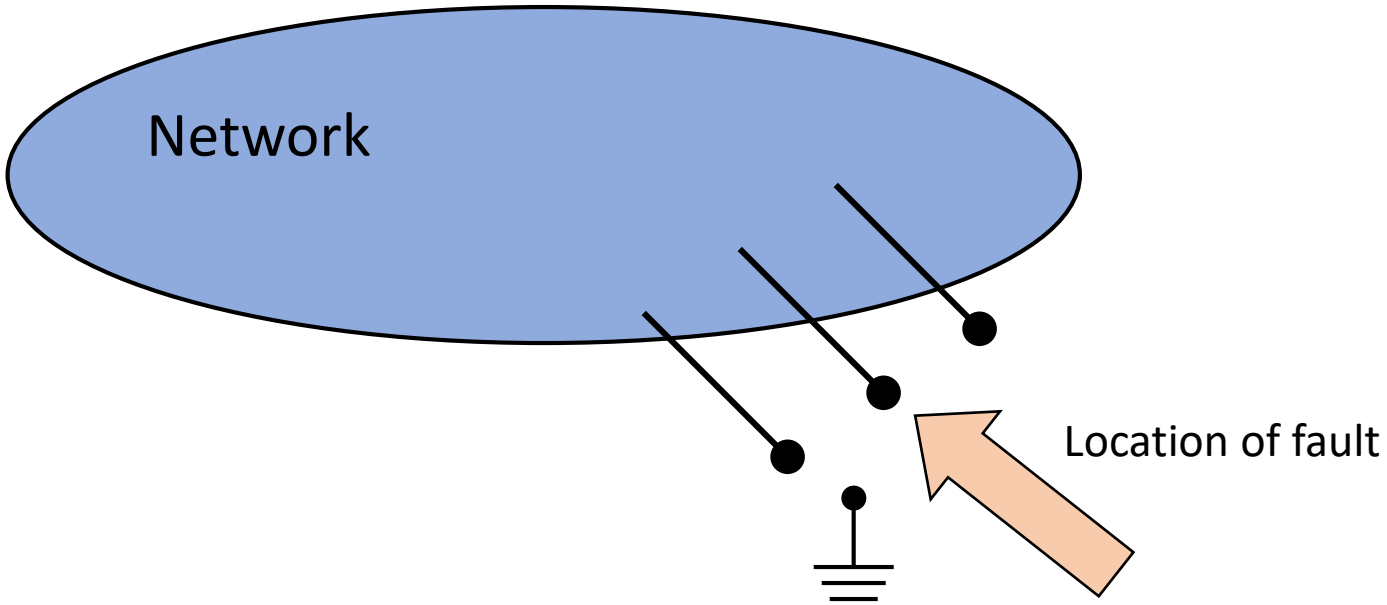
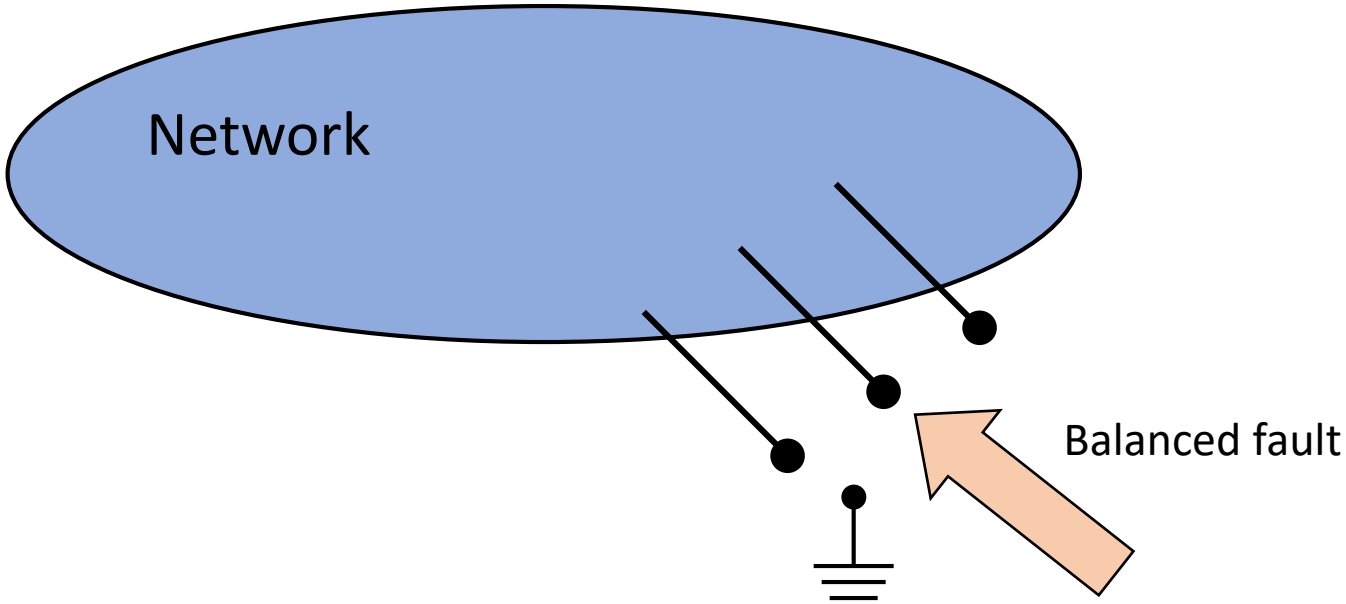
Fault current calculations

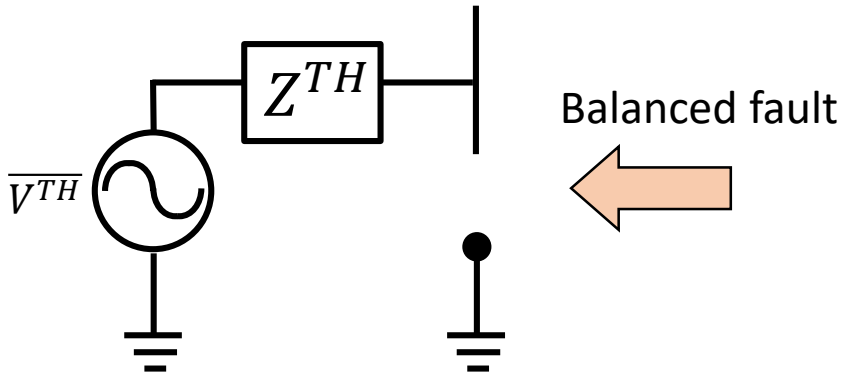


- Prior to the fault:
  - Currents stay in the intact network
- During the fault:
  - Currents flow from the network into the ground or into another phase of the network
- To calculate these currents, we need a model of the network as seen from the fault

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Balanced fault current calculations

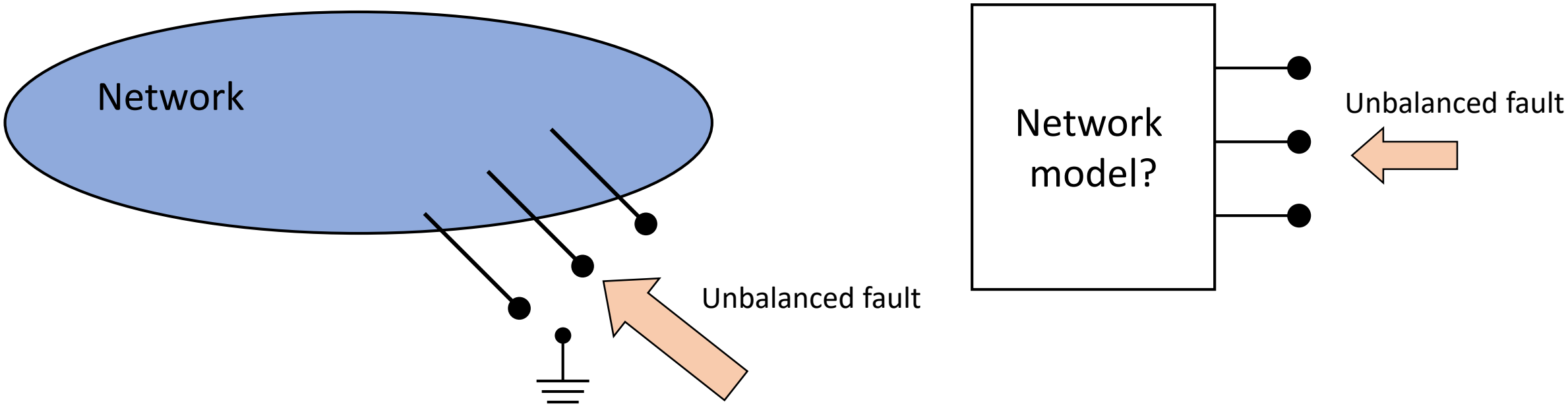




- A balanced three-phase fault affects all three phases equally
- It does not break the three-phase symmetry of the system
- The fault can be analyzed using a single-phase Thevenin equivalent

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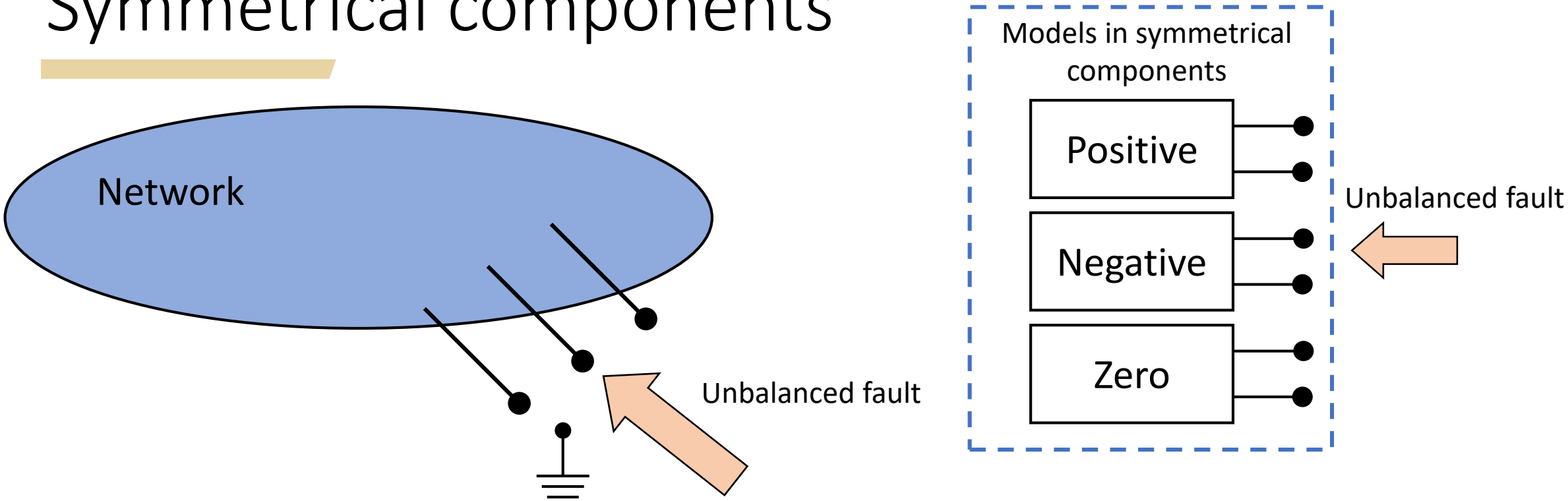
Unbalanced fault current calculations



- Unbalanced faults do not affect all three phases equally
- They break the three-phase symmetry of the system
- Unbalanced faults can not be analyzed using a single-phase model
- Analyzing unbalanced faults in three-phase variables provides little physical insight

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Symmetrical components



- Transform variables from three-phase to symmetrical components
- Decouples three-phase network model into three decoupled models
- Each model represents a symmetrical component
- Using these models simplifies analysis of unbalanced faults

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a notation

$a = 1\angle 120° = -0.5 + j0.866$

Multiplying a phasor by a rotates it by 120°

$a^2 = 1\angle 240° = -0.5 - j0.866$

Multiplying a phasor by a^2 rotates it by 240°

$a^3 = 1\angle 360° = 1\angle 0° = 1$

Multiplying a phasor by a^3 leaves it unchanged

$1 + a + a^2 = 0$

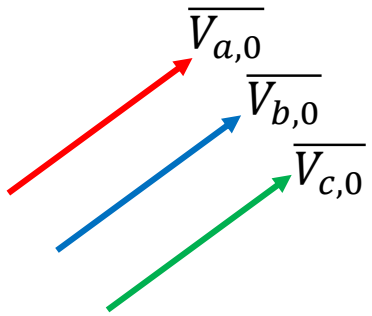
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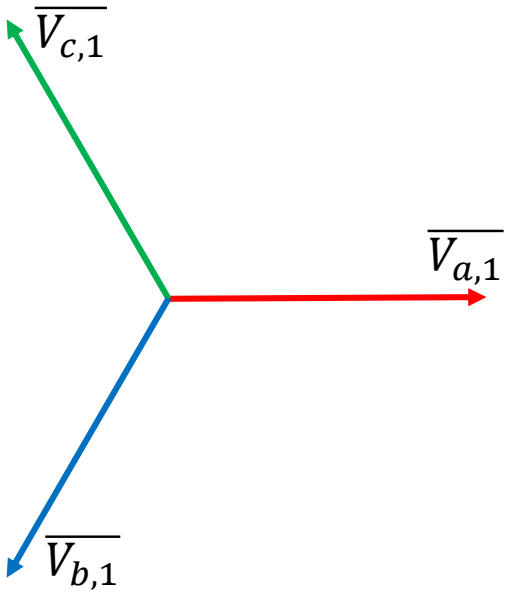
Concept of symmetrical components

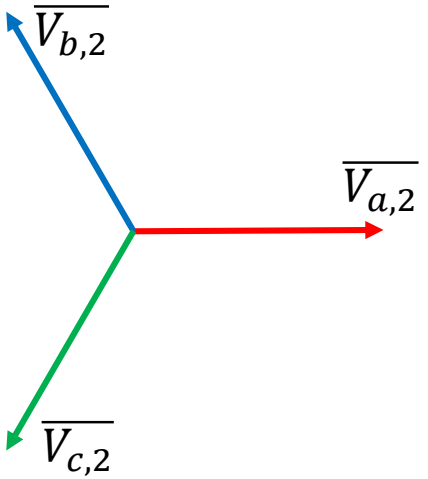
- Any unbalanced three-phase voltage or current can be decomposed into the sum of three balanced three-phase voltages or currents called sequence components:
  - A zero-sequence component where all three voltages or currents have the same phase
  - A positive-sequence component where the three voltages or currents are 120° out of phase with each other in the positive phase sequence (e.g., a-b-c)
  - A negative-sequence component where the three voltages or currents are 120° out of phase with each other but in the opposite phase sequence (e.g., a-c-b)

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Sequence components







Zero-sequence

Positive-sequence

Negative-sequence

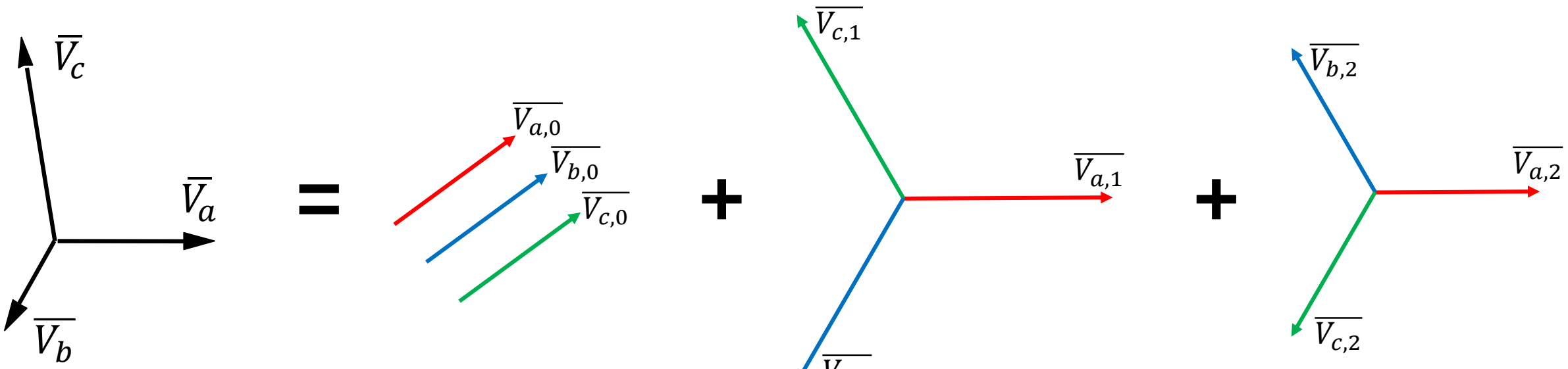
Subscript 0

Subscript 1

Subscript 2

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Concept of symmetrical components



Unbalanced voltages

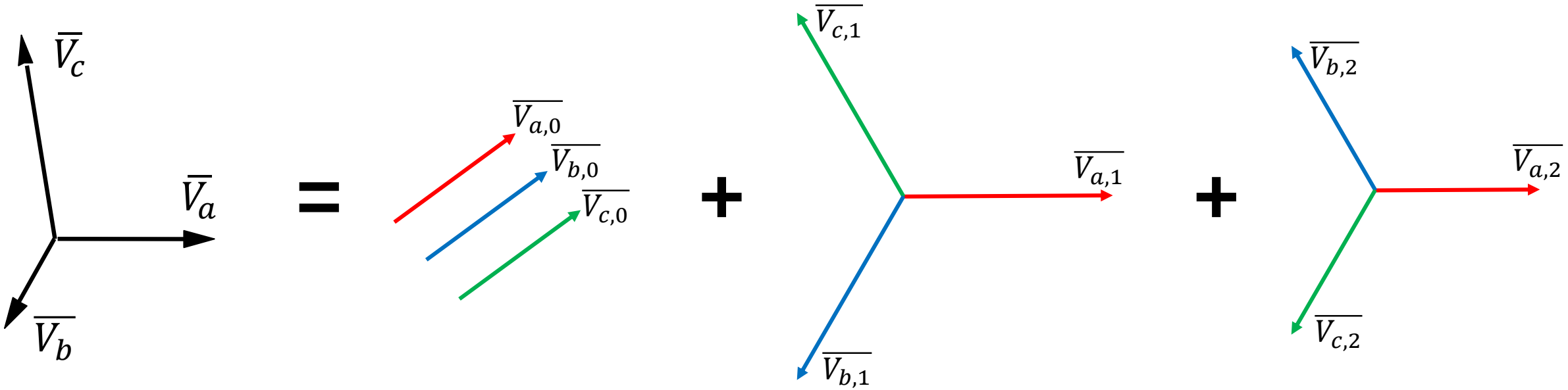
Zero-sequence component

Positive-sequence component

Negative-sequence component

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Concept of symmetrical components



$$\overline{V_a} = \overline{V_{a,0}} + \overline{V_{a,1}} + \overline{V_{a,2}}$$

$$\overline{V_b} = \overline{V_{b,0}} + \overline{V_{b,1}} + \overline{V_{b,2}}$$

$$\overline{V_c} = \overline{V_{c,0}} + \overline{V_{c,1}} + \overline{V_{c,2}}$$

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Concept of symmetrical components

- Each component is balanced
- Can represent each component by a single phasor

- Zero sequence: $\overline{V_0} = \overline{V_{a,0}} = \overline{V_{b,0}} = \overline{V_{c,0}}$

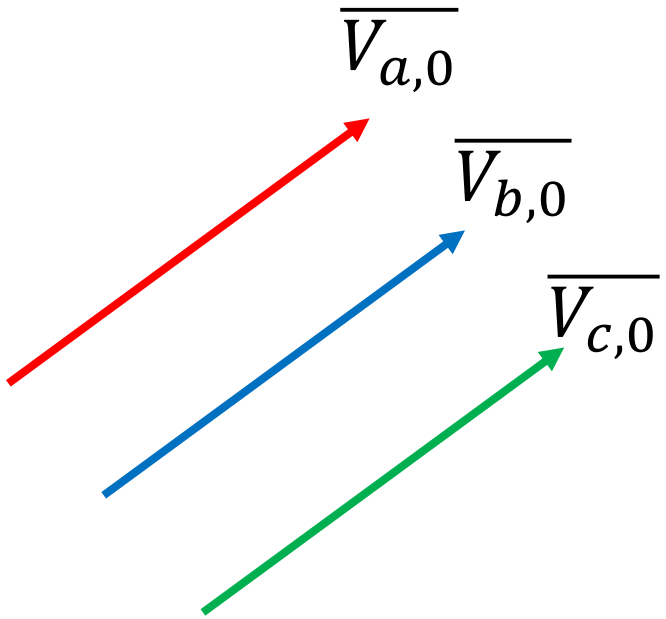
- Positive sequence: $\overline{V_1} = \overline{V_{a,1}}$; $\overline{V_{b,1}} = a^2\overline{V_1}$; $\overline{V_{c,1}} = a\overline{V_1}$

- Negative sequence: $\overline{V_2} = \overline{V_{a,2}}$; $\overline{V_{b,2}} = a\overline{V_2}$; $\overline{V_{c,2}} = a^2\overline{V_2}$

- This applies to currents as well as voltages

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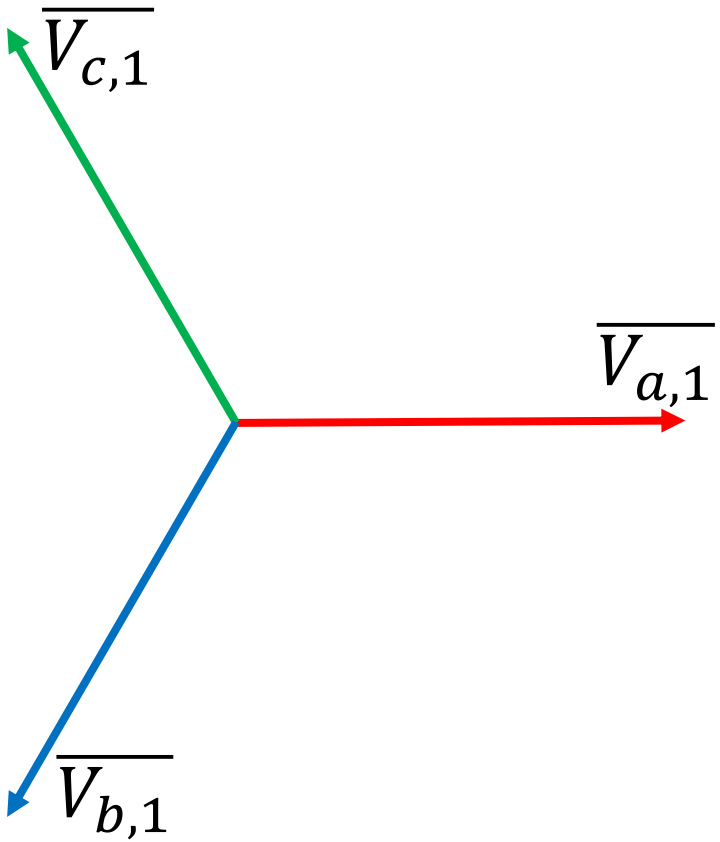
Zero sequence



$$\overline{V_0} = \overline{V_{a,0}} = \overline{V_{b,0}} = \overline{V_{c,0}}$$

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Positive sequence



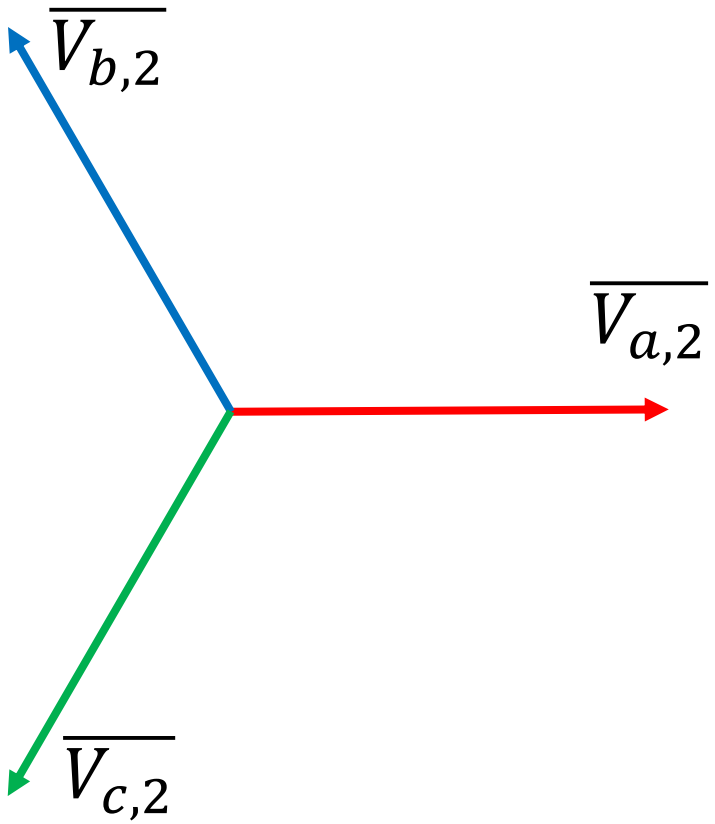
$$\overline{V_1} = \overline{V_{a,1}}$$

$$\overline{V_{b,1}} = a^2\overline{V_1}$$

$$\overline{V_{c,1}} = a\overline{V_1}$$

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Negative sequence



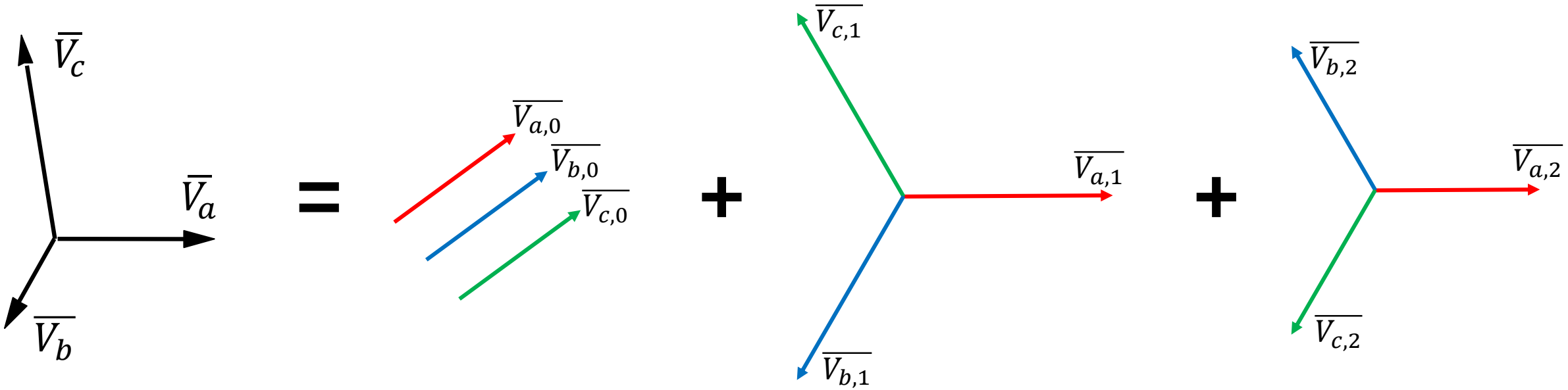
$$\overline{V_2} = \overline{V_{a,2}}$$

$$\overline{V_{b,2}} = a\overline{V_2}$$

$$\overline{V_{c,2}} = a^2\overline{V_2}$$

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Phase $\boldsymbol{a}$ as a function of symmetrical components



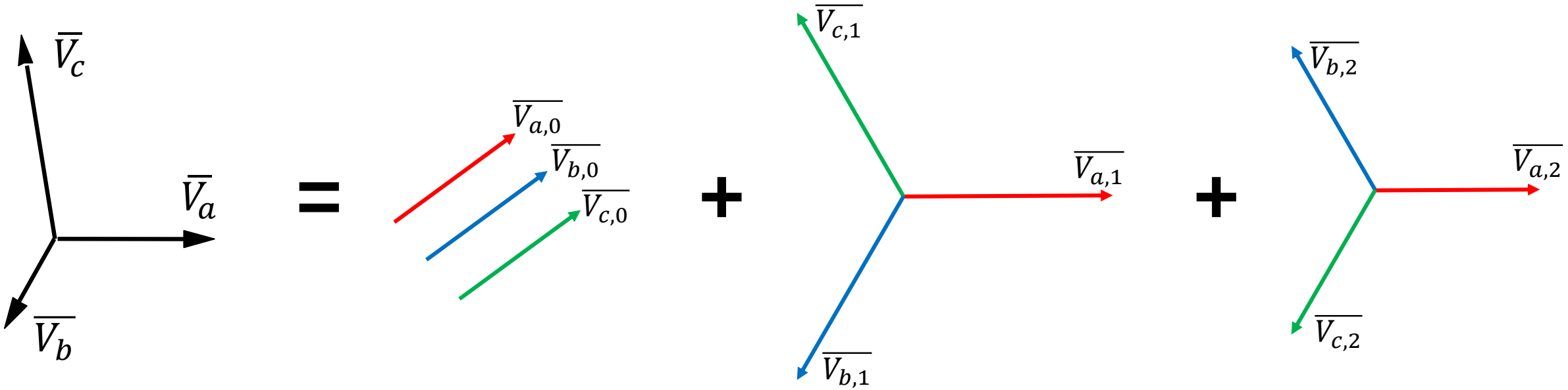
$$\overline{V_a} = \overline{V_{a,0}} + \overline{V_{a,1}} + \overline{V_{a,2}}$$

$$\begin{cases} \overline{V_{a,0}} = \overline{V_0} \\ \overline{V_{a,1}} = \overline{V_1} \\ \overline{V_{a,2}} = \overline{V_2} \end{cases}$$

$$\Rightarrow \quad \overline{V_a} = \overline{V_0} + \overline{V_1} + \overline{V_2}$$

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Phase *b* as a function of symmetrical components



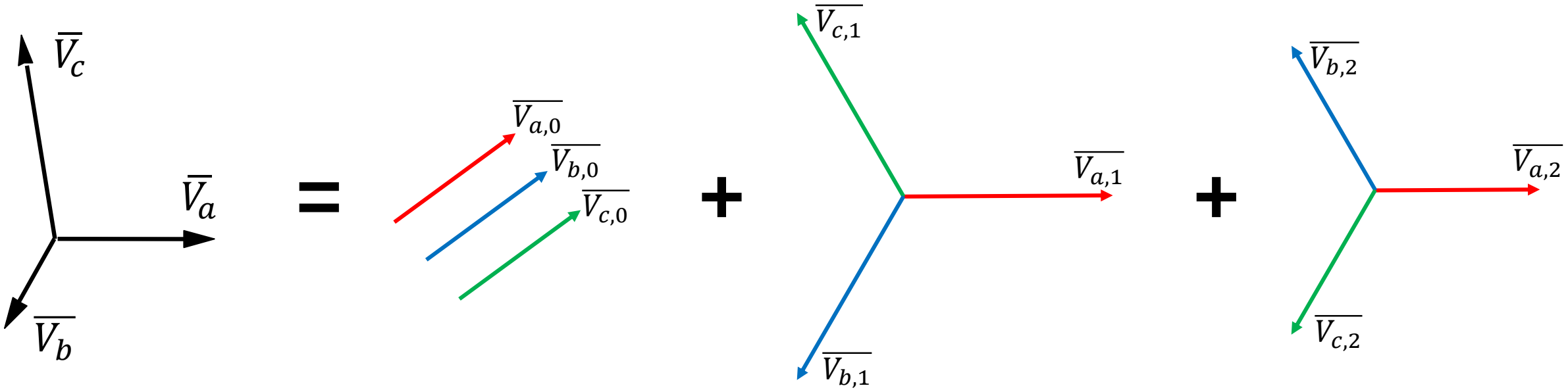
$$\overline{V_b} = \overline{V_{b,0}} + \overline{V_{b,1}} + \overline{V_{b,2}}$$

$$\begin{cases} \overline{V_{b,0}} = \overline{V_0} \\ \overline{V_{b,1}} = a^2\overline{V_1} \\ \overline{V_{b,2}} = a\overline{V_2} \end{cases}$$

$$\Rightarrow \quad \overline{V_b} = \overline{V_0} + a^2\overline{V_1} + a\overline{V_2}$$

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Phase *c* as a function of symmetrical components



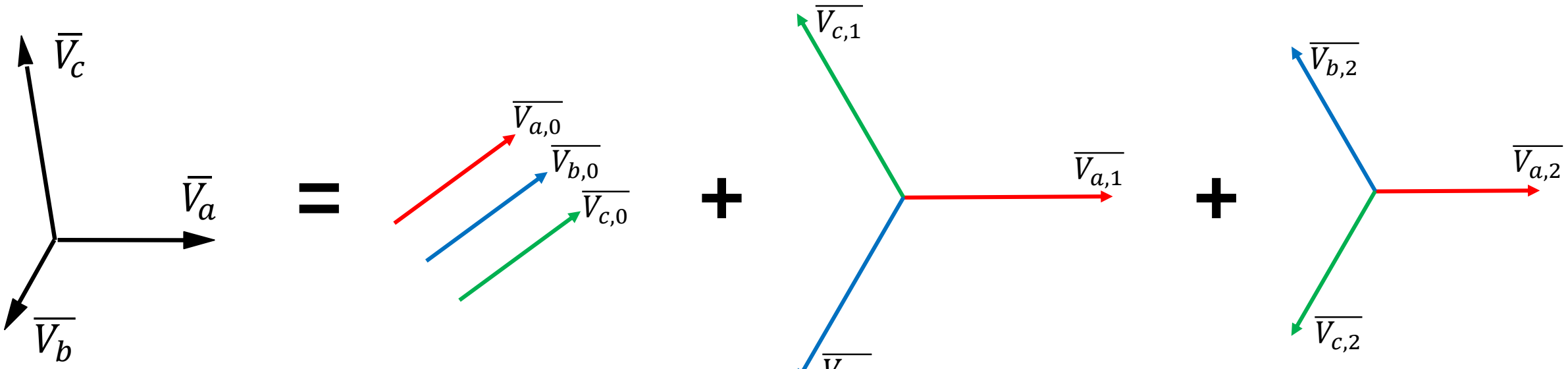
$$\overline{V_c} = \overline{V_{c,0}} + \overline{V_{c,1}} + \overline{V_{c,2}}$$

$$\begin{cases} \overline{V_{c,0}} = \overline{V_0} \\ \overline{V_{c,1}} = a\overline{V_1} \\ \overline{V_{c,2}} = a^2\overline{V_2} \end{cases}$$

$$\overline{V_c} = \overline{V_0} + a\overline{V_1} + a^2\overline{V_2}$$

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Phase variables from symmetrical components



$$\begin{cases} \overline{V_a} = \overline{V_0} + \overline{V_1} + \overline{V_2} \\ \overline{V_b} = \overline{V_0} + a^2\overline{V_1} + a\overline{V_2} \\ \overline{V_c} = \overline{V_0} + a\overline{V_1} + a^2\overline{V_2} \end{cases}$$

$$\begin{pmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} \overline{V_0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix}$$

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Phase variables from symmetrical components

$$\begin{pmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} \overline{V_0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix}$$

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$$\boldsymbol{V_P = AV_S}$$



Phase variables

Symmetrical components

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Symmetrical components from phase variables

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$$\boldsymbol{A^{-1}} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}$$

$$\begin{pmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}\begin{pmatrix} \overline{V_0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix}$$

$$\begin{pmatrix} \overline{V_0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{pmatrix}$$

$$\boldsymbol{V_P = AV_S}$$

$$\boldsymbol{V_S = A^{-1}V_P}$$

Symmetrical components

Phase variables

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Currents in symmetrical components

Transformation from phase quantities to symmetrical components applies to voltages as well as currents

$$\boldsymbol{I_P = AI_S}$$

$$\begin{pmatrix} \overline{I_a} \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} \overline{I_0} \\ \overline{I_1} \\ \overline{I_2} \end{pmatrix}$$

$$\boldsymbol{I_S = A^{-1} I_P}$$

$$\begin{pmatrix} \overline{I_0} \\ \overline{I_1} \\ \overline{I_2} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \begin{pmatrix} \overline{I_a} \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix}$$

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Example 10.1

Calculate the sequence components of the following set of balanced voltages:
$\overline{V_a} = 110\angle 0°\ V;\ \overline{V_b} = 110\angle 240°\ V;\ \overline{V_c} = 110\angle 120°V$

$$\overline{V_0} = \frac{1}{3}(110\angle 0° + 110\angle 240° + 110\angle 120°) = 0$$

$$\begin{aligned}\overline{V_1} &= \frac{1}{3}(110\angle 0° + 1\angle 120° \times 110\angle 240° + 1\angle 240° \times 110\angle 120°) \\ &= \frac{1}{3}(110\angle 0° + 110\angle 0° + 110\angle 0°) \\ &= 110\angle 0°\ V = \overline{V_a}\end{aligned}$$

$$\begin{aligned}\overline{V_2} &= \frac{1}{3}(110\angle 0° + 1\angle 240° \times 110\angle 240° + 1\angle 120° \times 110\angle 120°) \\ &= \frac{1}{3}(110\angle 0° + 110\angle 120° + 110\angle 240°) = 0\end{aligned}$$

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Example 10.2

Calculate the sequence components of the following set of balanced currents:

$\overline{I_a} = 30\angle 0°\ A;\ \overline{I_b} = 30\angle 120°\ A;\ \overline{I_c} = \ 30\angle 240°A$

$$\overline{I_0} = \frac{1}{3}(30\angle 0° + 30\angle 120° + 30\angle 240°) = 0$$

$$\overline{I_1} = \frac{1}{3}(30\angle 0° + 1\angle 120° \times 30\angle 120° + 1\angle 240° \times 30\angle 240°)$$
$$= \frac{1}{3}(30\angle 0° + 30\angle 240° + 30\angle 120°) = 0$$

$$\overline{I_2} = \frac{1}{3}(30\angle 0° + 1\angle 240° \times 30\angle 120° + 1\angle 120° \times 30\angle 240°)$$
$$= \frac{1}{3}(30\angle 0° + 30\angle 0° + 30\angle 0°)$$
$$= 30\angle 0°A = \overline{I_a}$$

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Example 10.3

Calculate the sequence components of the following set of unbalanced voltages:
$\overline{V_a} = 100\angle 0°\ V;\ \overline{V_b} = 90\angle 240°\ V;\ \overline{V_c} = \ 110\angle 120°\ \ V$

$$\overline{V_0} = \frac{1}{3}(100\angle 0° + 90\angle 240° + 110\angle 120°) = 5.77\angle 90°\ V$$

$$\overline{V_1} = \frac{1}{3}(100\angle 0° + 1\angle 120° \times 90\angle 240° + 1\angle 240° \times 110\angle 120°) = 100\angle 0°\ V$$

$$\overline{V_2} = \frac{1}{3}(100\angle 0° + 1\angle 240° \times 90\angle 240° + 1\angle 120° \times 110\angle 120°) = 5.77\angle -90°\ V$$

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Example 10.4

Calculate the phase voltages from the following sequence components:

$\overline{V_0} = 5\angle 10°\ V;\ \overline{V_1} = 50\angle 5°\ V;\ \overline{V_2} = \ 10\angle 30°V$

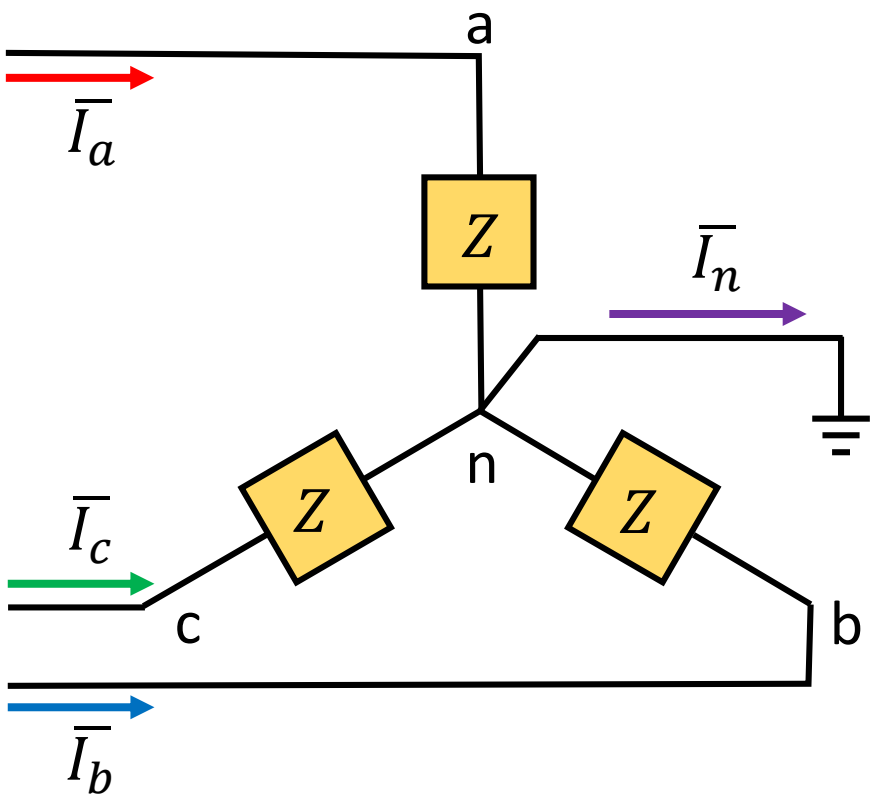
$\overline{V_a} = 5\angle 10° + 50\angle 5° + 10\angle 30° = 64.21\angle 9.16°\ V$

$\overline{V_b} = 5\angle 10° + 1\angle 240° \times 50\angle 5° + 1\angle 120° \times 10\angle 30° = 46.63\angle -122.23°\ V$

$\overline{V_c} = 5\angle 10° + 1\angle 120° \times 50\angle 5° + 1\angle 240° \times 10\angle 30° = 39.71\angle 126.74°\ V$

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Relation between the neutral and zero sequence currents



$$\bar{I}_n = \bar{I}_a + \bar{I}_b + \bar{I}_c$$

$$\begin{pmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{pmatrix}$$

$$\bar{I}_0 = \frac{1}{3}(\bar{I}_a + \bar{I}_b + \bar{I}_c)$$

$$\Rightarrow \bar{I}_0 = \frac{1}{3}\,\bar{I}_n$$

A zero-sequence current can occur only if the neutral current is not zero i.e., when the neutral is grounded

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Example 10.5: Zero-sequence and neutral currents

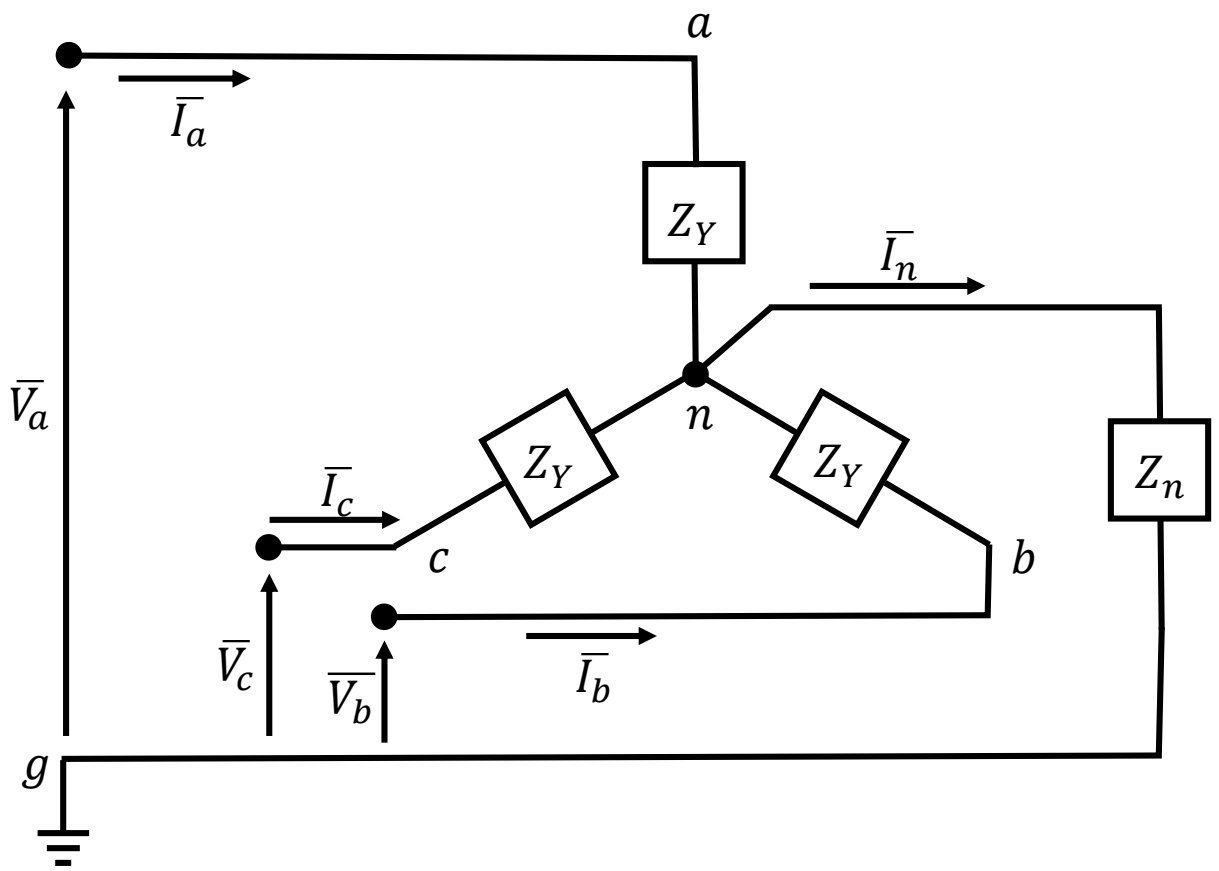
Given $\bar{I}_a = 60\angle 0° \; A$; $\bar{I}_b = 60\angle 240° \; A$; $\bar{I}_c = \; 0 \; \; A$, calculate the zero-sequence and neutral currents.

$$\bar{I}_0 = \frac{1}{3}(60\angle 0° + 60\angle 240° + 0) = 20\angle -60° \; A$$

$$\bar{I}_n = 3 \; \bar{I}_0 = 60\angle -60° \; A$$

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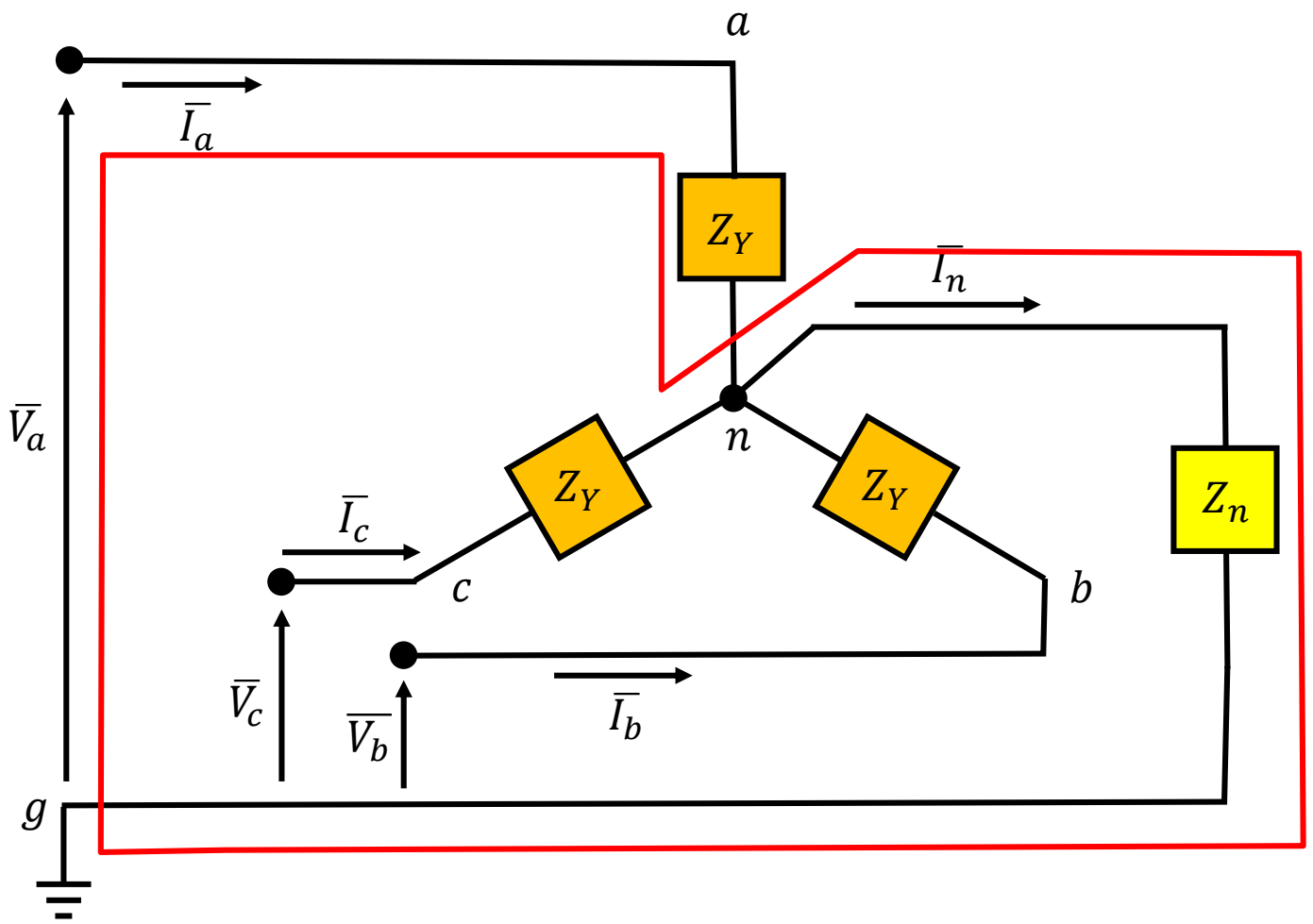
Sequence networks



- In a three-phase network, if the voltages or currents are not balanced, what happens in one phase affects what happens in the other phases
- This coupling makes analysis complex
- Using symmetrical components, we can replace three-phase networks by decoupled sequence networks

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Sequence networks representation of a load



Write KVL around the loop a-n-g

$$\overline{V_a} = \overline{I_a} Z_Y + \overline{I_n} Z_n$$

$$\overline{I_n} = \overline{I_a} + \overline{I_b} + \overline{I_c}$$

$$\overline{V_a} = \overline{I_a}(Z_Y + Z_n) + \overline{I_b} Z_n + \overline{I_c} Z_n$$

Similarly, around the loops b-n-g and c-n-g:

$$\overline{V_b} = \overline{I_a} Z_n + \overline{I_b}(Z_Y + Z_n) + \overline{I_c} Z_n$$

$$\overline{V_c} = \overline{I_a} Z_n + \overline{I_b} Z_n + \overline{I_c}(Z_Y + Z_n)$$

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Sequence networks representation of a load

$$\overline{V_a} = \overline{I_a}(Z_Y + Z_n) + \overline{I_b}Z_n + \overline{I_c}Z_n$$

$$\overline{V_b} = \overline{I_a}Z_n + \overline{I_b}(Z_Y + Z_n) + \overline{I_c}Z_n$$

$$\overline{V_c} = \overline{I_a}Z_n + \overline{I_b}Z_n + \overline{I_c}(Z_Y + Z_n)$$

$$\begin{pmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{pmatrix} = \begin{pmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{pmatrix} \begin{pmatrix} \overline{I_a} \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix}$$

$$\boldsymbol{V_P} = \overline{\boldsymbol{Z_P}}\boldsymbol{I_P}$$

Full matrix → coupling between the three phases if the currents are imbalanced

What happens if we convert to symmetrical components?

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Sequence networks representation of a load

$$\boldsymbol{V_P = Z_P I_P}$$

$$\boldsymbol{A^{-1} V_P = A^{-1} Z_P I_P}$$

$$\boldsymbol{V_S = A^{-1} V_P} \qquad \boldsymbol{I_P = A I_S}$$

$$\boldsymbol{V_S = A^{-1} Z_P A\, I_S}$$

$$\boldsymbol{V_S = Z_S\, I_S}$$

$$\boldsymbol{Z_S = A^{-1} Z_P A} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

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Sequence networks representation of a load

$$\boldsymbol{Z_S = A^{-1} Z_P A} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$$1 + a + a^2 = 0$$

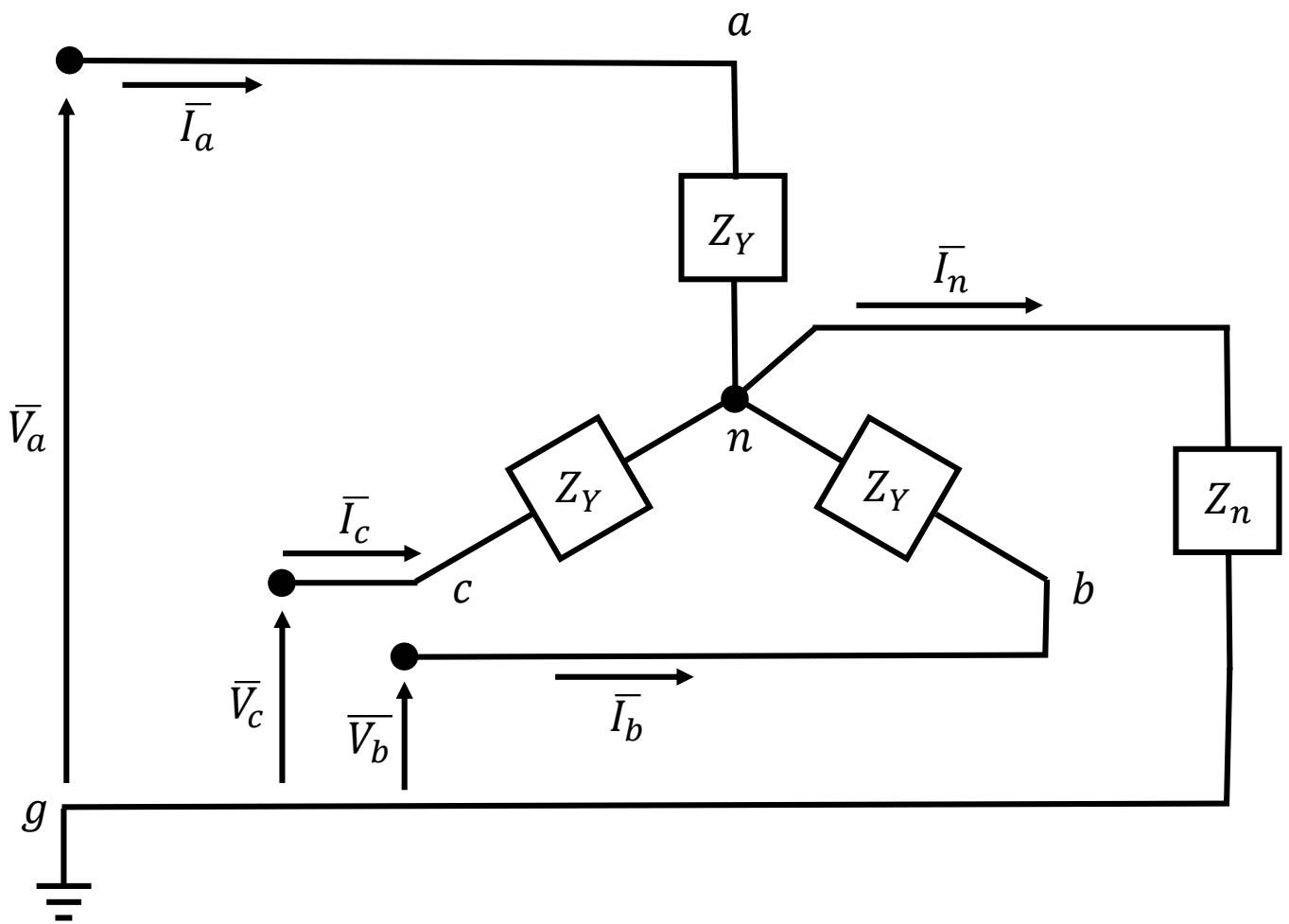
$$\boldsymbol{Z_S} = \begin{pmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{pmatrix}$$

$$\boldsymbol{V_S = Z_S\, I_S}$$

$$\begin{cases} \overline{V_0} = (Z_Y + 3Z_n)\overline{I_0} = Z_0\overline{I_0} \\ \overline{V_1} = Z_Y\,\overline{I_1} = Z_1\overline{I_1} \\ \overline{V_2} = Z_Y\,\overline{I_2} = Z_2\overline{I_2} \end{cases}$$

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Sequence networks representation of a load



$$\bar{V}_a = \bar{I}_a(Z_Y + Z_n) + \bar{I}_b Z_n + \bar{I}_c Z_n$$

$$\bar{V}_b = \bar{I}_a Z_n + \bar{I}_b(Z_Y + Z_n) + \bar{I}_c Z_n$$

$$\bar{V}_c = \bar{I}_a Z_n + \bar{I}_b Z_n + \bar{I}_c(Z_Y + Z_n)$$

$$\bar{V}_0 = (Z_Y + 3Z_n)\bar{I}_0 = Z_0 \bar{I}_0$$

$$\bar{V}_1 = Z_Y \bar{I}_1 = Z_1 \bar{I}_1$$

$$\bar{V}_2 = Z_Y \bar{I}_2 = Z_2 \bar{I}_2$$

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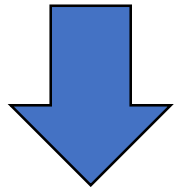
Sequence networks representation of a load

- Converting to symmetrical components decouples the equations
- Positive sequence currents
  → positive sequence voltages
- Negative sequence currents
  → negative sequence voltages
- Zero sequence currents
  → zero sequence voltages

$$\overline{V_a} = \overline{I_a}(Z_Y + Z_n) + \overline{I_b}Z_n + \overline{I_c}Z_n$$

$$\overline{V_b} = \overline{I_a}Z_n + \overline{I_b}(Z_Y + Z_n) + \overline{I_c}Z_n$$

$$\overline{V_c} = \overline{I_a}Z_n + \overline{I_b}Z_n + \overline{I_c}(Z_Y + Z_n)$$



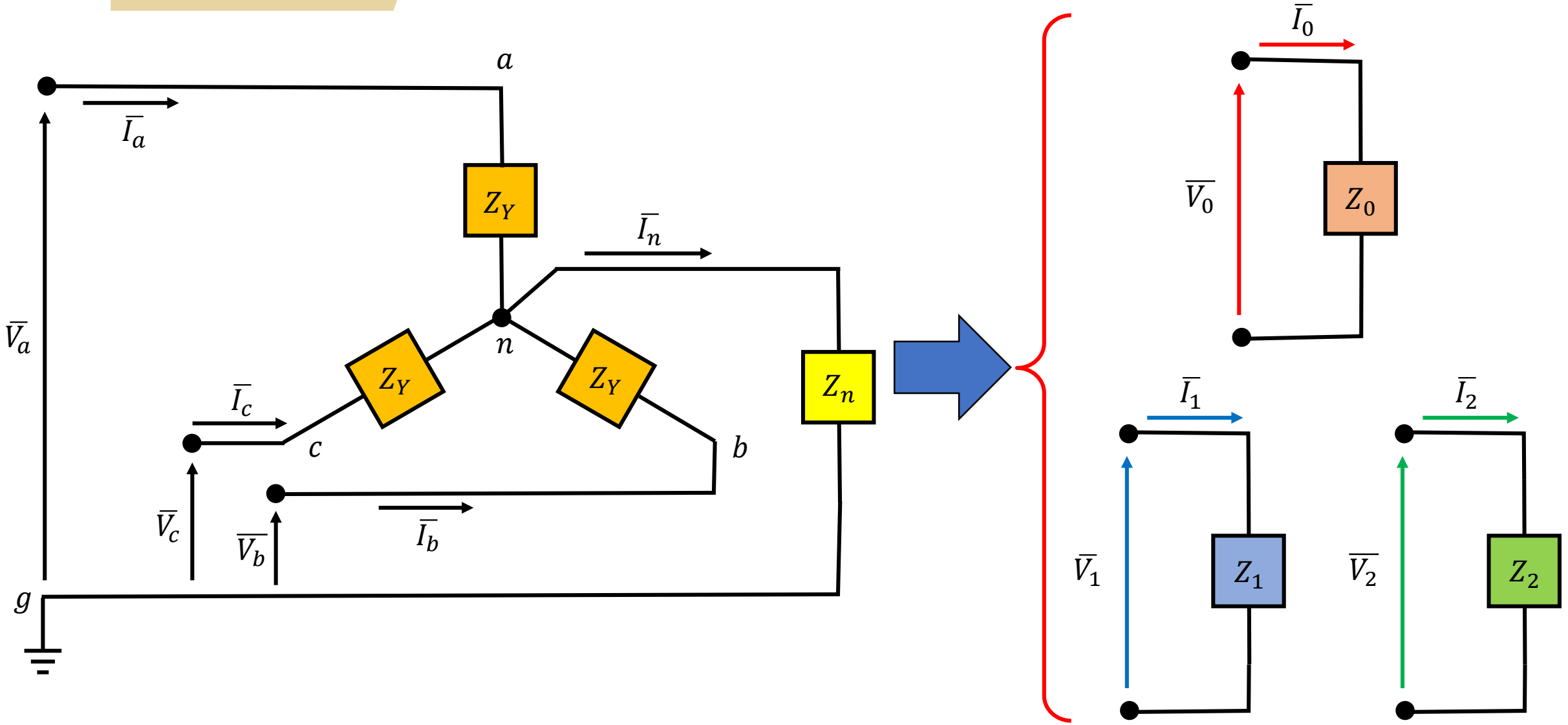
$$\overline{V_0} = (Z_Y + 3Z_n)\overline{I_0} = Z_0\overline{I_0}$$

$$\overline{V_1} = Z_Y\,\overline{I_1} = Z_1\overline{I_1}$$

$$\overline{V_2} = Z_Y\,\overline{I_2} = Z_2\overline{I_2}$$

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Sequence networks representation of a load



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Sequence networks representation of a load



$Z_1 = Z_Y$

$Z_2 = Z_Y$

Z_n does not appear in Z_1 or Z_2

$Z_0 = Z_Y + 3Z_n$

Because $\overline{I_n} = 3\overline{I_0}$

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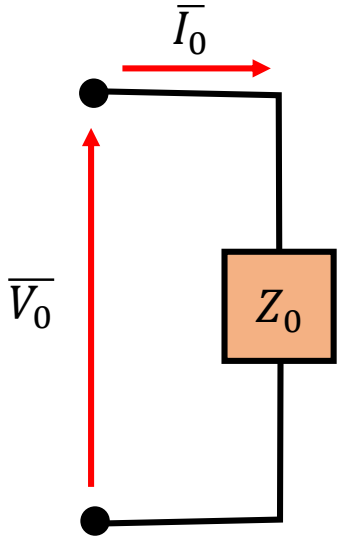
Ungrounded Y connection

Ungrounded Y connection

$Z_n = \infty$

$Z_0 = \infty$

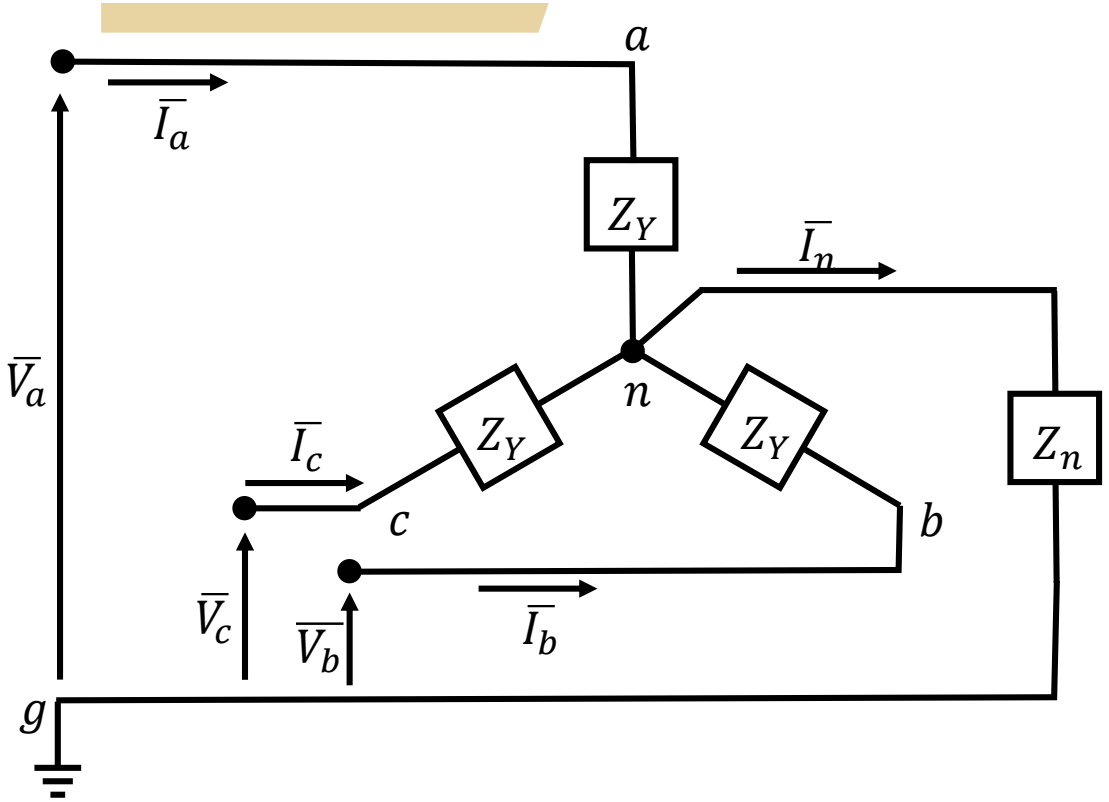
No zero-sequence current

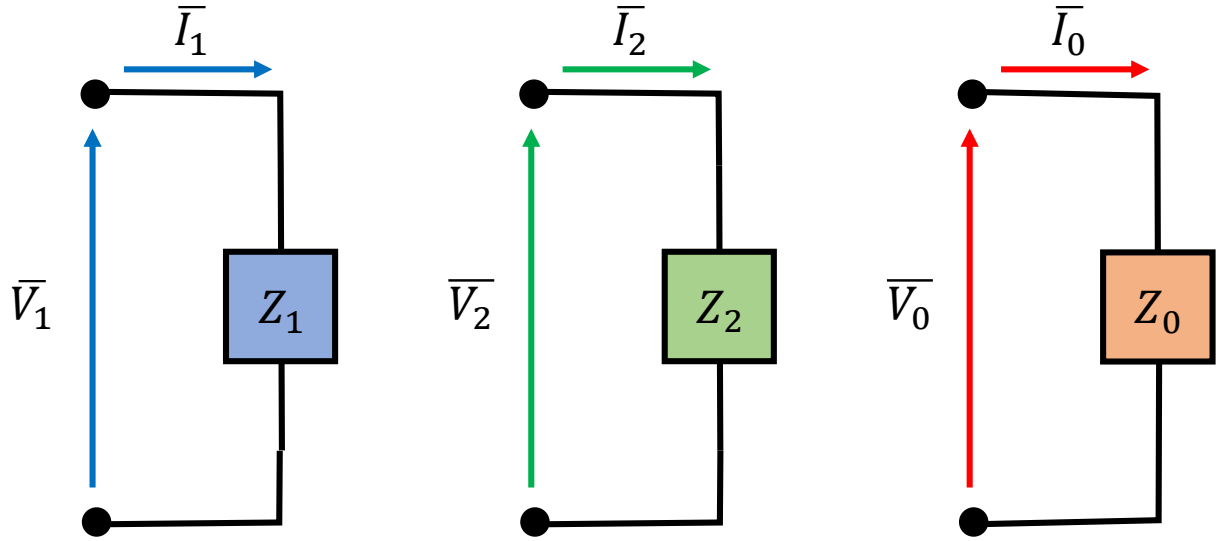


$Z_0 = Z_Y + 3Z_n$

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Balanced set of voltages in positive phase sequence



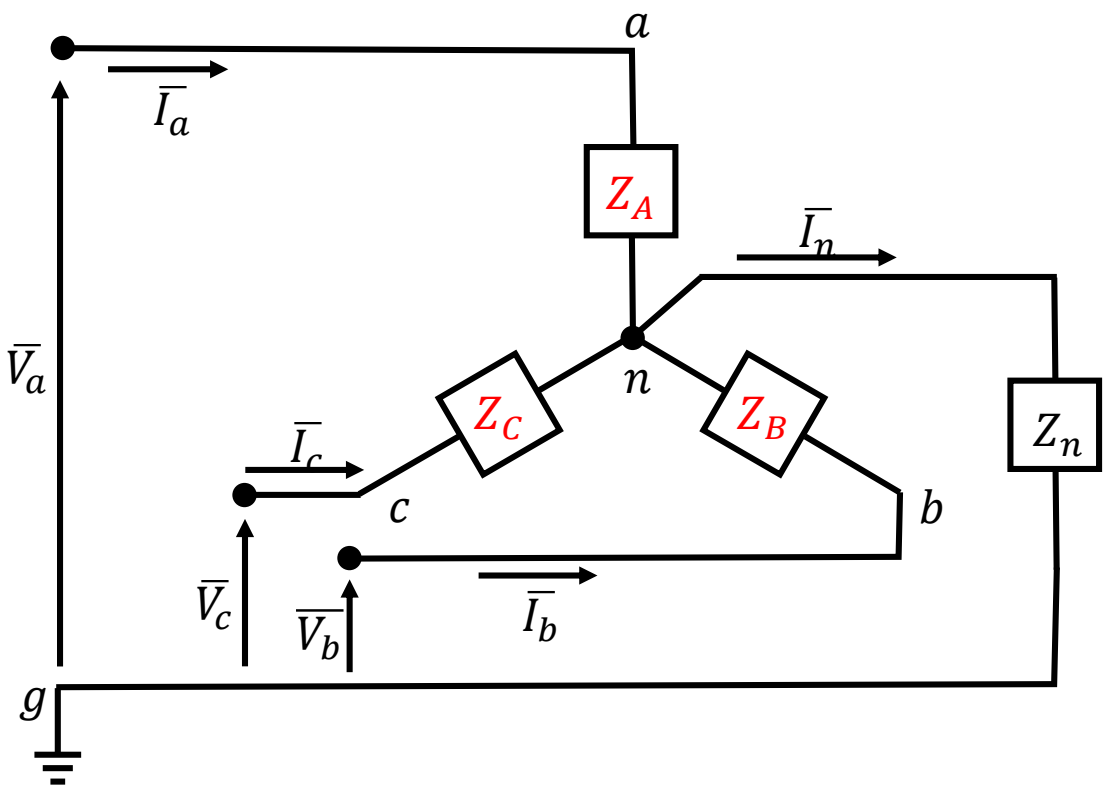


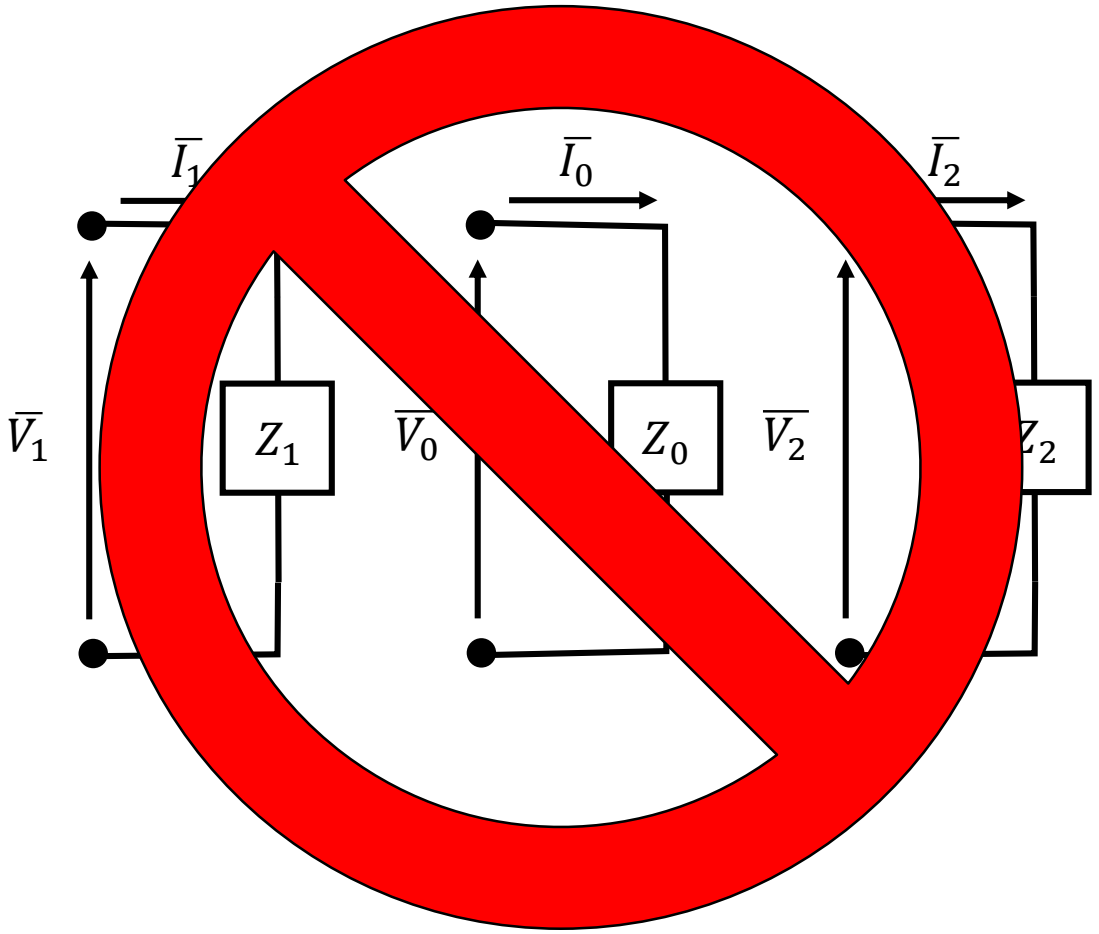
$$\bar{V}_1 \neq 0$$
$$\bar{V}_2 = 0$$
$$\bar{V}_0 = 0$$

Positive-sequence network is equivalent to the single-phase representation of a balanced three-phase system

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Unbalanced loads





If the load is not balanced, the sequence networks are not decoupled!

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Example 10.6: Applying imbalanced voltages to a Y-connected load

Each branch of a Y-connected load has an impedance of $9 + j3\Omega$. The neutral point of this load is connected to ground through an impedance of $j2\Omega$. The following set of unbalanced line-to-neutral voltages is applied to this load:

$\overline{V_a} = 100\angle 0°\ V$

$\overline{V_b} = 90\angle 240°\ V$

$\overline{V_c} = 110\angle 120°\ V$

Calculate the current in each phase and in the neutral connection.

$Z_0 = Z_Y + 3Z_n = 9 + j3 + 3 \times j2 = 9 + j9\Omega$

$Z_1 = Z_Y = 9 + j3\Omega$

$Z_2 = Z_Y = 9 + j3\Omega$

From Example 10.3:

$\overline{V_0} = 5.77\angle 90°\ V$

$\overline{V_1} = 100\angle 0°\ V$

$\overline{V_2} = 5.77\angle -90°\ V$

$\overline{I_0} = \frac{\overline{V_0}}{Z_0} = \frac{5.77\angle 90°}{9+j9} = 0.453\angle 45°\ A$

$\overline{I_1} = \frac{\overline{V_1}}{Z_1} = \frac{100\angle 0°}{9+j3} = 10.54\angle -18.43°\ A$

$\overline{I_2} = \frac{\overline{V_2}}{Z_3} = \frac{5.77\angle -90°}{9+j3} = 0.608\angle -108.43°\ A$

$\overline{I_a} = 10.75\angle -19.51°\ A$

$\overline{I_b} = 9.57\angle -136.77°\ A$

$\overline{I_c} = 11.32\angle +101.20°\ A$

$\overline{I_n} = 3\overline{I_0} = 1.359\angle 45°\ A$

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Example 10.7: Applying imbalanced voltages to a delta-connected load

Repeat the calculations of Example 10.5 assuming that the branch impedances are connected in delta. Replace the delta-connected load by its Y-connected equivalent.

$Z_Y = \frac{Z_\Delta}{3} = \frac{9+j3}{3} = 3 + j1\Omega$ and $Z_n = \infty$ because there is no neutral connection in a delta-connected load.

$Z_0 = \infty$

$Z_1 = 3 + j1\Omega$

$Z_2 = 3 + j1\Omega$

$\overline{I_0} = \frac{\overline{V_0}}{Z_0} = 0$

$\overline{I_1} = \frac{\overline{V_1}}{Z_1} = \frac{100\angle 0°}{3+j1} = 31.62\angle -18.43°\ A$

$\overline{I_2} = \frac{\overline{V_2}}{Z_3} = \frac{5.77\angle -90°}{3+j1} = 1.824\angle -108.43°\ A$

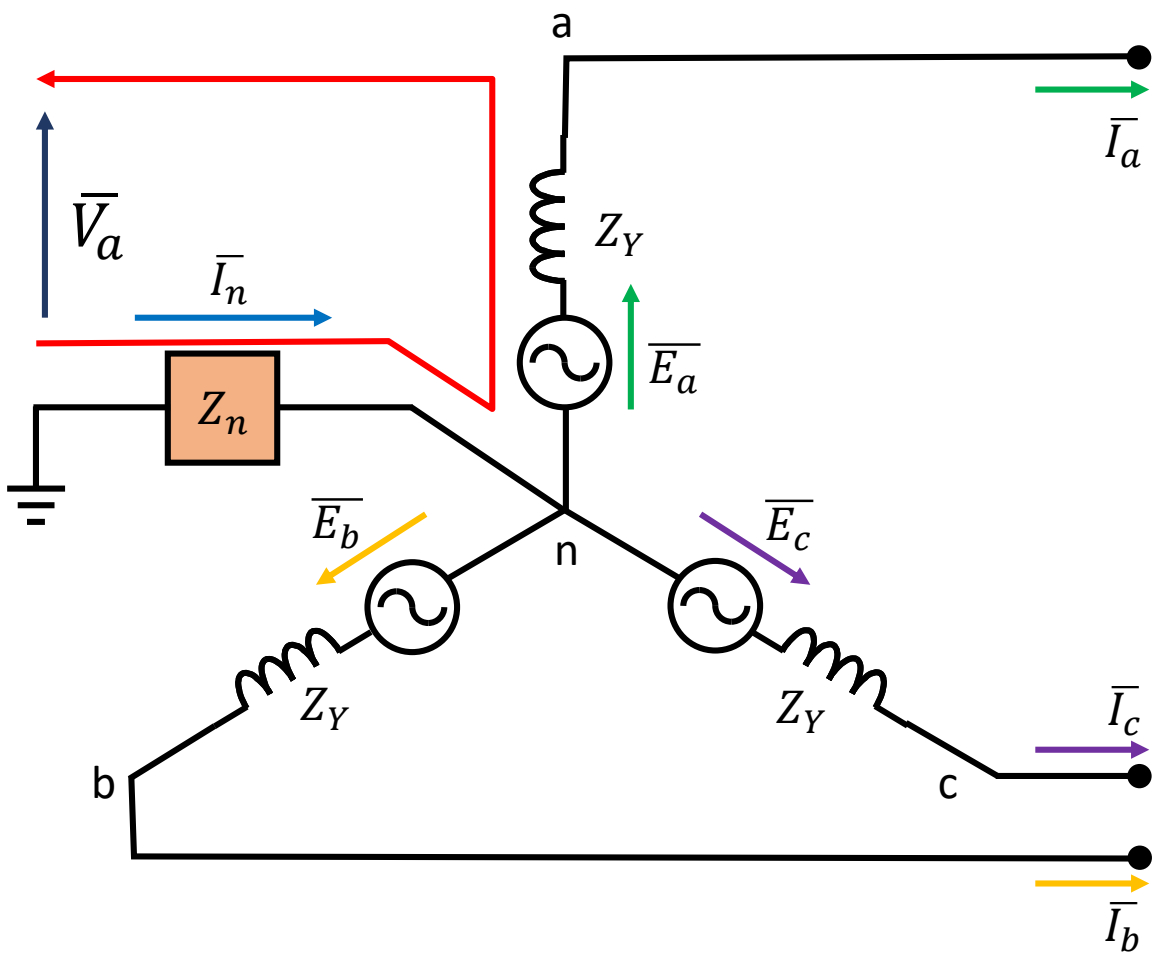
$\overline{I_a} = 31.67\angle -21.73°\ A$

$\overline{I_b} = 30.05\angle -136.69°\ A$

$\overline{I_c} = 33.21\angle +103.14°\ A$

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Sequence networks representation of generators



$$\overline{V_a} = \overline{E_a} - \overline{I_a}Z_Y - \overline{I_n}Z_n$$

$$\overline{V_a} = \overline{E_a} - \overline{I_a}Z_Y - (\overline{I_a} + \overline{I_b} + \overline{I_c})Z_n$$

$$\overline{V_a} = \overline{E_a} - \overline{I_a}(Z_Y+Z_n) - \overline{I_b}Z_n - \overline{I_c}Z_n$$

$$\overline{V_b} = \overline{E_b} - \overline{I_b}(Z_Y+Z_n) - \overline{I_a}Z_n - \overline{I_c}Z_n$$

$$\overline{V_c} = \overline{E_c} - \overline{I_c}(Z_Y+Z_n) - \overline{I_a}Z_n - \overline{I_c}Z_n$$

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Sequence networks representation of generators

$$\overline{V_a} = \overline{E_a} - \overline{I_a}(Z_Y + Z_n) - \overline{I_b}Z_n - \overline{I_c}Z_n$$

$$\overline{V_b} = \overline{E_b} - \overline{I_b}(Z_Y + Z_n) - \overline{I_a}Z_n - \overline{I_c}Z_n$$

$$\overline{V_c} = \overline{E_c} - \overline{I_c}(Z_Y + Z_n) - \overline{I_a}Z_n - \overline{I_b}Z_n$$

$$\begin{pmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{pmatrix} = \begin{pmatrix} \overline{E_a} \\ \overline{E_b} \\ \overline{E_c} \end{pmatrix} - \begin{pmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{pmatrix} \begin{pmatrix} \overline{I_a} \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix}$$

$$\boldsymbol{V_P = E_P - Z_P I_P}$$

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Sequence networks representation of generators

$$\boldsymbol{V_P = E_P - Z_P I_P}$$

$$\boldsymbol{A^{-1} V_P = A^{-1} E_P - A^{-1} Z_P I_P}$$

$$\boldsymbol{A^{-1} V_P = V_S}$$

$$\boldsymbol{I_P = A I_S}$$

$$\boldsymbol{V_S = A^{-1} E_P - A^{-1} Z_P A I_S}$$

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Sequence networks representation of generators

$$\boldsymbol{V_S = A^{-1}E_P - A^{-1}Z_P A I_S}$$

$$\boldsymbol{A^{-1}E_P} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} \overline{E_a} \\ \overline{E_b} \\ \overline{E_c} \end{pmatrix} = \begin{pmatrix} 0 \\ \overline{E_1} \\ 0 \end{pmatrix} = \boldsymbol{E_S}$$

⟸ Because the internal e.m.f. are a balanced set of voltages

$$\boldsymbol{A^{-1}Z_P A = Z_S} = \begin{pmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{pmatrix} \Longrightarrow \quad \boldsymbol{V_S = E_S - Z_S I_S}$$

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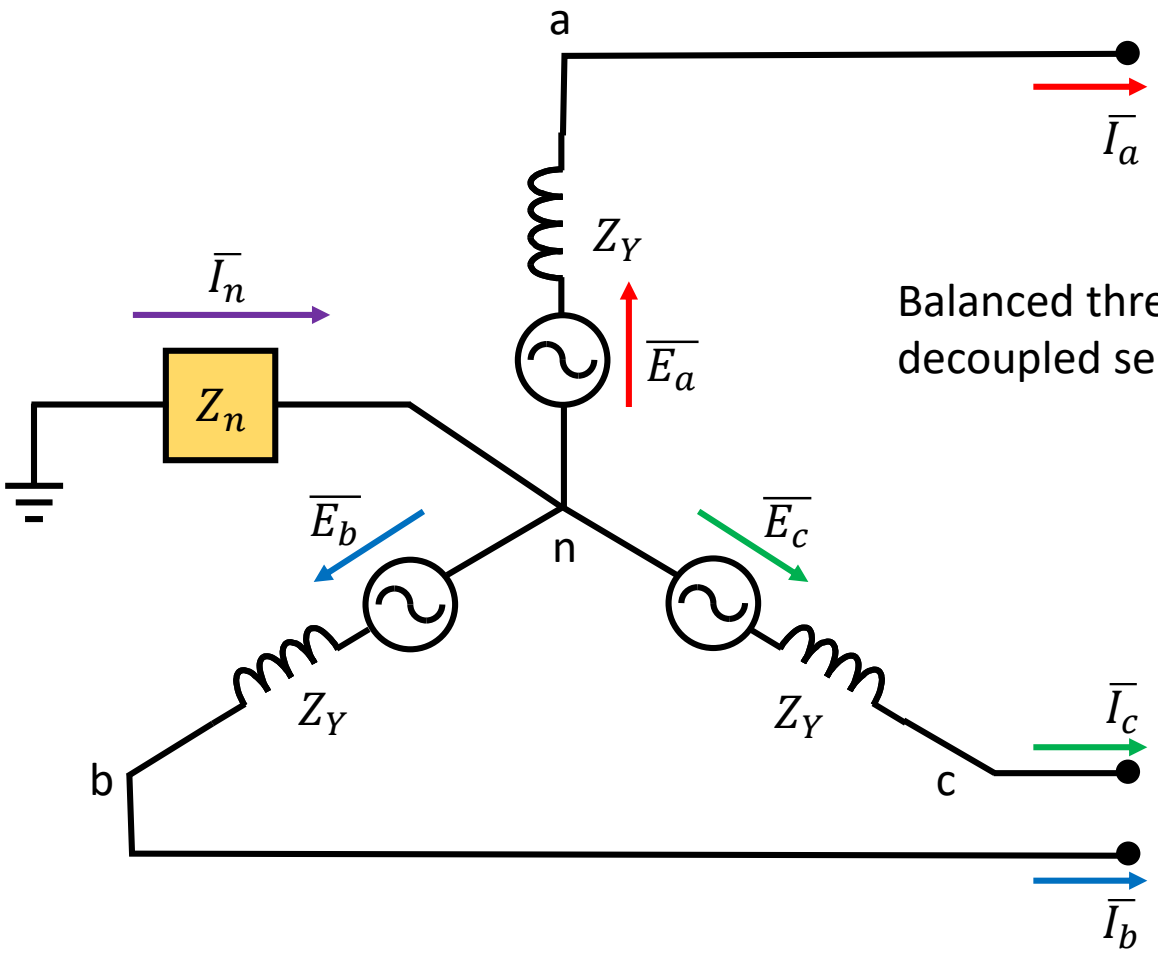
Sequence networks representation of generators

$$\boldsymbol{V_S = E_S - Z_S I_S}$$

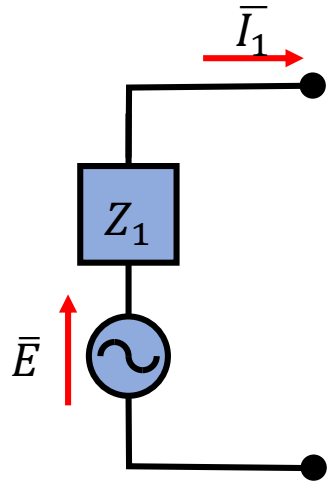
$$\begin{pmatrix} \overline{V_0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix} = \begin{pmatrix} 0 \\ \overline{E_1} \\ 0 \end{pmatrix} - \begin{pmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{pmatrix} \begin{pmatrix} \overline{I_0} \\ \overline{I_1} \\ \overline{I_2} \end{pmatrix}$$

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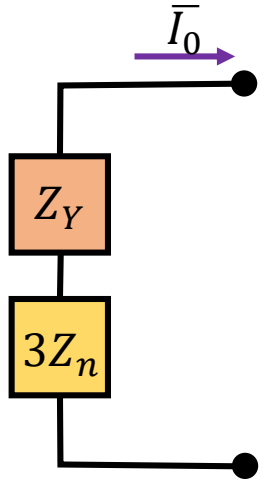
Sequence networks representation of generators



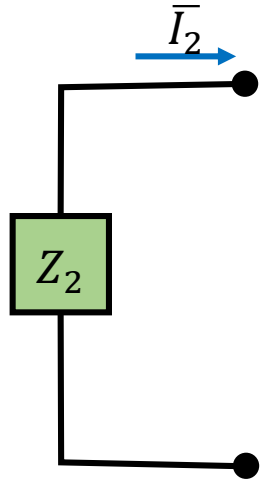
Balanced three-phases → decoupled sequence networks



Positive-sequence



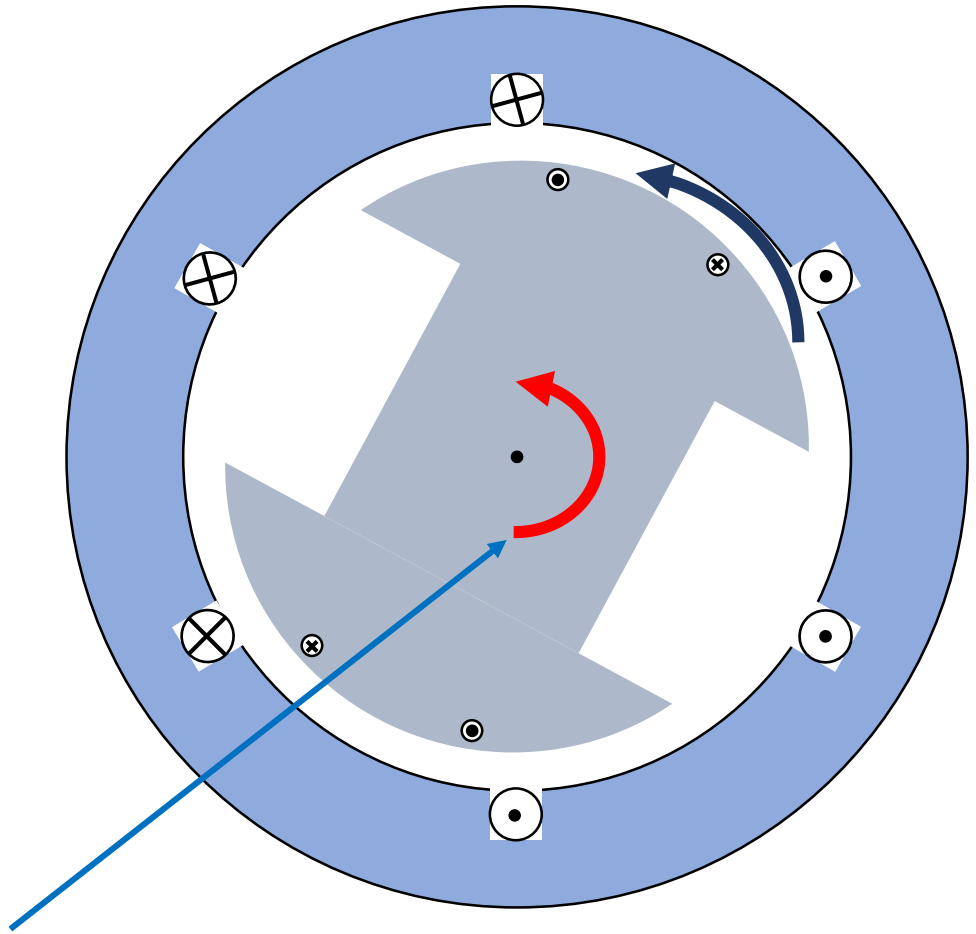
Zero-sequence



Negative-sequence

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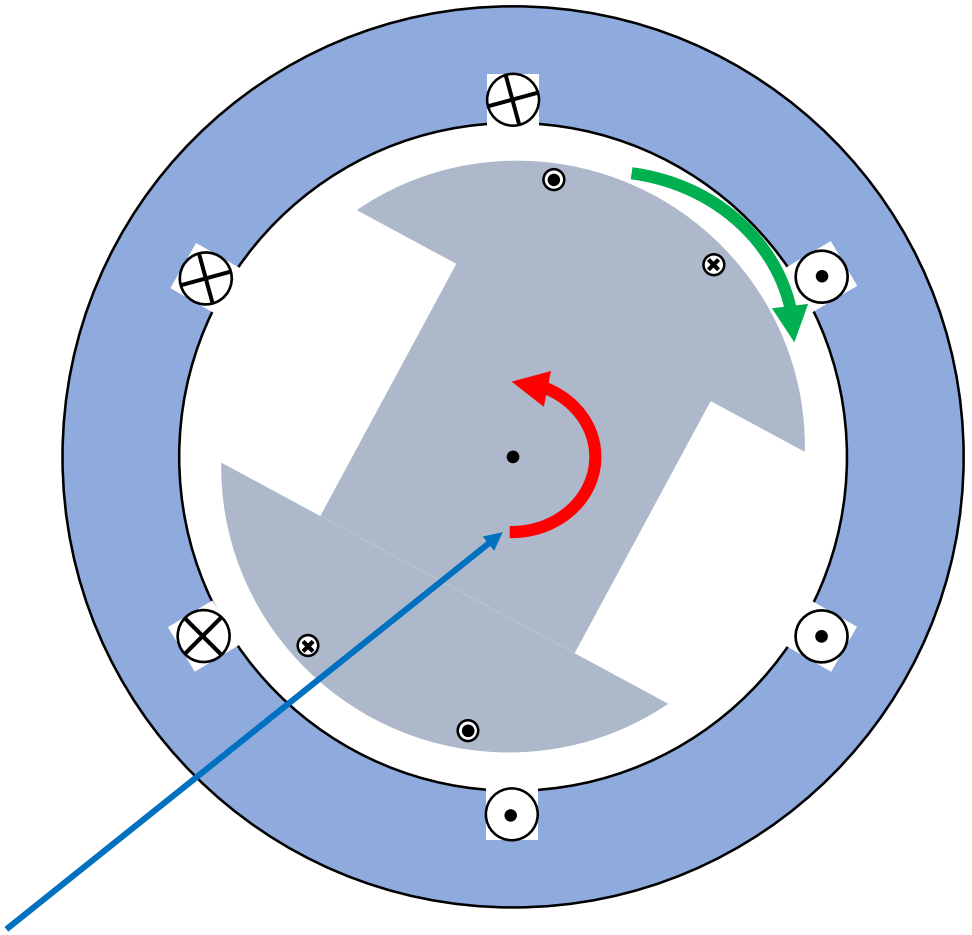
Positive sequence impedance of a rotating machine



- Apply positive sequence currents
- Create a flux wave that rotates in the same direction as the rotor
- Flux wave is in synchronism with the rotor
- Path of positive sequence magnetic flux is mostly through iron
- Low reluctance, high impedance
- Z_1

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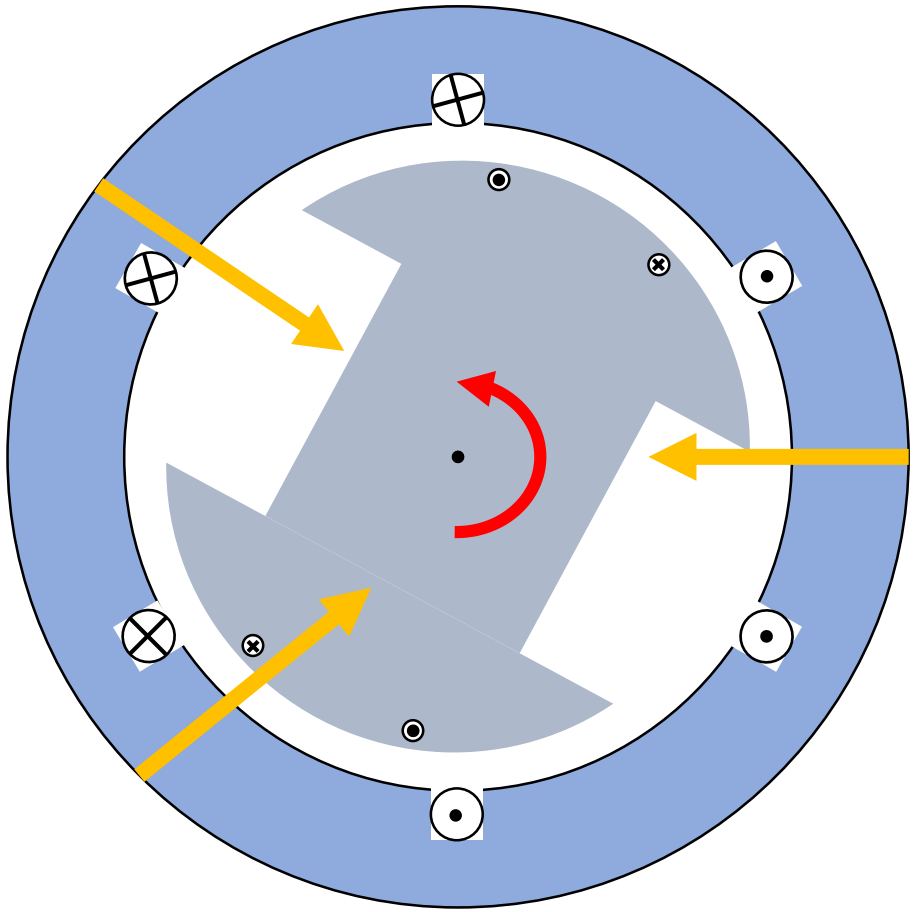
Negative sequence impedance of a rotating machine



- Apply negative sequence currents
- Create a flux wave that rotates in the opposite direction as the rotor
- Flux cannot penetrate the rotor
- Path of negative sequence magnetic path is mostly through the air
- High reluctance, low impedance
- $Z_2 < Z_1$

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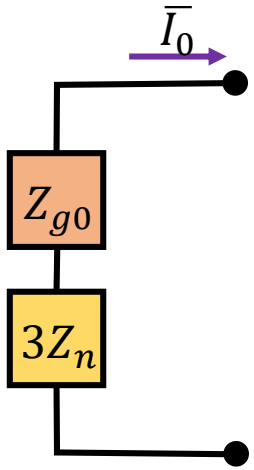
Zero sequence impedance of a rotating machine



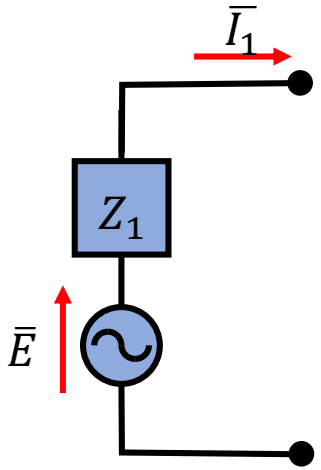
- Apply zero sequence currents
- Flux created by all three phases are in phase
- Resulting flux cannot penetrate the rotor
- Zero sequence magnetic flux path is mostly through the air
- High reluctance, low impedance
- $Z_{g0} < Z_2 < Z_1$

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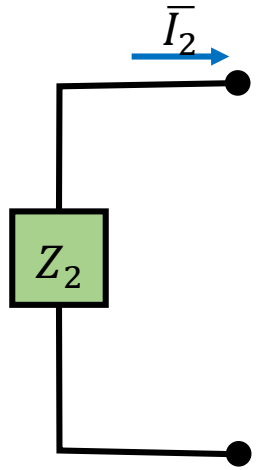
Generators sequence impedances



Zero-sequence



Positive-sequence



Negative-sequence

- Depending on the time frame considered, Z_1 is the synchronous, transient, or subtransient impedance
- $Z_0 = Z_{g0} + 3Z_n$ → zero sequence current can flow through a generator only if it is Y-connected and the neutral is connected to ground

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Sequence impedances of lines and cables

- Reactance of lines and cables reflects their magnetic field
- Depends on the distance and configuration of the conductors
- If the phases are arranged symmetrically → $Z_1 = Z_2$



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Sequence impedances of lines and cables



- Unequal distance between the phases

→ Unequal positive and negative sequence magnetic fields

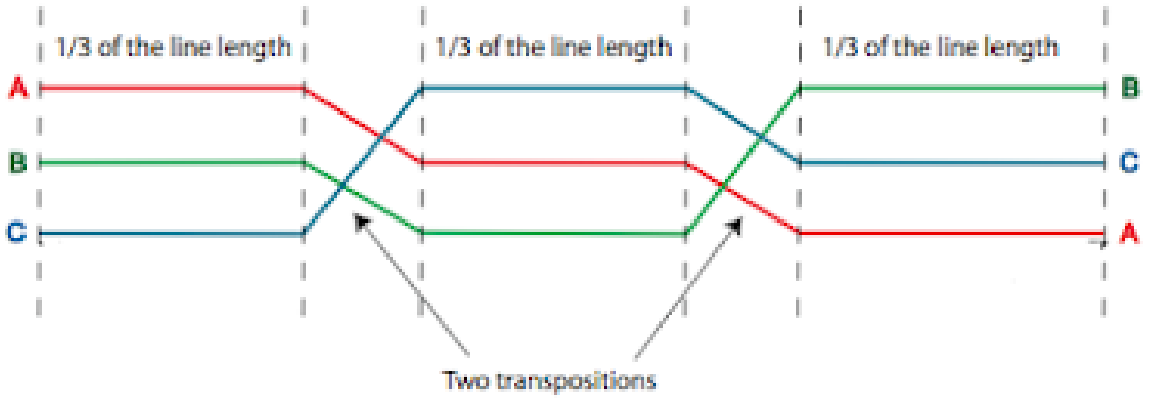
→ $Z_1 \neq Z_2$

- Need regular transposition to restore balance

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Transposition





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Sequence impedances of lines and cables

- Zero-sequence impedance depends on zero-sequence magnetic field
- Very different from magnetic fields created by the positive- or negative-sequence currents

→ Lines have a much lower zero-sequence impedance

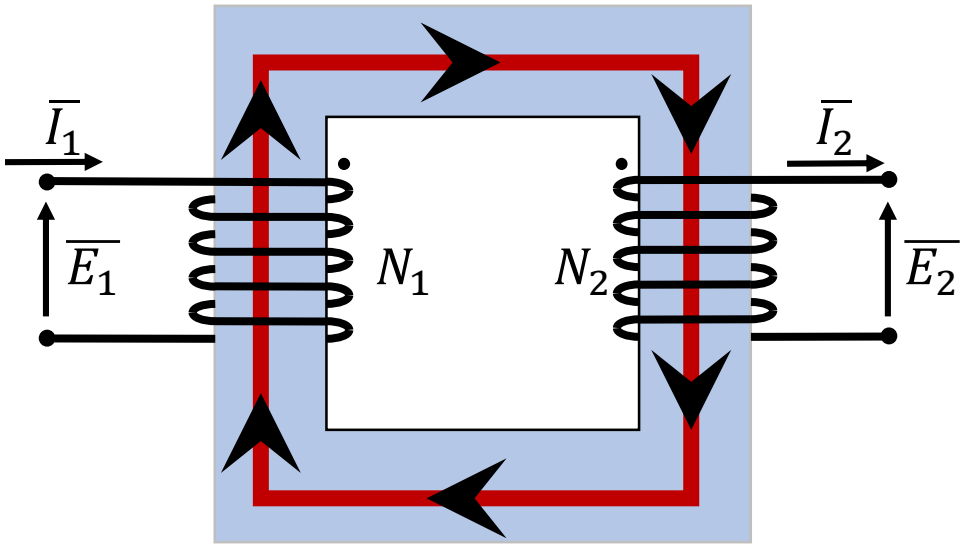
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Sequence impedances of transformers

- Ideal transformer:

$$\frac{\overline{I_1}}{\overline{I_2}} = \frac{N_2}{N_1}$$

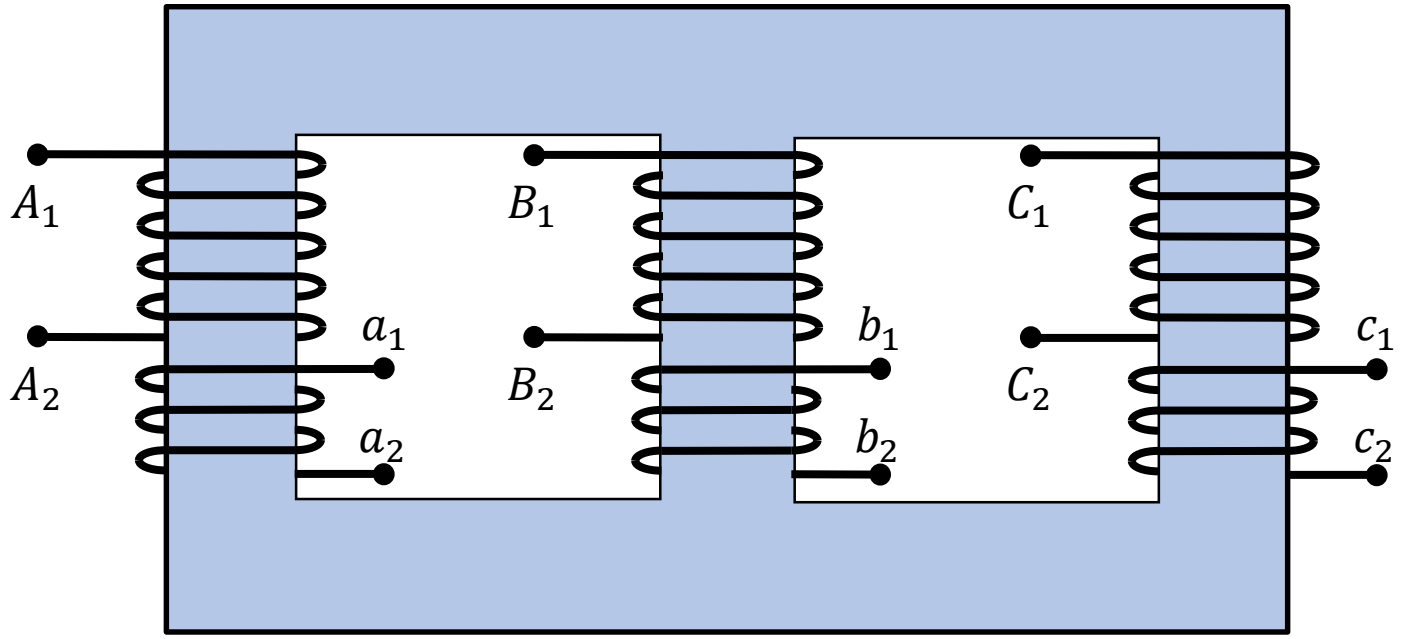
- Primary and secondary currents are in phase
- Secondary current must balance the primary current



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Sequence impedances of transformers

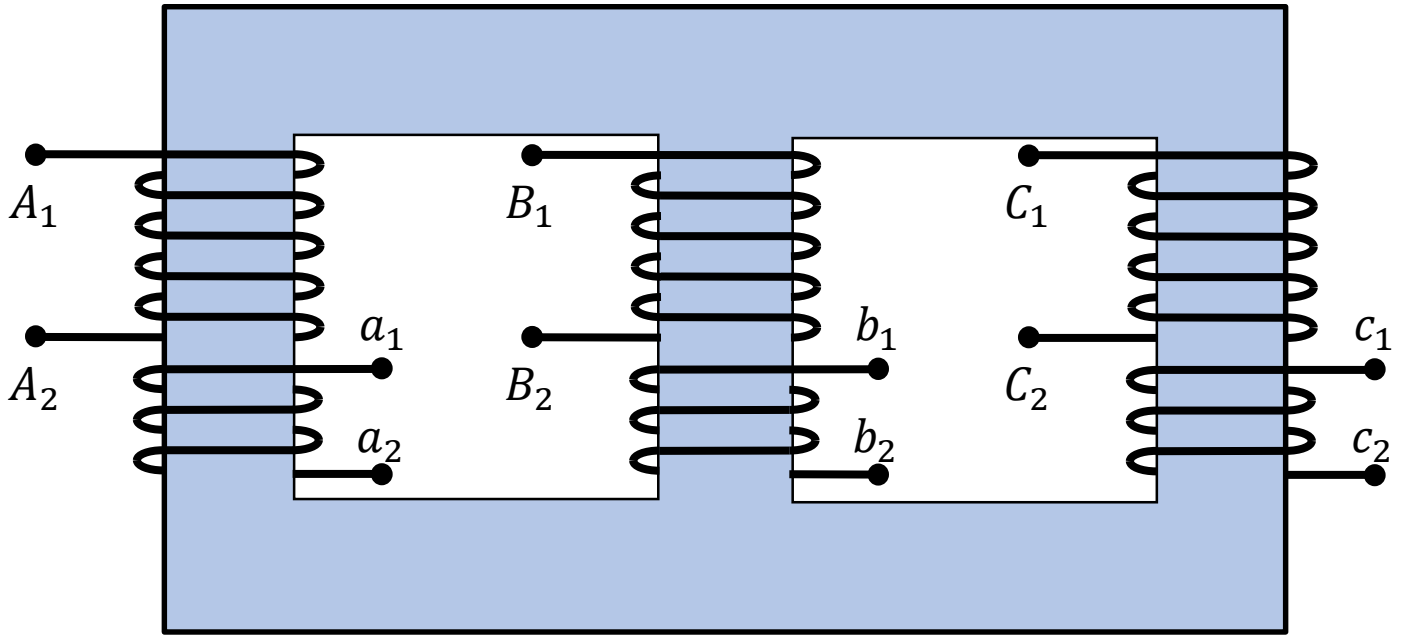
- Primary and secondary currents must be in balance in each phase
- Positive-sequence primary current must be balanced by a positive sequence secondary current
- Same for the negative- and zero-sequence currents



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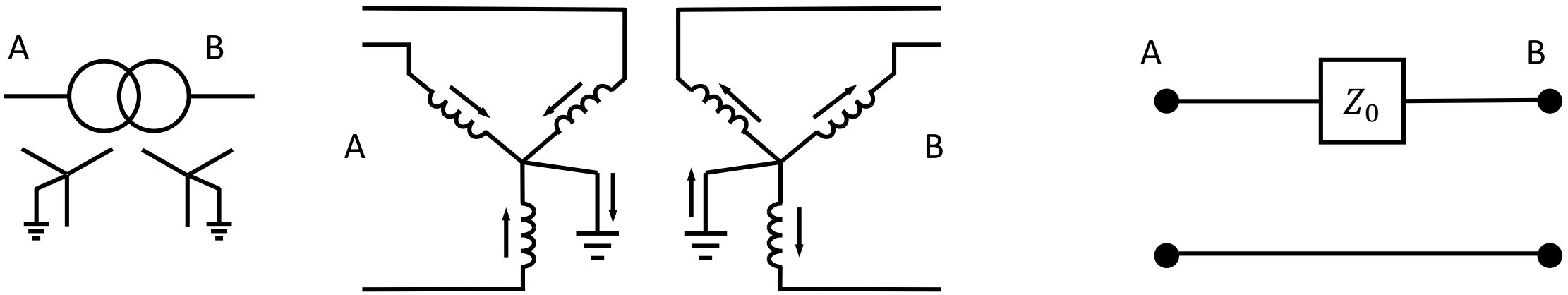
Sequence impedances of transformers

- Balancing the primary and secondary positive and negative sequence currents is always possible
- Depending on the connection of the primary and secondary windings, this may or may not be possible for the zero-sequence.



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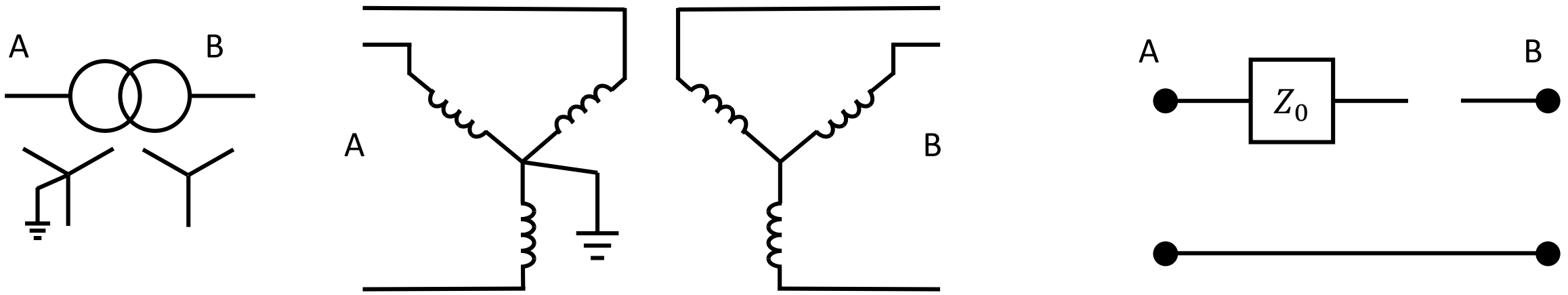
Grounded-Y / Grounded-Y connection



Primary zero-sequence current can be balanced by a secondary zero sequence current.
There is thus a path for zero-sequence currents through this transformer.

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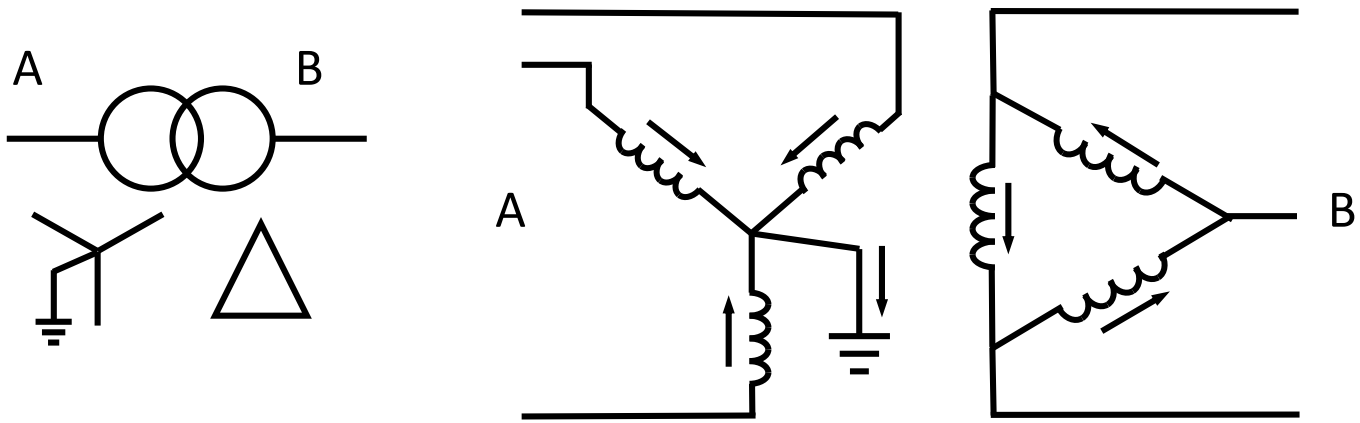
Grounded-Y / Ungrounded-Y connection

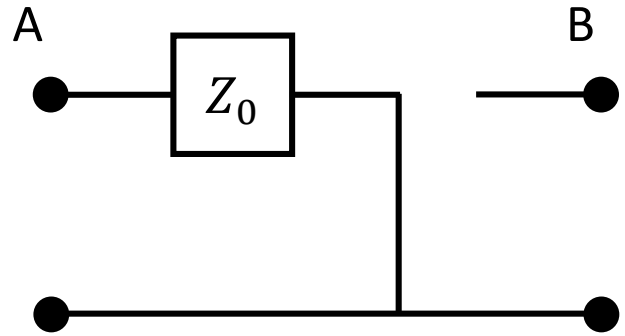


Primary zero-sequence current cannot be balanced by a secondary zero sequence current. Therefore, no zero-sequence current can flow in or out of this transformer.

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Grounded-Y / Delta connection

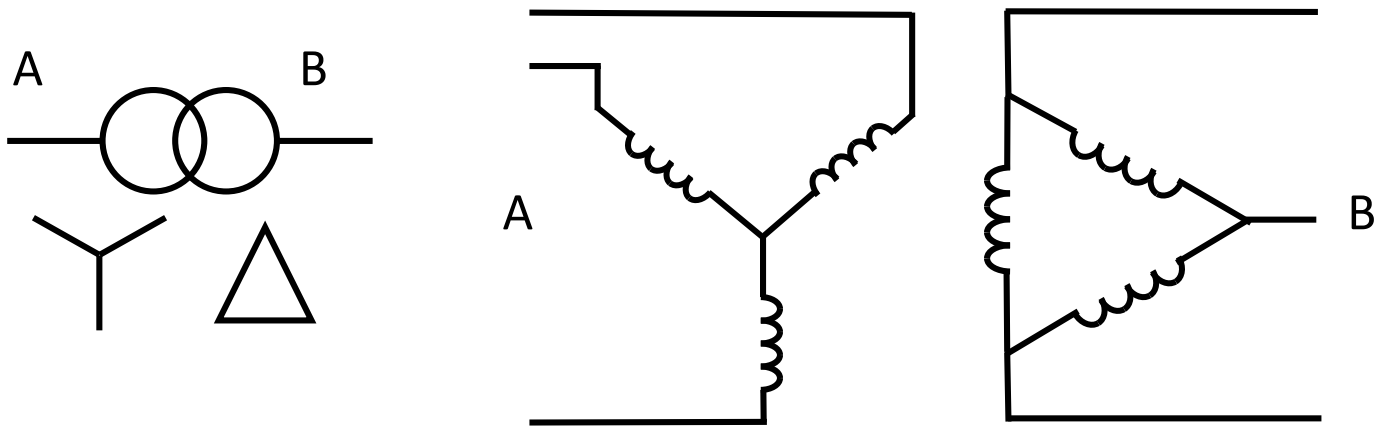


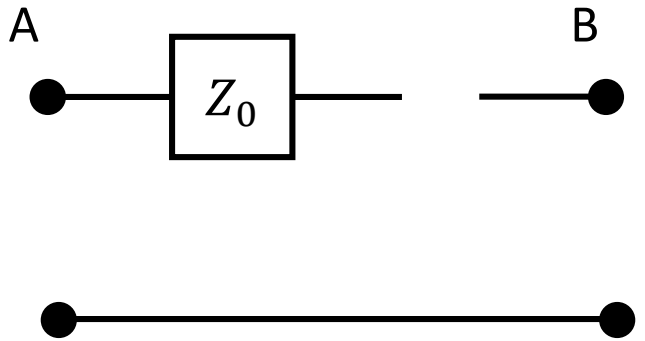


Primary zero-sequence current can be balanced by a circulating zero sequence in the secondary. There is thus a path for zero sequence currents on the grounded-Y side, but not on the delta side.

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Ungrounded-Y / Delta connection

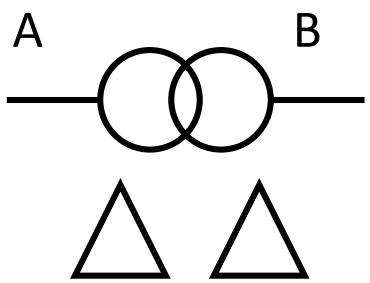


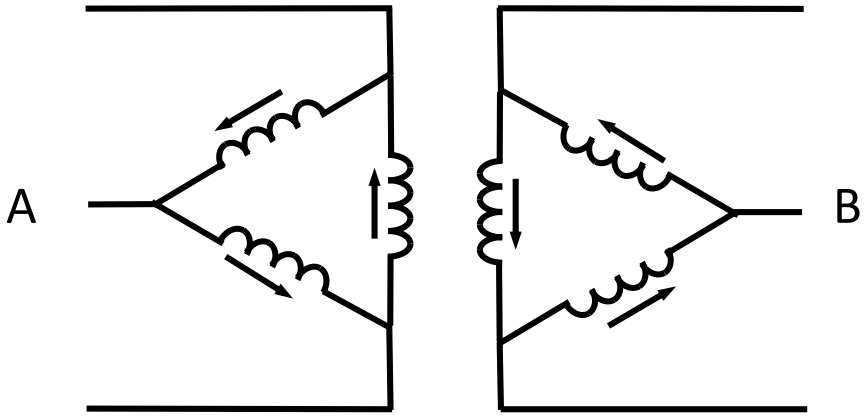


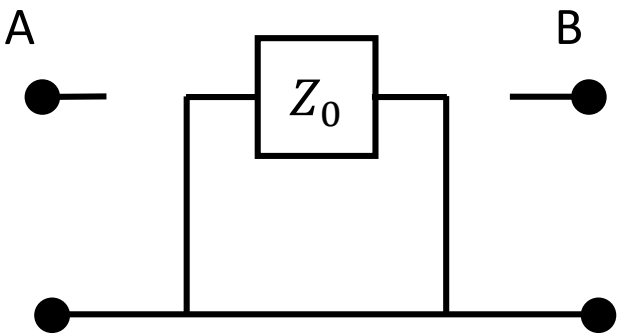
There is no path for a zero-sequence current on the primary side. No zero-sequence current can therefore flow in or out of this transformer.

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Delta / Delta connection



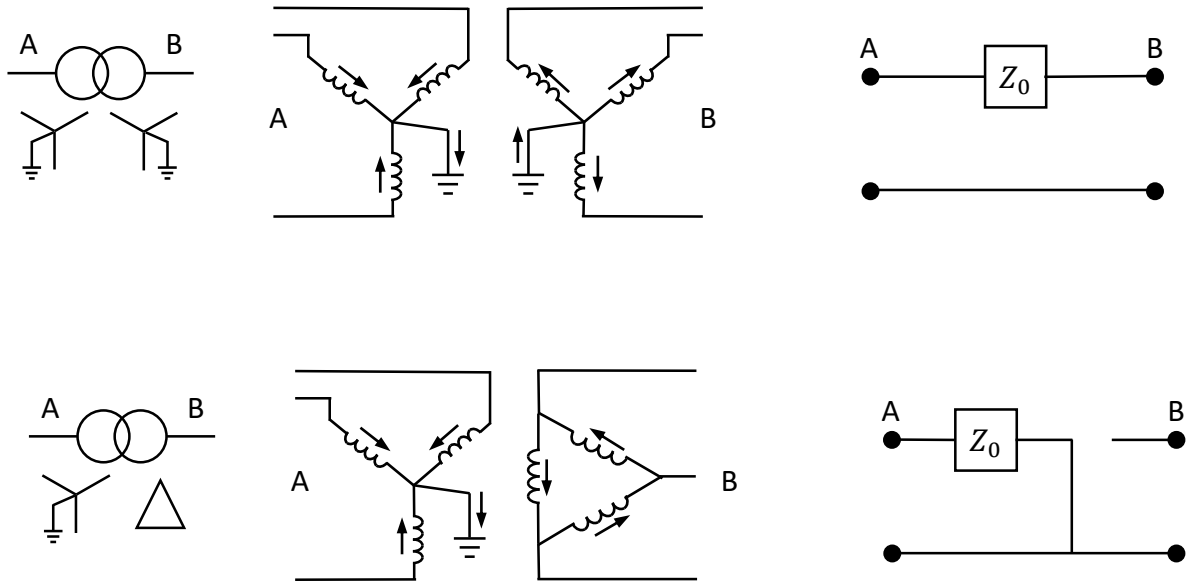


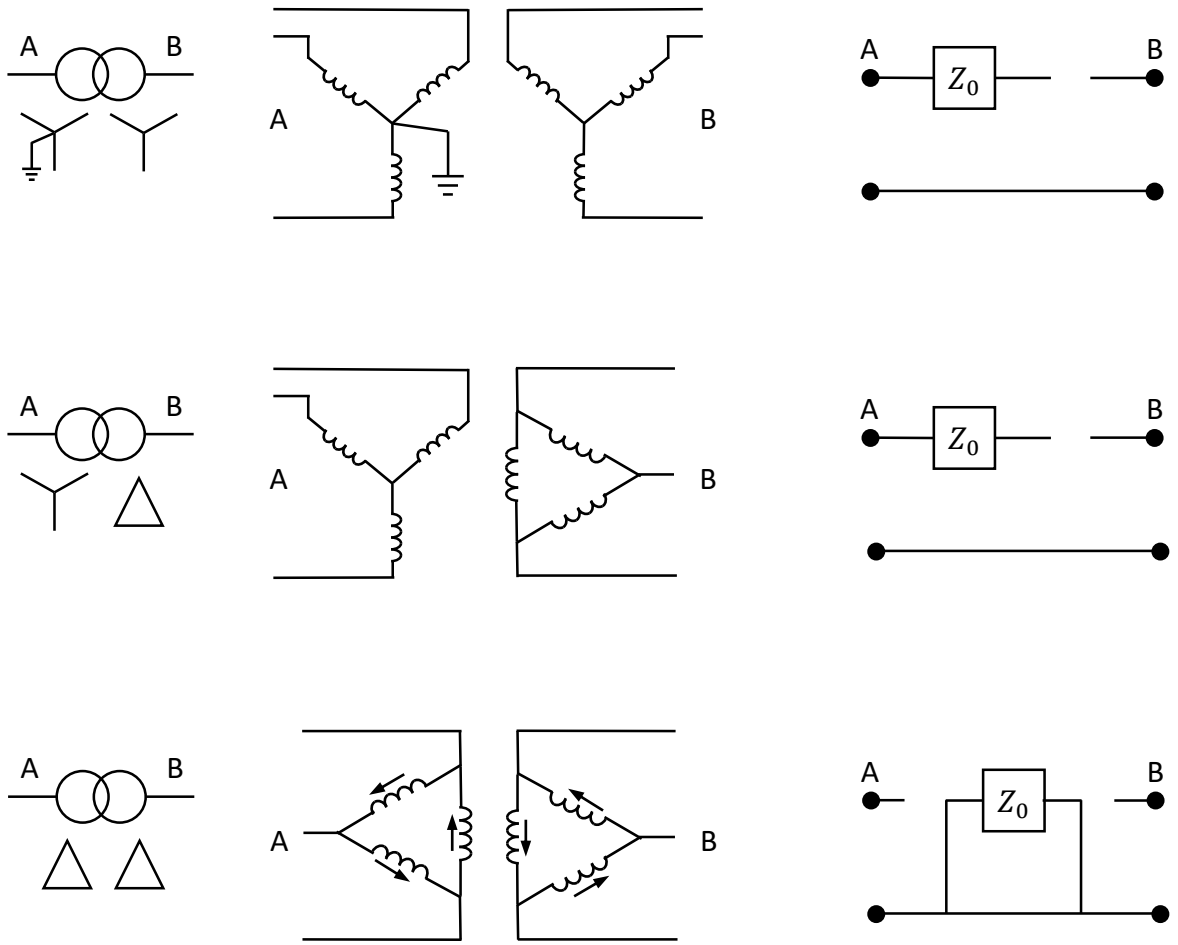


While balanced zero-sequence currents can circulate within the windings, they cannot flow in or out of this transformer.

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Effect of transformer connections on zero-sequence





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Summary

- Variable transformation from phase voltages and currents to positive-, negative-, and zero-sequence voltages and currents
- Applying this transformation to balanced three-phase devices results in decoupled positive-, negative-, and zero-sequence models of these devices.
- Each device (line, load, generator, transformer) has a positive-, a negative-, and a zero-sequence impedance

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Sequence impedances

- Positive-sequence impedance Z_1
  - Same as impedance of single-phase representation of three-phase circuits
- Negative-sequence impedance Z_2
  - Equal to positive-sequence impedance for components that do not differentiate between the phase sequences
  - $Z_2 = Z_1$ for passive loads, transformers and transposed lines
  - $Z_2 \neq Z_1$ for generators, motors and un-transposed lines

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Sequence impedances

- Zero-sequence impedance Z_0
  - Usually different from Z_1 and Z_2
  - Because $\overline{I_n} = 3\overline{I_0}$, Z_0 is infinite if there is no connection between neutral and ground
  - Because primary and secondary currents must be balanced, the connection of a transformer windings affects its zero-sequence model
  - Some transformer connections block zero sequence currents

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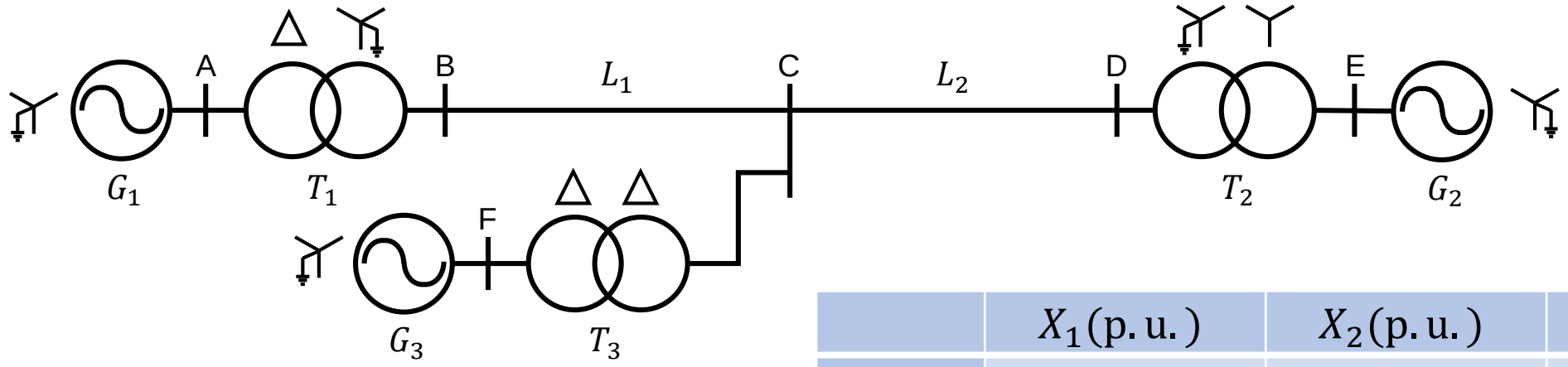
Building sequence networks

- Gather decoupled models of all network devices in terms of symmetrical components
- Combine these models according to the topology of the network

→ Decoupled positive-, negative-, and zero-sequence networks

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Example 10.8: Sequence networks

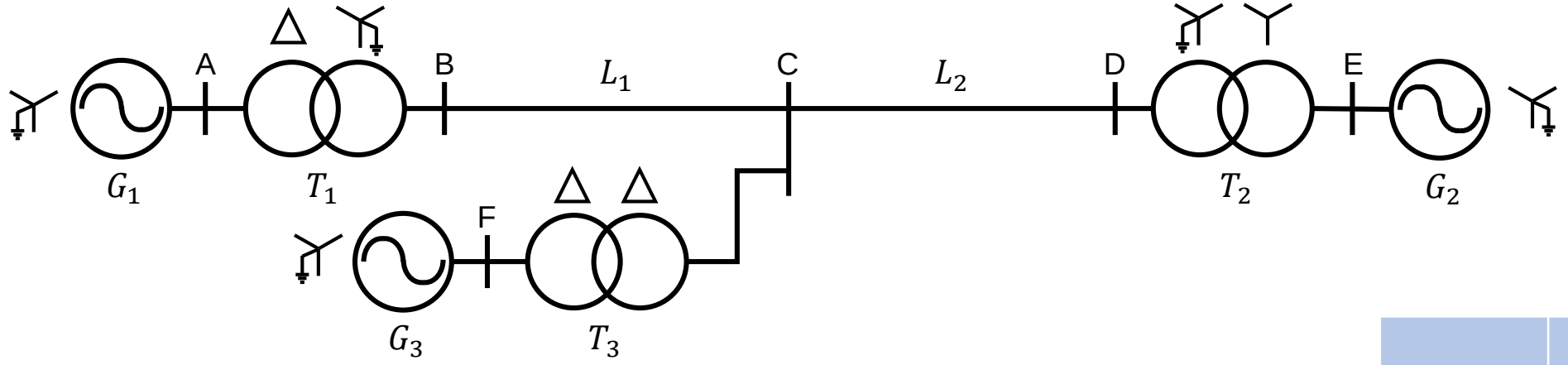


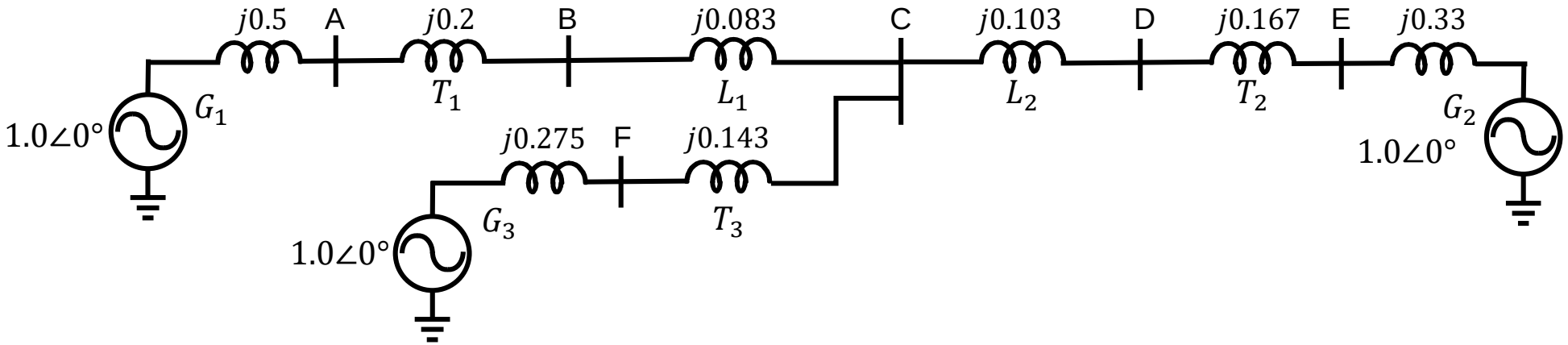
	X_1(p.u.)	X_2(p.u.)	X_0 (p.u.)
G_1	0.5	0.2	0.18
G_2	0.33	0.15	0.15
G_3	0.275	0.1	0.12
T_1	0.2	0.2	0.2
T_2	0.167	0.167	0.167
T_3	0.143	0.143	0.143
L_1	0.083	0.083	0.4
L_2	0.103	0.103	0.6

System is assumed unloaded and operating at nominal voltage

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Example 10.8: Positive sequence network

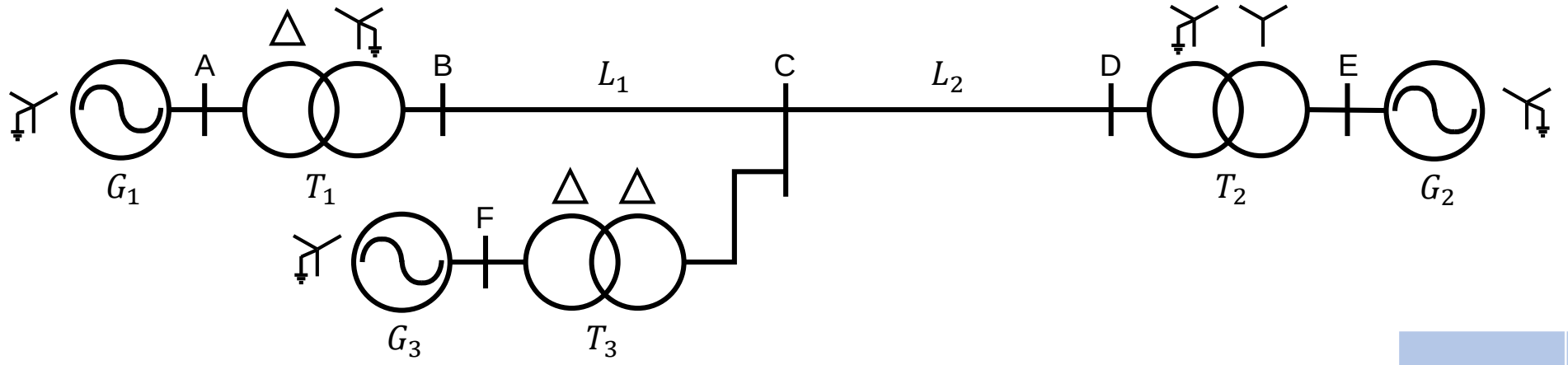


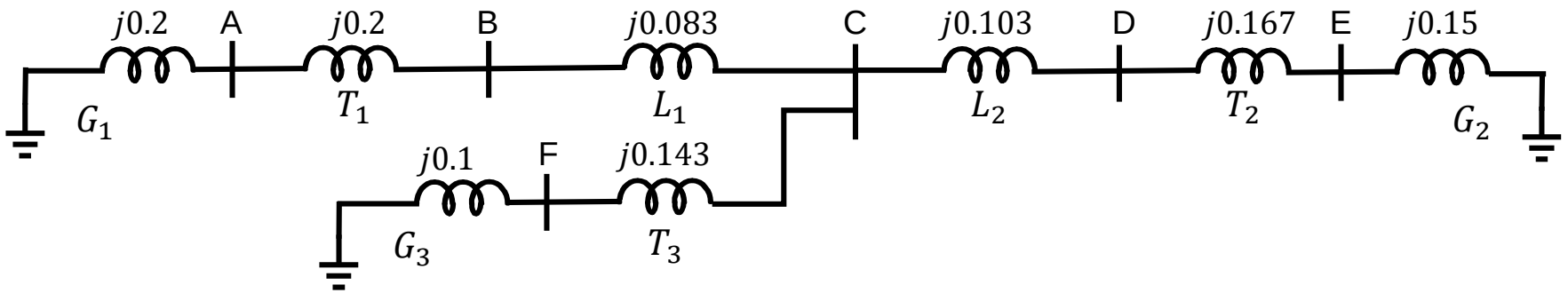


	X_1(p.u.)
G_1	0.5
G_2	0.33
G_3	0.275
T_1	0.2
T_2	0.167
T_3	0.143
L_1	0.083
L_2	0.103

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Example 10.8: Negative sequence networks

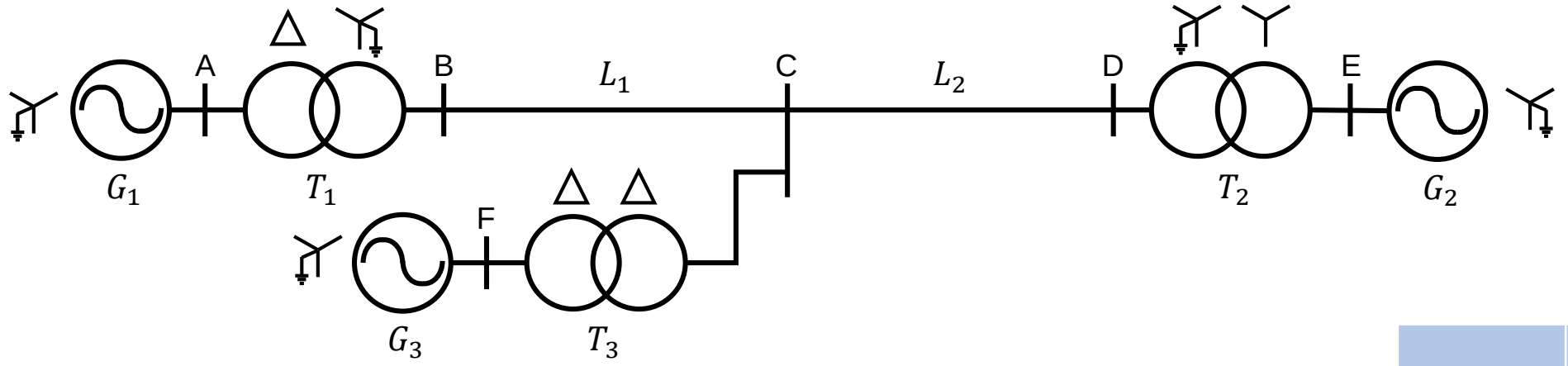


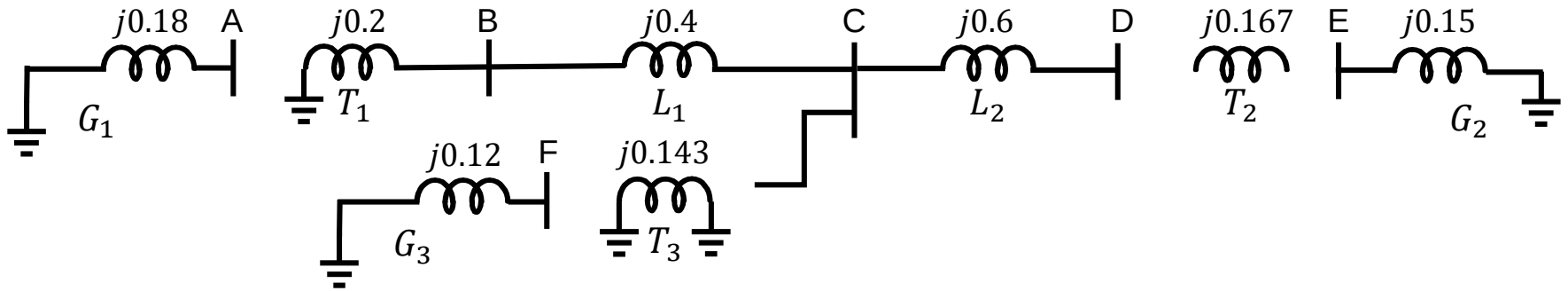


	$X_2(\text{p.u.})$
G_1	0.2
G_2	0.15
G_3	0.1
T_1	0.2
T_2	0.167
T_3	0.143
L_1	0.083
L_2	0.103

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Example 10.8: Zero sequence networks





	X_0 (p.u.)
G_1	0.18
G_2	0.15
G_3	0.12
T_1	0.2
T_2	0.167
T_3	0.143
L_1	0.4
L_2	0.6

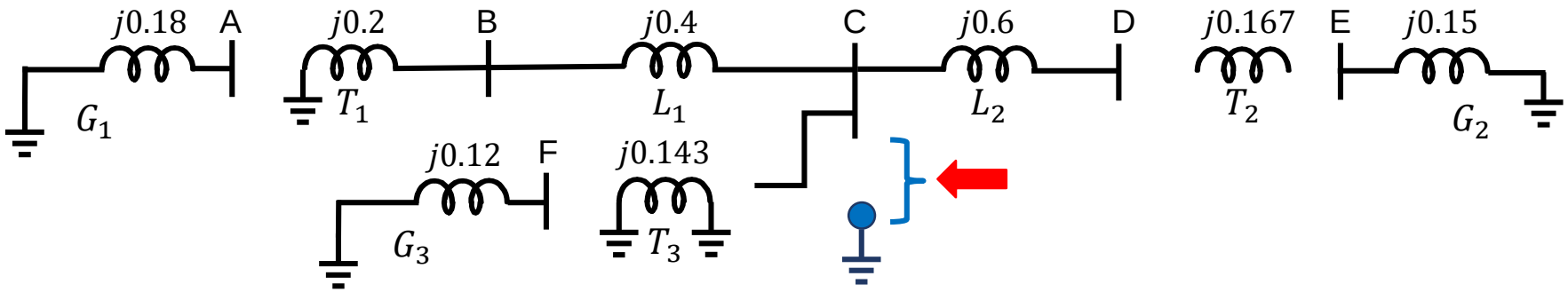
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Thevenin equivalents of sequence networks

- To study balanced faults, we used Thevenin equivalents of the single-phase representation of the network
- To study unbalanced faults, we will use Thevenin equivalents of the positive-, negative-, and zero-sequence networks

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Example 10.9: Thevenin equivalents of sequence networks



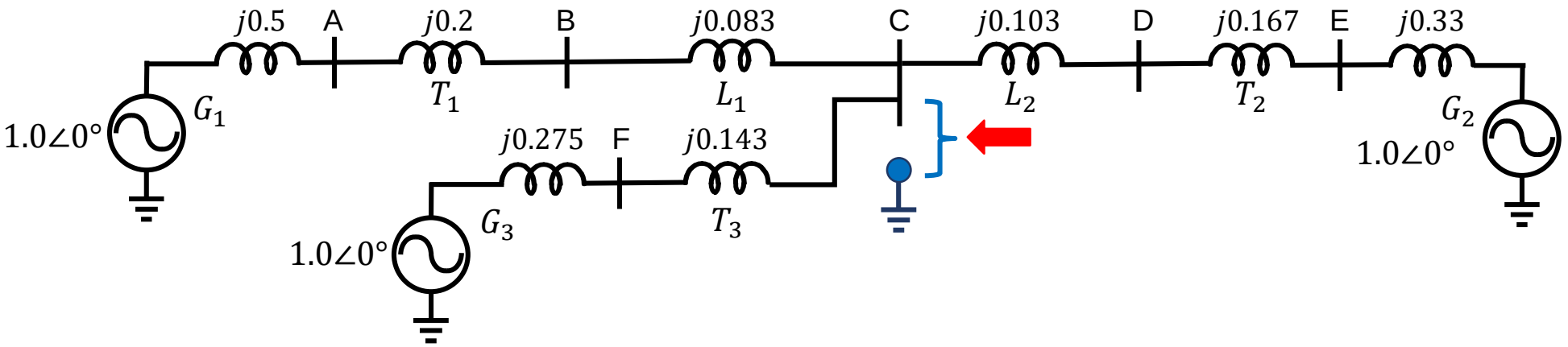
Zero-sequence Thevenin equivalent at node C:

$$\overline{V_C^{TH,0}} = 0$$

$$Z_C^{TH,0} = j(0.2 + 0.4) = j0.6\ p.u.$$

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Example 10.9: Thevenin equivalents of sequence networks



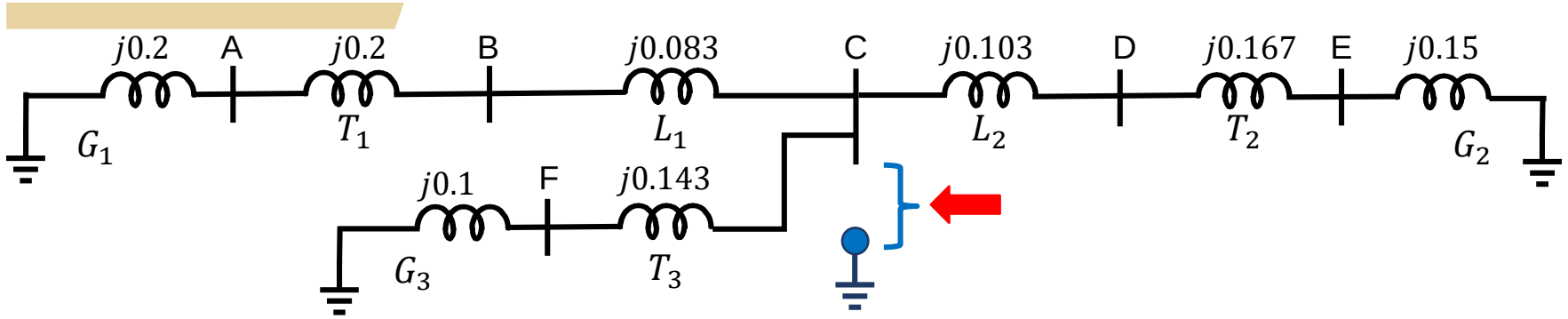
Positive-sequence Thevenin equivalent at node C:

$$\overline{V_C^{TH,1}} = 1.0 \text{ p. u.}$$

$$Z_C^{TH,1} = j(0.33 + 0.167 + 0.103) \parallel j(0.5 + 0.2 + 0.083) \parallel j(0.275 + 0.143)$$
$$= j0.187\ p.u.$$

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Example 10.9: Thevenin equivalents of sequence networks



Negative-sequence Thevenin equivalent at node C:

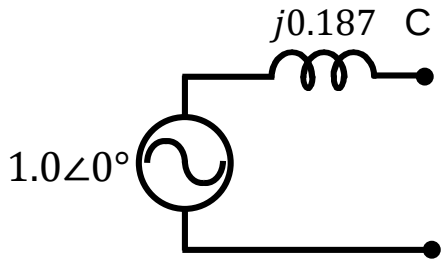
$$\overline{V_C^{TH,2}} = 0$$

$$Z_C^{TH,2} = j(0.15 + 0.167 + 0.103) \parallel j(0.2 + 0.2 + j0.083) \parallel j(0.1 + 0.143)$$
$$= j0.117\ p.u.$$

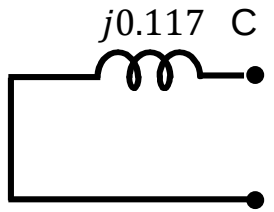
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Example 10.9: Thevenin equivalents of sequence networks

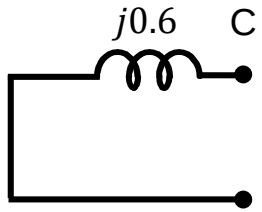
Positive-sequence Thevenin equivalent:



Negative-sequence Thevenin equivalent:

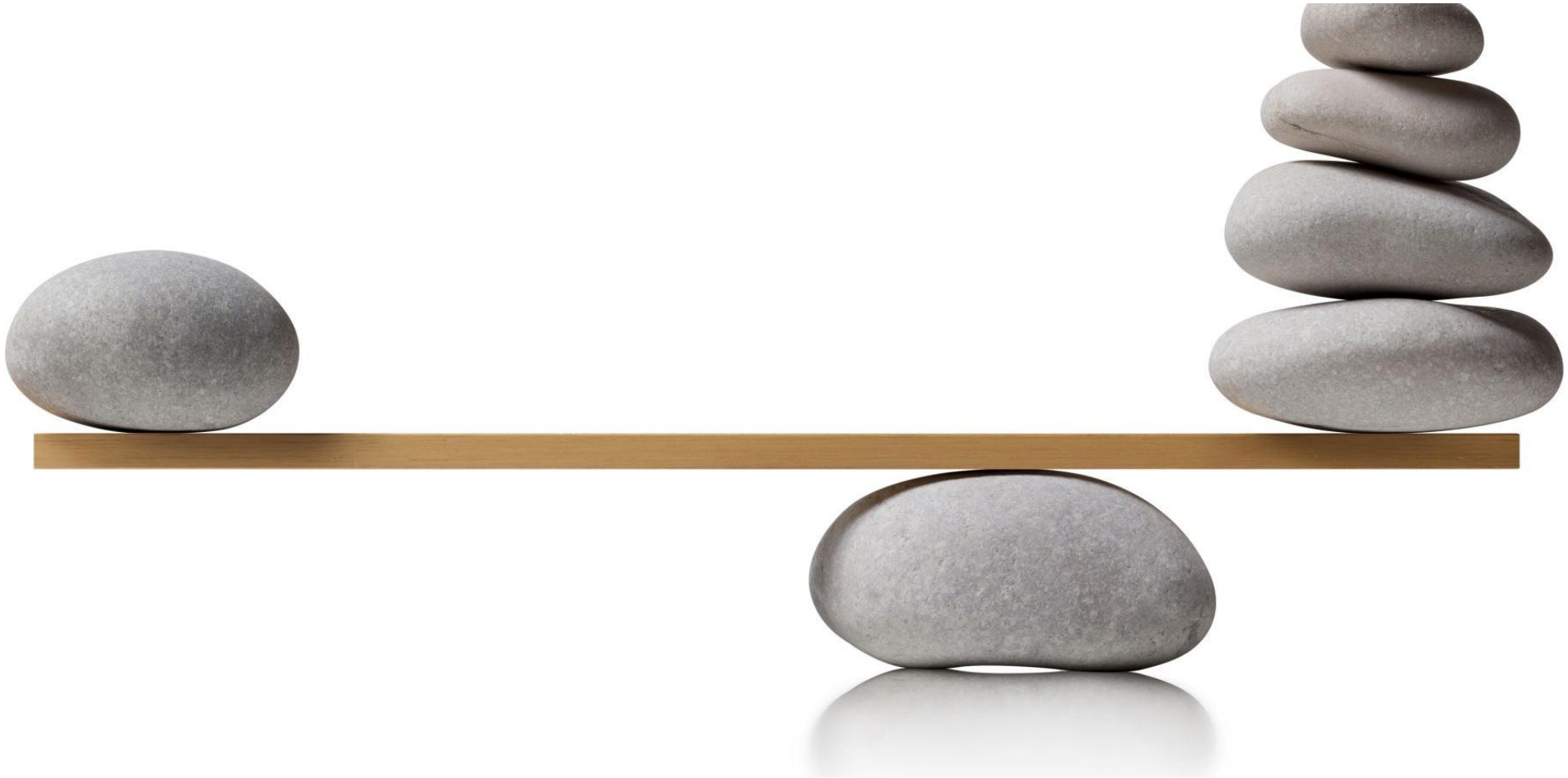


Zero-sequence Thevenin equivalent:



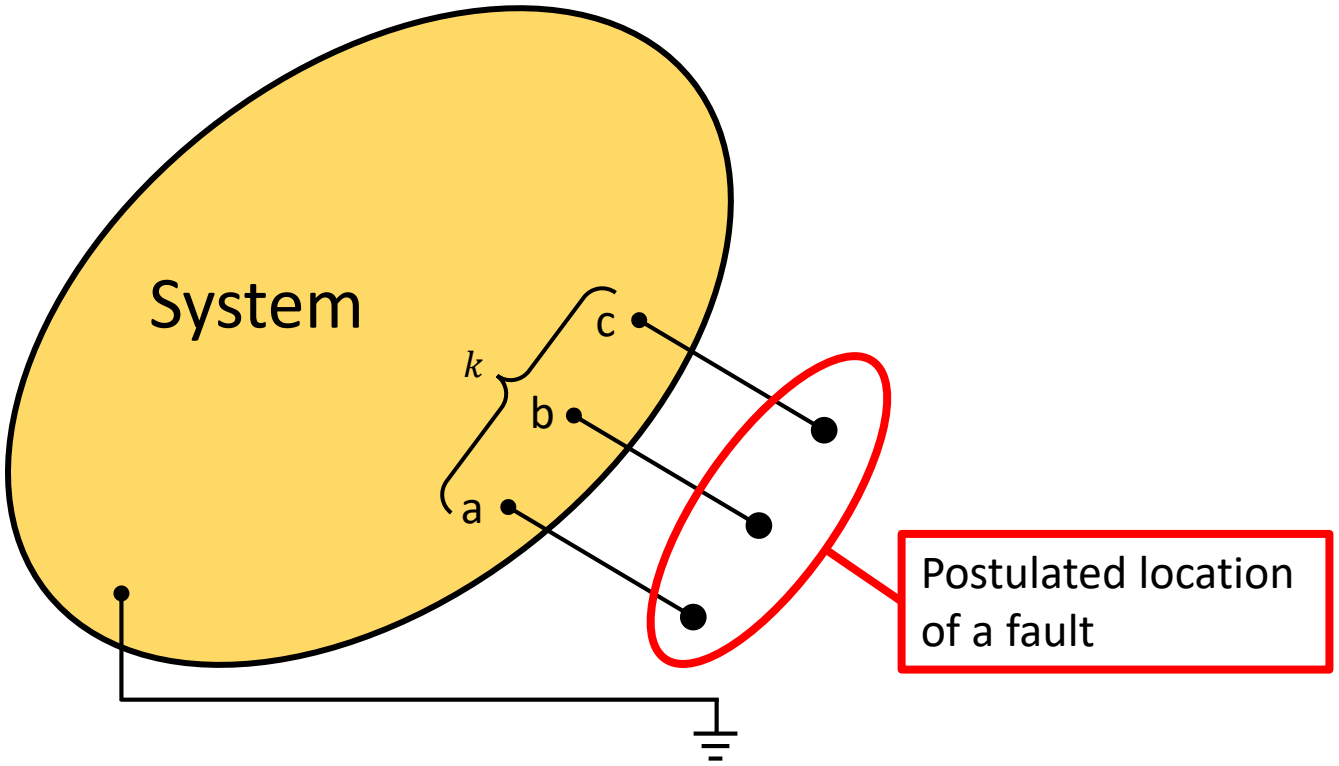
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Analyzing unbalanced faults



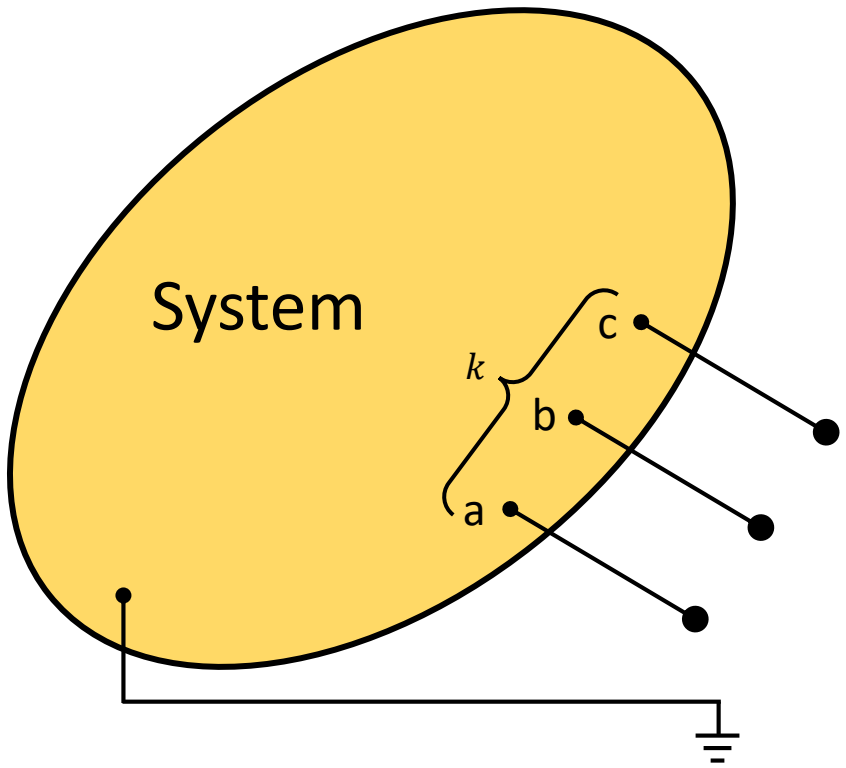
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Analyzing unbalanced faults



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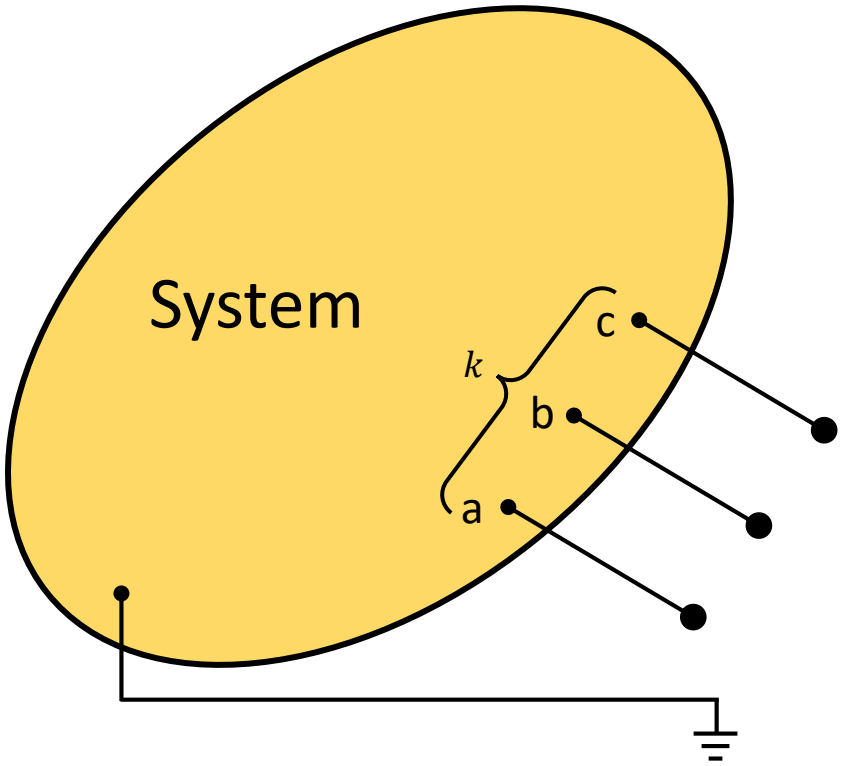
Pre-fault conditions



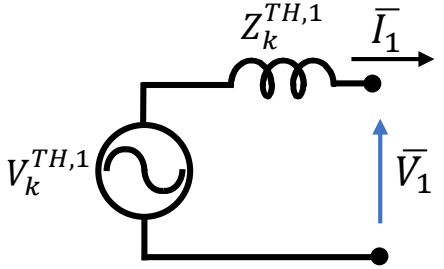
- System is balanced before the fault
  - Symmetrical component networks will be coupled only at the point of the fault
- Neglect pre-fault load currents
  - All voltages are equal to the pre-fault voltage at the point of the fault
- Neglect resistances, shunt admittances, rotor saliency, saturation
  - Commercial-grade fault calculation programs do not make these assumptions

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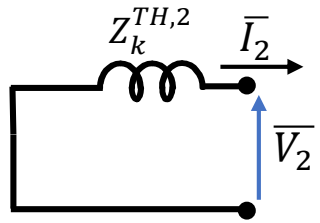
Pre-fault conditions



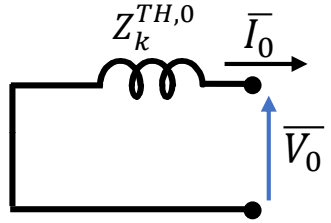
Thevenin equivalent looking into the network from node k:



Positive sequence



Negative sequence

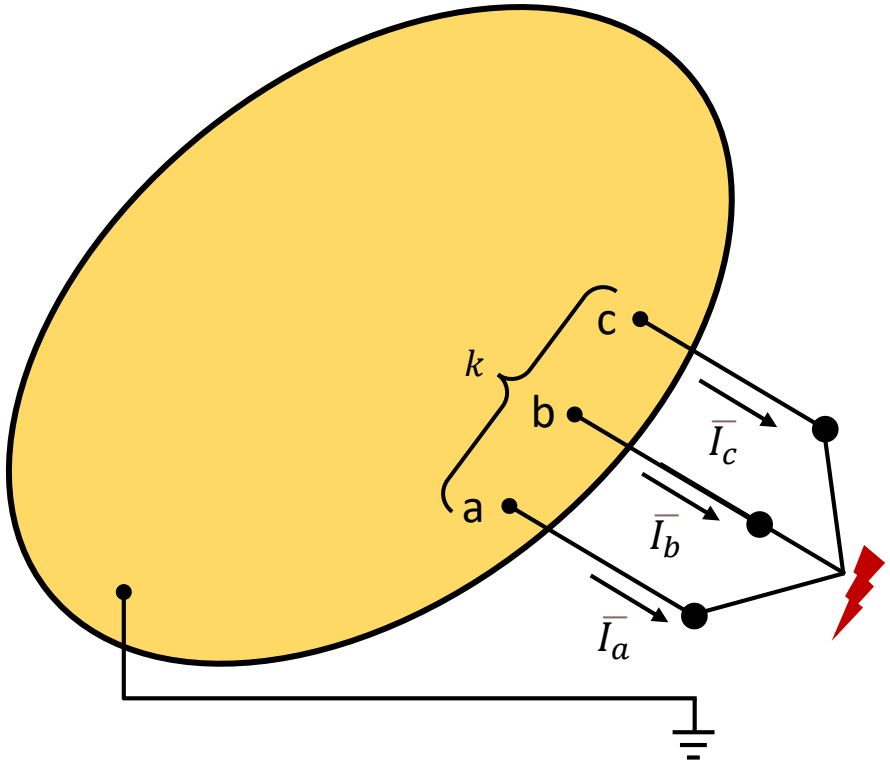


Zero sequence

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Analyzing balanced faults using symmetrical components

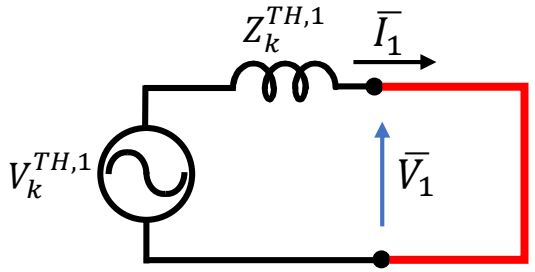
- Balanced three-phase fault at node k:
- $\overline{V_a} = \overline{V_b} = \overline{V_c} = 0$



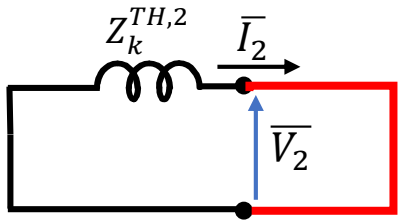
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Analyzing balanced faults using symmetrical components

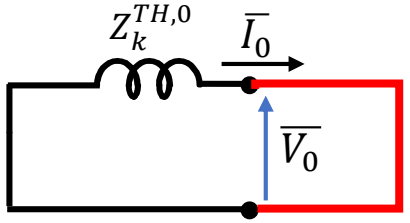
- Balanced three-phase fault at node k:
- $\overline{V_a} = \overline{V_b} = \overline{V_c} = 0$
- $\begin{pmatrix} \overline{V_0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{pmatrix}$
- $\begin{pmatrix} \overline{V_0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$



$$\overline{I_1} = \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1}}$$



$$\overline{I_2} = 0$$



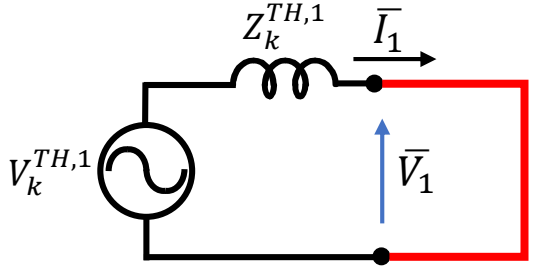
$$\overline{I_0} = 0$$

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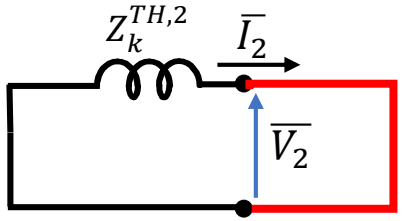
Analyzing balanced faults using symmetrical components

Balanced three-phase fault at node k:

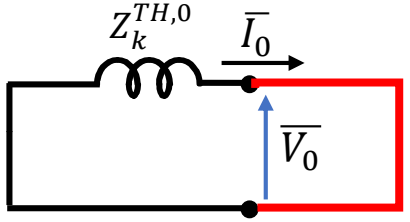
$$\begin{pmatrix} \overline{I_a} \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} 0 \\ \overline{I_1} \\ 0 \end{pmatrix} = \begin{pmatrix} \overline{I_1} \\ a^2\overline{I_1} \\ a\overline{I_1} \end{pmatrix}$$



$$\overline{I_1} = \frac{\overline{V_1}}{Z_k^{TH,1}}$$



$$\overline{I_2} = 0$$



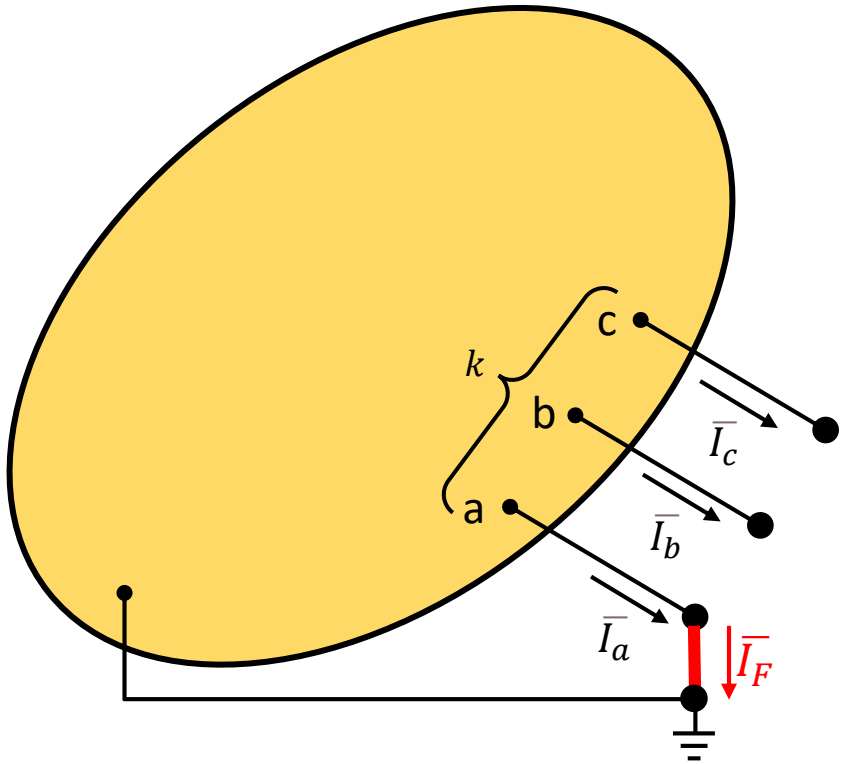
$$\overline{I_0} = 0$$

A balanced fault creates balanced fault currents

We already knew that, but it is nice to check it using symmetrical components

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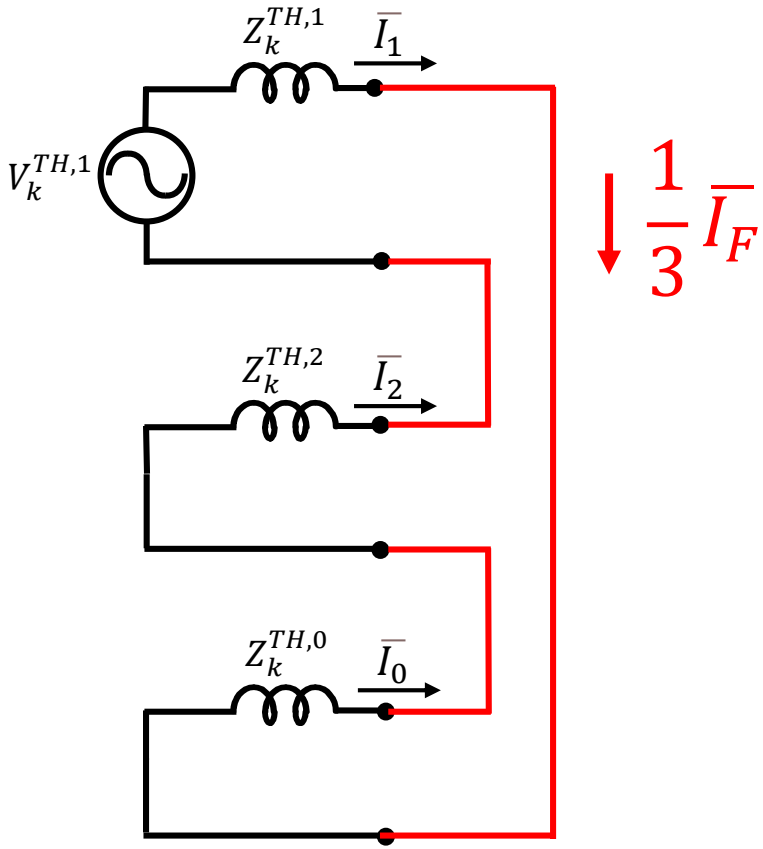
Single-line-to-ground fault at node k



- Pre-fault: $\bar{I}_a = \bar{I}_b = \bar{I}_c = 0$
- Without loss of generality, assume that the fault is on phase a
- $\bar{I}_a = \bar{I}_F \neq 0$
- $\bar{I}_b = \bar{I}_c = 0$
- $$\begin{pmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} \bar{I}_F \\ 0 \\ 0 \end{pmatrix}$$
- $\bar{I}_0 = \bar{I}_1 = \bar{I}_2 = \frac{1}{3}\bar{I}_F$

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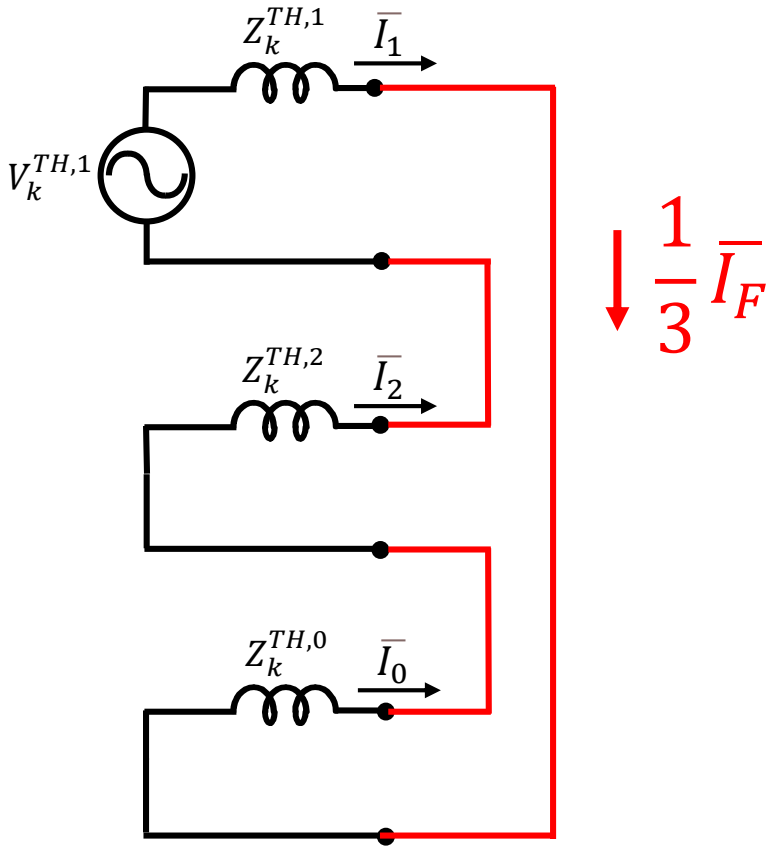
Single-line-to-ground fault at node k



- $\bar{I}_0 = \bar{I}_1 = \bar{I}_2 = \frac{1}{3}\bar{I}_F$
- A single-line-to-ground fault connects the three sequence networks in series
- $\bar{I}_0 = \bar{I}_1 = \bar{I}_2 = \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1}+Z_k^{TH,2}+Z_k^{TH,0}}$
- $\bar{I}_F = \frac{3\overline{V_k^{TH,1}}}{Z_k^{TH,1}+Z_k^{TH,2}+Z_k^{TH,0}}$

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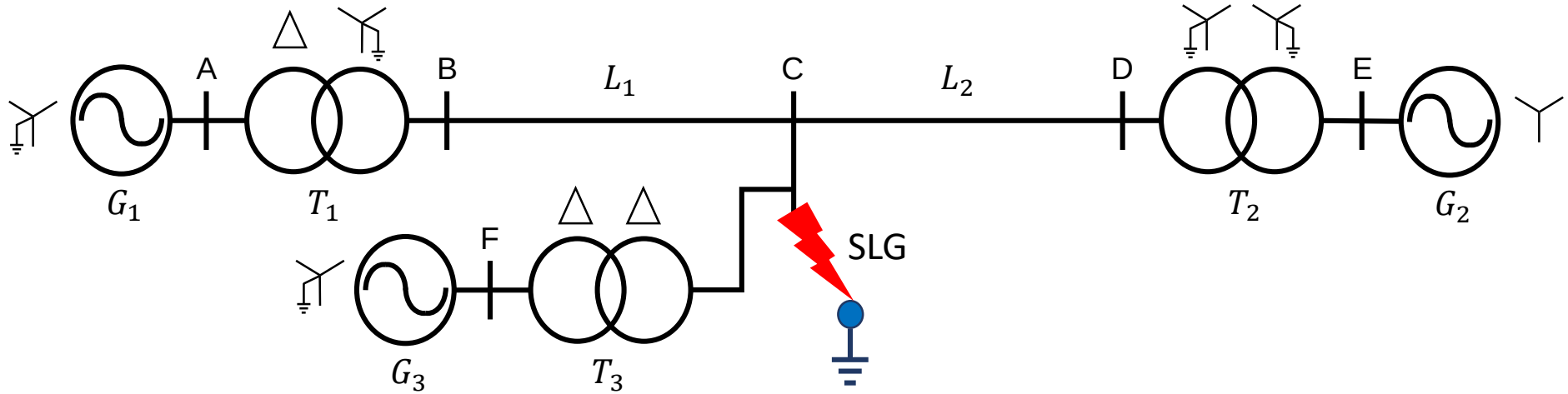
Single-line-to-ground fault at node k



- $\overline{I_0} = \overline{I_1} = \overline{I_2} = \frac{1}{3}\overline{I_F}$
- $\overline{I_F} = \frac{3\overline{V_k^{TH,1}}}{Z_k^{TH,1}+Z_k^{TH,2}+Z_k^{TH,0}}$
- $\begin{pmatrix} \overline{I_a} \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} \frac{1}{3}\overline{I_F} \\ \frac{1}{3}\overline{I_F} \\ \frac{1}{3}\overline{I_F} \end{pmatrix}$
- $\overline{I_a} = \overline{I_F}$
- $\overline{I_b} = \overline{I_c} = 0$

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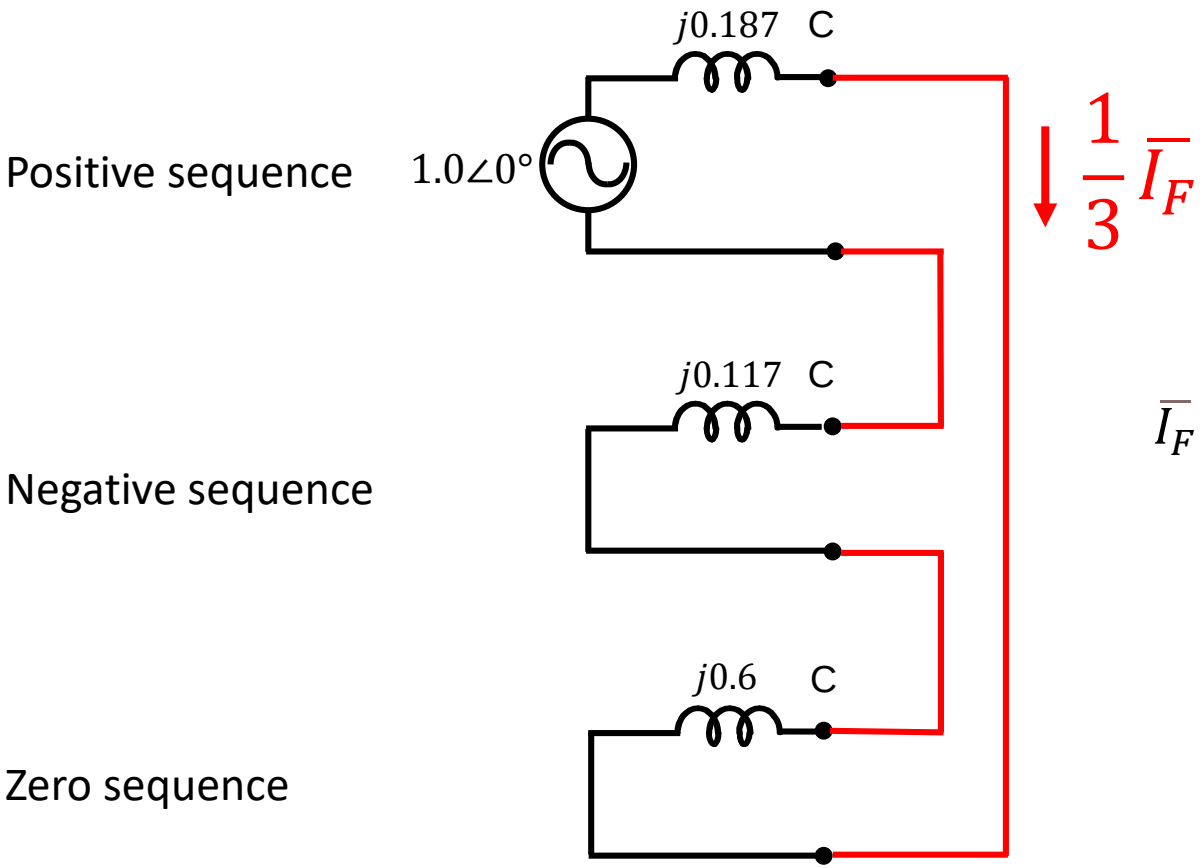
Example 10.10: Single-line-to-ground at bus C



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Example 10.10: Single-line-to-ground at bus C

From Example 10.9:

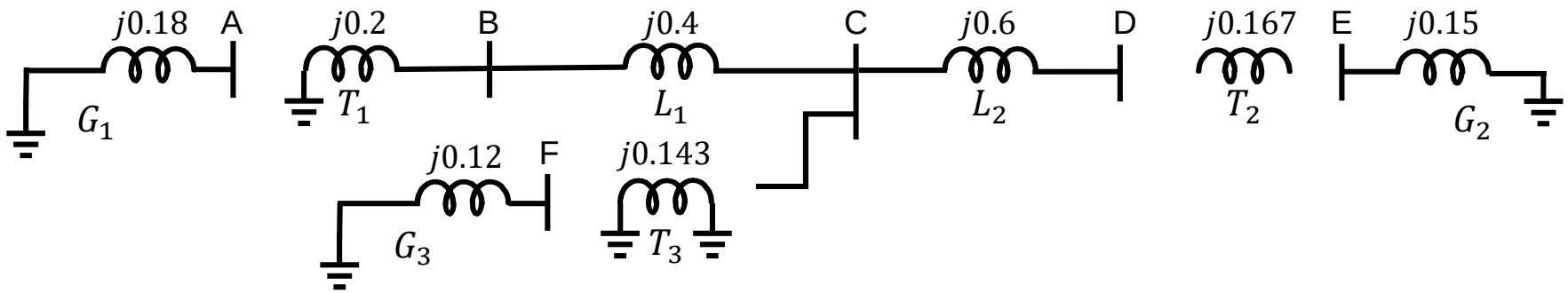


$$\bar{I}_F = \frac{3 \times 1.0\angle 0°}{j(0.6 + 0.187 + 0.117)} = 3.32\angle -90° p.u.$$

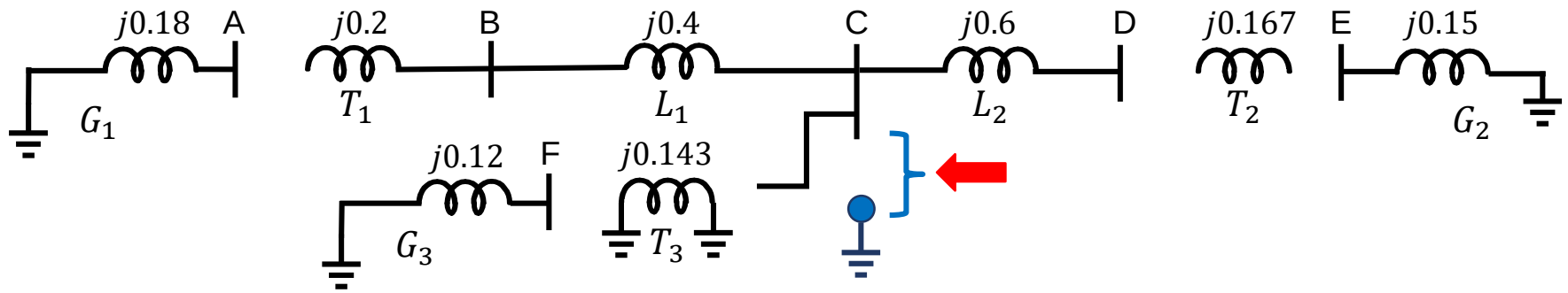
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Importance of ground connections

Zero sequence circuit with T_1 delta-grounded Y:



Zero sequence circuit with T_1 delta-Y:



$$Z_C^{TH,0} = \infty \quad \Rightarrow \bar{I}_F = \frac{3\overline{V_k^{TH,1}}}{Z_k^{TH,1} + Z_k^{TH,2} + Z_k^{TH,0}} = 0!$$

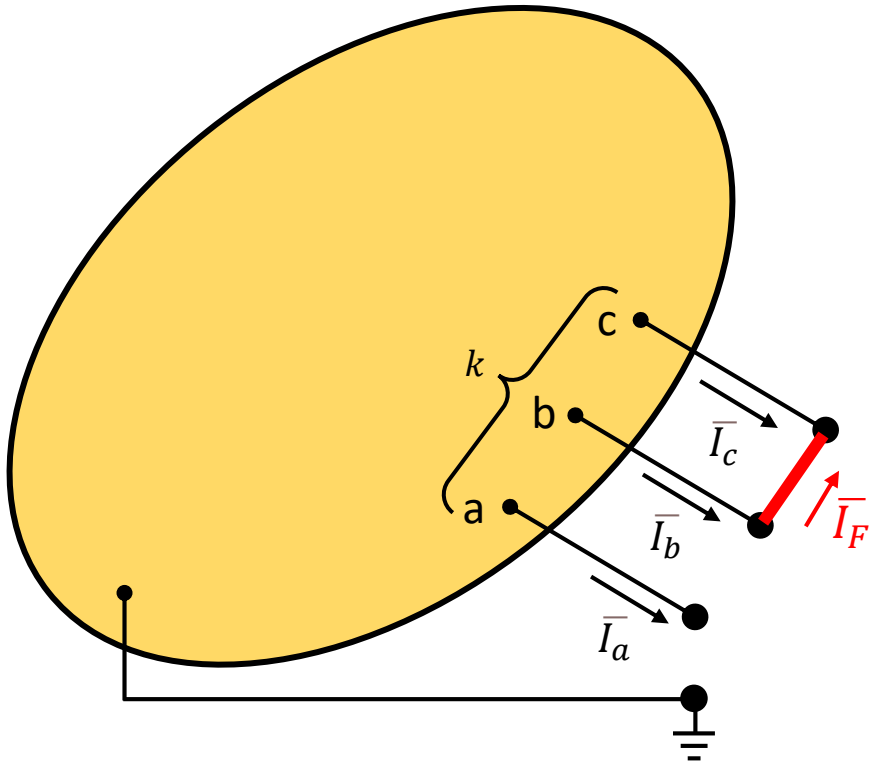
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Importance of ground connections

- Is having a zero fault current a good thing or a bad thing?
  - For the protection system to operate, there must be a large enough fault current
  - Undetected faults represent a safety hazard because a normal action can then have unexpected consequences

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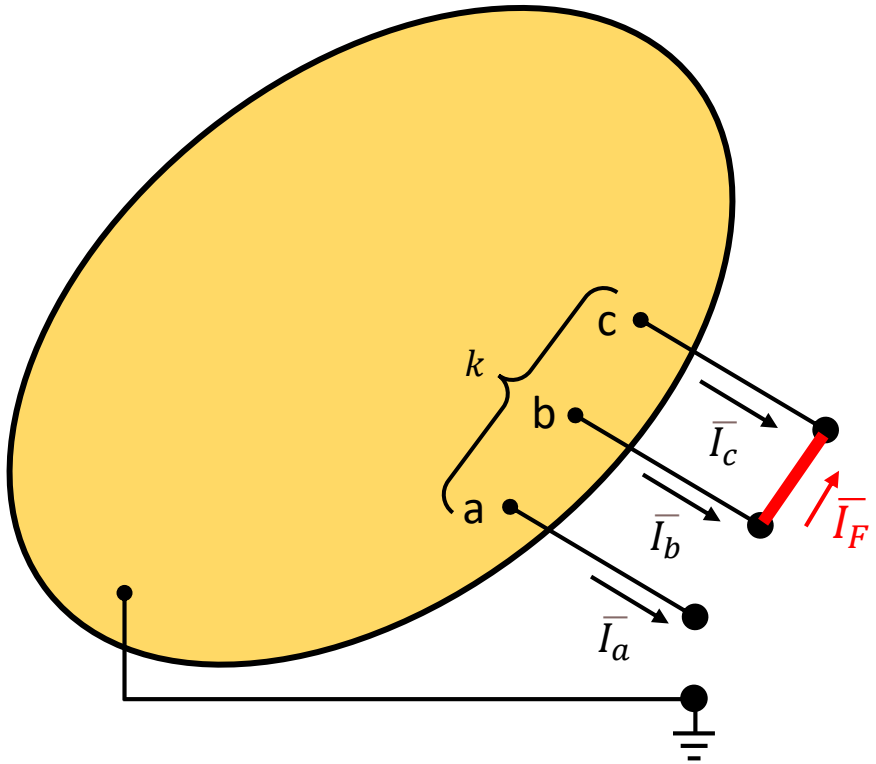
Line-to-line fault at node k



- Pre-fault: $\bar{I}_a = \bar{I}_b = \bar{I}_c = 0$
- Without loss of generality, assume that the fault is between phases b and c
- $\bar{I}_a = 0$
- $\bar{I}_b = -\bar{I}_c = \bar{I}_F$

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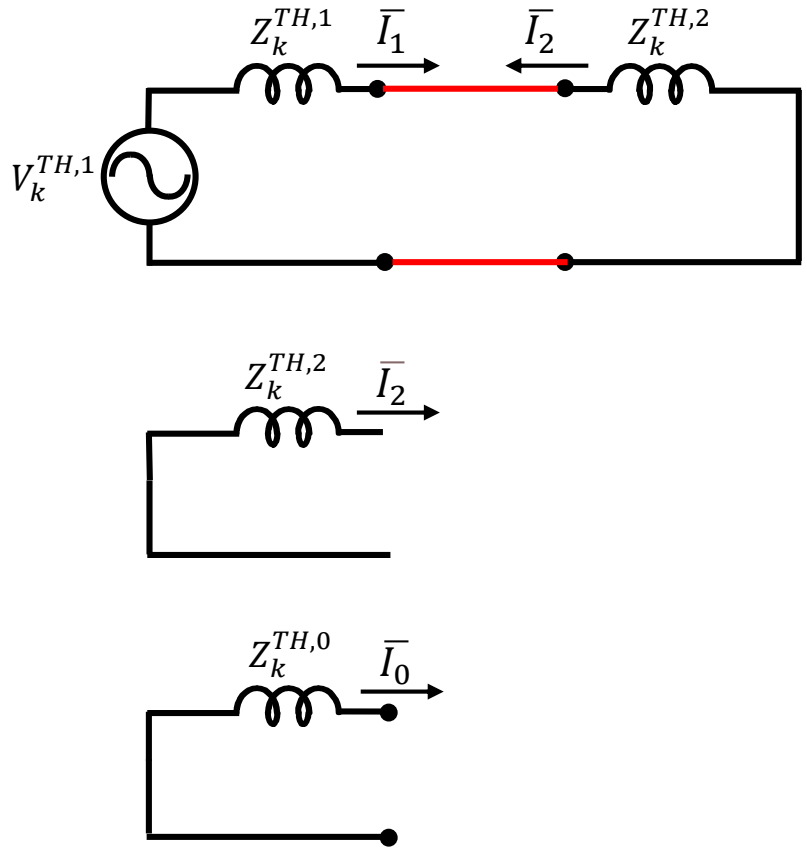
Line-to-line fault at node k



- $\bar{I}_a = 0$
- $\bar{I}_b = -\bar{I}_c = \bar{I}_F$
- $\begin{pmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} 0 \\ \bar{I}_F \\ -\bar{I}_F \end{pmatrix}$
- $\bar{I}_0 = \frac{1}{3}(\bar{I}_F - \bar{I}_F) = 0$
- $\bar{I}_1 = \frac{1}{3}(a - a^2)\bar{I}_F$
- $\bar{I}_2 = \frac{1}{3}(a^2 - a)\bar{I}_F = -\bar{I}_1$

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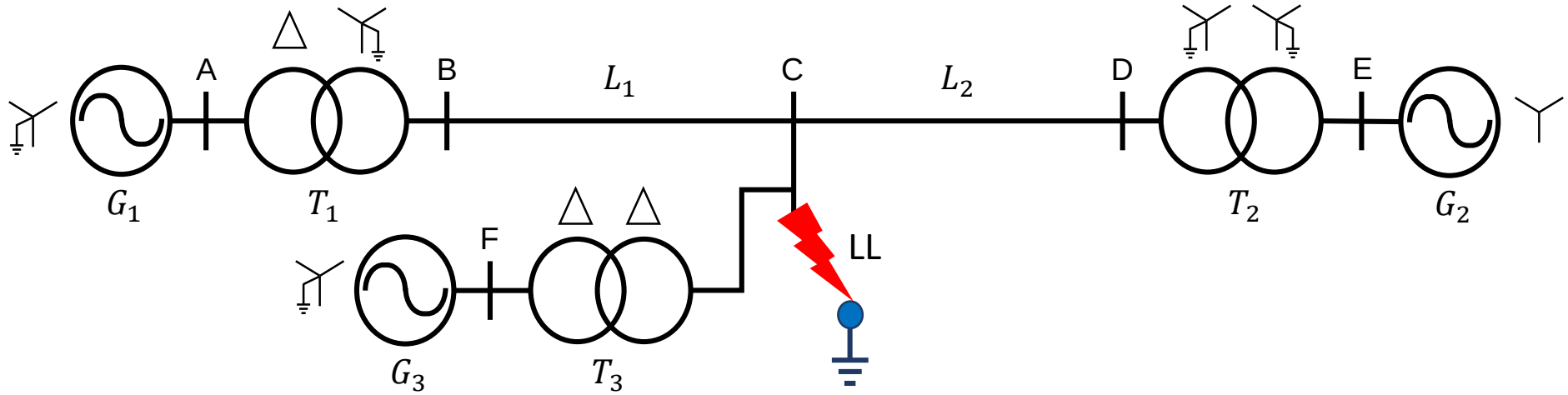
Line-to-line fault at node k



- $\bar{I}_0 = 0$
- $\bar{I}_1 = \frac{1}{3}(a - a^2)\bar{I}_F$
- $\bar{I}_2 = \frac{1}{3}(a^2 - a)\bar{I}_F = -\bar{I}_1$
- $\bar{I}_1 = -\bar{I}_2 = \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1} + Z_k^{TH,2}}$
- $a - a^2 = 1\angle 120° - 1\angle 240° = j\sqrt{3}$
- $\bar{I}_F = \frac{3}{j\sqrt{3}}\bar{I}_1 = -j\sqrt{3} \times \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1} + Z_k^{TH,2}}$

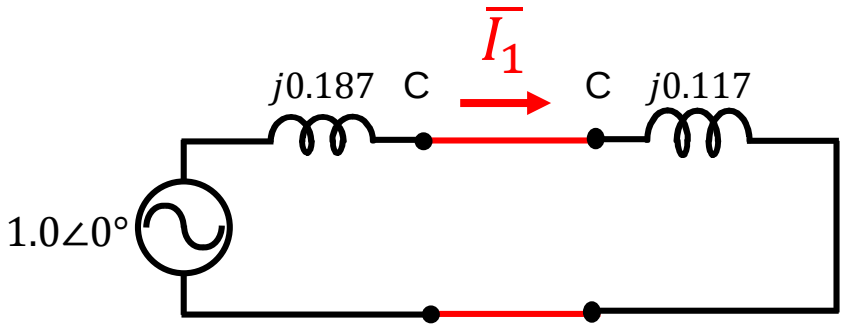
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Example 10.11: Line-to-line (LL) fault

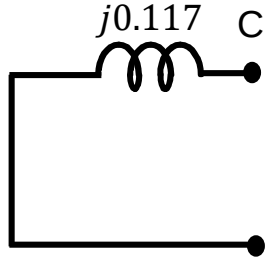


From Example 10.9:

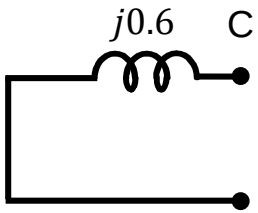
Positive sequence



Negative sequence

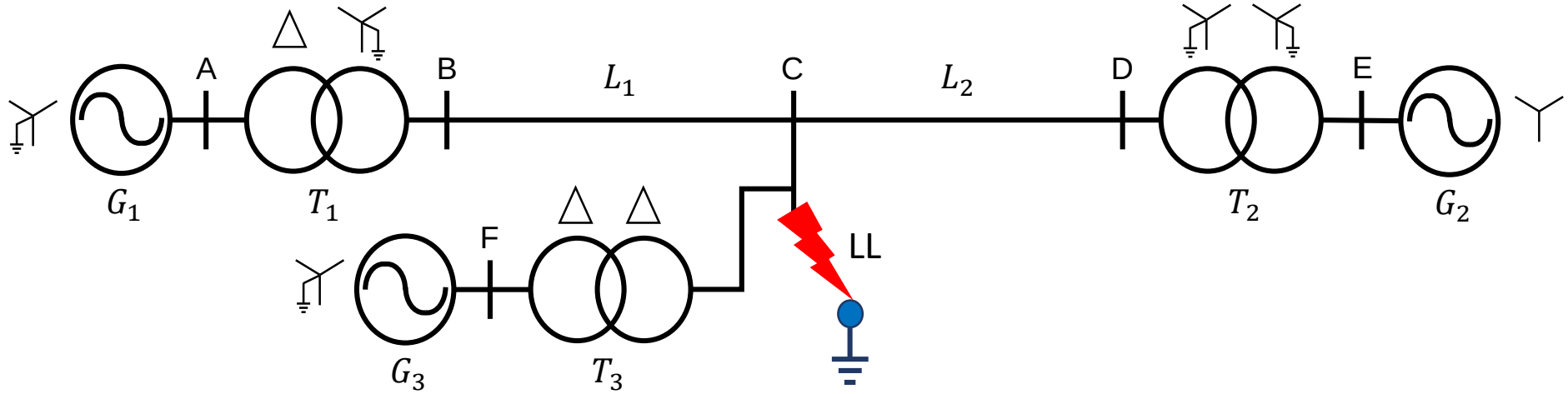


Zero sequence



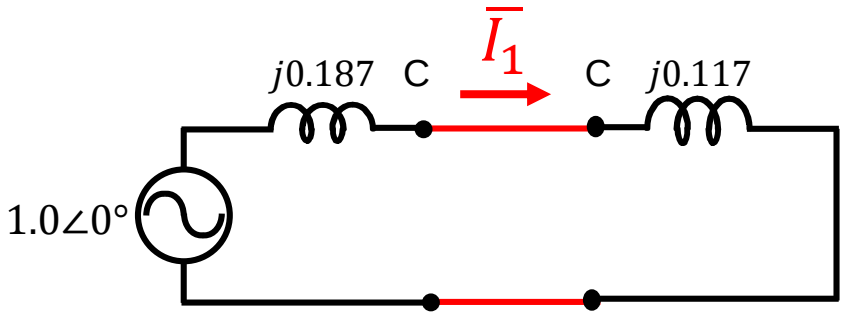
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Example 10.11: Line-to-line (LL) fault

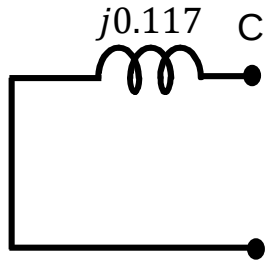


From Example 10.9:

Positive sequence



Negative sequence

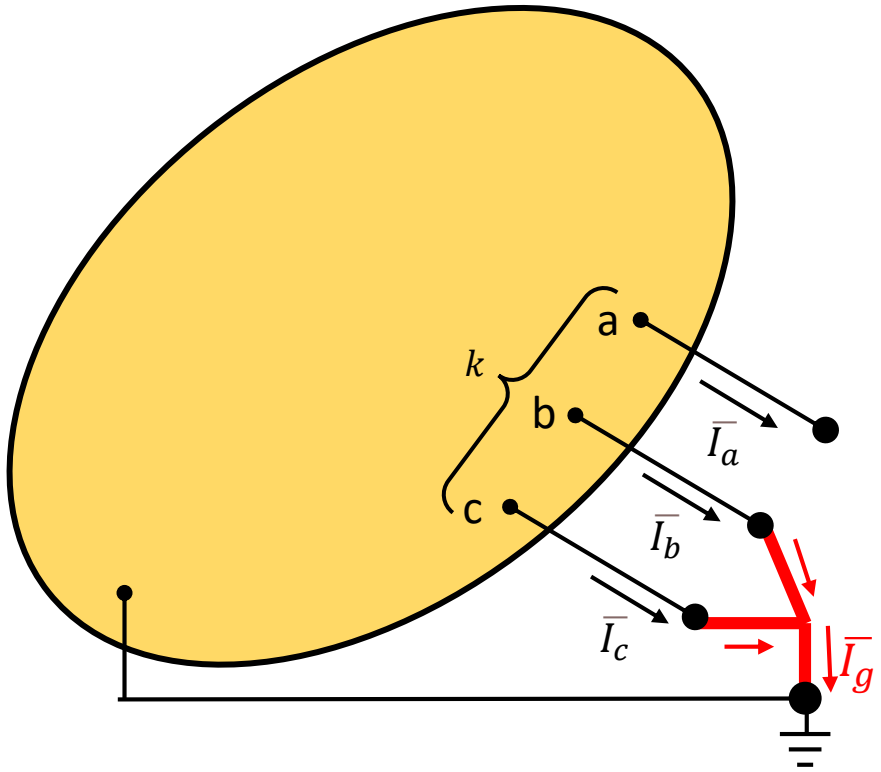


$$\overline{I_F} = \frac{3}{j\sqrt{3}}\overline{I_1} = -j\sqrt{3} \times \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1} + Z_k^{TH,2}}$$

$$\overline{I_F} = -j\sqrt{3} \times \frac{1.0\angle 0°}{j(0.187 + 0.117)} = -5.70\angle 0°\ p.u.$$

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Double line-to-line to ground fault at node k

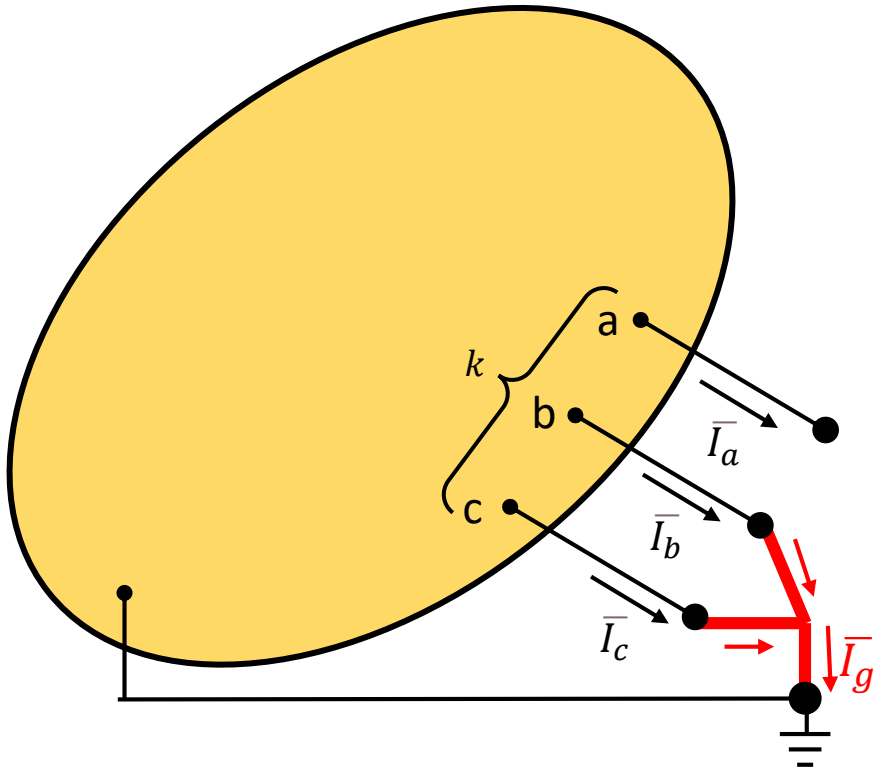


- Pre-fault: $\bar{I}_a = \bar{I}_b = \bar{I}_c = 0$
- Without loss of generality, assume that the fault is between phases b and c and ground
- $\bar{I}_a = 0$
- $\bar{I}_g = \bar{I}_b + \bar{I}_c$

- $\bar{V}_b = \bar{V}_c = 0$
- $\bar{V}_a \neq 0$

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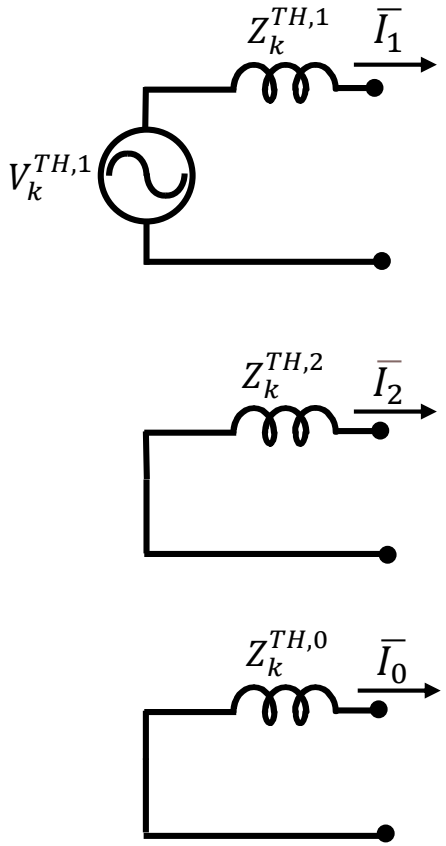
Double line-to-line to ground fault at node k



- $\overline{V_b} = \overline{V_c} = 0$
- $\begin{pmatrix} \overline{V_0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} \overline{V_a} \\ 0 \\ 0 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} \overline{V_a} \\ \overline{V_a} \\ \overline{V_a} \end{pmatrix}$
- $\overline{V_0} = \overline{V_1} = \overline{V_2} = \frac{1}{3}\overline{V_a}$
- $\overline{I_a} = 0$
- $\begin{pmatrix} 0 \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}\begin{pmatrix} \overline{I_0} \\ \overline{I_1} \\ \overline{I_2} \end{pmatrix}$
- $\overline{I_0} + \overline{I_1} + \overline{I_2} = 0$

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Double line-to-line to ground fault at node k

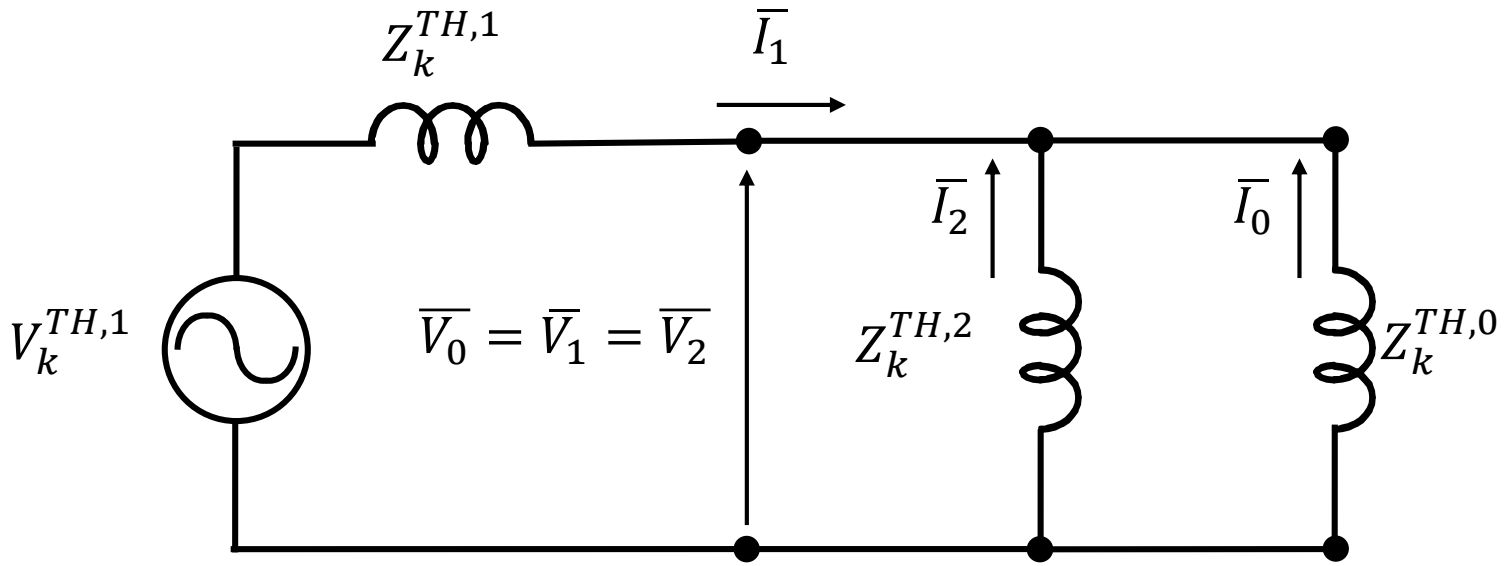


What connection of the sequence networks models:

- $\overline{V_0} = \overline{V_1} = \overline{V_2} = \frac{1}{3}\overline{V_a}$
- $\overline{I_0} + \overline{I_1} + \overline{I_2} = 0$

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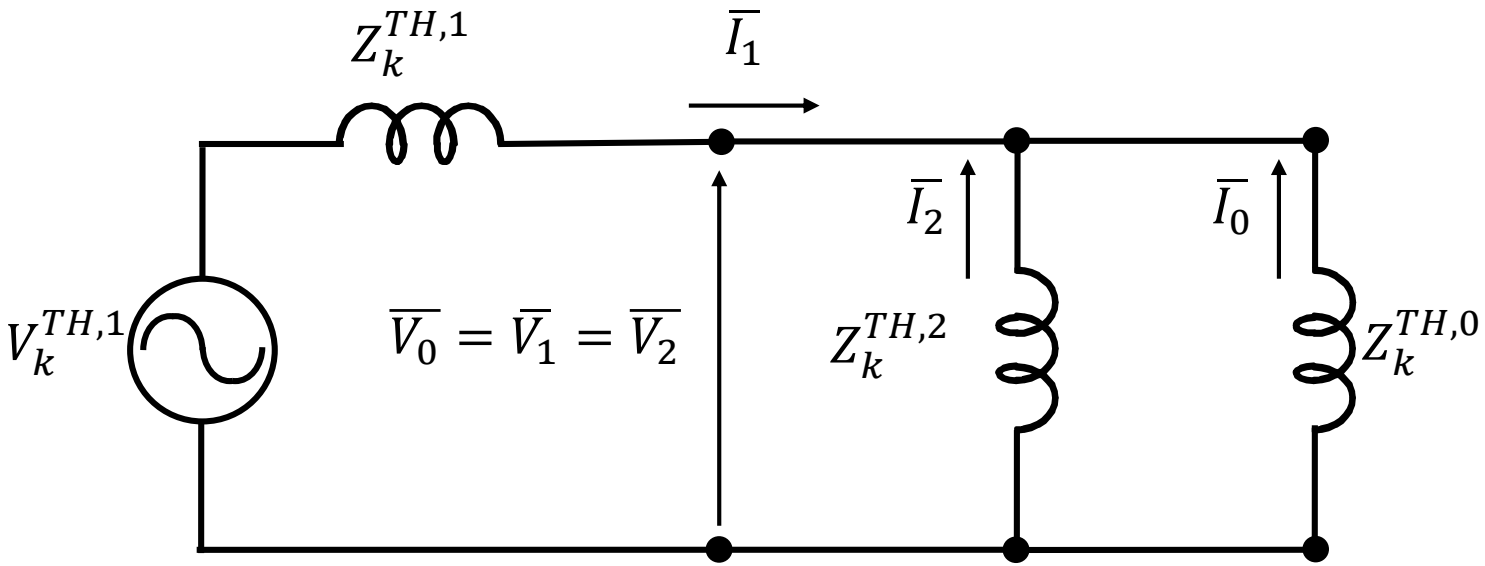
Double line-to-line to ground fault at node k



- $\overline{V_0} = \overline{V_1} = \overline{V_2} = \frac{1}{3}\overline{V_a}$
- $\overline{I_0} + \overline{I_1} + \overline{I_2} = 0$

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Double line-to-line to ground fault at node k

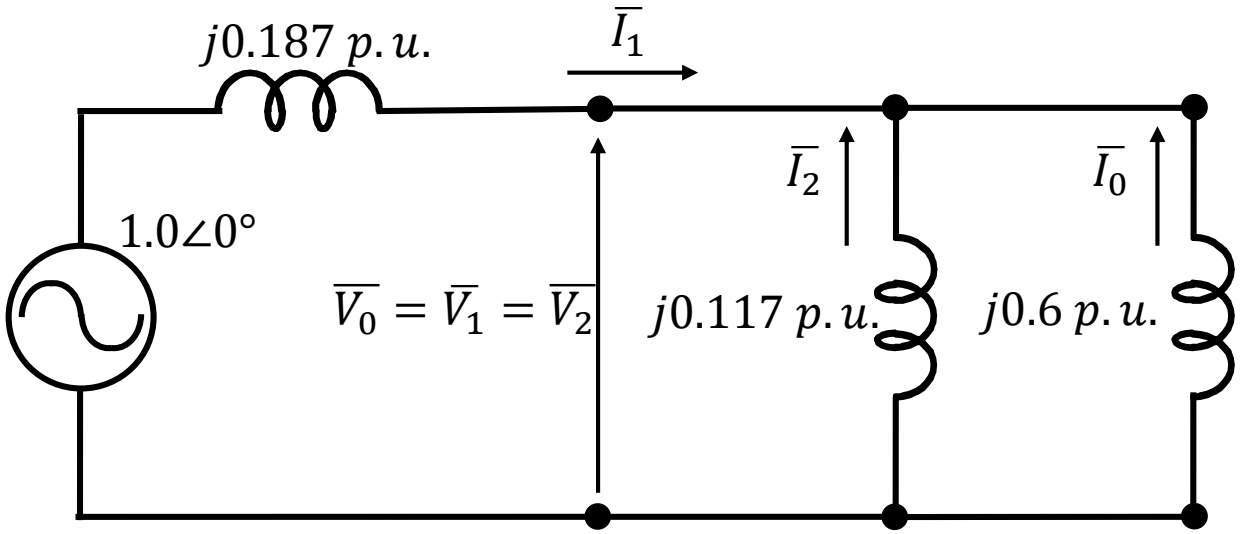


- $\overline{I_1} = \dfrac{\overline{V_k^{TH,1}}}{Z_k^{TH,1} + \dfrac{Z_k^{TH,2} Z_k^{TH,0}}{Z_k^{TH,2} + Z_k^{TH,0}}}$
- $\overline{V_0} = \overline{V_1} = \overline{V_2} = \overline{V_k^{TH,1}} - Z_k^{TH,1} \overline{I_1}$
- $\overline{I_2} = -\dfrac{\overline{V_2}}{Z_k^{TH,2}}$
- $\overline{I_0} = -\dfrac{\overline{V_0}}{Z_k^{TH,0}}$
- $\begin{pmatrix} 0 \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} \overline{I_0} \\ \overline{I_1} \\ \overline{I_2} \end{pmatrix}$
- $\overline{I_g} = \overline{I_b} + \overline{I_c}$

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Example 10.12

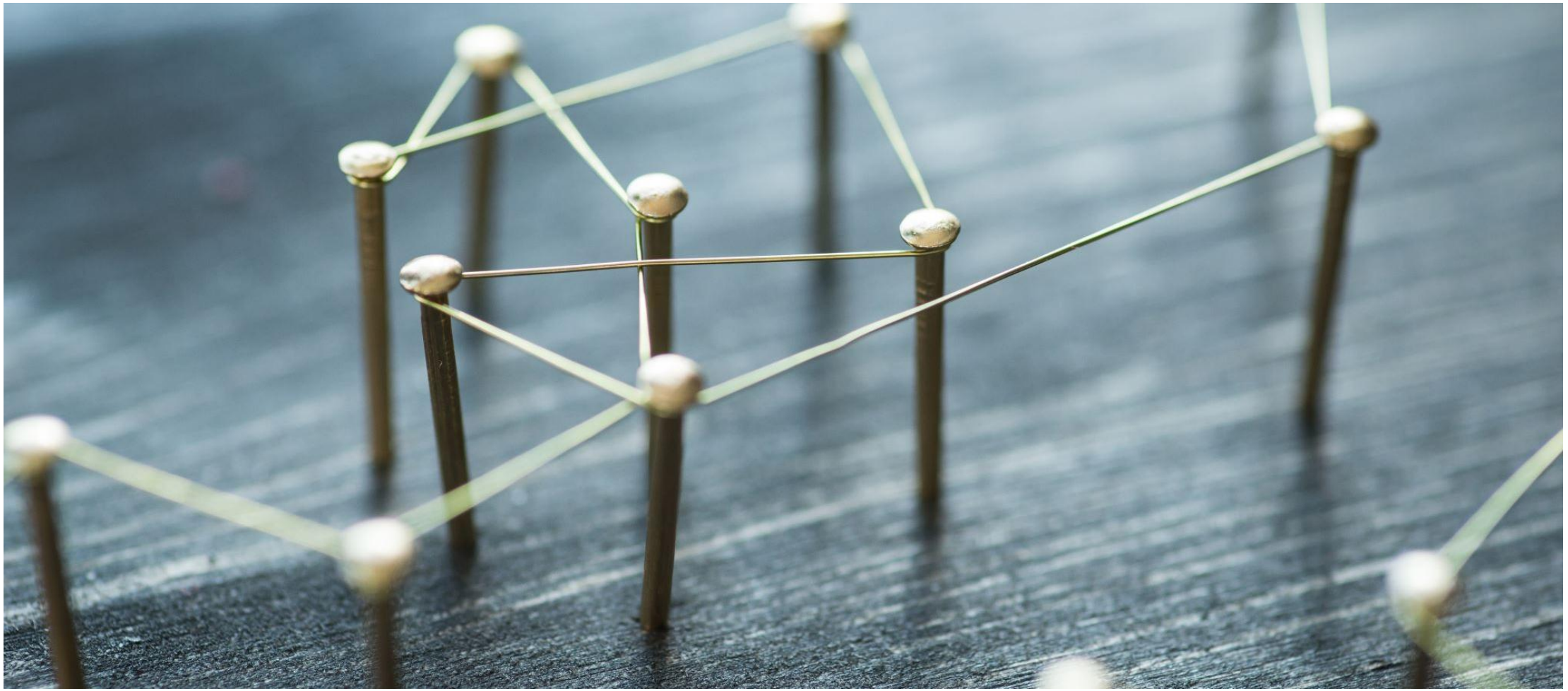
From Example 10.9:



- $\bar{I}_1 = \frac{1.0\angle 0°}{j\left(0.187 + \frac{0.117 \times 0.6}{0.117 + 0.6}\right)} = 3.51\angle -90°\ p.u.$
- $\overline{V_0} = \overline{V_1} = \overline{V_2} = 1.0\angle 0° - j\ 0.187\ \times 3.51\angle -90° = 0.343\angle 0°\ p.u.$
- $\bar{I}_2 = -\frac{0.343\angle 0°}{j0.117} = 2.93\angle 90°\ p.u.$
- $\bar{I}_0 = -\frac{0.343\angle 0°}{j0.6} = 0.572\angle 90°\ p.u.$
- $\begin{pmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} 0.572\angle 90° \\ 3.51\angle -90° \\ 2.93\angle 90° \end{pmatrix} = \begin{pmatrix} 0 \\ 5.64\angle 171.21° \\ 5.64\angle 8.79° \end{pmatrix}$
- $\bar{I}_g = 5.64\angle 171.21° + 5.64\angle 8.79° = 1.72\angle 90°\ p.u.$

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Unbalanced fault calculations in large networks



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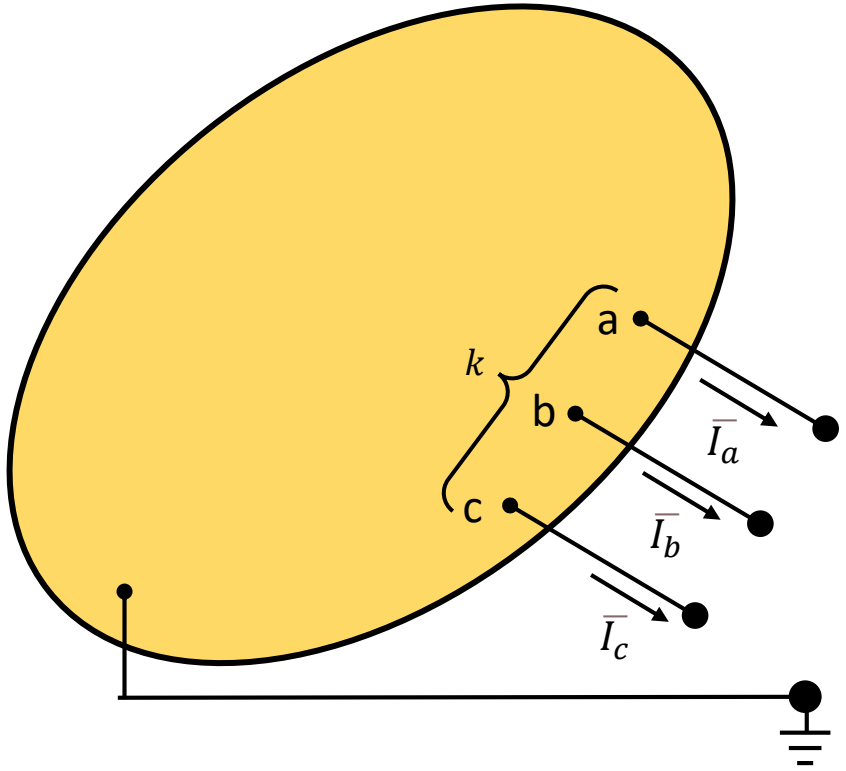
Unbalanced faults in large networks

- Small networks → calculate Thevenin equivalents by hand
- Large networks → need systematic scalable technique

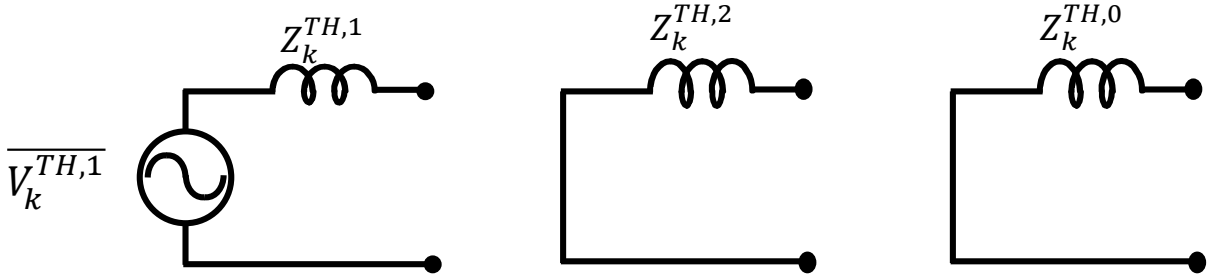
1. Build the admittance matrices of the decoupled, pre-fault positive-, negative, and zero-sequence networks
2. Invert these matrices to obtain the corresponding impedance matrices
3. The diagonal elements of these impedance matrices are the Thevenin equivalent impedances of the sequence networks
4. Connect these Thevenin equivalent impedances as before to calculate the fault currents

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Unbalanced faults at node k in a large network

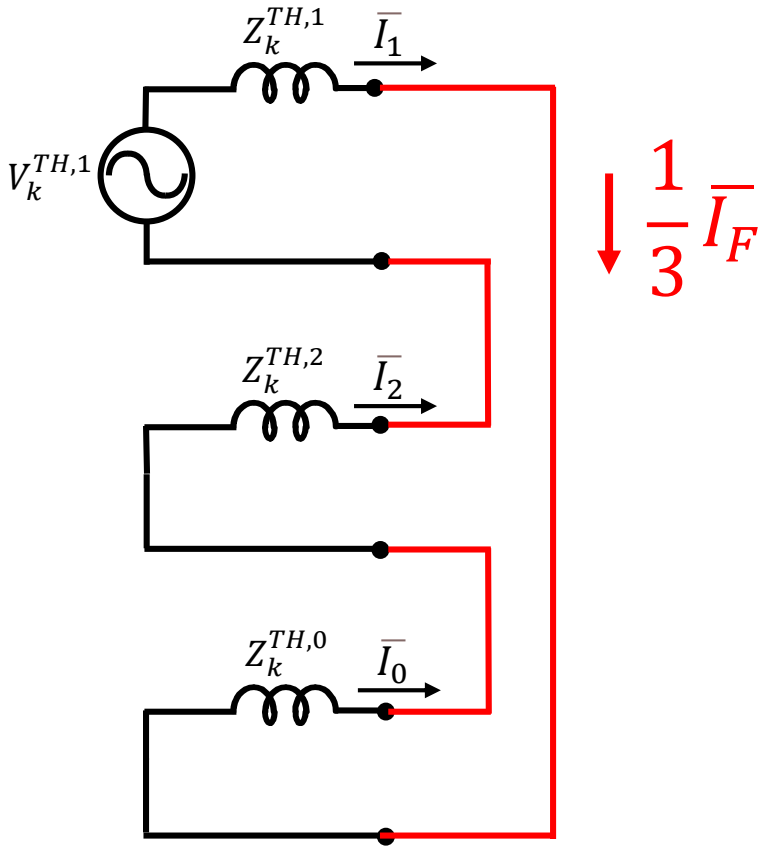


- $Z_k^{TH,1} = Z_1\,(k,k)$
- $Z_k^{TH,2} = Z_2\,(k,k)$
- $Z_k^{TH,0} = Z_0\,(k,k)$



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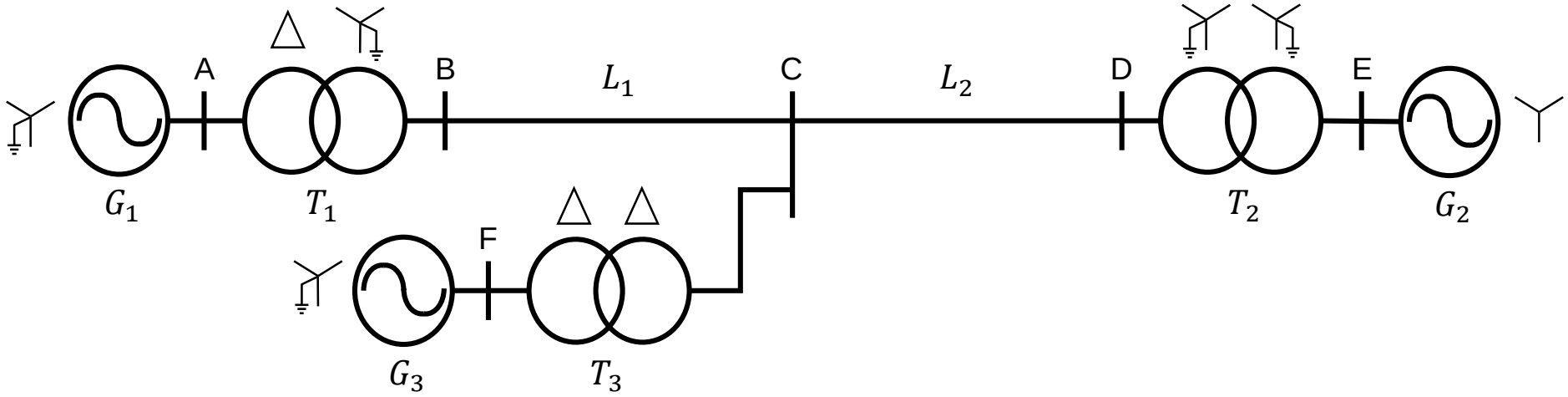
Single-line-to-ground fault at node k



- $Z_k^{TH,1} = Z_1\,(k,k)$
- $Z_k^{TH,2} = Z_2\,(k,k)$
- $Z_k^{TH,0} = Z_0\,(k,k)$
- $\bar{I}_F = \dfrac{3\overline{V_k^{TH,1}}}{Z_1(k,k)+Z_2(k,k)+Z_0(k,k)}$

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Example 10.13

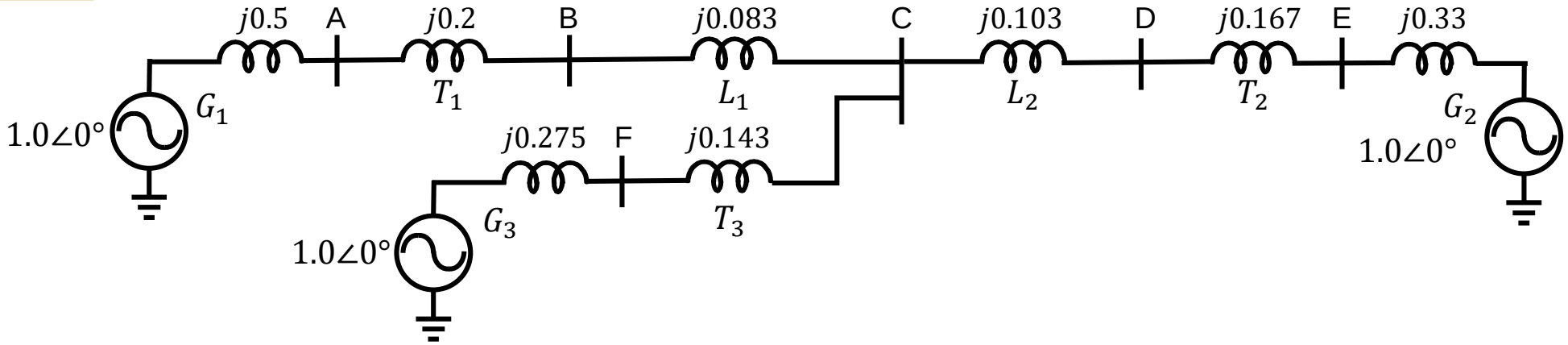


Single-line-to-ground fault at every node of this network?

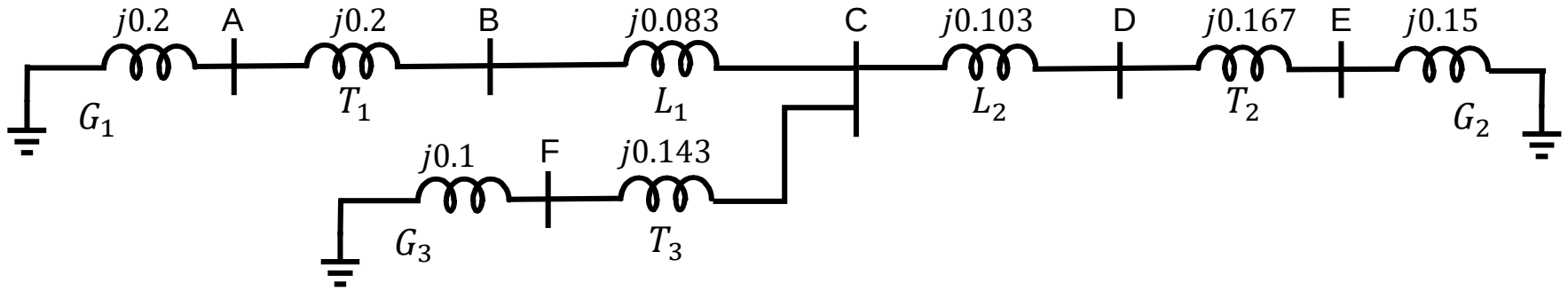
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Example 10.13: Impedance diagrams

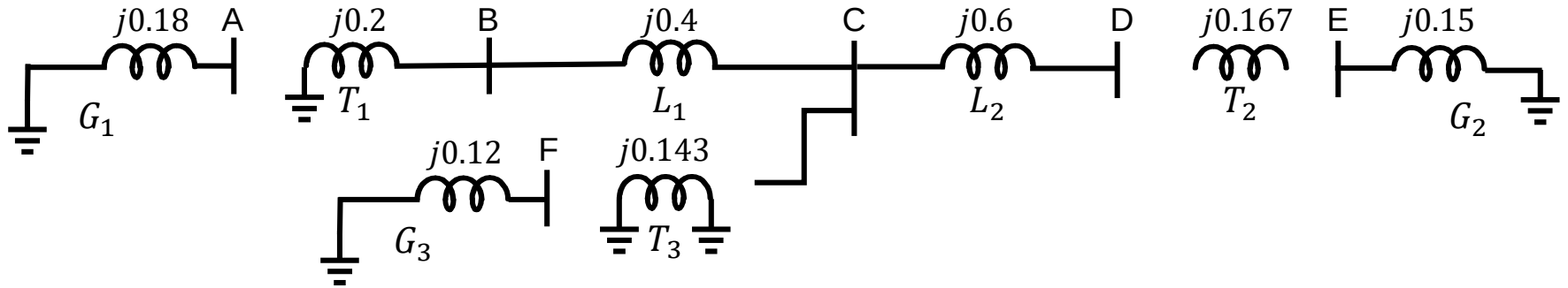
Positive sequence



Negative sequence

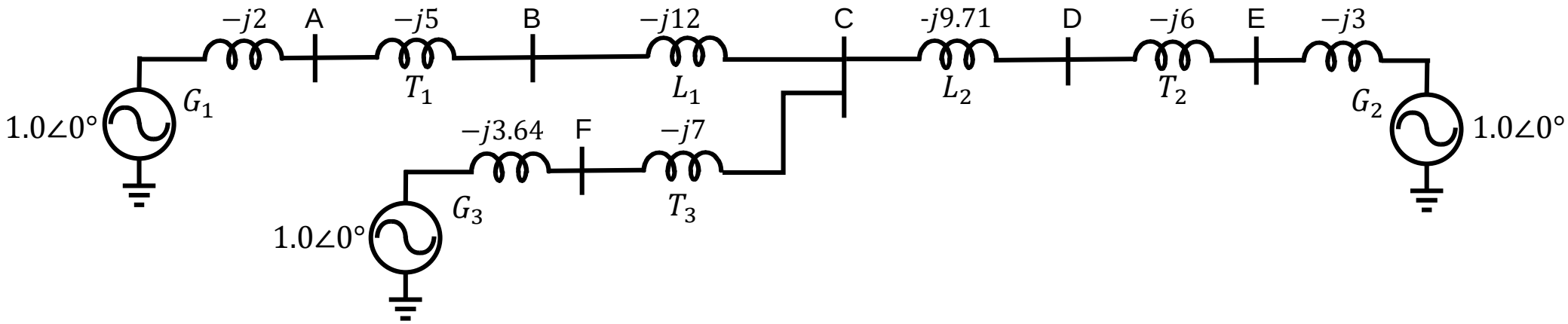


Zero sequence



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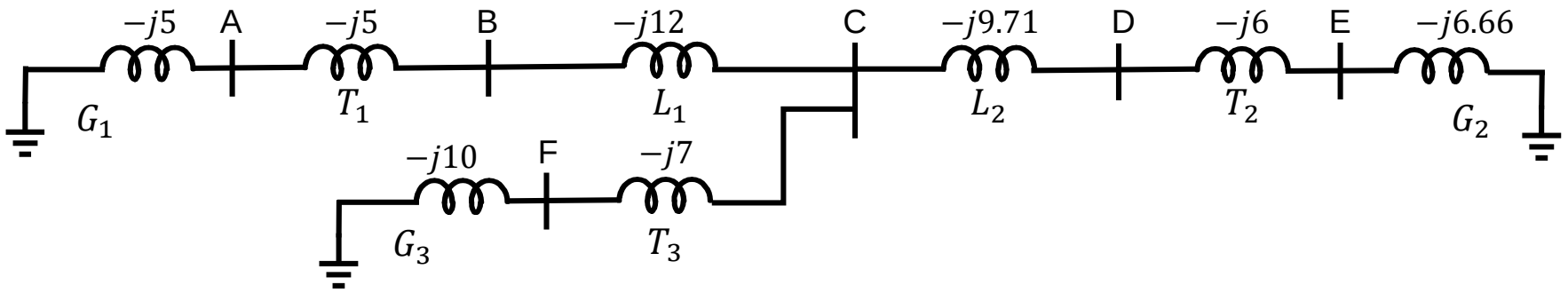
Example 10.13: Positive-sequence admittances



$$Y_1 = j\begin{pmatrix} -7 & 5 & 0 & 0 & 0 & 0 \\ 5 & -17 & 12 & 0 & 0 & 0 \\ 0 & 12 & -28.71 & 9.71 & 0 & 7 \\ 0 & 0 & 9.71 & -15.71 & 6 & 0 \\ 0 & 0 & 0 & 6 & -9 & 0 \\ 0 & 0 & 7 & 0 & 0 & -10.64 \end{pmatrix}$$

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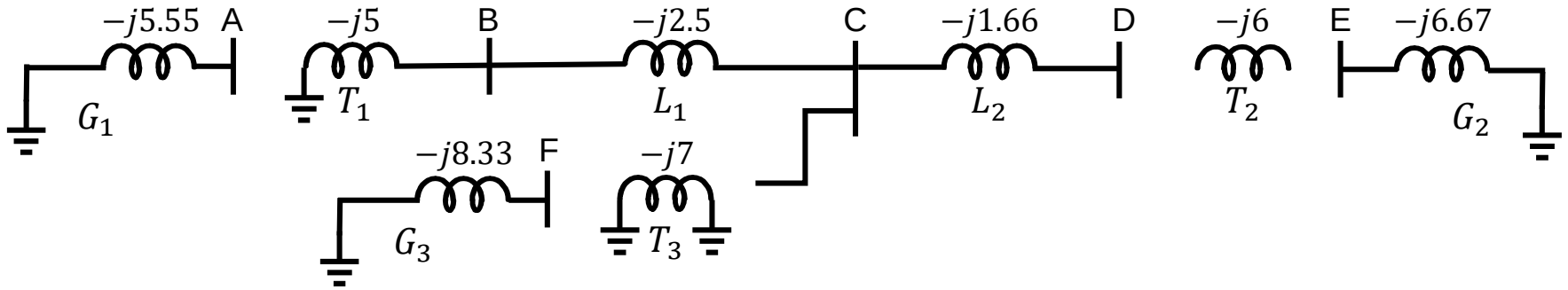
Example 10.13: Negative-sequence admittances



$$Y_2 = j\begin{pmatrix} -10 & 5 & 0 & 0 & 0 & 0 \\ 5 & -17 & 12 & 0 & 0 & 0 \\ 0 & 12 & -28.75 & 9.71 & 0 & 7 \\ 0 & 0 & 9.71 & -15.71 & 6 & 0 \\ 0 & 0 & 0 & 6 & -12.66 & 0 \\ 0 & 0 & 7 & 0 & 0 & -17 \end{pmatrix}$$

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Example 10.13: Zero-sequence admittances



$$Y_0 = j\begin{pmatrix} -5.55 & 0 & 0 & 0 & 0 & 0 \\ 0 & -7.5 & 2.5 & 0 & 0 & 0 \\ 0 & 2.5 & -4.16 & 1.66 & 0 & 0 \\ 0 & 0 & 1.66 & -1.66 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & -8.33 \end{pmatrix}$$

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Example 10.13: Impedance matrices

Positive sequence

$$Z_1 = j\begin{pmatrix} 0.257 & 0.160 & 0.120 & 0.099 & 0.066 & 0.079 \\ 0.160 & 0.224 & 0.168 & 0.139 & 0.093 & 0.110 \\ 0.120 & 0.168 & 0.187 & 0.156 & 0.104 & 0.123 \\ 0.099 & 0.139 & 0.156 & 0.214 & 0.143 & 0.102 \\ 0.066 & 0.093 & 0.104 & 0.143 & 0.206 & 0.068 \\ 0.079 & 0.110 & 0.123 & 0.102 & 0.068 & 0.175 \end{pmatrix}$$

Negative sequence

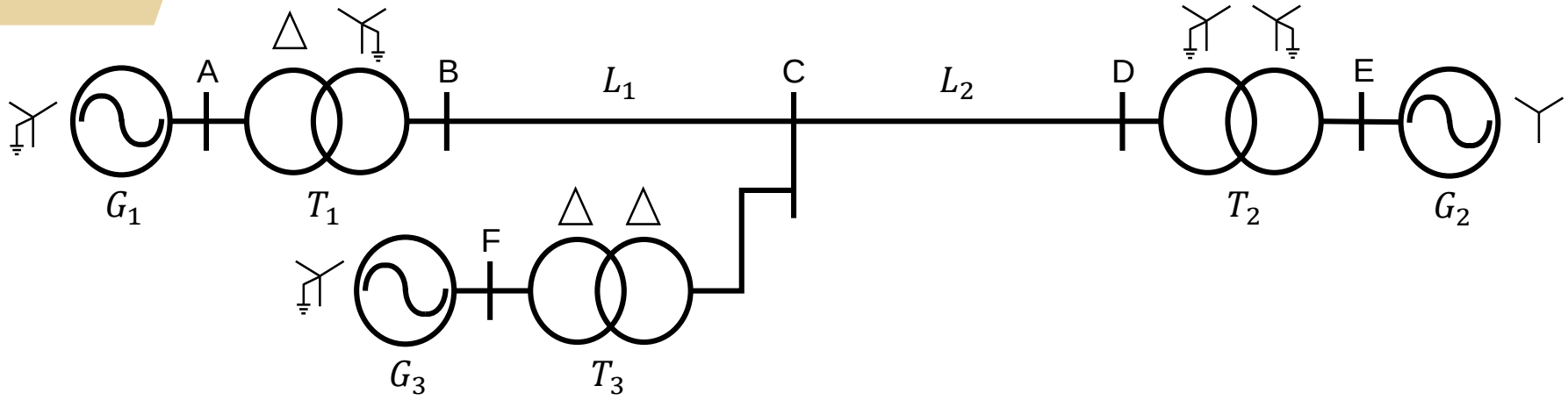
$$Z_2 = j\begin{pmatrix} 0.137 & 0.074 & 0.048 & 0.036 & 0.017 & 0.020 \\ 0.074 & 0.149 & 0.096 & 0.073 & 0.034 & 0.040 \\ 0.048 & 0.096 & 0.116 & 0.088 & 0.042 & 0.048 \\ 0.036 & 0.073 & 0.088 & 0.144 & 0.068 & 0.036 \\ 0.017 & 0.034 & 0.042 & 0.068 & 0.111 & 0.017 \\ 0.020 & 0.040 & 0.048 & 0.036 & 0.017 & 0.079 \end{pmatrix}$$

Zero sequence

$$Z_0 = j\begin{pmatrix} 0.180 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.200 & 0.200 & 0.200 & 0.000 & 0.000 \\ 0.000 & 0.200 & 0.600 & 0.600 & 0.000 & 0.000 \\ 0.000 & 0.200 & 0.600 & 1.202 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.500 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.120 \end{pmatrix}$$

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Example 10.13: SLG and LLL faults at all buses



Node	$Z_0(k,k)$	$Z_1(k,k)$	$Z_2(k,k)$	Single-phase fault	Three-phase fault
A	0.18	0.257	0.137	5.226	3.891
B	0.2	0.224	0.149	5.236	4.464
C	0.6	0.187	0.116	3.322	5.348
D	1.202	0.214	0.144	1.923	4.673
E	1.500	0.206	0.111	1.651	4.854
F	0.120	0.175	0.079	8.021	5.714

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