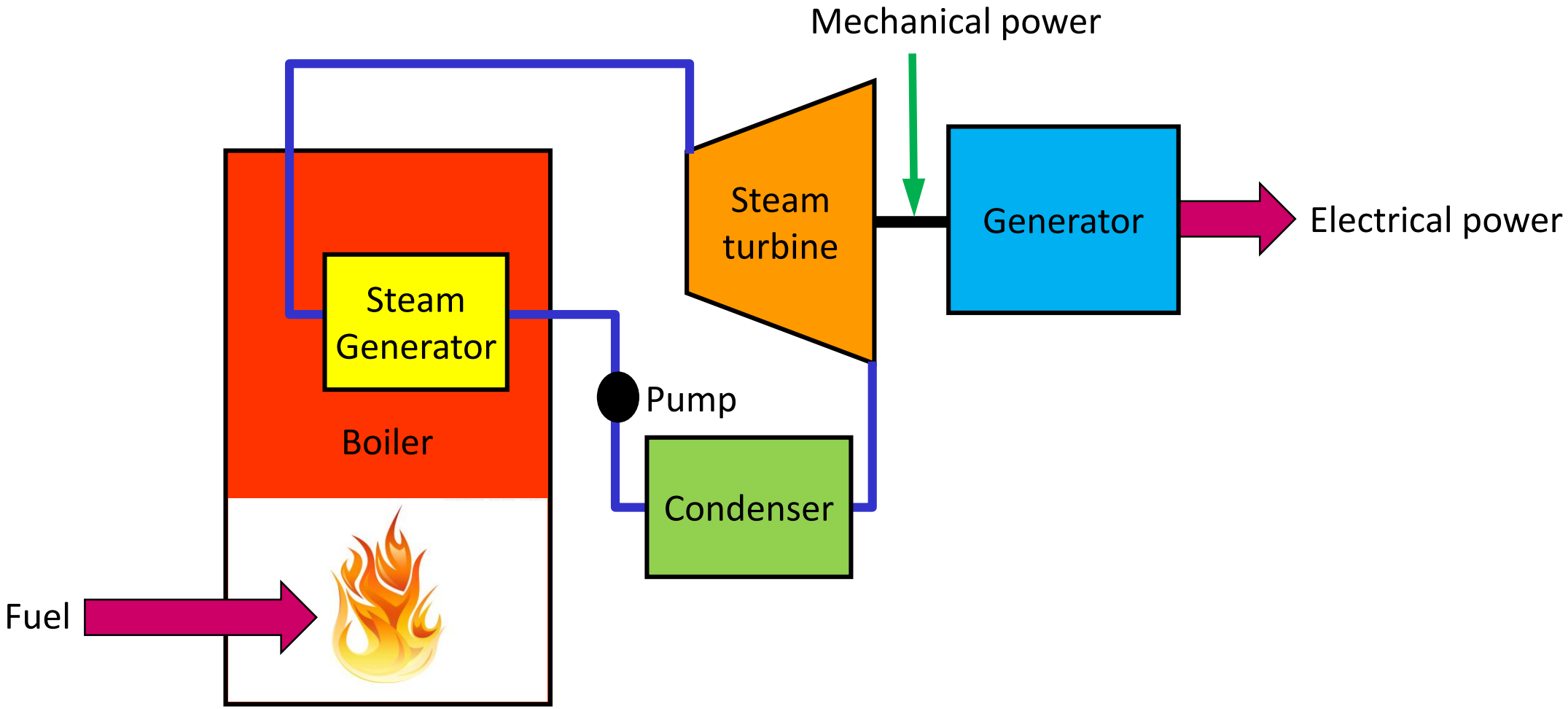
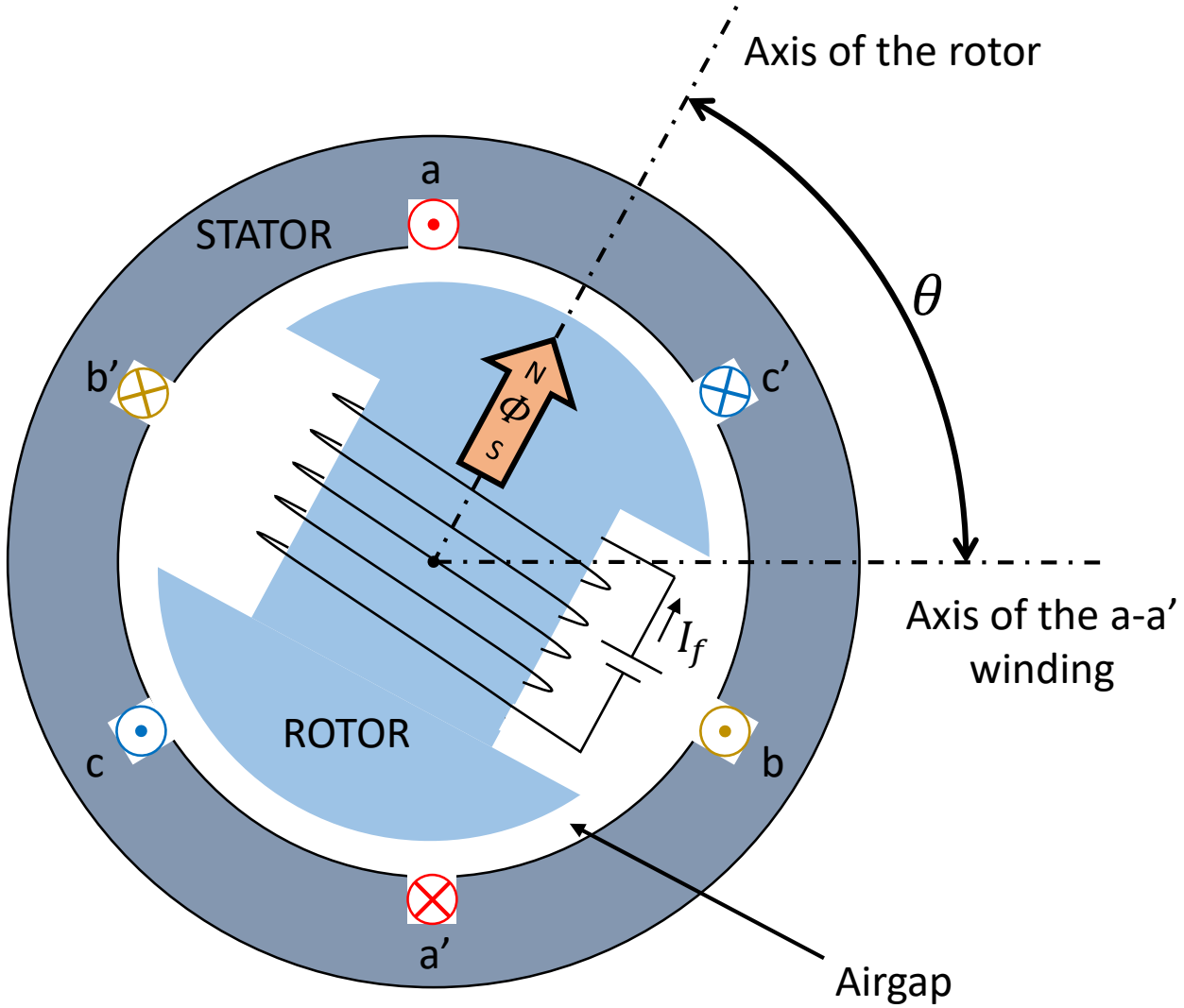
Example: steam plant

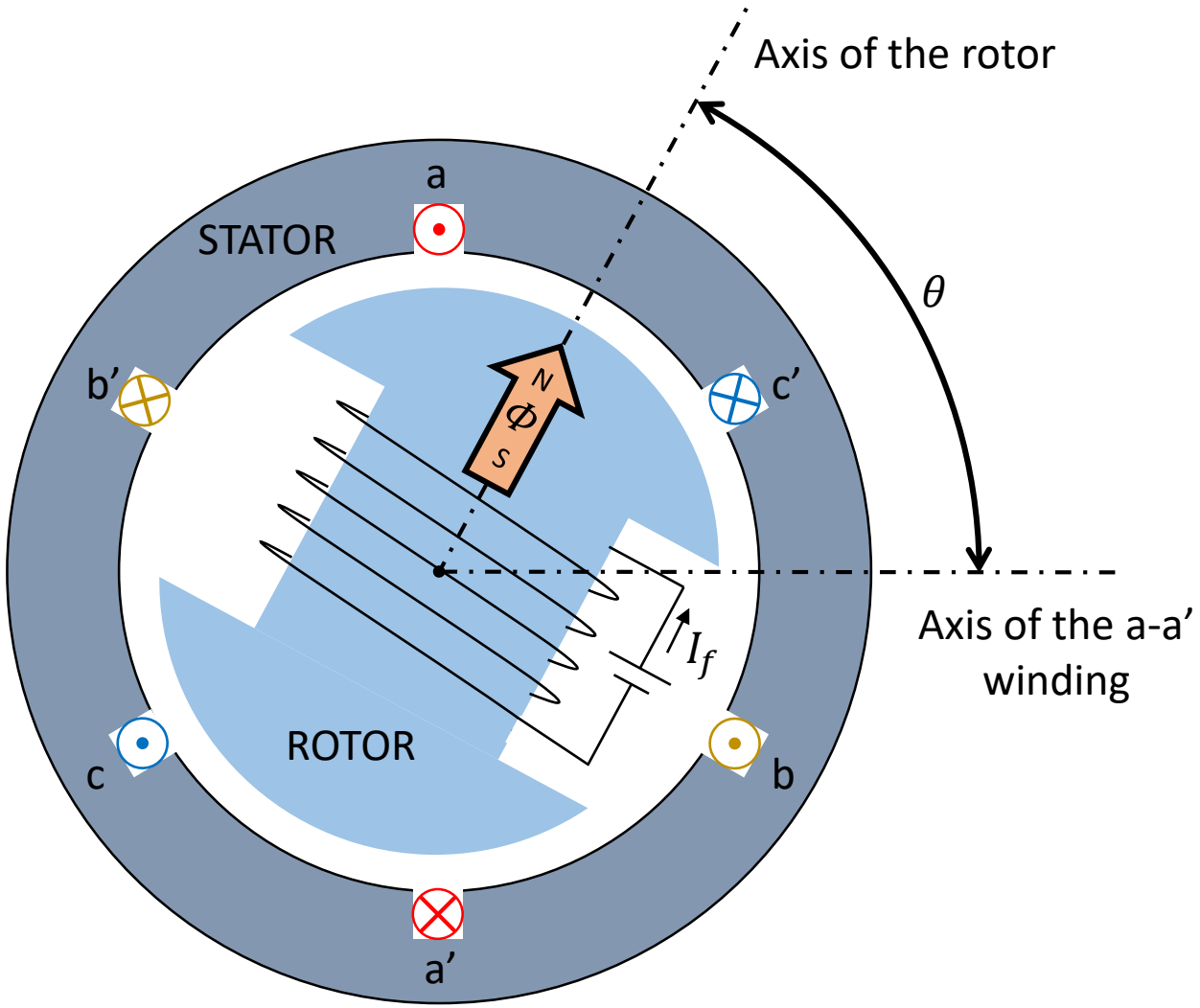


Structure of a generator



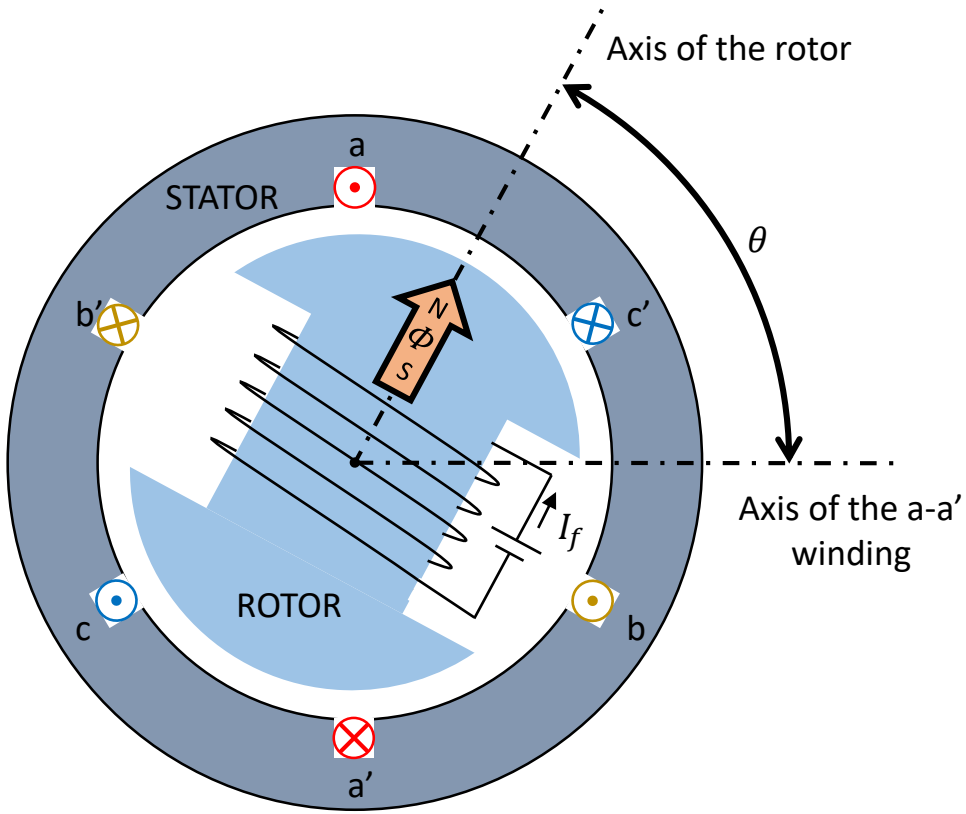
Magnetic fluxes linking the stator windings

- Stator windings are 120° apart
- $\lambda_{a-a'} = \mathcal{N}\Phi\cos\theta$
- $\lambda_{b-b'} = \mathcal{N}\Phi\cos(\theta - 120°)$
- $\lambda_{c-c'} = \mathcal{N}\Phi\cos(\theta + 120°)$
- λ: flux linkage
- Φ: rotor flux
- $\mathcal{N}$: effective number of turns

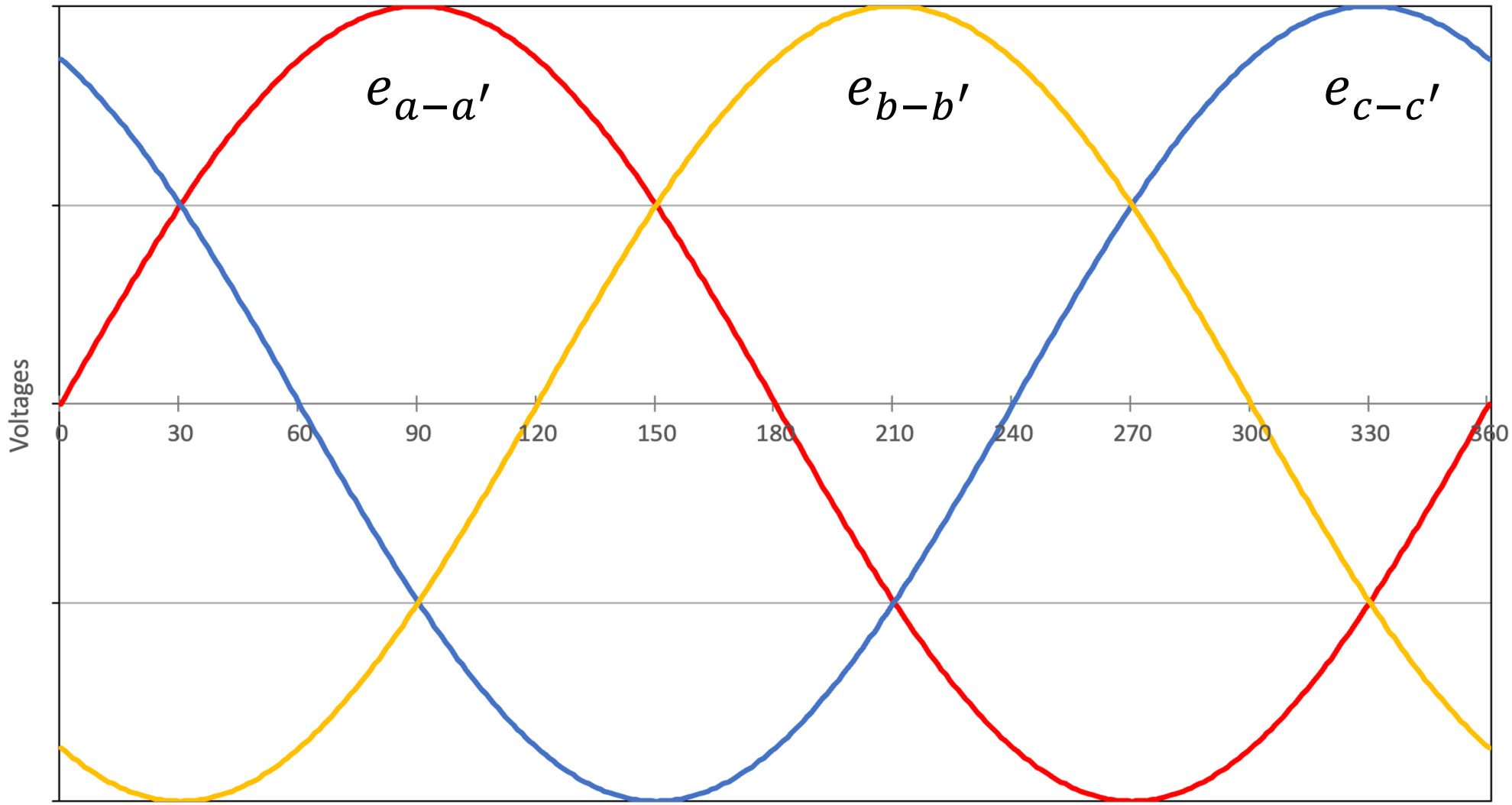


Voltages induced in the stator windings

- Rotor rotates at constant speed $\omega \rightarrow \theta = \omega t + \theta_0$
- Stator flux linkages are time varying
- $\lambda_{a-a'} = \mathcal{N}\Phi \cos(\omega t + \theta_0)$
- $\lambda_{b-b'} = \mathcal{N}\Phi \cos(\omega t + \theta_0 - 120°)$
- $\lambda_{c-c'} = \mathcal{N}\Phi \cos(\omega t + \theta_0 + 120°)$
- Lenz's law: $e(t) = \frac{d\phi(t)}{dt}$
- $e_{a-a'}(t) = \frac{d\lambda_{a-a'}(t)}{dt} = -\mathcal{N}\Phi\omega \sin(\omega t + \theta_0)$
- $e_{b-b'}(t) = \frac{d\lambda_{b-b'}(t)}{dt} = -\mathcal{N}\Phi\omega \sin(\omega t + \theta_0 - 120°)$
- $e_{c-c'}(t) = \frac{d\lambda_{c-c'}(t)}{dt} = -\mathcal{N}\Phi\omega \sin(\omega t + \theta_0 + 120°)$



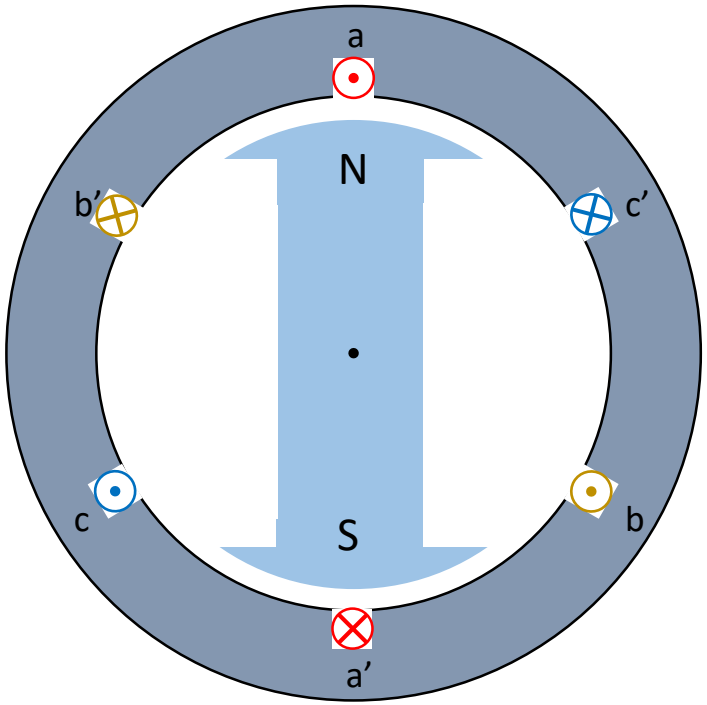
Voltages induced in the stator windings



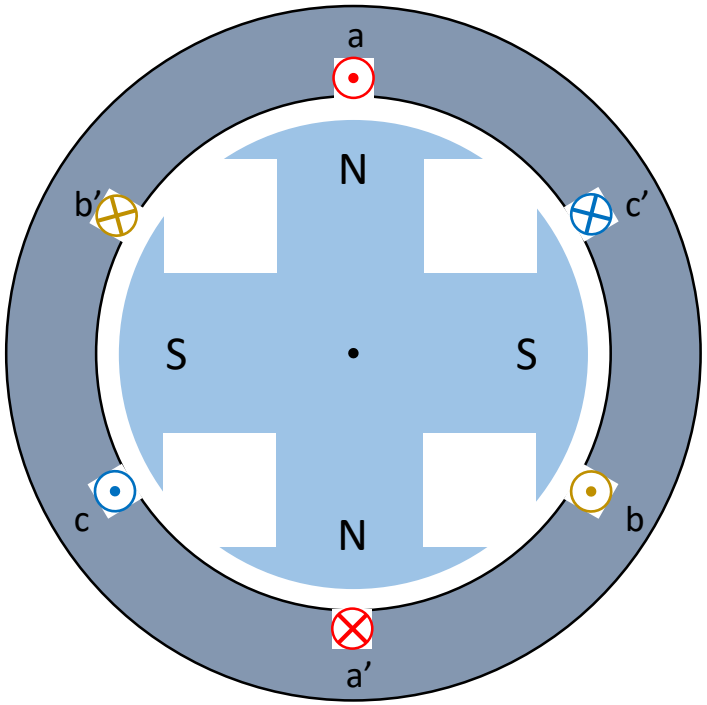
Voltages induced in the stator windings

- Have the same magnitude
  - Proportional to the rotor flux Φ
- Have the same angular frequency ω
  - Determined by the mechanical speed of rotation
- Are exactly 120° out of phase in time
  - Because the windings are shifted by 120° in space

Number of magnetic poles

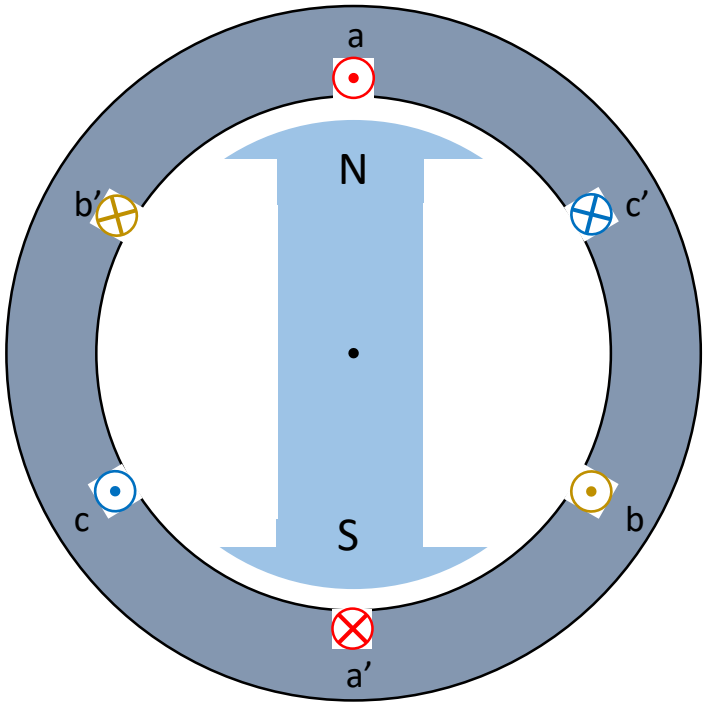


Rotor with two magnetic poles
(or one pole pair)

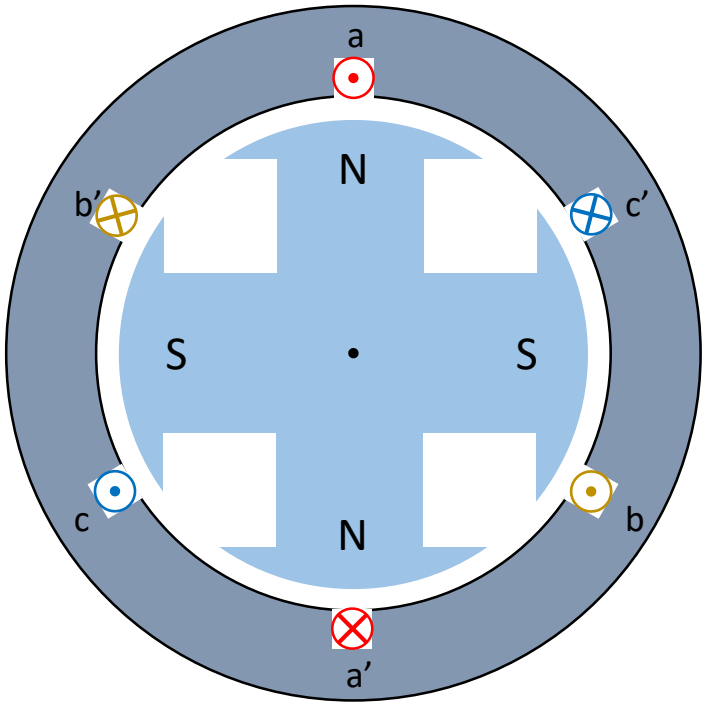


Rotor with four magnetic poles
(or two pole pairs)

Frequency of induced voltages



One mechanical rotation
induces one electrical period



One mechanical rotation induces two electrical periods
because each stator winding ”sees” two North poles and two south poles

Frequency of induced voltages

- Generalization to p rotor magnetic poles:

$$\omega = \frac{p}{2}\omega_m$$

- $\omega = 2\pi f$: electrical angular frequency
- $\omega_m = 2\pi \frac{N}{60}$: mechanical angular frequency
- N: rotational speed in revolutions per minute (rpm)

$$f = \frac{p}{120}N$$

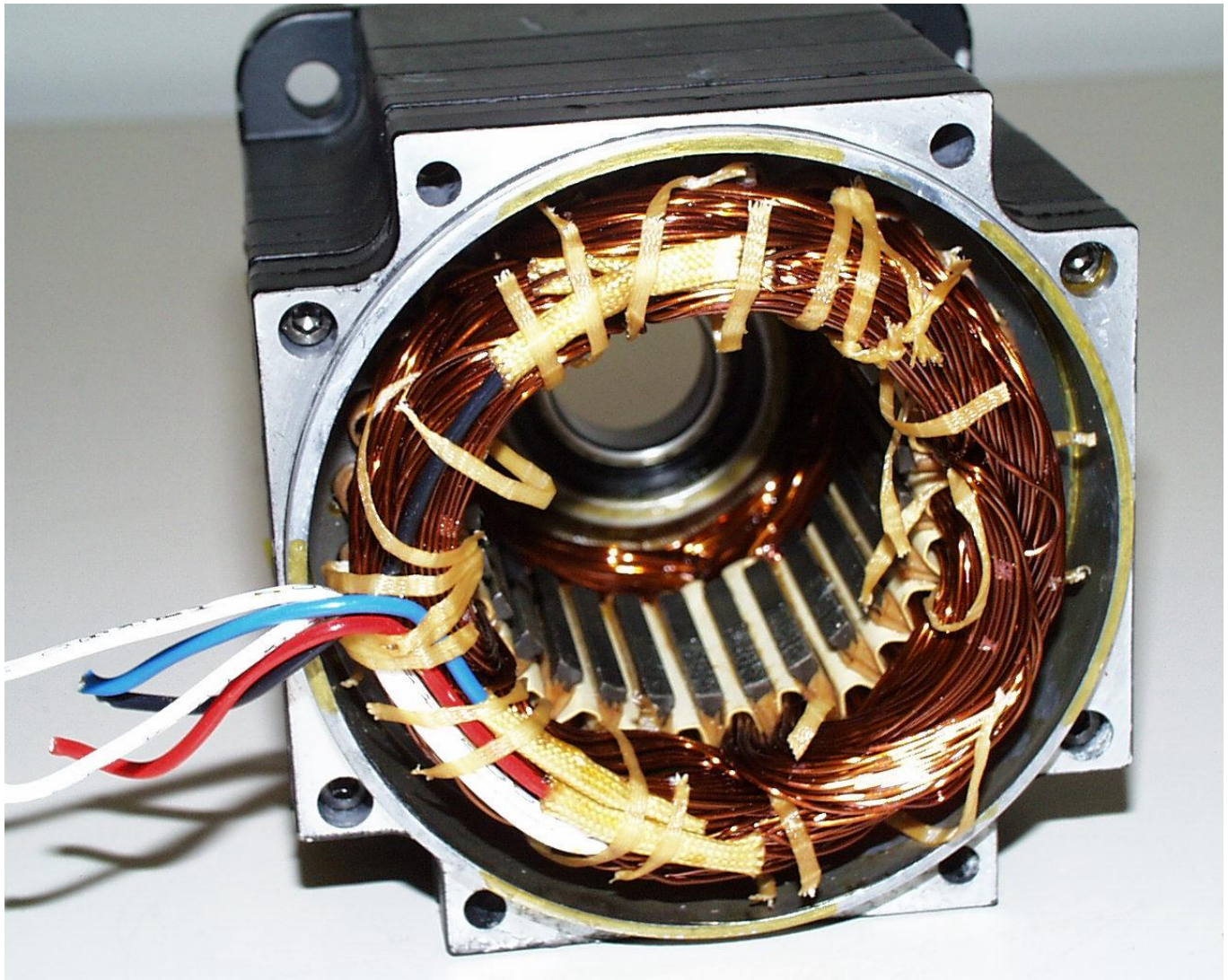
Typical rotational speed of generator

- Steam and gas turbines are most efficient at high speed
  - $p = 2$ or $p = 4$
  - $f = 60\ Hz$
  - $N = \frac{120}{p} f$

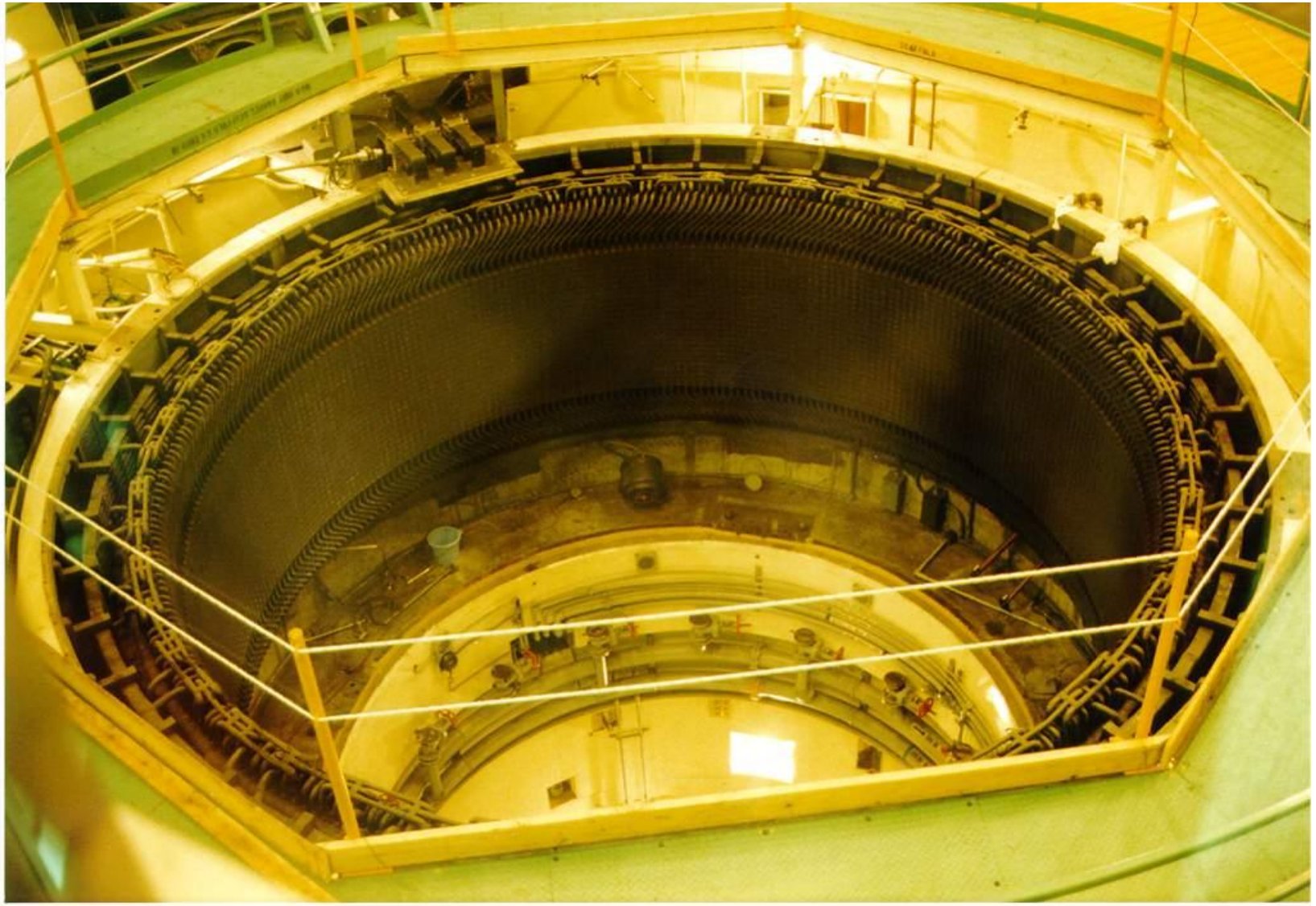
  $\Rightarrow N = 3600\ RPM\ or\ N = 1800\ RPM$
- Hydro turbines are most efficient at low speed
  - $p = 100$
  - $f = 60\ Hz$
  - $N = \frac{120}{p} f$

  $\Rightarrow N = 72\ RPM$
- Wind turbines rotate at speeds between 5 and 20 rpm
  - Would require too many poles
  - Use mechanical gearbox or power electronics

Stator of a small generator



Stator of a large hydro generator



Installing the stator windings of a hydro generator



Rotor of the generator for a thermal power plant



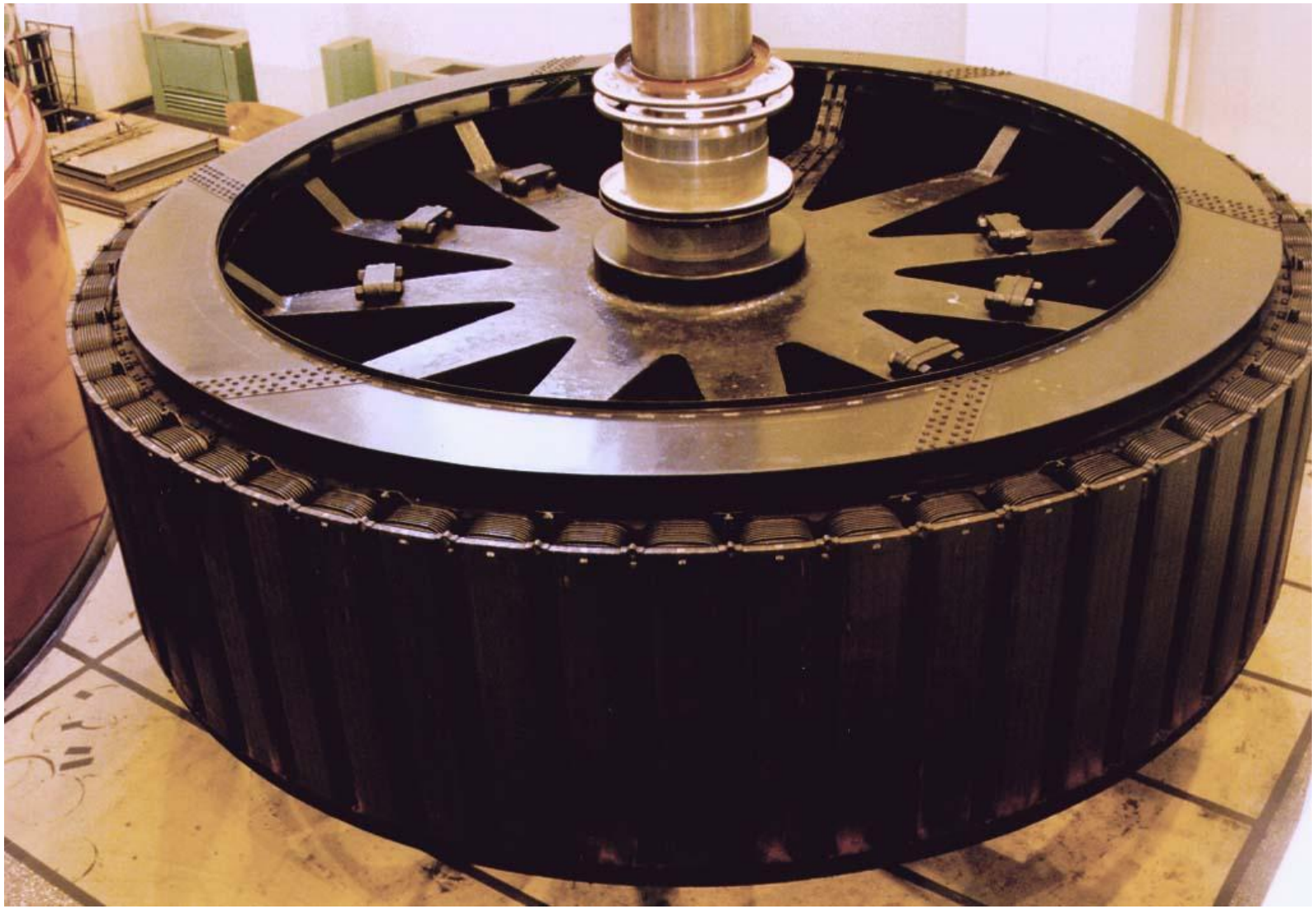
Picture: Ansaldo Energia

Cutout of the generator for a thermal power plant

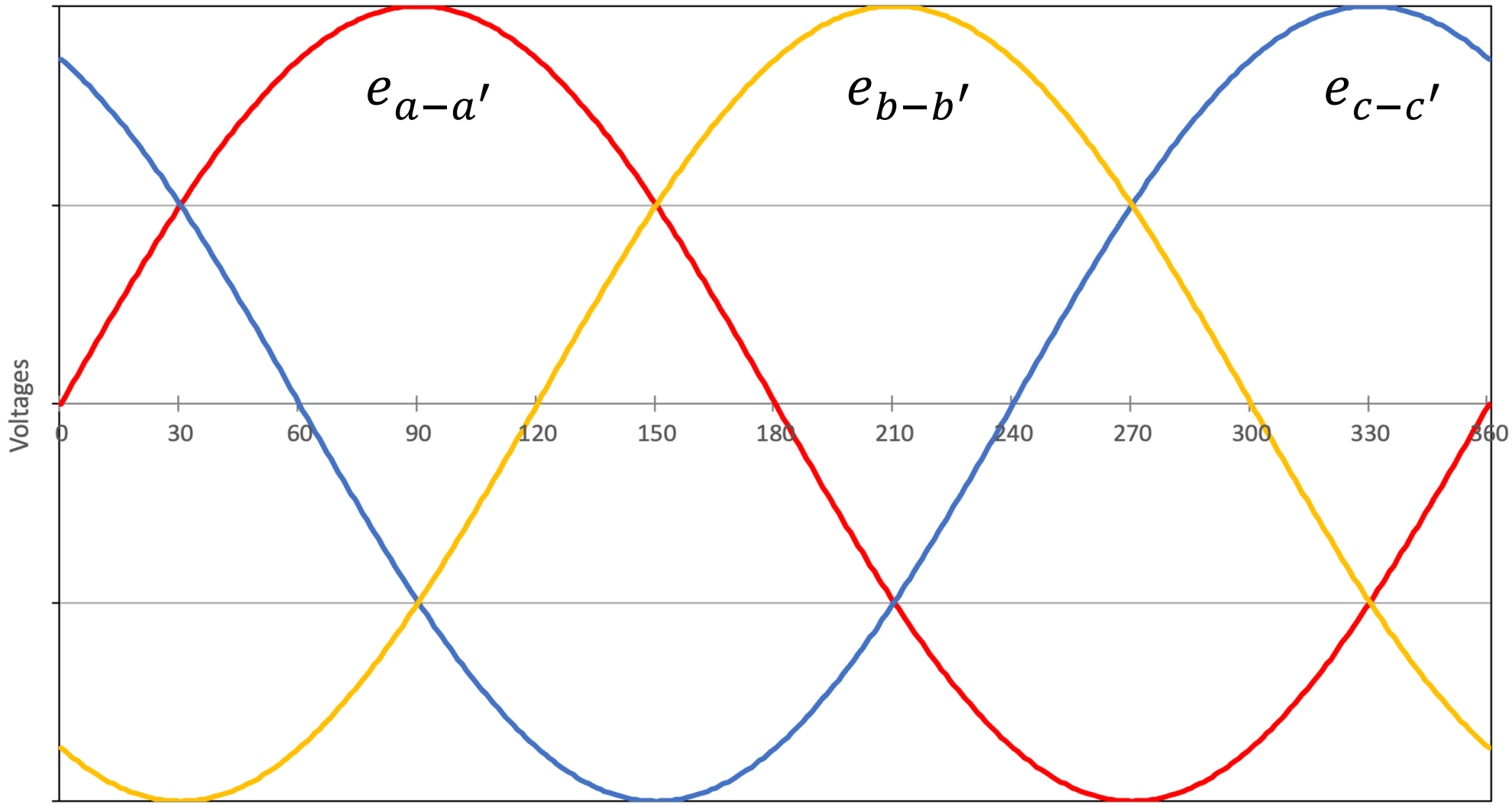


Picture: Ansaldo Energia

Rotor of a large hydro generator



Three-phase systems

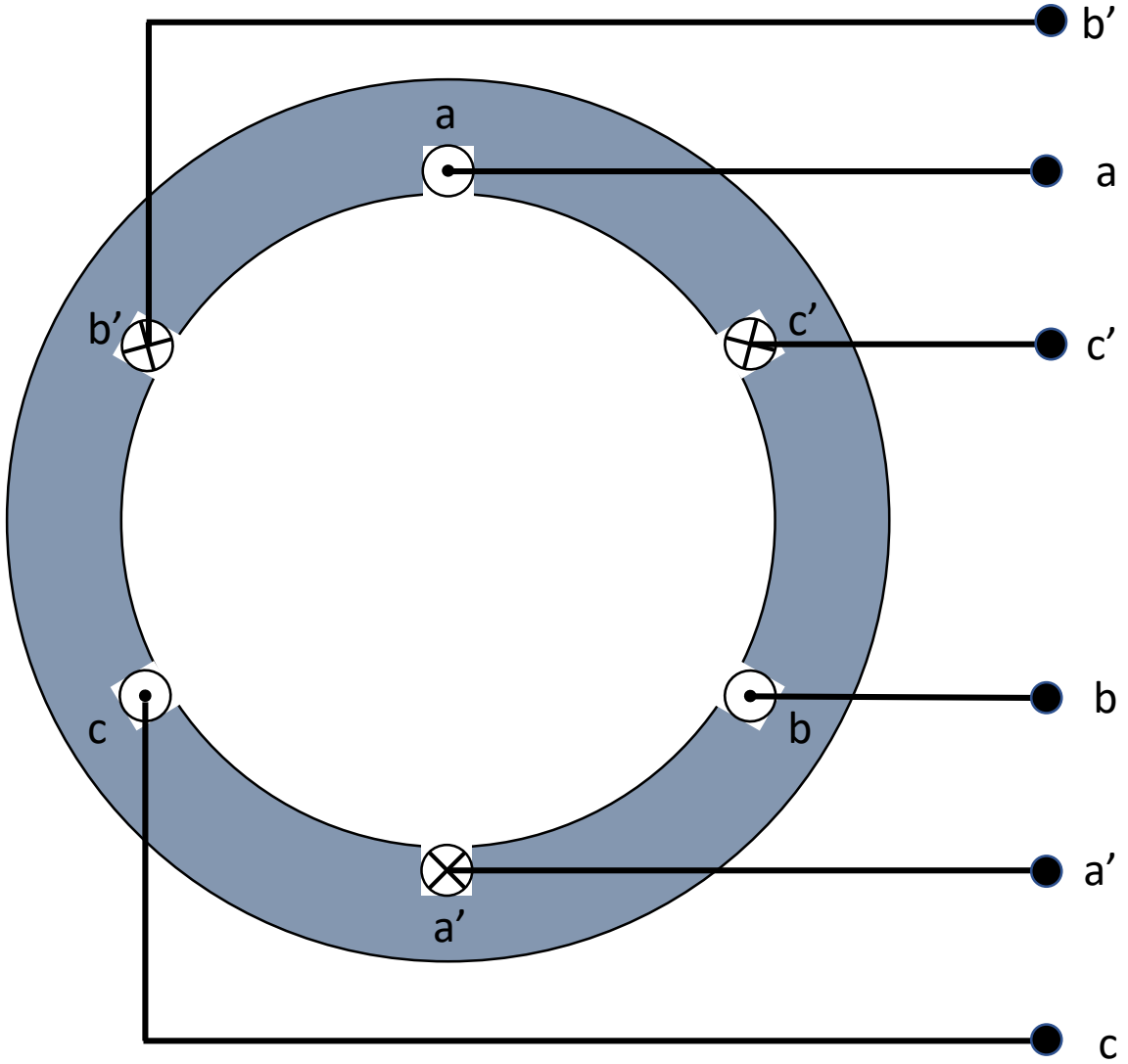


Why use three-phase systems?

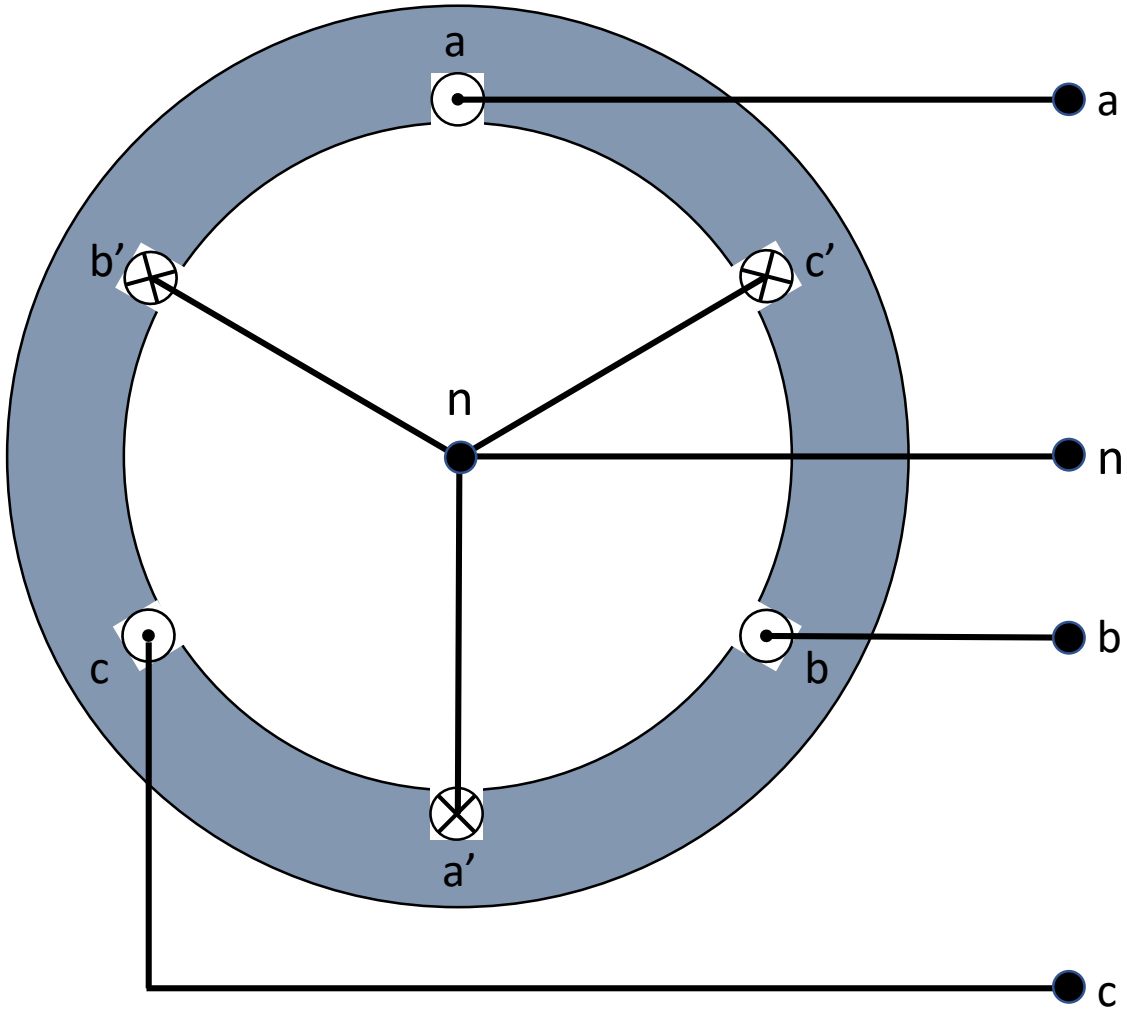
- Three-phase systems can produce a rotating magnetic field
  - Three-phase motors start without the need for extra equipment
- For the same physical size, a three-phase generator produces more power than a single-phase generator
- Three-phase lines transmit more power
- Power in a three-phase system does not pulsate at twice the frequency as it does in a single-phase system

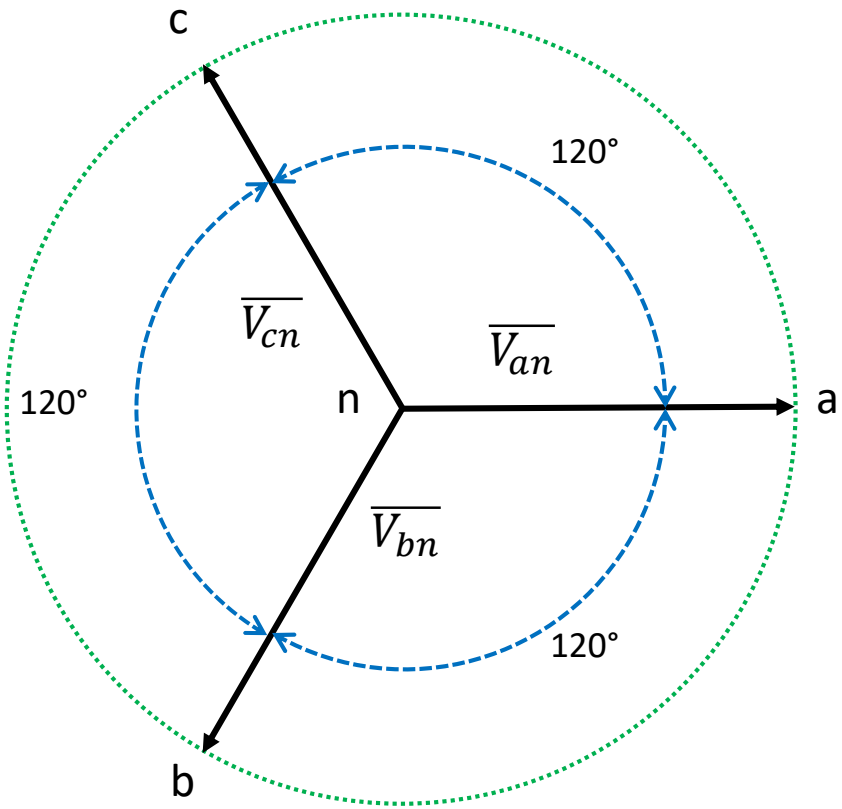
Three-phase sources

- 3 sets of windings → 6 terminals
- Connecting 6 wires over a long distance is expensive
- We don’t need 6 wires to have a three- phase system
- Y connection → use three or four wires
- Delta connection → use three wires



Y-connection





Line-ot-neutral voltages

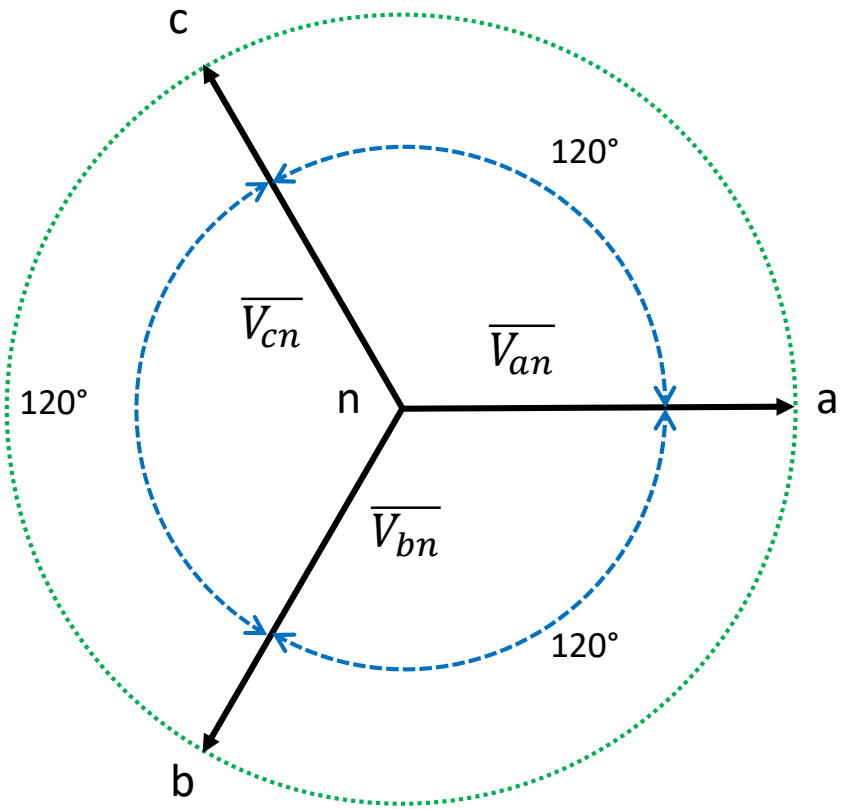
$$\overline{V_{an}} = V\angle 0°$$

$$\overline{V_{bn}} = V\angle -120°$$

$$\overline{V_{cn}} = V\angle +120°$$

Balanced three-phase voltage phasors

- Equal magnitude
- Equal 120° phase differences



Line-to-neutral voltages

$$\overline{V_{an}} + \overline{V_{bn}} + \overline{V_{cn}}$$

$$= V\angle 0° + V\angle -120° + V\angle 120°$$

$$= V +$$

$$V\cos(-120°) + jV\sin(-120°) +$$

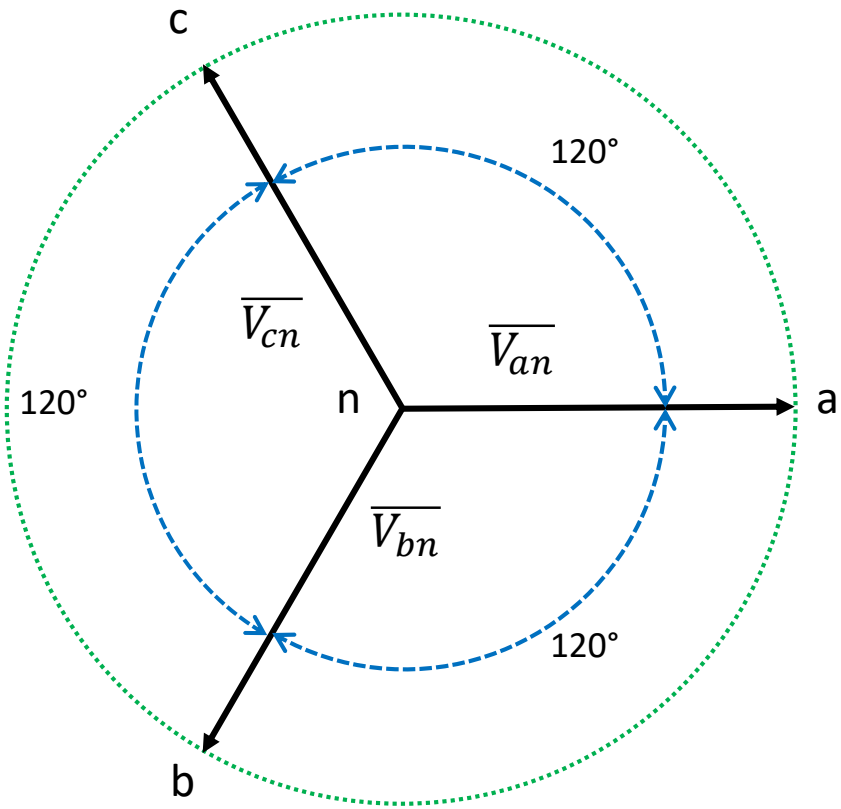
$$V\cos(120°) + jV\sin(120°)$$

$$= V[1 + \cos(-120°) + \cos(120°)] +$$

$$\mathrm{j}V[\sin(120°) + \sin(-120°)]$$

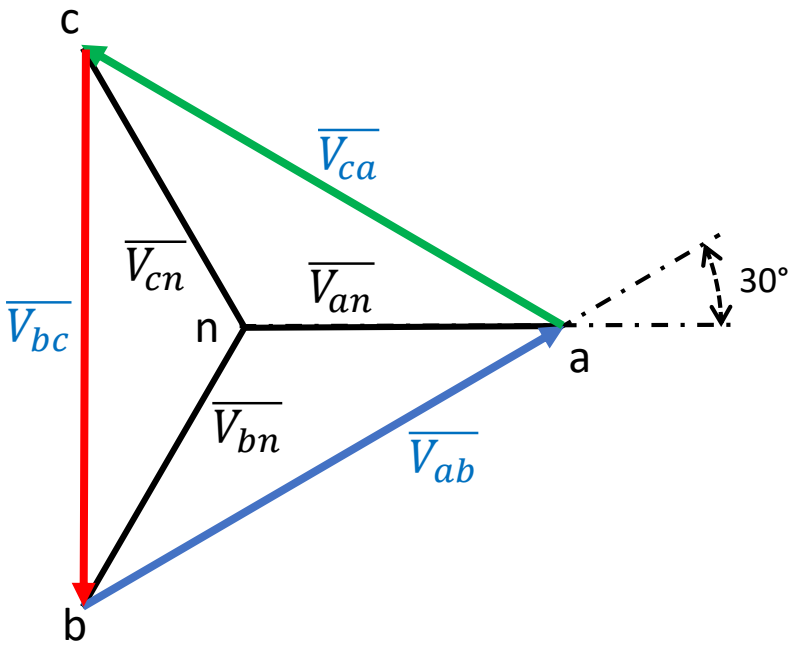
$$= 0$$

The sum of any set of balanced phasors is zero



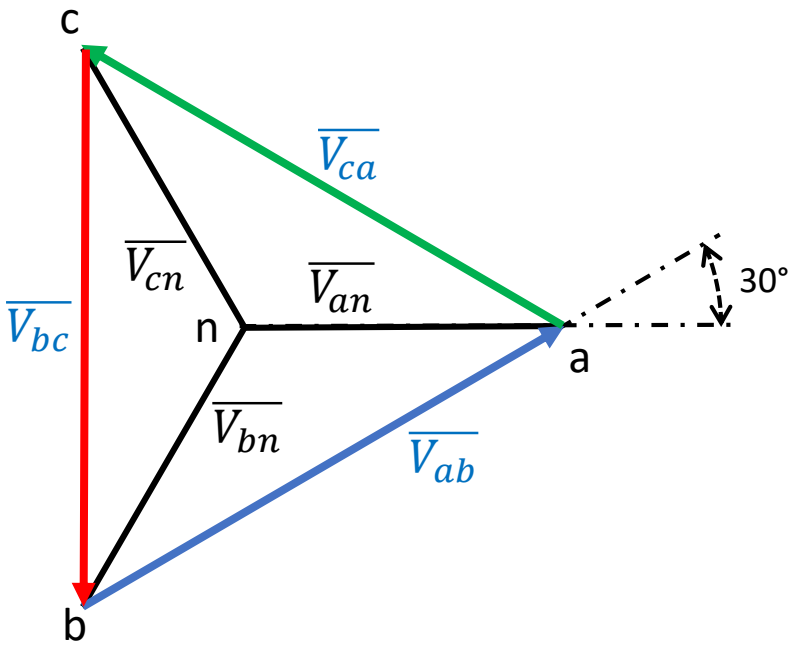
Line-to-line voltages

$$\overline{V_{ab}} = \overline{V_{nb}} + \overline{V_{an}}$$
$$= \overline{V_{an}} - \overline{V_{bn}}$$
$$= V\angle 0° - V\angle -120°$$
$$= \sqrt{3}V\angle 30°$$



Line-to-line voltages

$$\overline{V_{ca}} = \overline{V_{na}} + \overline{V_{cn}}$$
$$= \overline{V_{cn}} - \overline{V_{an}}$$
$$= V\angle 120° - V\angle 0°$$
$$= \sqrt{3}V\angle 150°$$

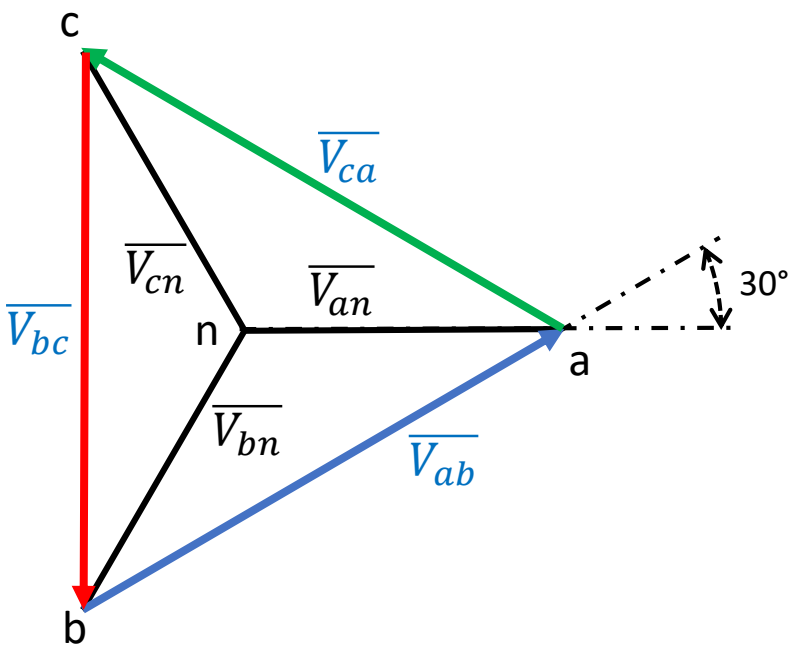


Line-to-line voltages

$$\overline{V_{bc}} = \overline{V_{nc}} + \overline{V_{bn}}$$
$$= \overline{V_{bn}} - \overline{V_{cn}}$$
$$= V\angle -120° - V\angle +120°$$
$$= \sqrt{3}V\angle 270° = \sqrt{3}V\angle -90°$$

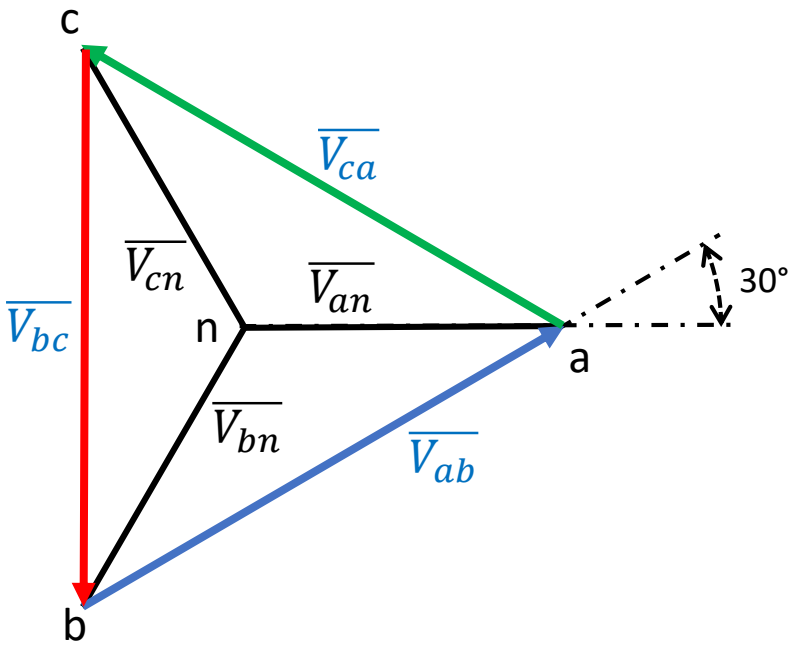
Line-to-line voltages:

- Balanced
- $\sqrt{3}$ times the magnitude of the line-to-neutral voltages

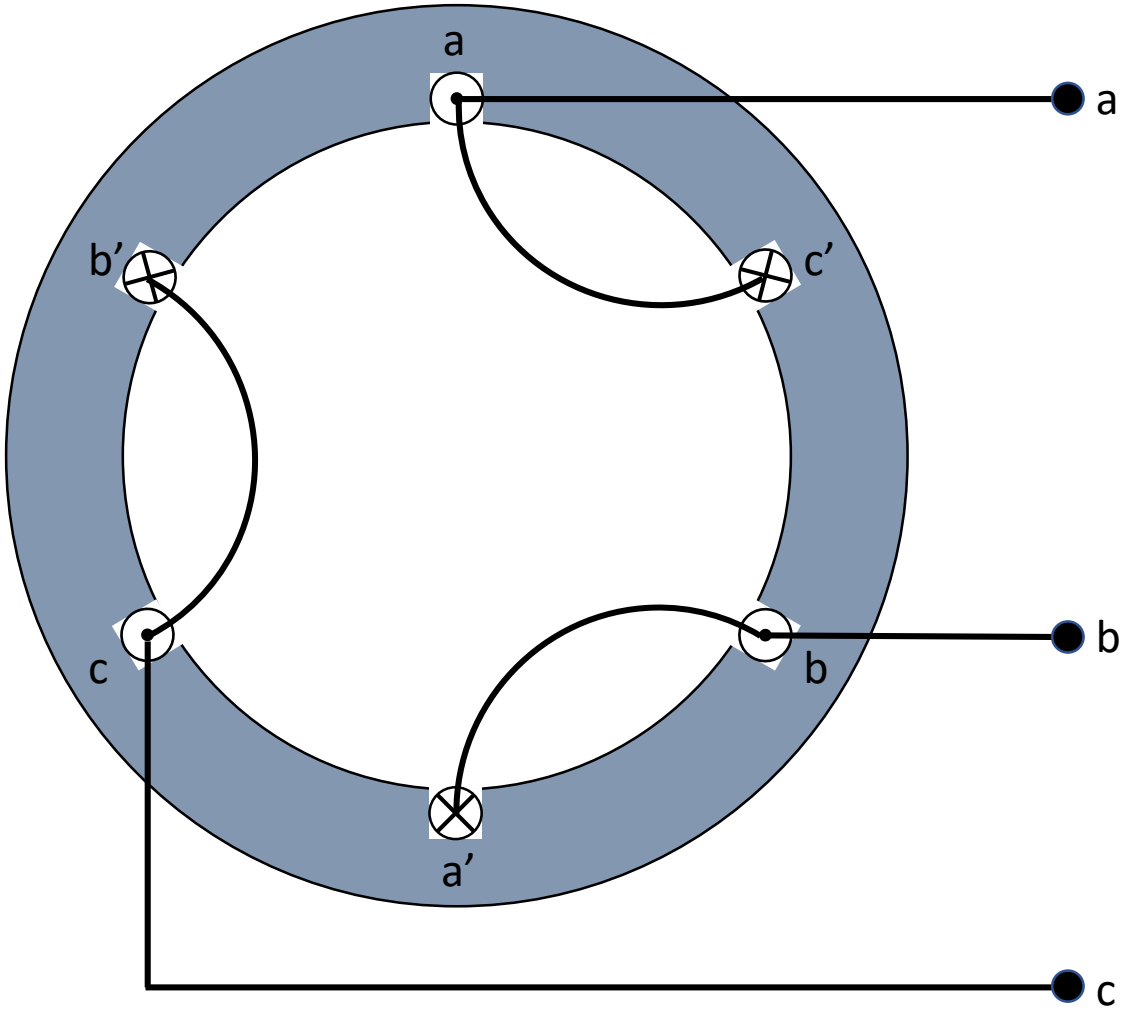


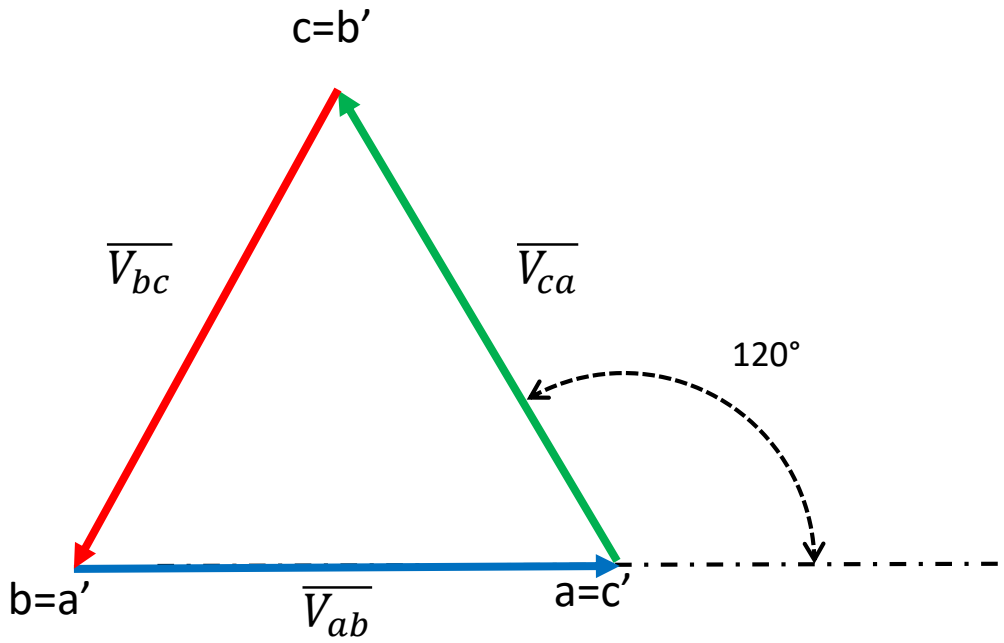
Line-to-line voltages

Because the neutral point n is not always accessible, three-phase voltages are usually specified in terms of their line-to-line magnitude



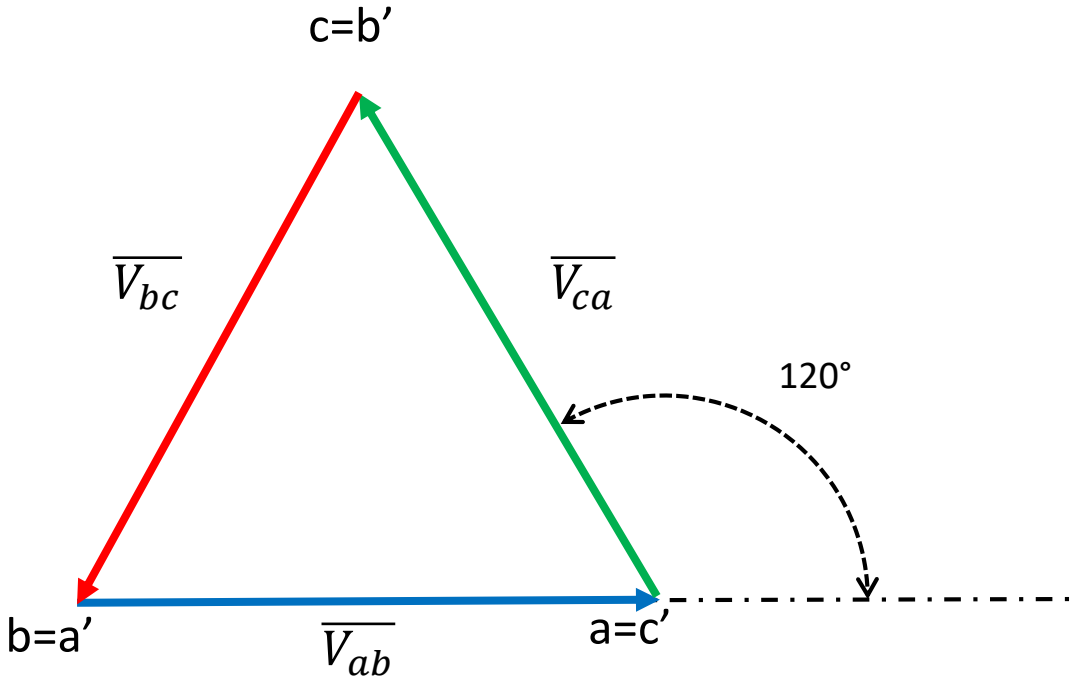
Delta connection



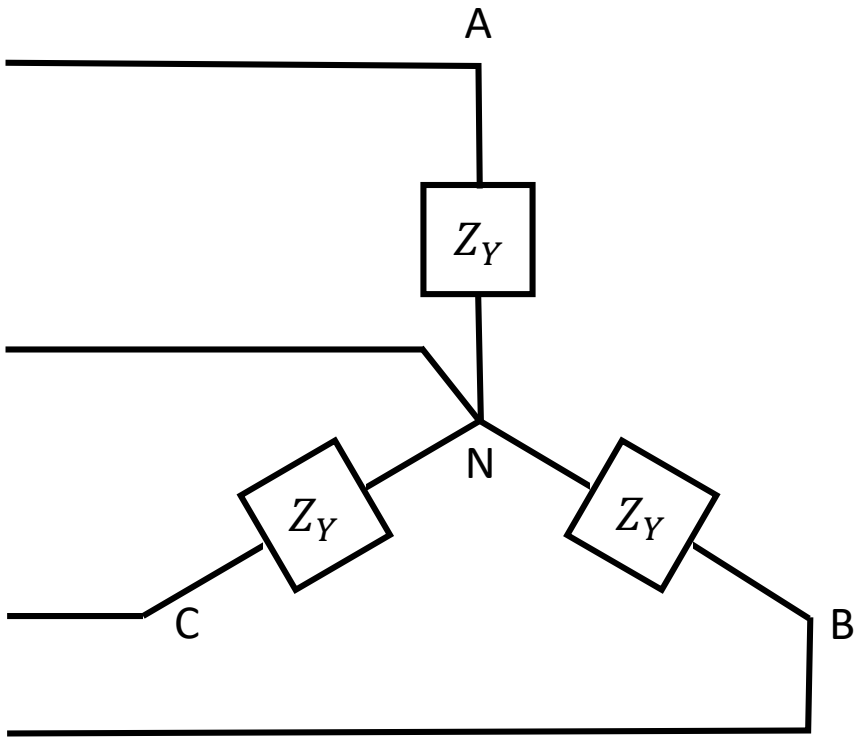


Delta connection

$$\overline{V_{ab}} = V\angle 0°$$
$$\overline{V_{ca}} = V\angle +120°$$
$$\overline{V_{bc}} = V\angle -120°$$

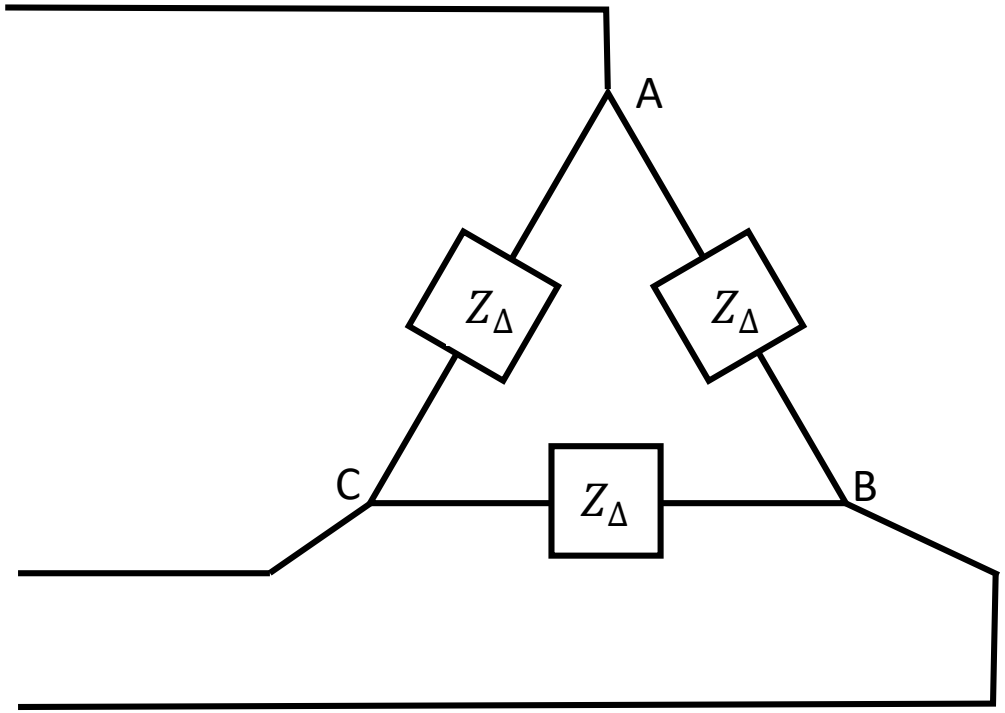


Three-phase loads



Y-connected load

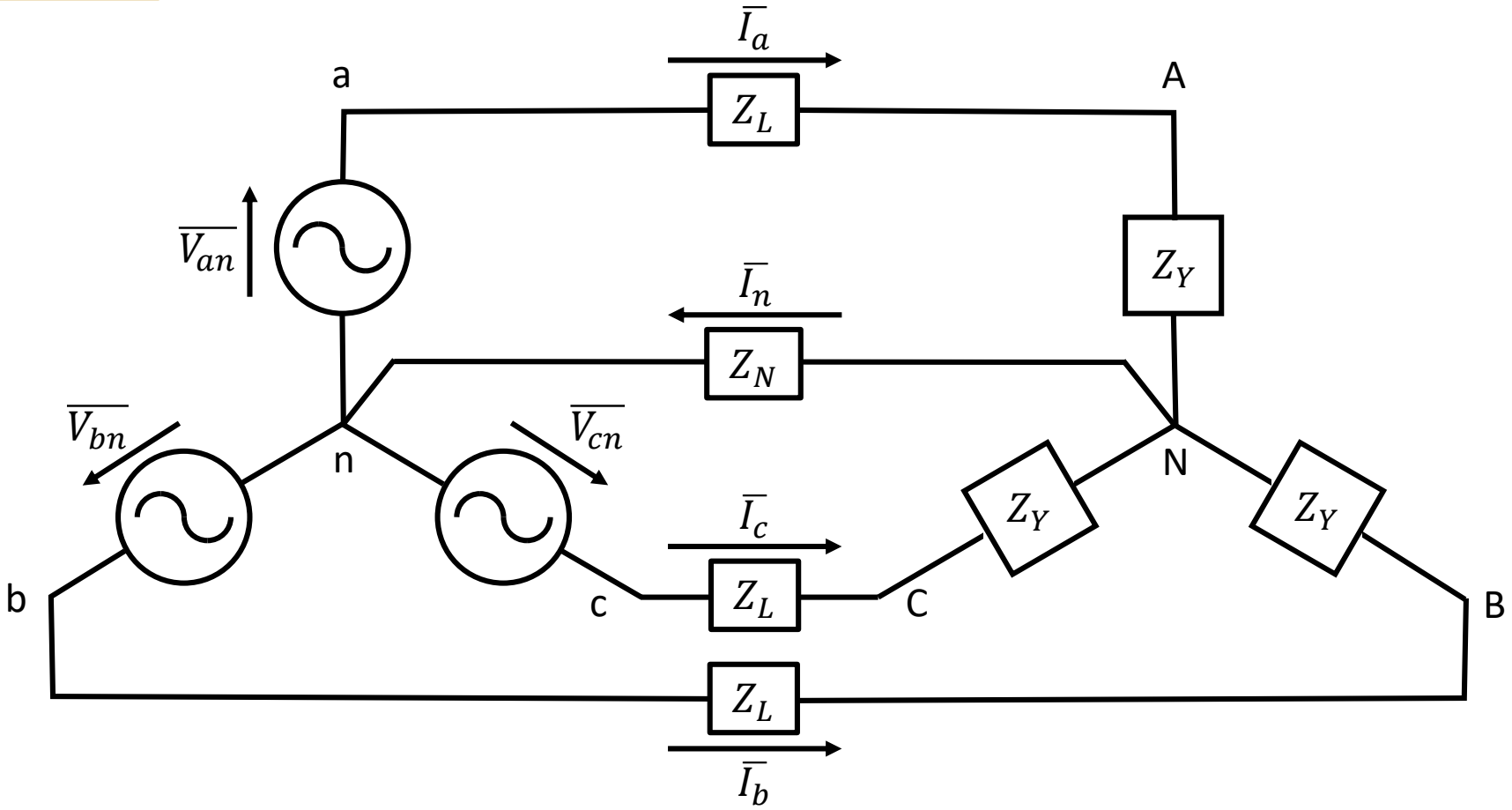
Balanced because same Z_Y in each phase



Delta-connected load

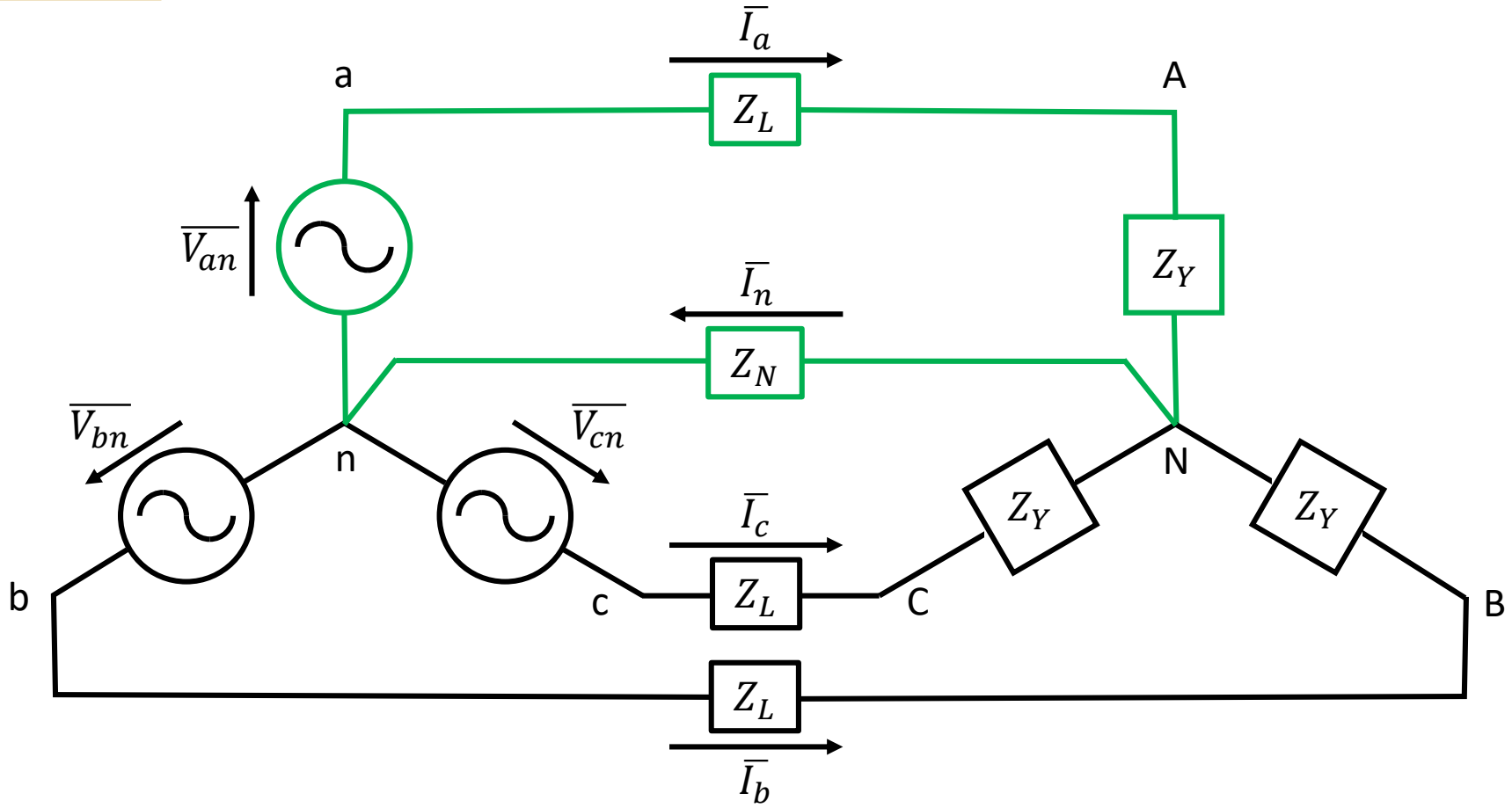
Balanced because same Z_Δ in each phase

Y-connected load connected to Y-connected source



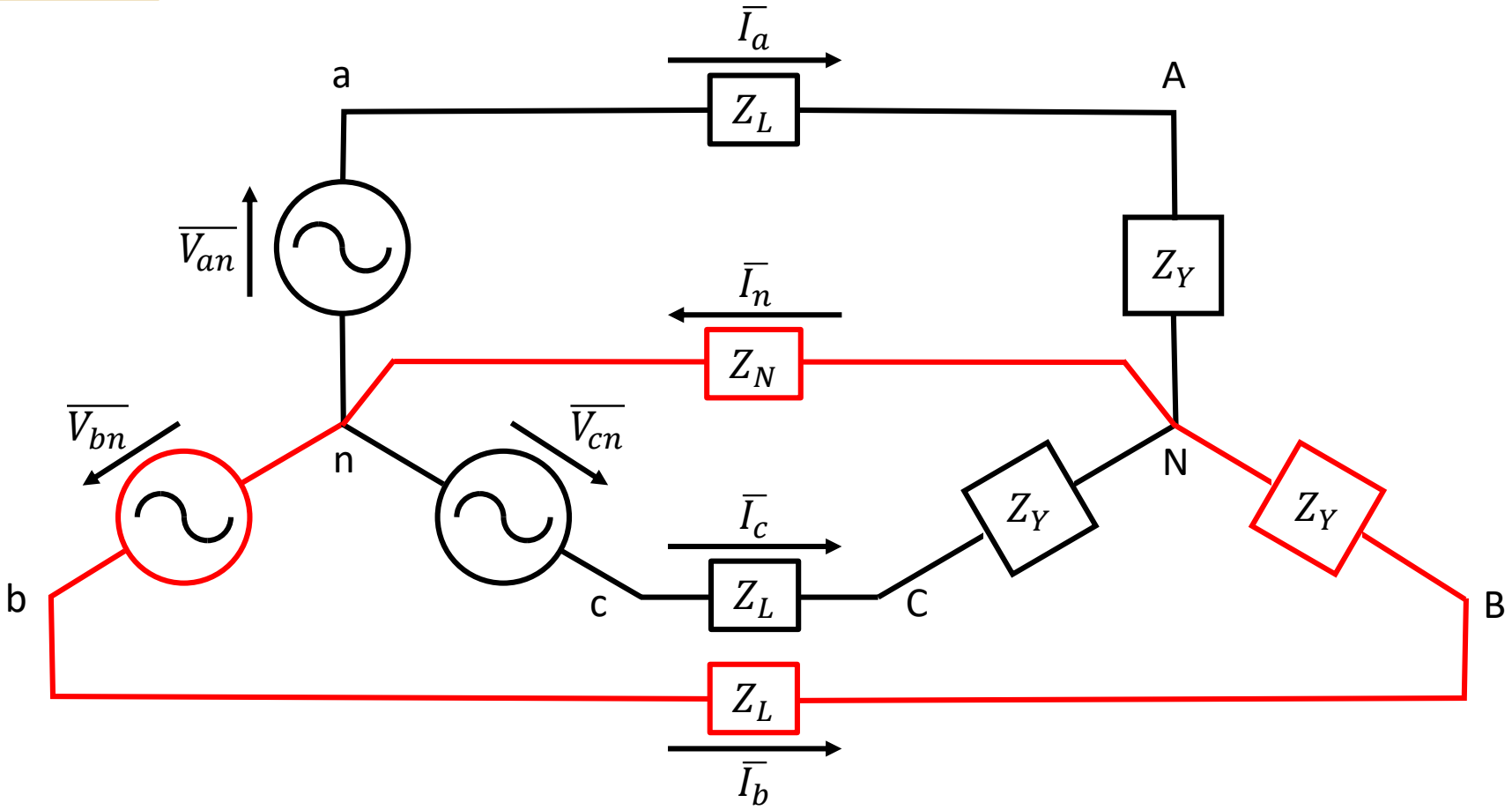
KCL at node n or N: $\overline{I_n} = \overline{I_a} + \overline{I_b} + \overline{I_c}$

KVL around the a-A-N-n-a loop



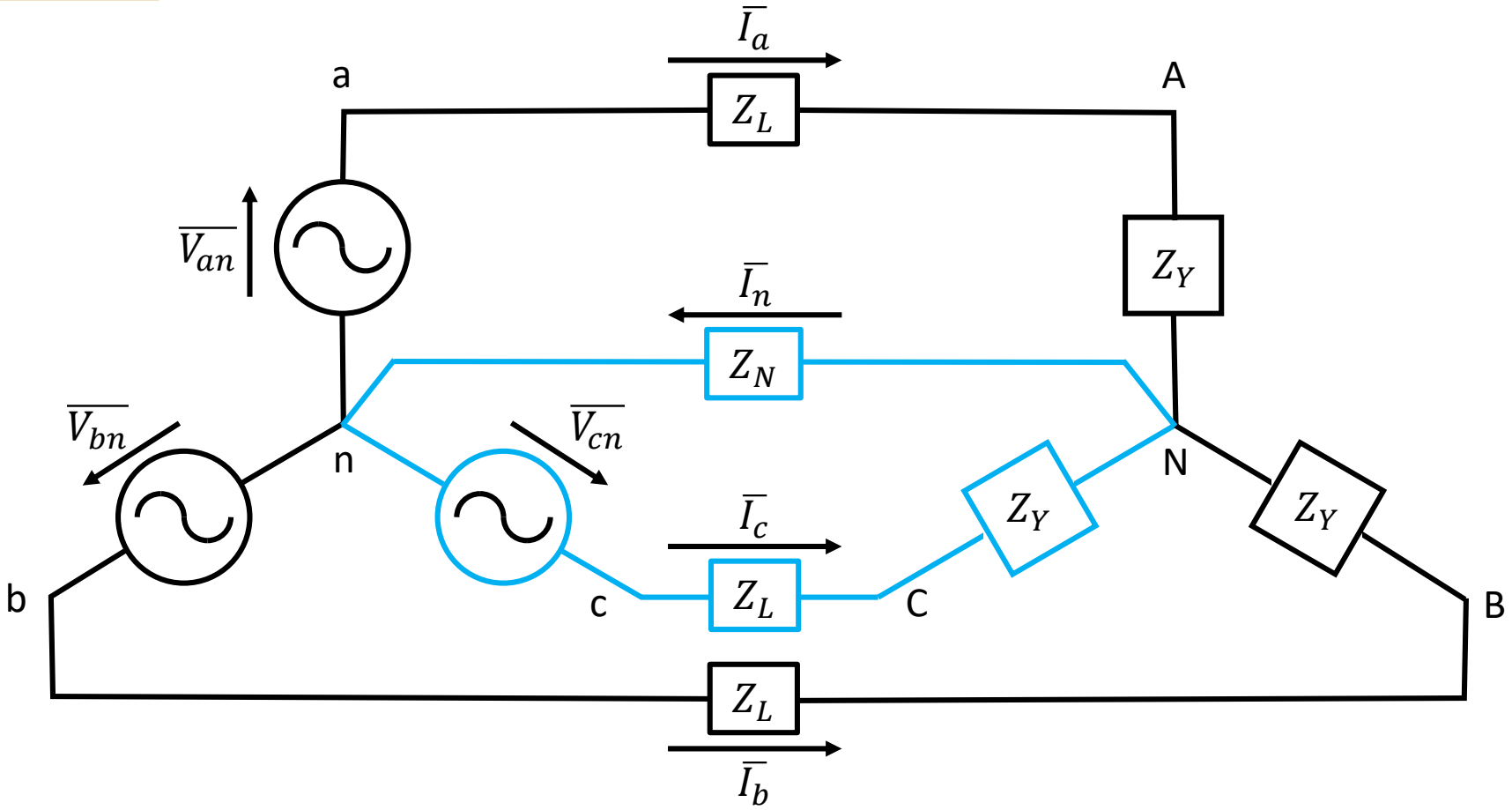
$$\overline{V_{an}} = \overline{I_a}(Z_Y + Z_L) + \overline{I_n}Z_N$$

KVL around the b-B-N-n-b loop



$$\overline{V_{bn}} = \overline{I_b}(Z_Y + Z_L) + \overline{I_n}Z_N$$

KVL around the c-C-N-n-c loop



$$\overline{V_{cn}} = \overline{I_c}(Z_Y + Z_L) + \overline{I_n}Z_N$$

Y-connected three-phase system

$$\overline{V_{an}} = \overline{I_a}(Z_Y + Z_L) + \overline{I_n}Z_N$$
$$\overline{V_{bn}} = \overline{I_b}(Z_Y + Z_L) + \overline{I_n}Z_N$$
$$\overline{V_{cn}} = \overline{I_c}(Z_Y + Z_L) + \overline{I_n}Z_N$$

$$\overline{V_{an}} + \overline{V_{bn}} + \overline{V_{cn}} = (\overline{I_a} + \overline{I_b} + \overline{I_c})(Z_Y + Z_L) + 3\overline{I_n}Z_N$$

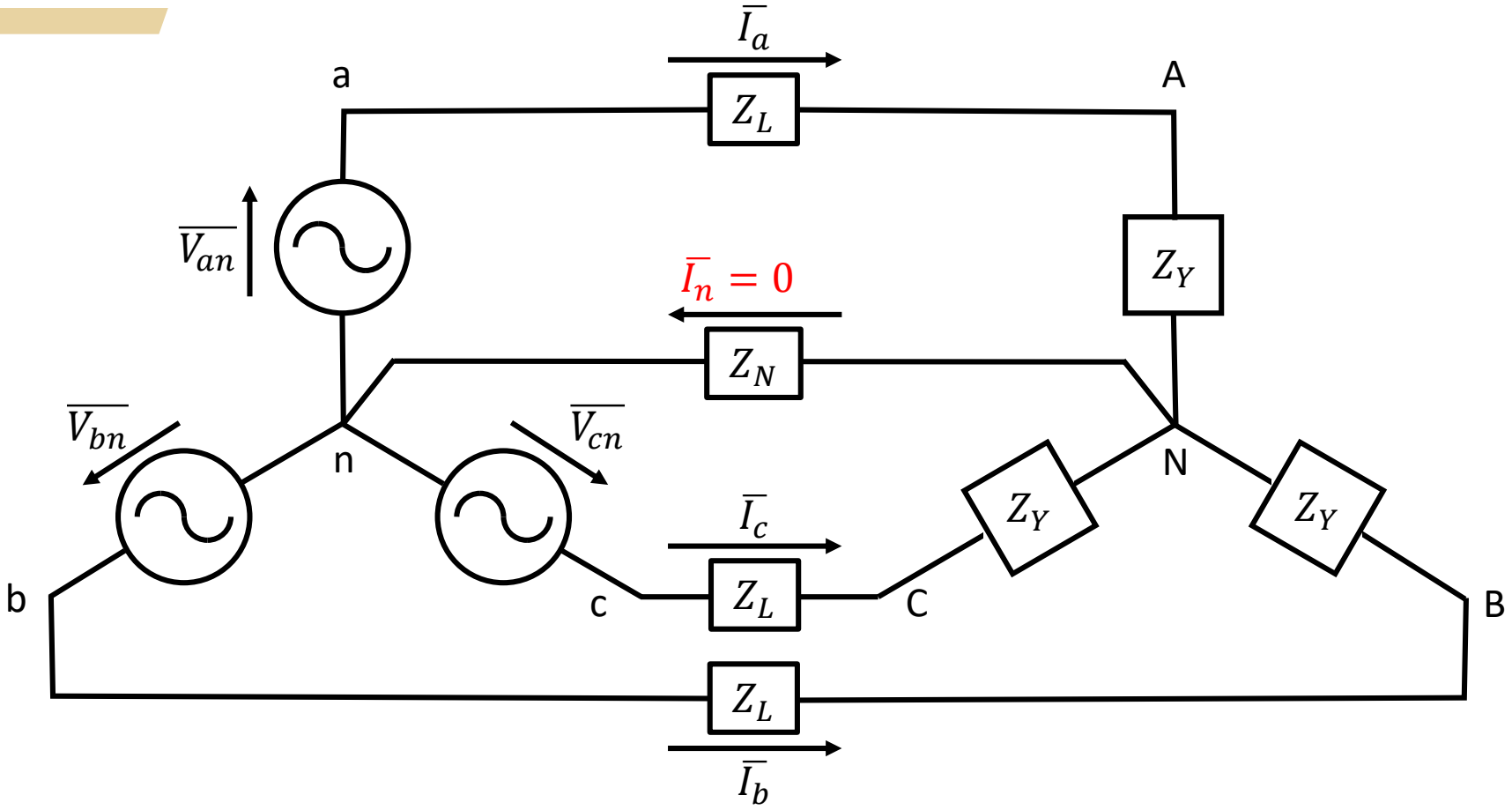
$= 0$ because source is balanced

$= \overline{I_a} + \overline{I_b} + \overline{I_c}$

$$0 = (\overline{I_a} + \overline{I_b} + \overline{I_c})(Z_Y + Z_L + 3Z_N)$$

$$\overline{I_a} + \overline{I_b} + \overline{I_c} = \overline{I_n} = 0$$

Neutral current



KCL at node n or N: $\overline{I_n} = 0 \rightarrow \overline{V_n} = \overline{V_N}$

Neutral connection is not required if the system is balanced

Unbalanced three-phase systems

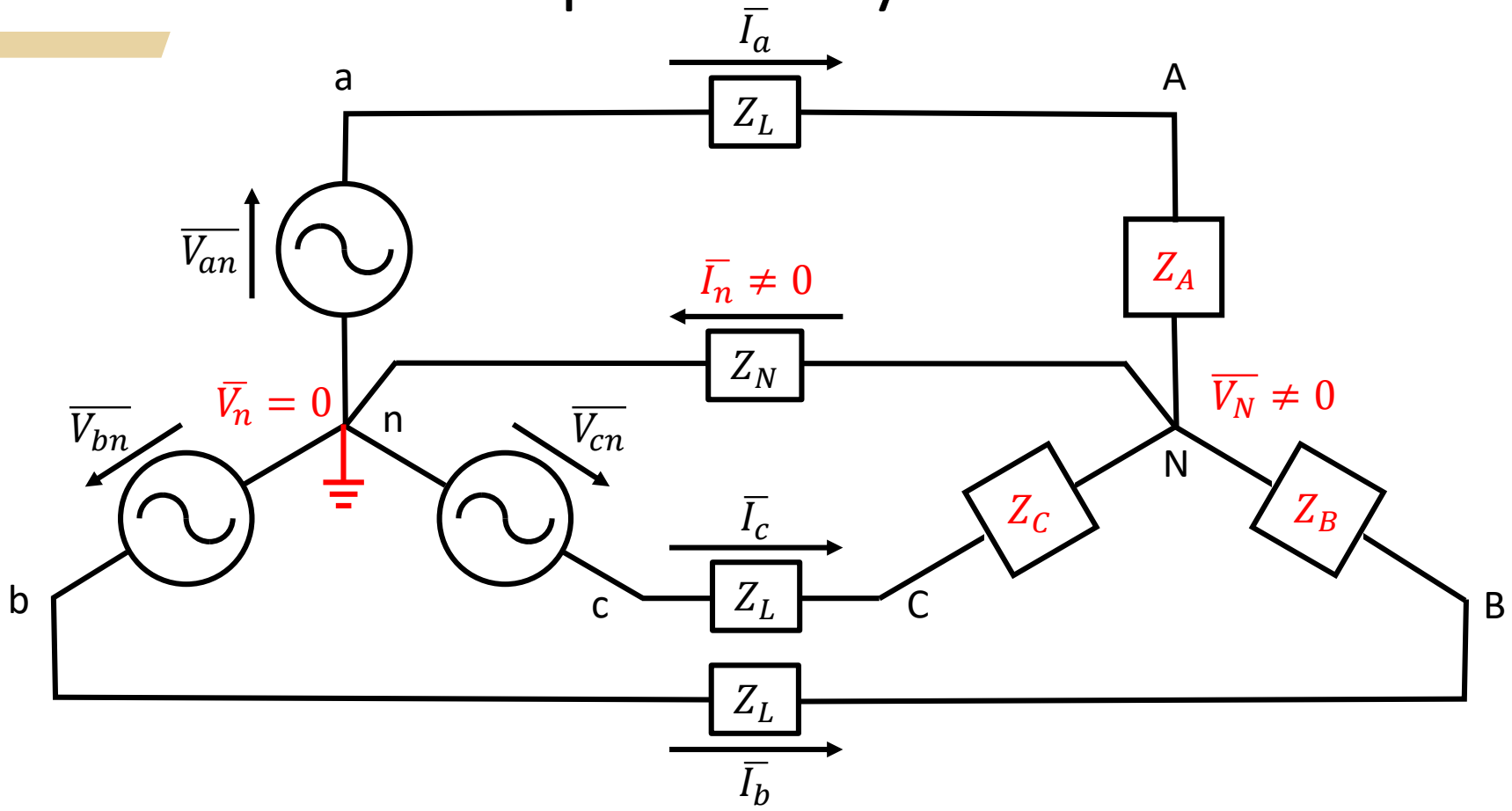
- Unbalanced source or unbalanced load

→ $\overline{I_n} \neq 0$ if there is a connection between the neutrals

→ $\overline{V_n} \neq \overline{V_N}$

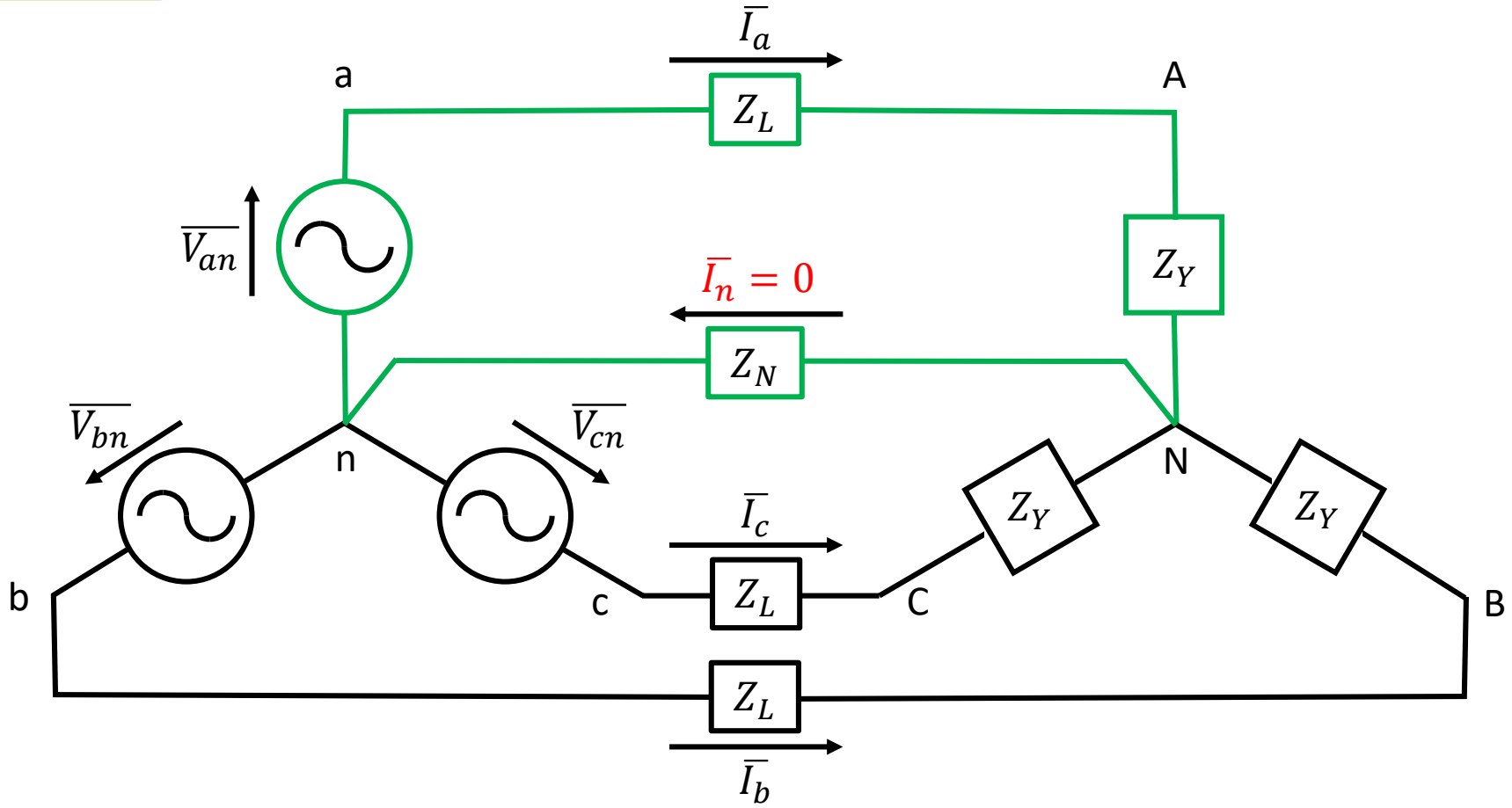
- Actual systems are usually not perfectly balanced
  - Single phase loads create unbalances
  - Distribution networks are unbalanced because they serve single-phase loads
  - Aggregating many unbalanced loads makes the system roughly balanced
- Chapter 10 discusses techniques for analyzing unbalanced systems

Unbalanced three-phase systems



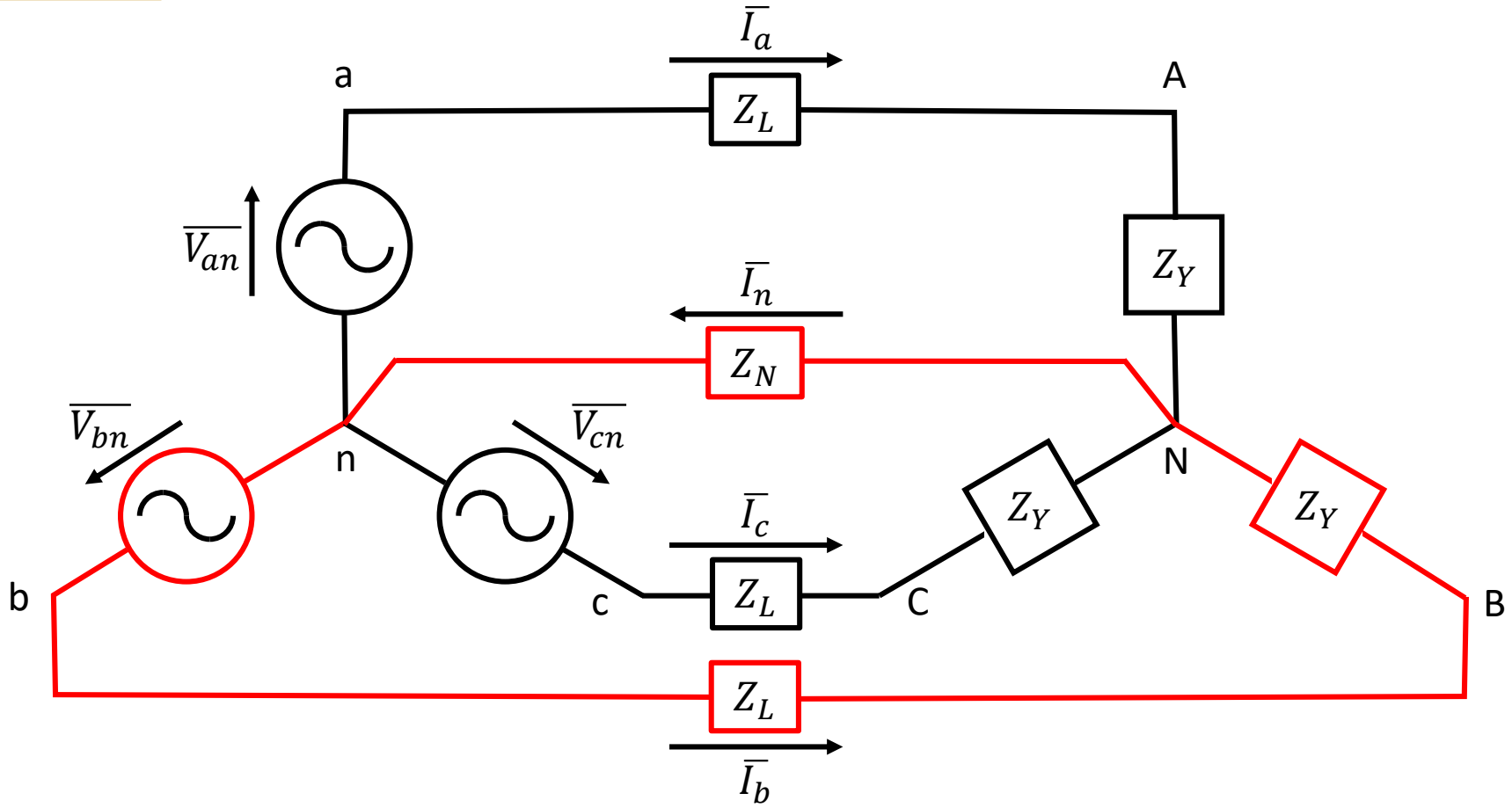
- Some neutral points are connected to ground ("grounded")
- However, never assume that $|\overline{V_n}| = 0$ or $|\overline{V_N}| = 0$!

Phase a of balanced system, $\bar{I}_n = 0, \bar{V}_n = \bar{V}_N$



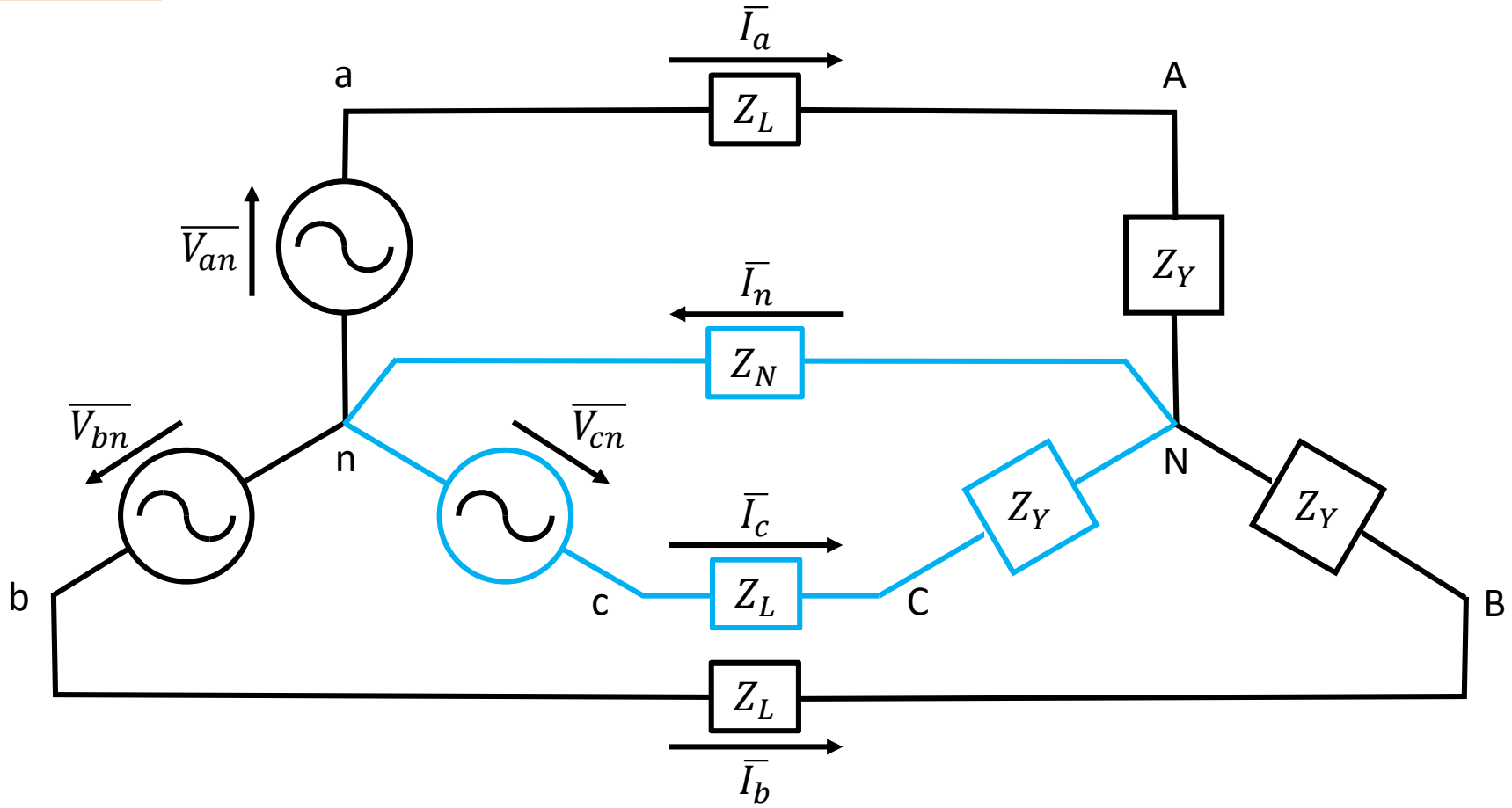
$$\overline{V_{an}} = \bar{I}_a(Z_Y + Z_L) \qquad \bar{I}_a = \frac{\overline{V_{an}}}{Z_Y + Z_L} = \frac{V\angle 0°}{|Z_Y + Z_L|\angle\theta} = I\angle(-\theta)$$

Phase b of balanced system, $\bar{I}_n = 0, \bar{V}_n = \overline{V_N}$



$$\overline{V_{bn}} = \bar{I}_b(Z_Y + Z_L) \qquad \bar{I}_b = \frac{\overline{V_{bn}}}{Z_Y + Z_L} = \frac{V\angle -120°}{|Z_Y + Z_L|\angle\theta} = I\angle(-120° - \theta)$$

Phase c of balanced system, $\bar{I}_n = 0, \bar{V}_n = \overline{V_N}$



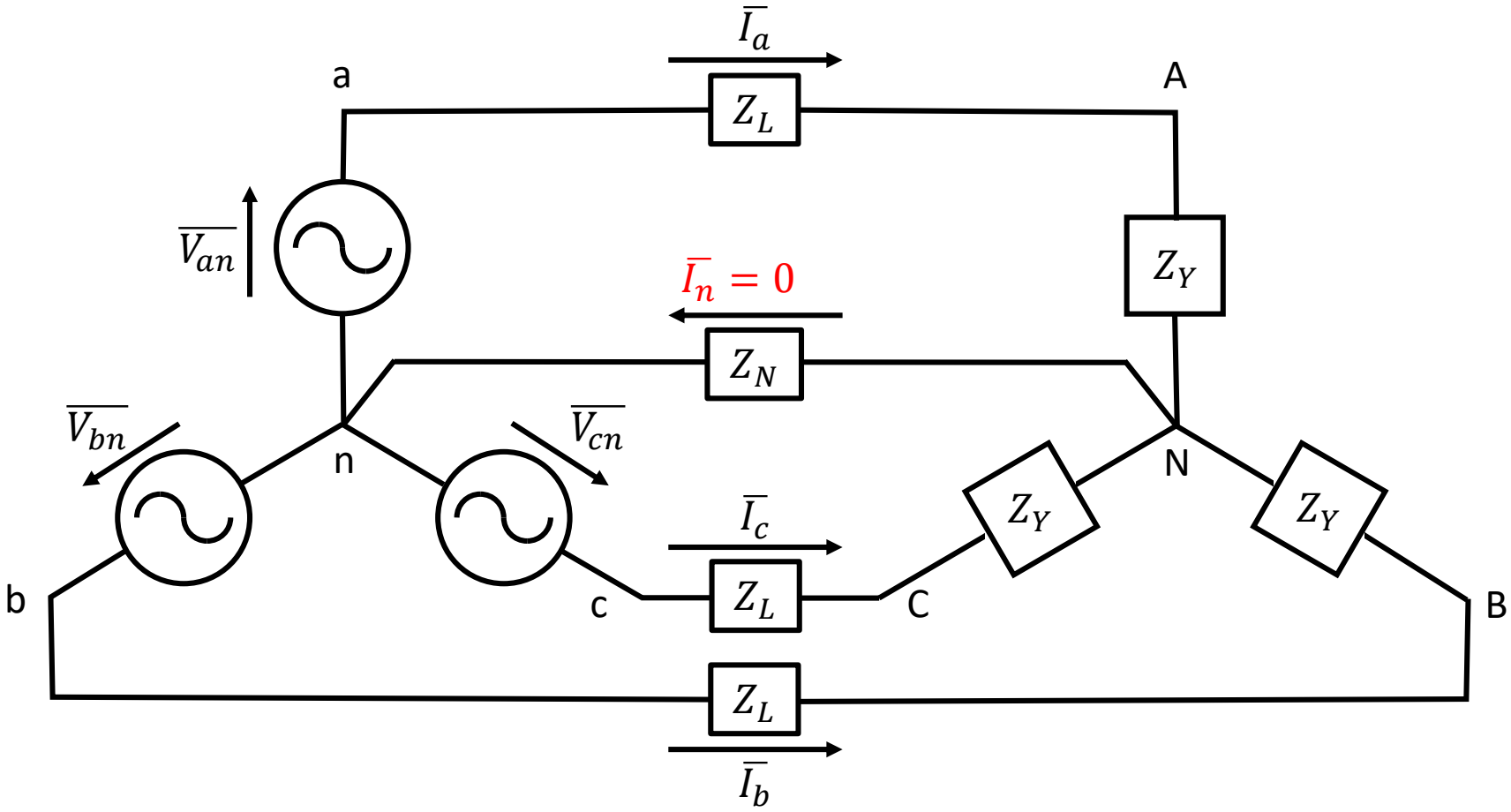
$$\overline{V_{cn}} = \bar{I}_c(Z_Y + Z_L) \qquad \bar{I}_c = \frac{\overline{V_{cn}}}{Z_Y + Z_L} = \frac{V\angle +120°}{|Z_Y + Z_L|\angle\theta} = I\angle(+120° - \theta)$$

Currents in a balanced system

$$\overline{I_a} = I\angle(-\theta)$$

$$\overline{I_b} = I\angle(-120° - \theta)$$

$$\overline{I_c} = I\angle(+120° - \theta)$$

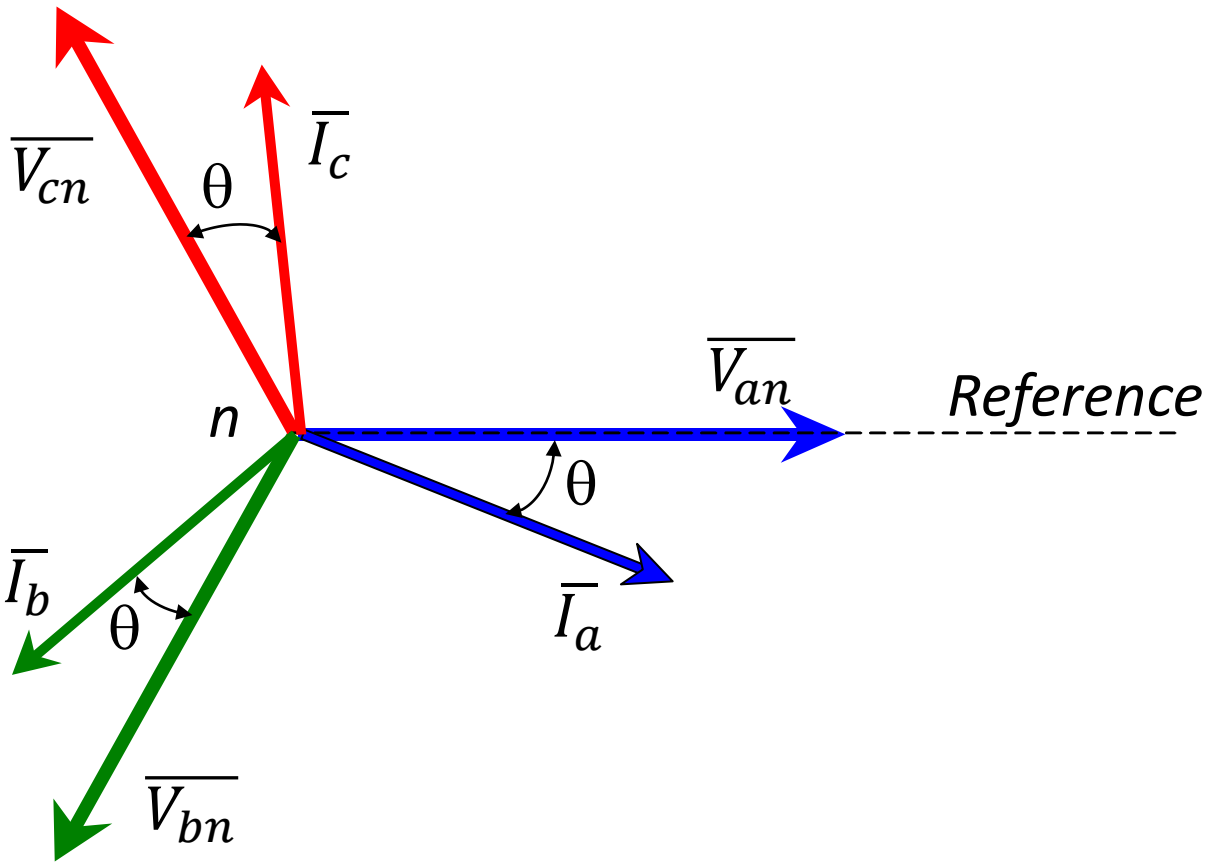


Currents in a balanced system

$$\overline{I_a} = I\angle(-\theta)$$

$$\overline{I_b} = I\angle(-120° - \theta)$$

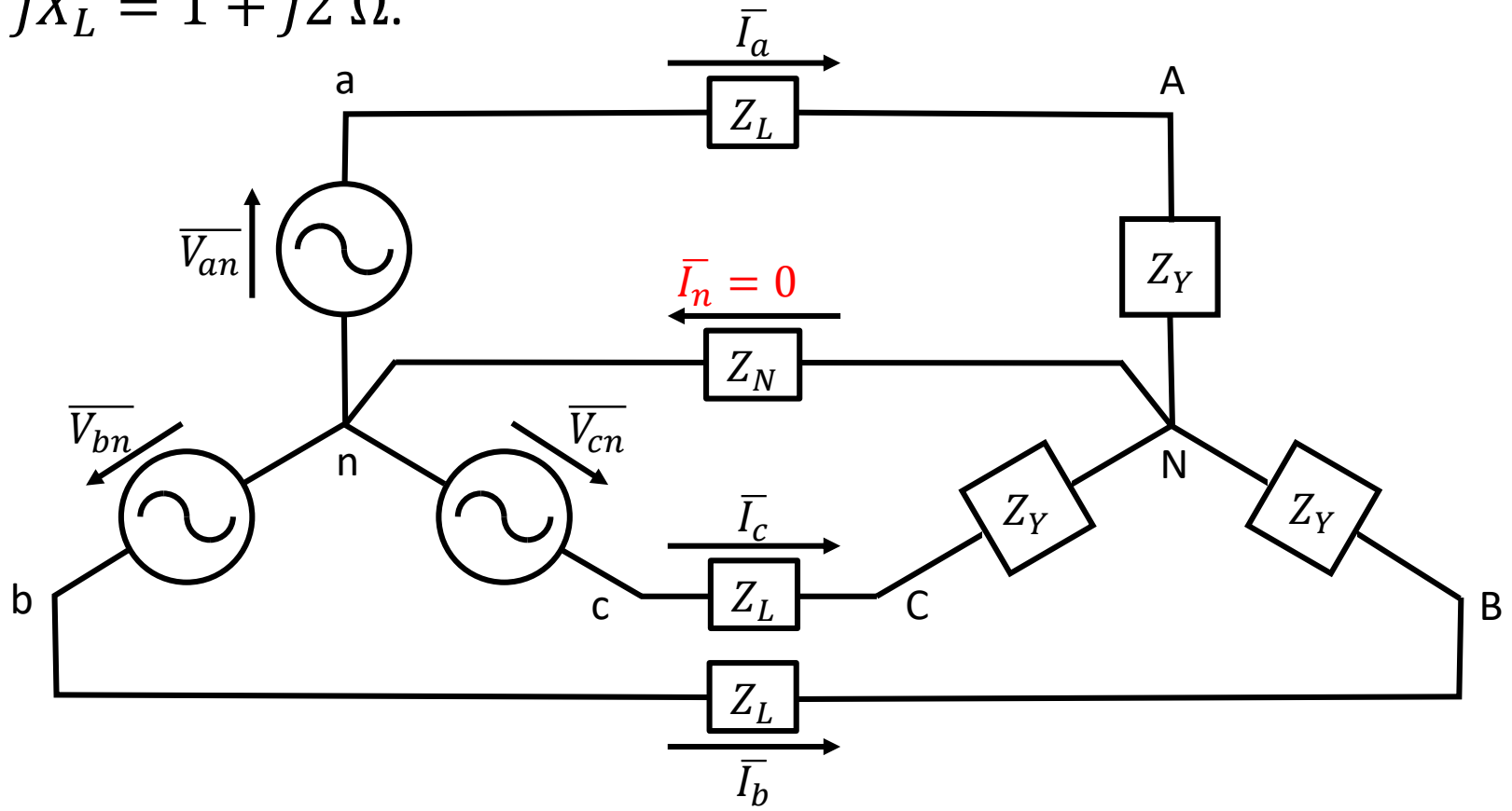
$$\overline{I_c} = I\angle(+120° - \theta)$$



A balanced Y-connected load draws a balanced set of currents

Example 4.1: Y-connected three-phase load

A balanced three-phase Y-connected load is supplied from a balanced three-phase voltage source with a line-to-line voltage of 480V. The impedance of each phase of the load is $Z_Y = 9 + j4\ \Omega$ and the impedance of each line connecting the source to the load is $Z_L = R_L + jX_L = 1 + j2\ \Omega$.



Example 4.1: Y-connected three-phase load

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$$\overline{V_{ab}} = 480\angle 0°\ \text{V}$$

$$\overline{V_{an}} = \frac{\overline{V_{ab}}}{\sqrt{3}}\angle -30° = 277.13\angle -30°\ \text{V}$$

$$\overline{I_a} = \frac{\overline{V_{an}}}{Z_Y + Z_L} = \frac{277.13\angle -30°}{10 + j6} = 23.764\angle -60.96°\ \text{A}$$

$$\overline{V_{bn}} = 277.13\angle -150° \qquad \overline{V_{cn}} = 277.13\angle +90°$$

$$\overline{I_b} = 23.764\angle -180.96° \qquad \overline{I_c} = 23.764\angle +59.04°$$

Example 4.1: Y-connected three-phase load

$\overline{V_{AN}} = \overline{V_{an}} - Z_L\overline{I_a} = 277.13\angle -30° - 23.764\angle -60.96° \times (1 + j2) = 234.05\angle -37.0°$

$\overline{V_{BN}} = 234.05\angle -157.0°$

$\overline{V_{CN}} = 234.05\angle 83.0°$

The complex power supplied by phase a of the source is:

$\overline{S_a} = \overline{V_{an}}\,\overline{I_a}^{*} = 277.13\angle -30° \times 23.764\angle +60.96° = 5647 + \text{j}3388$ VA

$P_a = 5.647$ kW

$Q_a = 3.388$ kvar

$P_{Source} = 3 \times P_a = 16.941$ kW

$Q_{Source} = 3 \times Q_a = 10.164$ kvar

The complex consumed in phase A of the load is:

$\overline{S_A} = P_A + jQ_A = \overline{V_{AN}}\,\overline{I_a}^{*} = 234.05\angle -37.0° \times 23.764\angle +60.96° = 5083 + \text{j}2258$ VA

$P_{Load} = 3 \times P_A = 15.248$ kW

$Q_{Load} = 3 \times Q_A = 6.775$ kvar

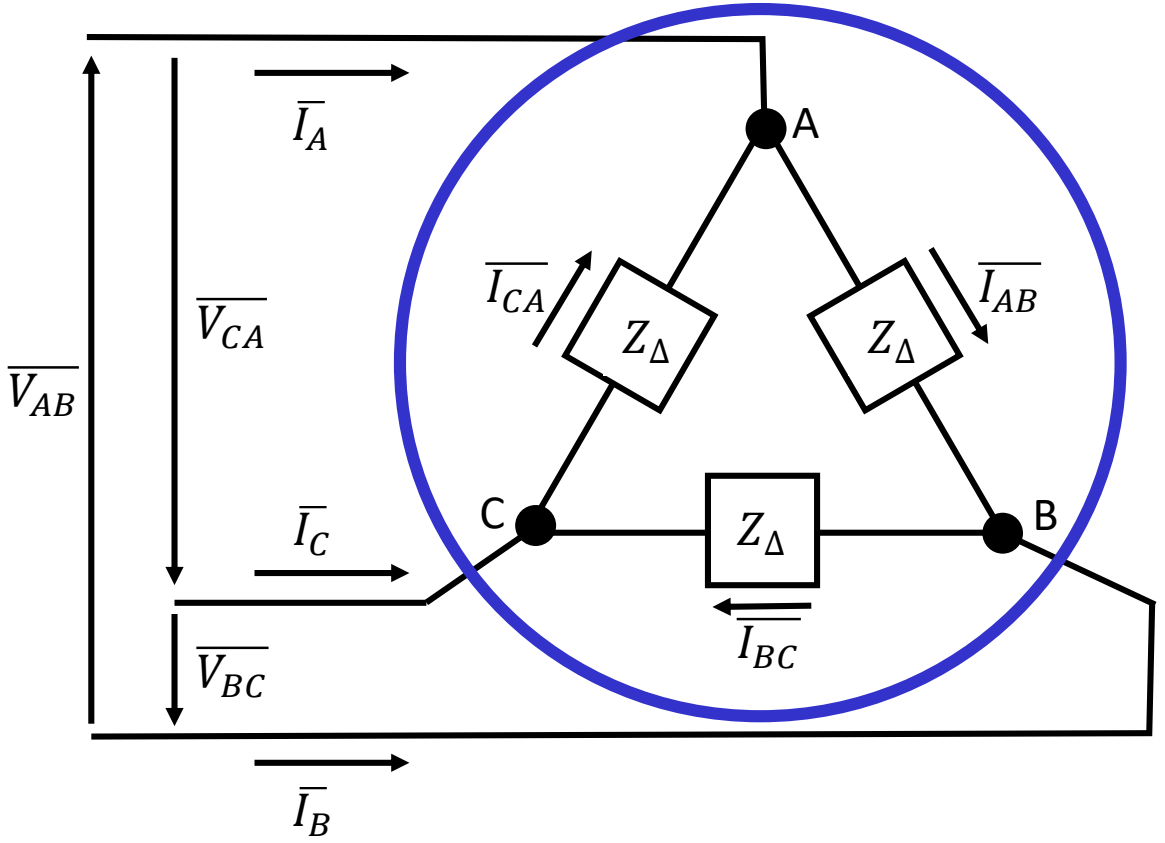
$P_{losses} = P_{Source} - P_{Load} = 1.693$ kW

$Q_{losses} = Q_{Source} - Q_{Load} = 3.389$ kvar

Delta-connected load

Apply KCL around the load:

$$\overline{I_A} + \overline{I_B} + \overline{I_C} = 0$$

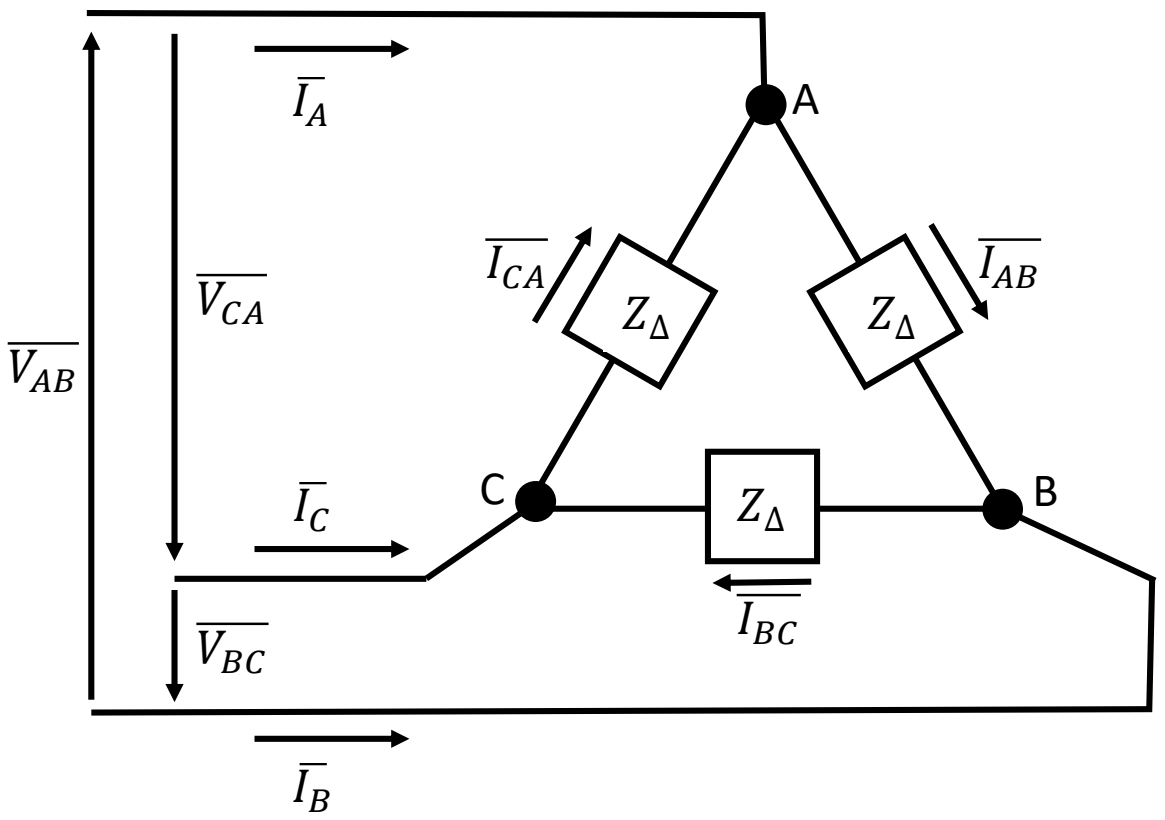


Delta-connected load

$$\overline{I_{AB}} = \frac{\overline{V_{AB}}}{Z_\Delta} = \frac{V\angle 0°}{|Z_\Delta|\angle\theta} = I\angle(-\theta)$$

$$\overline{I_{BC}} = \frac{\overline{V_{BC}}}{Z_\Delta} = \frac{V\angle -120°}{|Z_\Delta|\angle\theta} = I\angle(-120° - \theta)$$

$$\overline{I_{CA}} = \frac{\overline{V_{CA}}}{Z_\Delta} = \frac{V\angle +120°}{|Z_\Delta|\angle\theta} = I\angle(+120° - \theta)$$



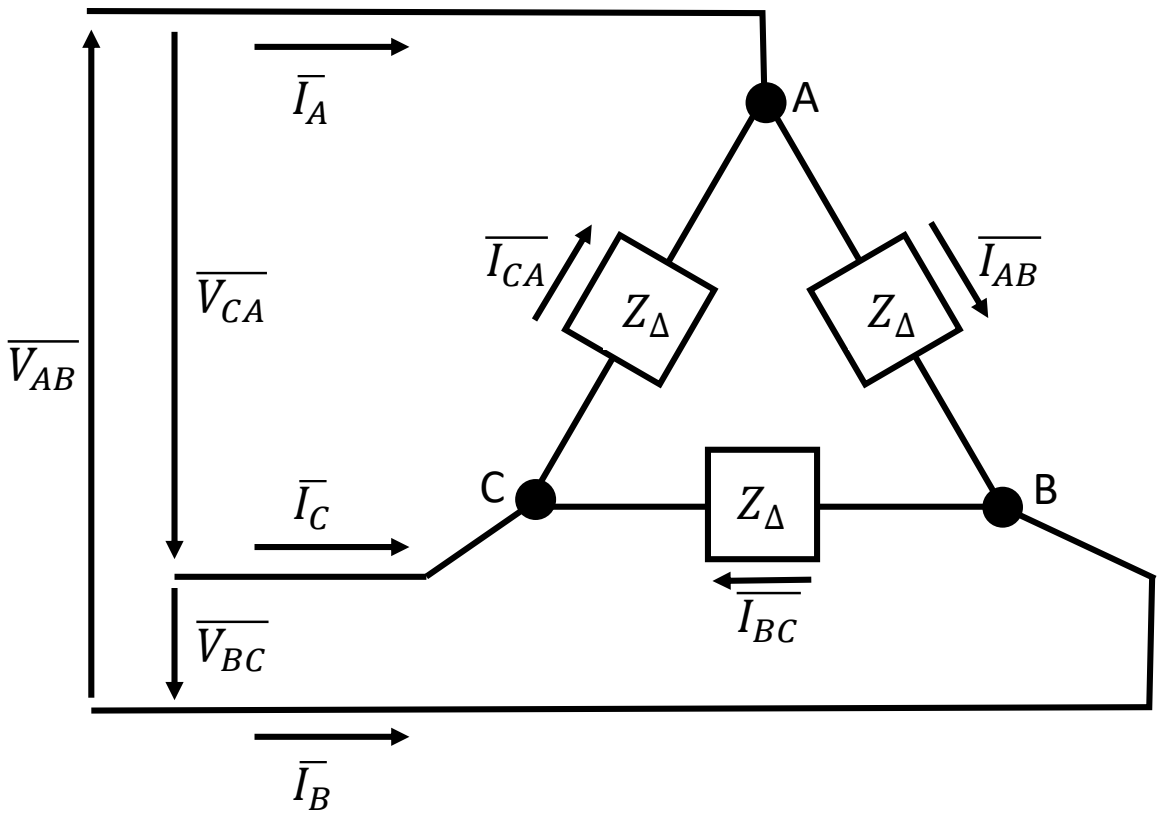
Delta-connected load

KCL at nodes A, B, and C:

$$\begin{aligned}\overline{I_A} &= \overline{I_{AB}} - \overline{I_{CA}} \\ &= I\angle(-\theta) - I\angle(+120° - \theta) \\ &= \sqrt{3}I\angle(-\theta - 30°)\end{aligned}$$

$$\begin{aligned}\overline{I_B} &= \overline{I_{BC}} - \overline{I_{AB}} \\ &= I\angle(-120° - \theta) - I\angle(-\theta) \\ &= \sqrt{3}I\angle(-\theta - 150°)\end{aligned}$$

$$\begin{aligned}\overline{I_C} &= \overline{I_{CA}} - \overline{I_{BC}} \\ &= I\angle(+120° - \theta) - I\angle(-120° - \theta) \\ &= \sqrt{3}I\angle(-\theta + 90°)\end{aligned}$$



A balanced delta-connected load draws a balanced set of currents

Example 4.2: Delta-connected three-phase load

A balanced three-phase Δ-connected load is supplied directly from a balanced three-phase voltage source with a line-to-line voltage of 480V. The impedance of each phase of the load is $Z_\Delta = 10 + j6\ \Omega$.

$\overline{V_{AB}} = 480\angle 0°$ V

$\overline{I_{AB}} = \frac{\overline{V_{AB}}}{Z_\Delta} = \frac{480\angle 0°}{10+j6} = 41.16\angle -30.96°$ A

$\overline{I_A} = \sqrt{3}\,\overline{I_{AB}}\angle(-30°) = 71.29\angle -60.96°$ A

$\overline{S_{AB}} = \overline{V_{AB}}\,\overline{I_{AB}}^{*} = 480\angle 0° \times 41.16\angle +30.96° = 16.942 + j10.163$ kVA

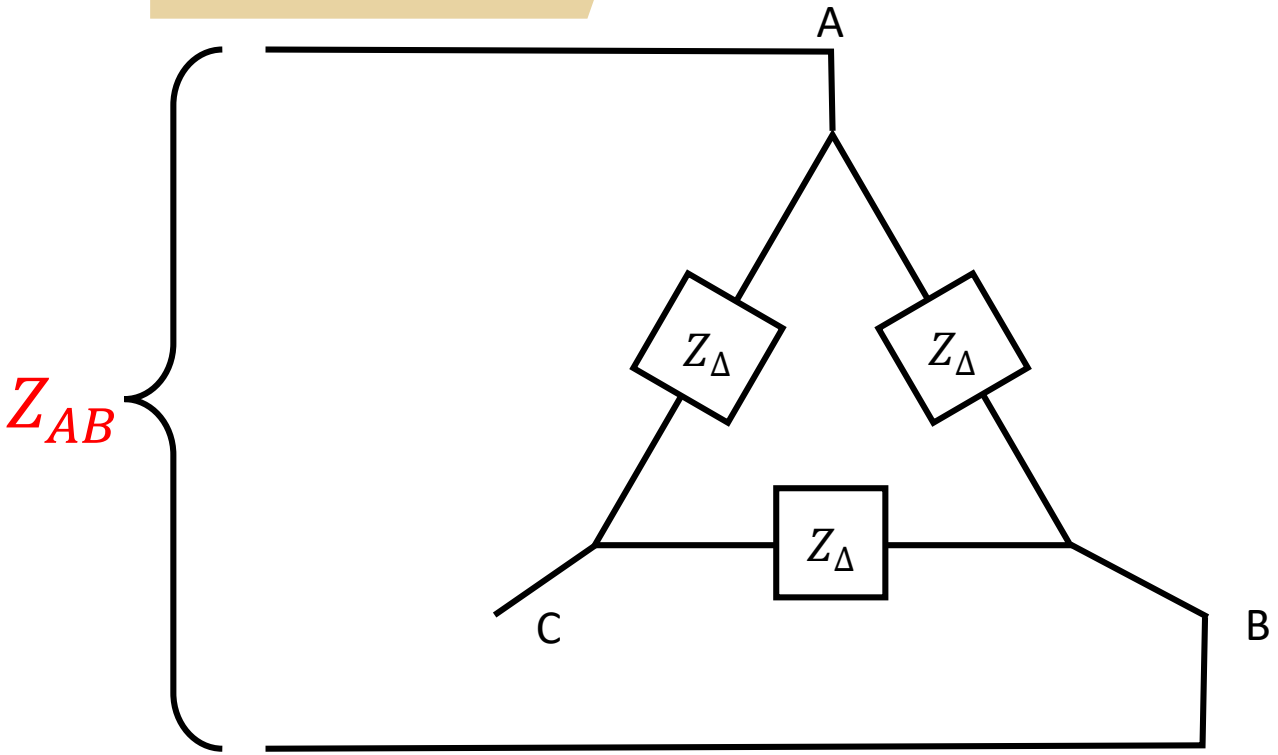
$P_{AB} = 16.942$ kW

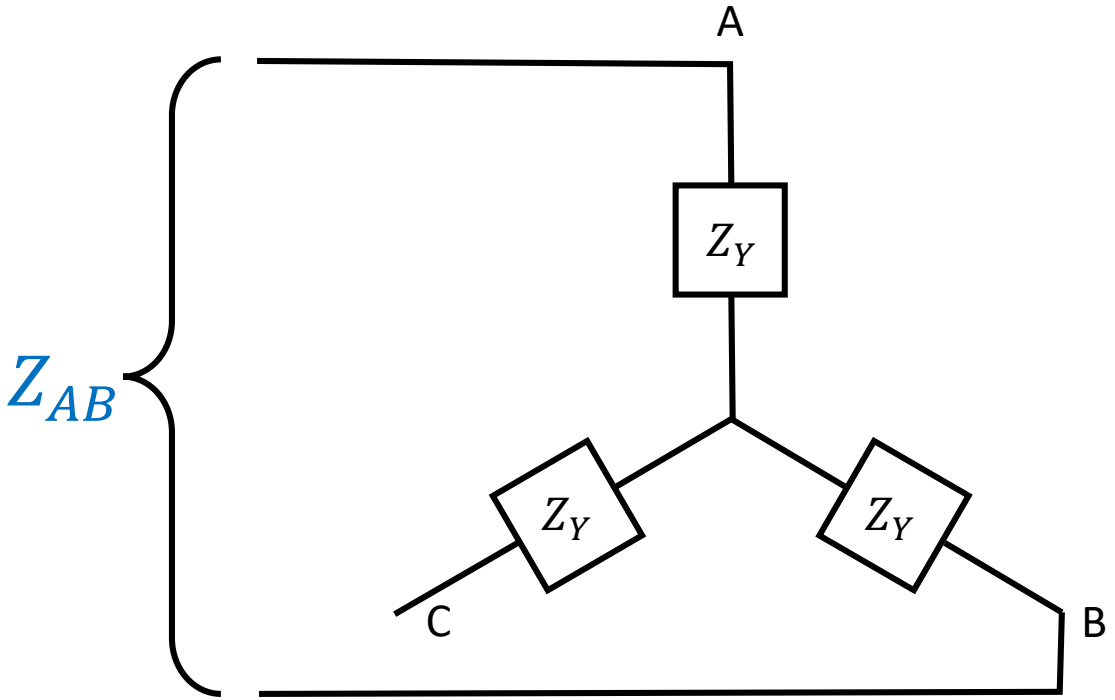
$Q_{AB} = 10.163$ kvar

$P = 3 \times P_{AB} = 50.826$ kW

$Q = 3 \times Q_{AB} = 30.489$ kvar

Δ-Y equivalence



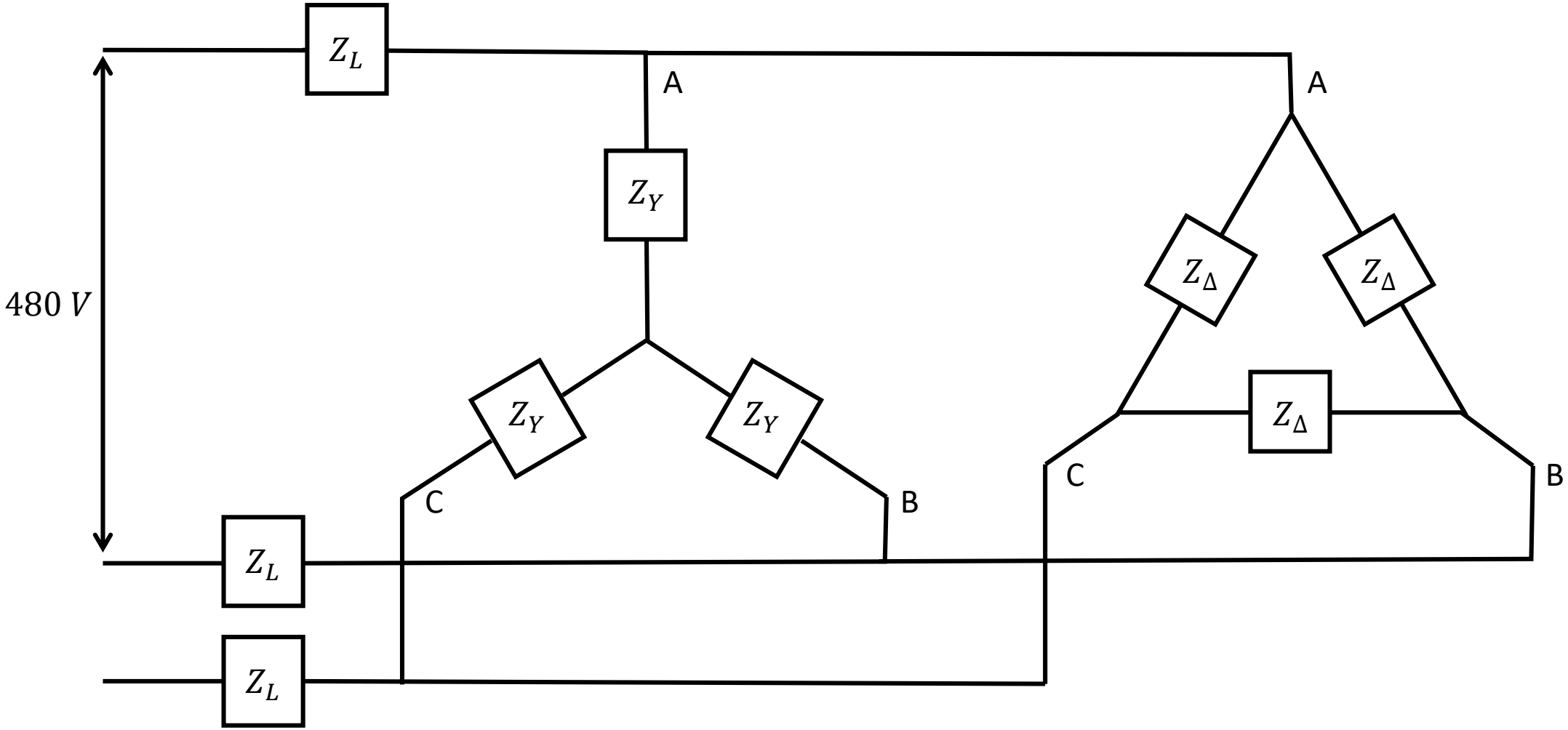


$$Z_{AB} = Z_\Delta \parallel 2Z_\Delta = \frac{2}{3} Z_\Delta$$

$$Z_{AB} = Z_Y + Z_Y = 2Z_Y$$

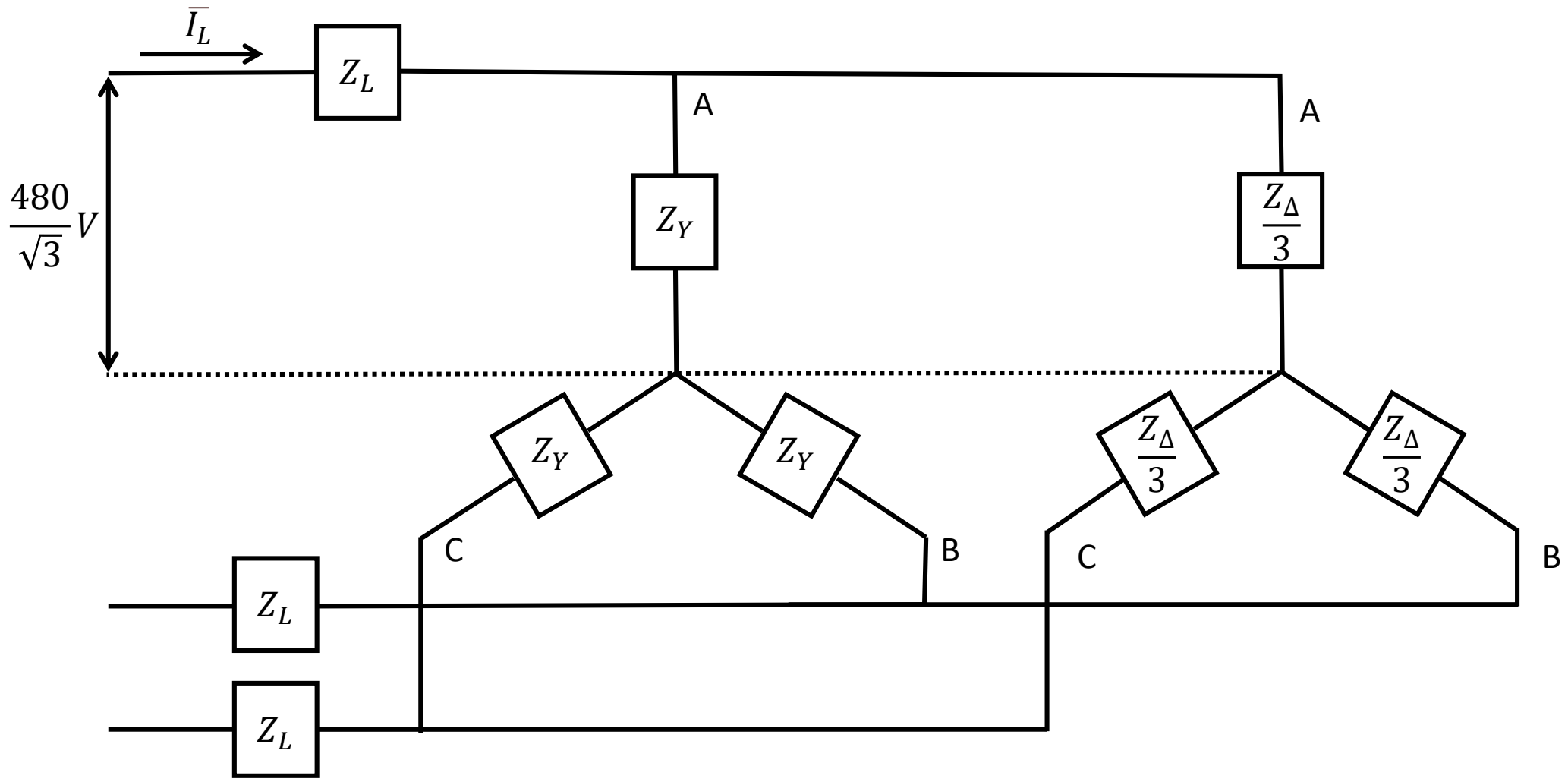
Equivalent if $Z_{AB} = Z_{AB} \rightarrow Z_Y = \frac{1}{3} Z_\Delta$

Example 4.3: Parallel Y- and Δ-connected loads



$$Z_L = 0.2 + j1\Omega, Z_Y = 10 + j2\Omega, Z_\Delta = 12 + j3\Omega$$

Example 4.3: Y-Δ equivalent



$$Z_L = 0.2 + j1\Omega, Z_Y = 10 + j2\Omega, Z_\Delta = 12 + j3\Omega$$

Example 4.3: Per-phase analysis

Impedance in each phase:

$$Z = Z_L + \left(Z_Y \parallel \frac{Z_\Delta}{3}\right) = 0.2 + j1 + \frac{(10+j2)(4+j1)}{(10+j2)+(4+j1)} = 3.486\angle 28.68°\,\Omega$$

$$\overline{I_L} = \frac{\overline{V_{LN}}}{Z} = \frac{\frac{480}{\sqrt{3}}\angle 0°}{3.486\angle 28.68°} = 79.49\angle -28.68°\ \text{A}$$

$$S_{1\phi} = \overline{V_{LN}}\,\overline{I_L^*} = 19.326 + j10.572\ \text{kVA}$$

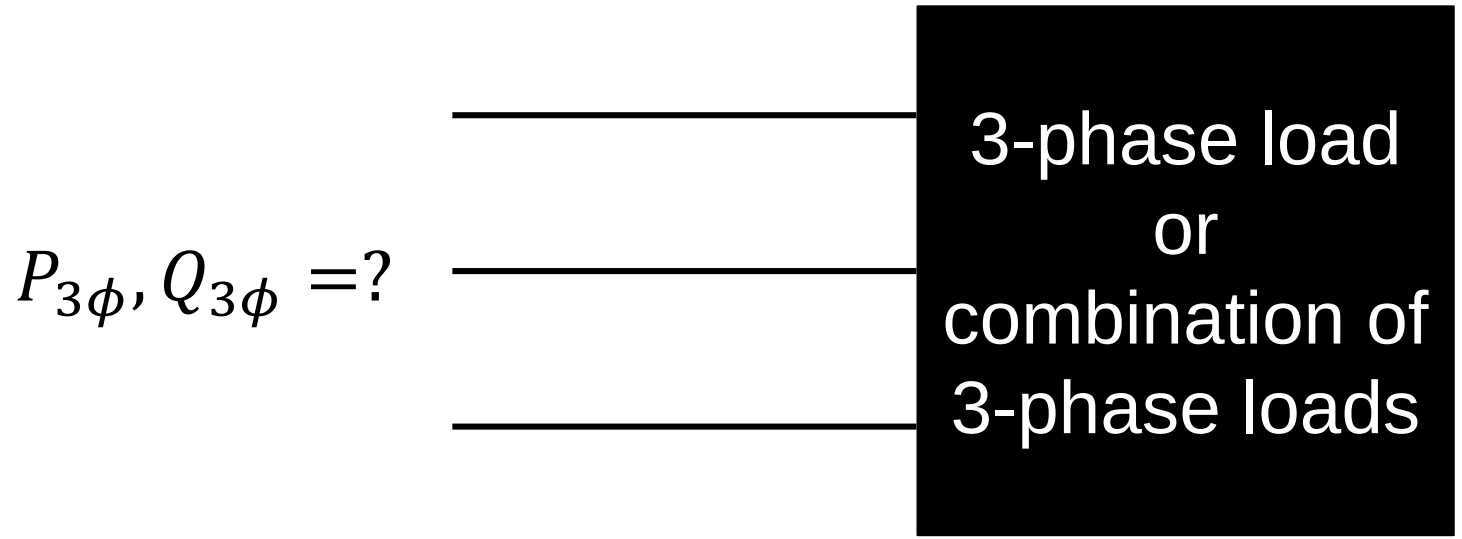
$$P_{3\phi} = 3 \times P_{1\phi} = 57.979\ \text{kW}$$

$$Q_{3\phi} = 3 \times Q_{1\phi} = 31.717\ \text{kvar}$$

Powers in balanced three-phase systems

- Power in each phase:
  - $P_{1\phi} = V_{phase}\, I_{phase} \cos\theta$
  - $Q_{1\phi} = V_{phase}\, I_{phase} \sin\theta$
- Power in balanced three-phase system
  - $P_{3\phi} = 3\, V_{phase}\, I_{phase} \cos\theta$
  - $Q_{3\phi} = 3\, V_{phase}\, I_{phase} \sin\theta$
- V_{phase} and I_{phase} depends on whether the load is Y- or Δ-connected
- Need uniform way of calculating the powers
- Express the powers in terms of line-to-line voltage and line current
- Those quantities are easily accessible

Powers in balanced three-phase systems



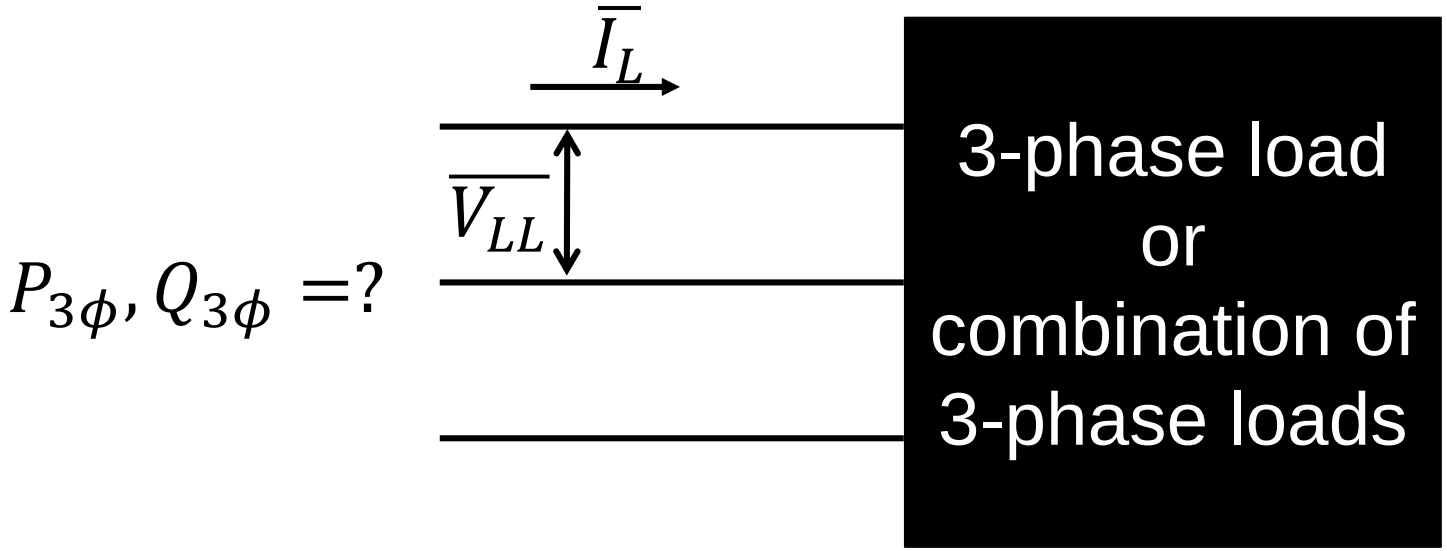
$$P_{3\phi} = 3\, V_{phase}\, I_{phase}\, cos\theta$$

$$Q_{3\phi} = 3 V_{phase}\, I_{phase}\, sin\theta$$

$$V_{phase} = ?$$

$$I_{phase} = ?$$

Powers in balanced three-phase systems



Y-connected load → $V_{phase} = \frac{1}{\sqrt{3}} V_{LL}$ and $I_{phase} = I_L$

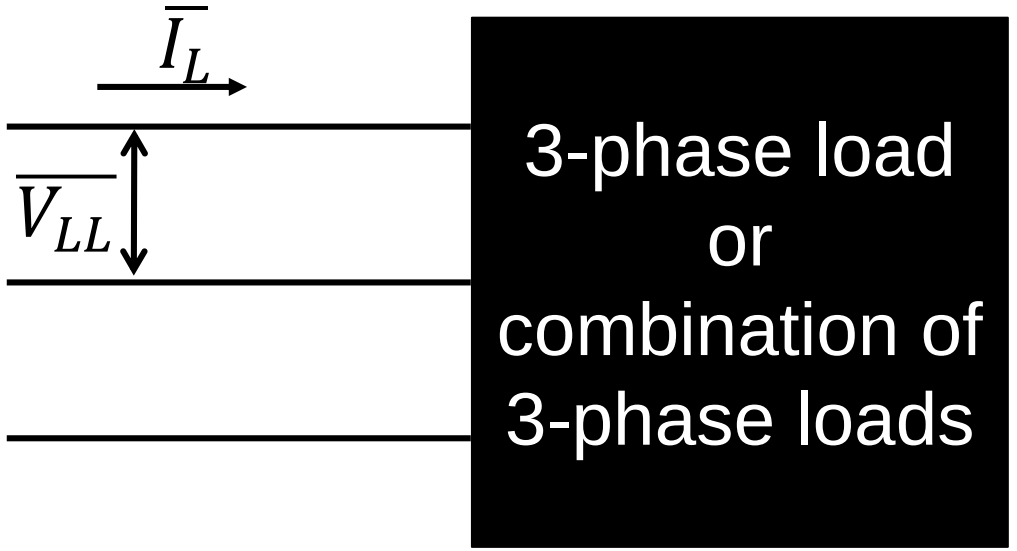
Δ-connected load → $V_{phase} = V_{LL}$ and $I_{phase} = \frac{1}{\sqrt{3}} I_L$

$$P_{3\phi} = 3\, V_{phase}\, I_{phase}\, cos\theta \quad \Rightarrow \quad P_{3\phi} = \sqrt{3}\, V_{LL}\, I_L\, cos\theta$$

$$Q_{3\phi} = 3 V_{phase}\, I_{phase}\, sin\theta \quad \Rightarrow \quad Q_{3\phi} = \sqrt{3}\, V_{LL}\, I_L\, sin\theta$$

Powers in balanced three-phase systems

$P_{3\phi}, Q_{3\phi} = ?$



$$P_{3\phi} = \sqrt{3}\, V_{LL}\, I_L\, cos\theta$$

$$Q_{3\phi} = \sqrt{3}\, V_{LL}\, I_L\, sin\theta$$

$$S_{3\phi} = \sqrt{{P_{3\phi}}^2 + {Q_{3\phi}}^2} = \sqrt{3}\, V_{LL}\, I_L$$

Example 4.4: Power in a three-phase load

Calculate the line current drawn by a 10 kVA unity power factor three-phase load supplied from a 480V source

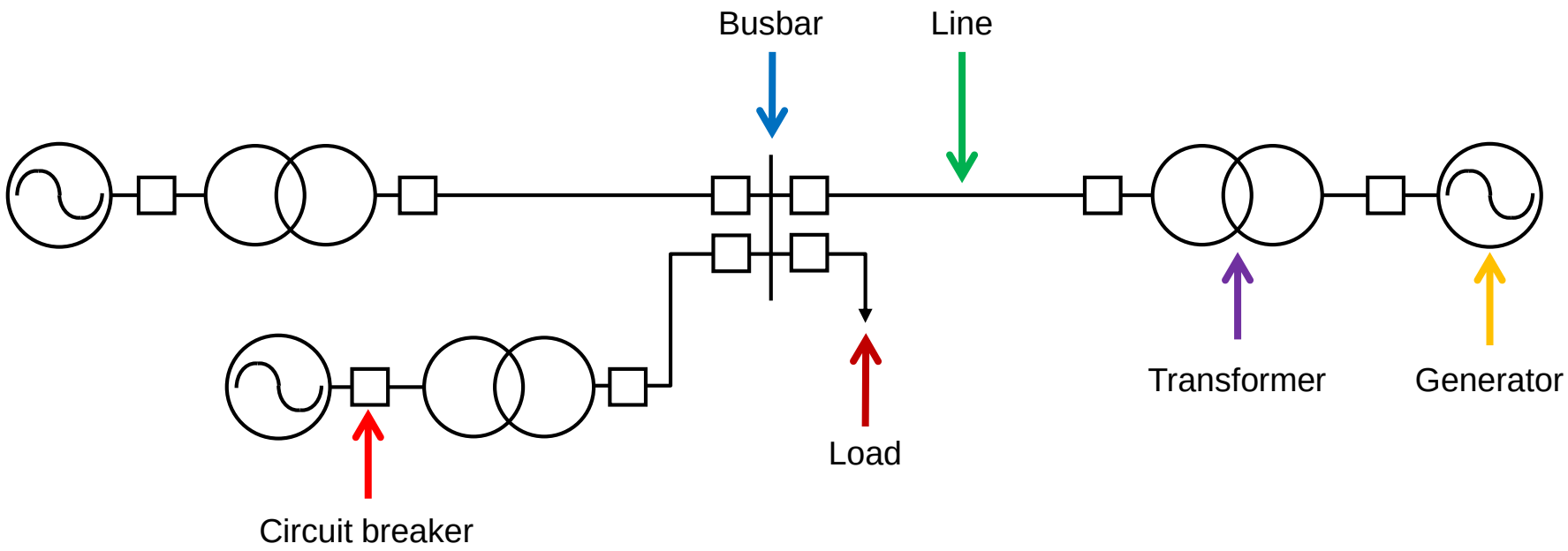
$$I_L = \frac{S}{\sqrt{3}\, V_{LL}} = \frac{10000}{\sqrt{3}\ \times 480} = 12.03\ \text{A}$$

Single-phase representation

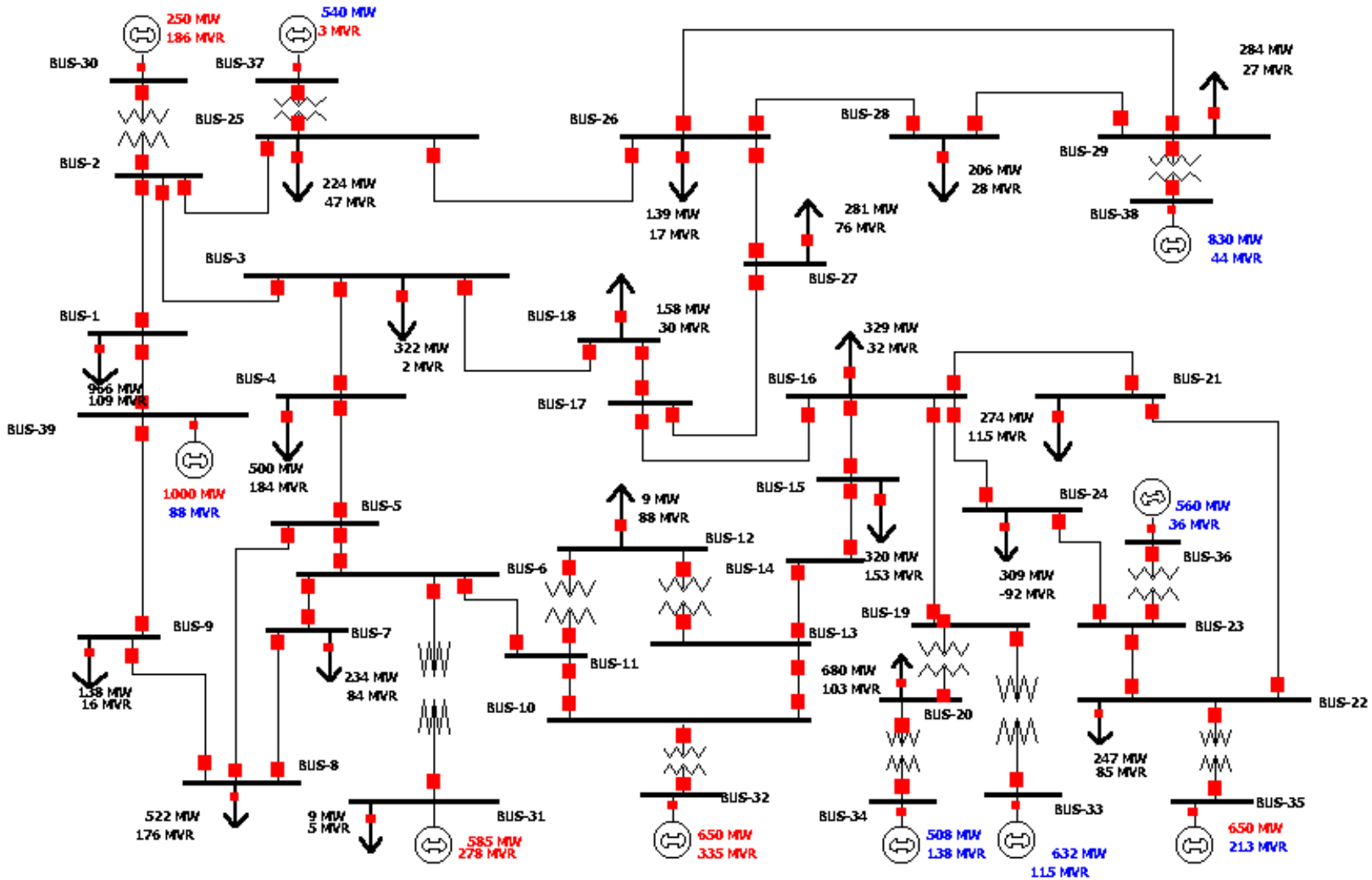
- Balanced system:
  - Balanced source
  - Balanced load
  - Balanced connection between source and load
- Symmetrical behavior in each phase
  - “Three-phase” symmetry
- Need to study only one phase

One-line diagram

- Graphical extension of single-phase representation
- Every item represents a three-phase component
- Balanced three-phase system → no need for a “return conductor”



One-line diagram for a 39-bus system



Per-unit system

- Normalization of the various quantities

$$per\ unit\ quantity = \frac{quantity\ in\ SI\ units}{base\ value\ of\ quantity\ in\ SI\ units}$$

- Simplifies the analysis
  - Removes most $\sqrt{3}$ factors
  - No difference between single-phase and three-phase

Example 4.5: Per-unit system

- Voltage at a particular node is $127\angle 30°$ kV
- Nominal or base value for voltages at that node is 132 kV
- Per unit value of this voltage:

$$V^{pu} = \frac{127\angle 30° kV}{132\ kV} = 0.962\ \angle 30°\ pu$$

- Per unit quantities are dimensionless
- Normalization affects only magnitude of complex numbers

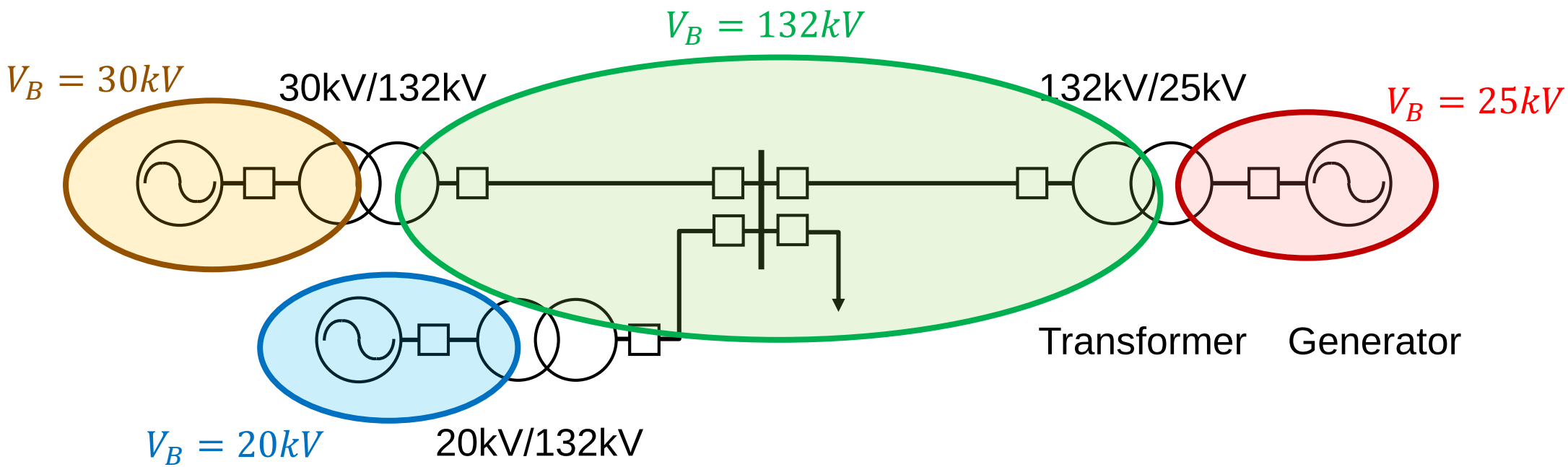
Choosing the base quantities – **Single-phase** system

1. Choose a common base value S_B for the powers
   - Applies to the active, reactive, complex and apparent powers
   - Applies to the entire system
   - Arbitrary but typically a company standard applies e.g., 100 MVA
   - Will be specified in homework assignments and exams

2. Choose the nominal or rated voltage for each area of the system as the base V_B for all voltages in that area
   - Different areas will have different base voltages
   - If a voltage is close to its nominal value, its per unit voltage is close to 1.0 p.u.

Example

$$S_B = 100\ MVA$$



Choosing the other base quantities – **Single-phase** system

Principle:

Relation between pu quantities = relation between SI quantities

3. Base current I_B
   - In SI quantities we have: $S = V\ I$
   - In p.u. we want: $S^{pu} = V^{pu}\ I^{pu}$
   - Which can be written: $\frac{S}{S_B} = \frac{V}{V_B}\ \frac{I}{I_B}$
   - Hence: $I_B = \frac{S_B}{V_B}$

Choosing the other base quantities – **Single-phase** system

4. Base impedance Z_B
   - Ohm's law in SI quantities: $V = I\ Z$
   - In pu we want: $V^{pu} = I^{pu}\ Z^{pu}$
   - Which implies: $Z_B = \frac{V_B}{I_B} = \frac{V_B^2}{S_B}$
   - $R_B = X_B = Z_B$
5. Base admittance: $Y_B = \frac{1}{Z_B}$

Summary of base quantities for **single-phase** systems

Quantity	Symbol	Rule
Apparent power	S_B	Arbitrary choice of one value for the entire system
Active power	P_B	$P_B = S_B$
Reactive power	Q_B	$Q_B = S_B$
Voltage	V_B	Nominal voltage at each location
Current	I_B	$I_B = \frac{S_B}{V_B}$
Impedance	Z_B	$Z_B = \frac{V_B^2}{S_B}$
Admittance	Y_B	$Y_B = \frac{1}{Z_B}$

Choosing the base quantities – **three-phase** system

Like single-phase but with a few twists

1. Choose a common base value S_B for the powers

2. Choose the nominal or rated voltage for each area of the system as the base V_B for all voltages in that area

   - We also want: $V_{LL}^{pu} = V_{LN}^{pu}$
   - Which means: $\frac{V_{LL}}{V_{B,LL}} = \frac{V_{LN}}{V_{B,LN}}$
   - Since $V_{LN} = \frac{1}{\sqrt{3}} V_{LL}$, we must have: $V_{B,LN} = \frac{V_{B,LL}}{\sqrt{3}}$

Choosing the base quantities – **three-phase** system

3. Base current I_B
   - In pu we want: $S^{pu} = V^{pu}\ I^{pu}$
   - Which can be written: $\frac{S}{S_B} = \frac{V}{V_B}\ \frac{I}{I_B}$
   - In SI quantities we have in a three-phase system: $S = \sqrt{3}\ V_{LL}\ I_L$
   - Which gives $\frac{\sqrt{3}\ V_{LL}\ I_L}{S_B} = \frac{V_{LL}}{V_{B,LL}}\ \frac{I_L}{I_B}$
   - Hence: $I_B = \frac{S_B}{\sqrt{3}\ V_{B,LL}}$

Choosing the base quantities – **three-phase** system

4. Base impedance Z_B

- Balanced Y-connected load: $V_{LN} = I_L \, Z_\phi$
- We would like to write: $V_{LN}^{pu} = I_L^{pu} \, Z_\phi^{pu}$
- Which implies: $\frac{V_{LN}}{V_{B,LN}} = \frac{I_L}{I_B} \frac{Z_\phi}{Z_B} \Rightarrow \frac{1}{V_{B,LN}} = \frac{1}{I_B} \frac{1}{Z_B} \Longrightarrow \frac{\sqrt{3}}{V_{B,LL}} = \frac{\sqrt{3} \, V_{B,LL}}{S_B} \frac{1}{Z_B}$
- Hence: $Z_B = \frac{V_{B,LN}}{I_B} = \frac{V_{B,LL}^2}{S_B}$

Summary of base quantities – **Three-phase** systems

Quantity	Symbol	Rule
Apparent power	S_B	Arbitrary choice of one value for the entire system
Active power	P_B	$P_B = S_B$
Reactive power	Q_B	$Q_B = S_B$
Line-to-line voltage	$V_{B,LL}$	Nominal line-to-line voltage at each location
Line-to-neutral voltage	$V_{B,LN}$	$V_{B,LN} = \frac{V_{B,LL}}{\sqrt{3}}$
Current	I_B	$I_B = \frac{S_B}{\sqrt{3}\, V_{B,ll}}$
Impedance	Z_B	$Z_B = \frac{V_{B,LL}^2}{S_B}$
Admittance	Y_B	$Y_B = \frac{1}{Z_B}$

Example 4.6: Base quantities

A small three-phase power system consists of two areas connected by a transformer. The nominal voltages is 132 kV in area Blue and 69 kV in area Red. Select base values for all quantities in both areas.

Let us choose $S_B = 100\ MVA$. This choice applies to both areas.

$V_B^{Blue} = 132\ kV$

$V_B^{Red} = 69\ kV$

$I_B^{Blue} = \frac{S_B}{\sqrt{3}\, V_B^{Blue}} = 437.39\ A$

$I_B^{Red} = \frac{S_B}{\sqrt{3}\, V_B^{Red}} = 836.74\ A$

$Z_B^{Blue} = \frac{\left(V_B^{Blue}\right)^2}{S_B} = 174.24\ \Omega$

$Z_B^{Red} = \frac{\left(V_B^{Red}\right)^2}{S_B} = 47.61\ \Omega$

Example 4.7: Per unit currents

A load in area Blue of Example 4.6 draws 0.2 pu of apparent power at a voltage of 0.97 pu.

Calculate the line current drawn by this load in Amps.

$$S^{pu} = V^{pu}\, I^{pu} = 0.2$$

Since $V^{pu} = 0.97$, we have:

$$I^{pu} = \frac{0.2}{0.97} = 0.206$$

$$I = I^{pu} \times I_B^{Blue} = 0.206 \times 437.39 = 90.184\ A$$

Converting per unit impedances

- Manufacturers give the values of the impedances in per unit calculated on the basis of the rating of the equipment
- Must be converted to the per unit system used for system studies
- Impedance values in Ohms don't depend on the per unit basis:

$$Z\ (\Omega) = Z_{new}^{pu} Z_{B,new} = Z_{old}^{pu} Z_{B,old} \quad \Rightarrow \quad Z_{new}^{pu} = Z_{old}^{pu} \frac{Z_{B,old}}{Z_{B,new}}$$

$$Z_{new}^{pu} = Z_{old}^{pu} \frac{\frac{V_{B,old}^2}{S_{B,old}}}{\frac{V_{B,new}^2}{S_{B,new}}} = Z_{old}^{pu} \frac{S_{B,new}}{S_{B,old}} \frac{V_{B,old}^2}{V_{B,new}^2}$$

Usually $V_{B,new} = V_{B,old}$

$$Z_{new}^{pu} = Z_{old}^{pu} \frac{S_{B,new}}{S_{B,old}}$$

Example 4.8: Base impedance conversion

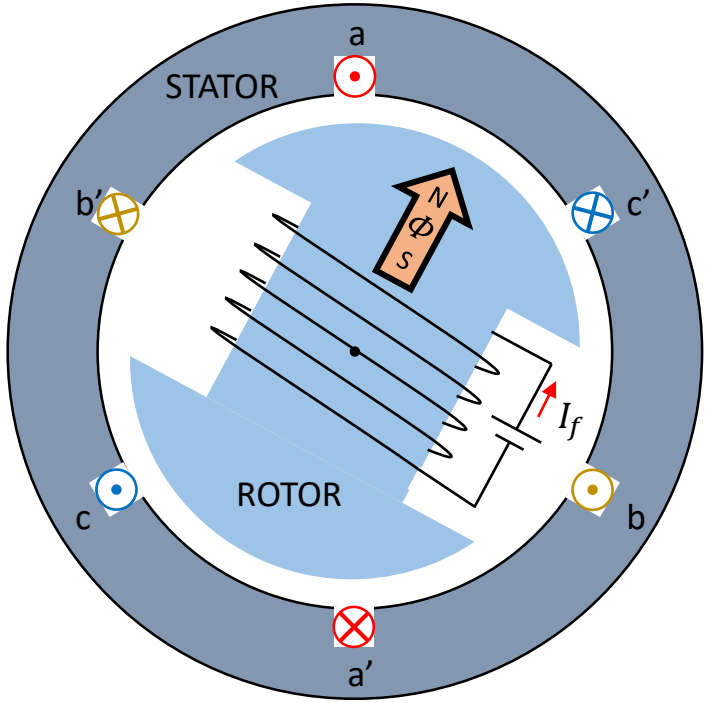
According to the data provided by its manufacturer, a device rated at 50 MVA has an impedance of 0.3 pu. This device must be studied as part of a system for which the base value for power is $S_B = 100\ MVA$. What value of impedance should be used for these system studies?

$$Z_{new}^{pu} = Z_{old}^{pu} \frac{S_{B,new}}{S_{B,old}} = 0.3 \times \frac{100}{50} = 0.6\ p.u.$$

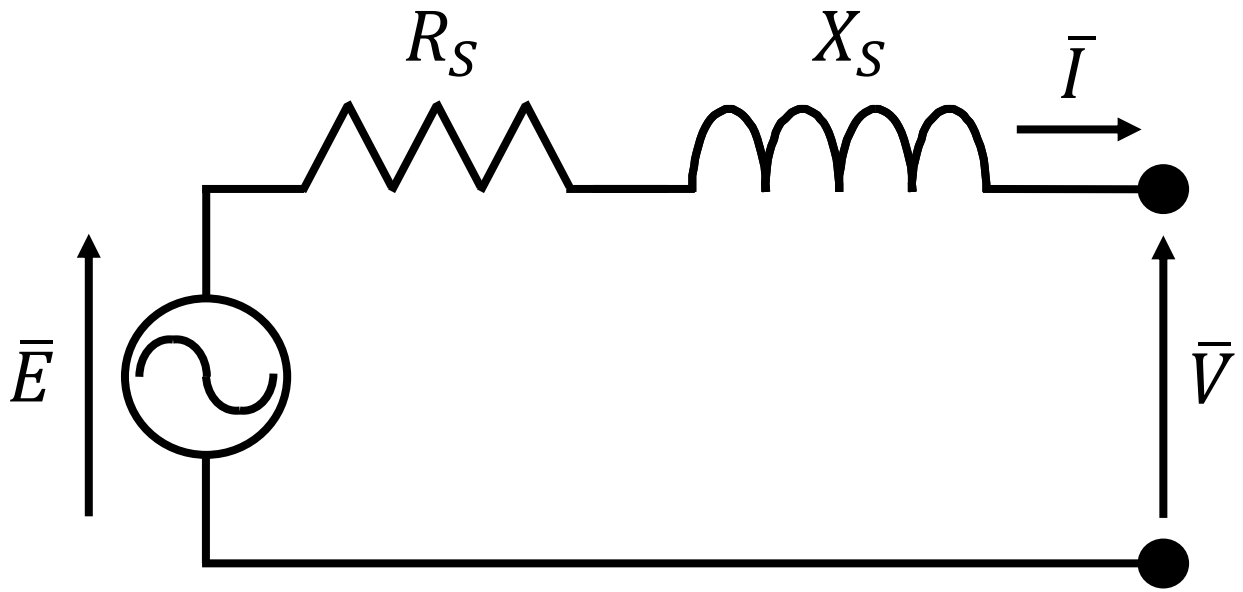
Synchronous generator model

- Rotor flux Φ induces three-phase e.m.f. $\bar{E}$ in the stator windings
- Φ is a function of I_f
- $\rightarrow |\bar{E}| = E = f(I_f)$
- The stator windings have a resistance R_S and an inductive reactance X_S
- The voltage at the terminal of the generator is:

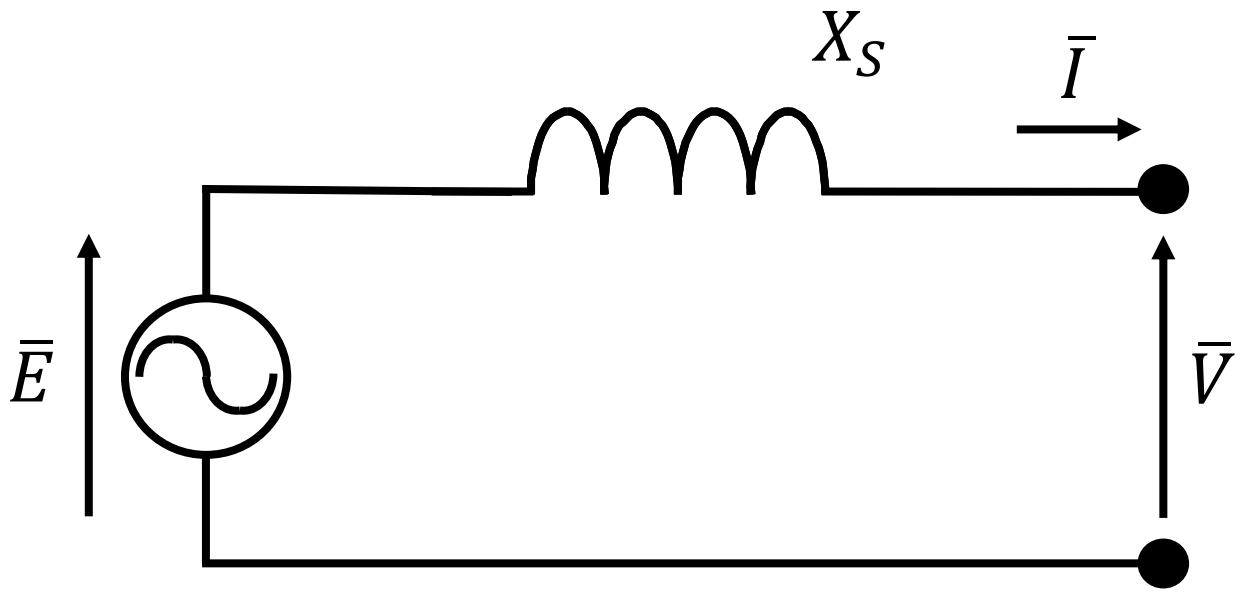
$$\bar{V} = \bar{E} - (R_S + jX_S)\bar{I}$$



Single-phase model of a synchronous generator



Single-phase model of a synchronous generator

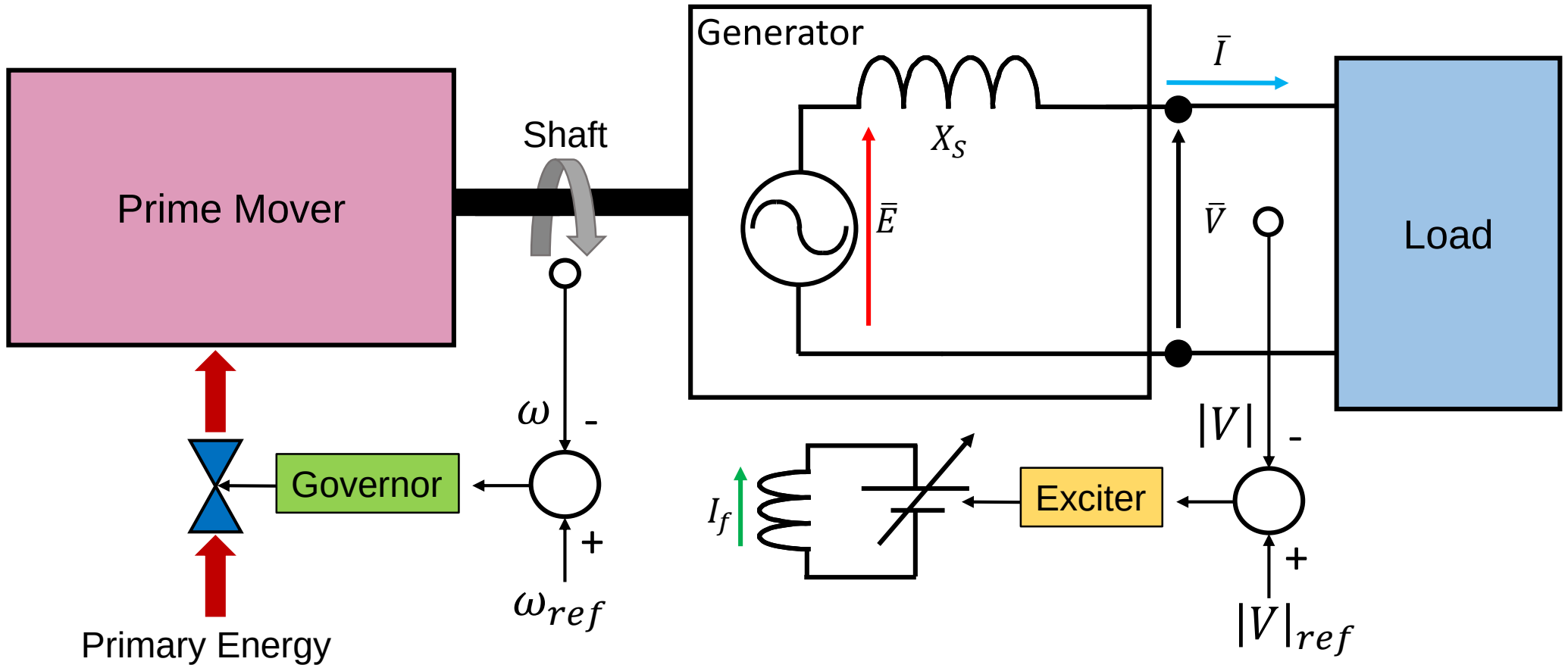


Usually, $X_S \gg R_S \Longrightarrow \bar{V} = \bar{E} - jX_S\bar{I}$

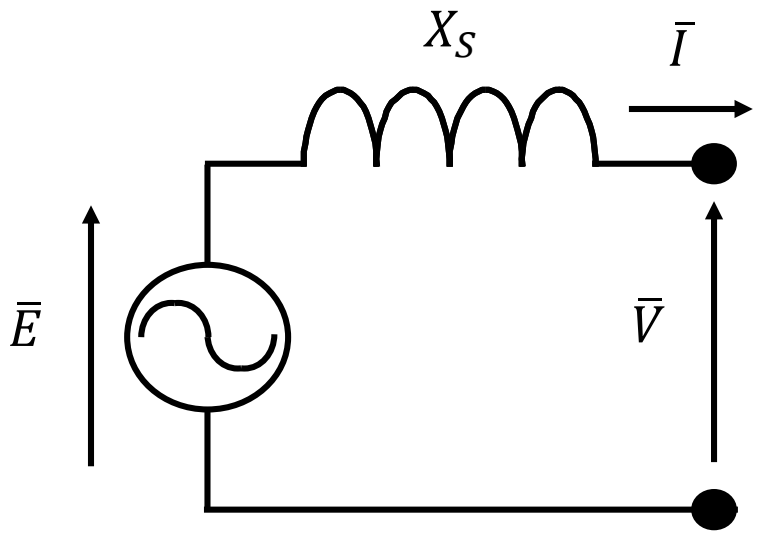
Control of an isolated generator

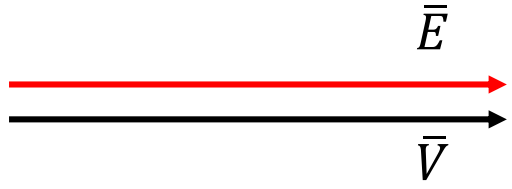


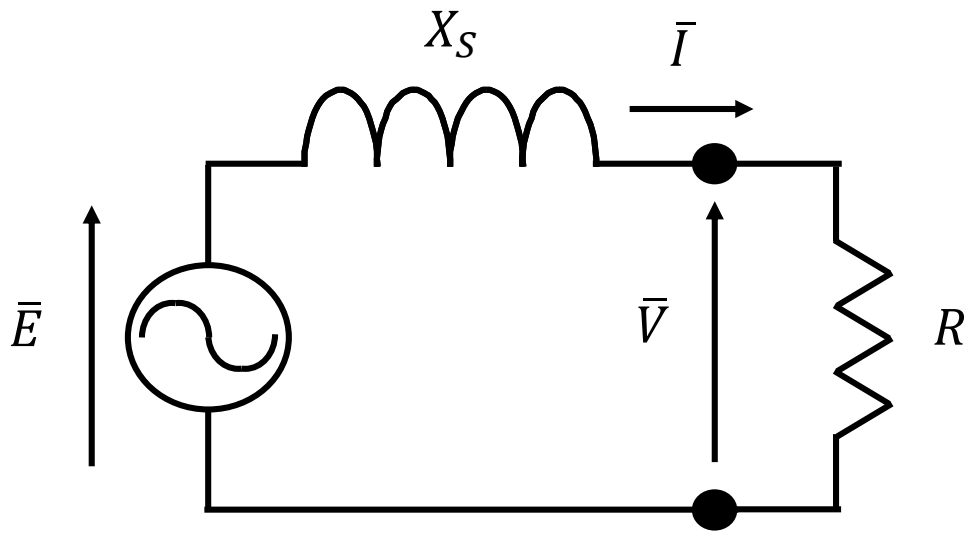
Control loops of an isolated generator

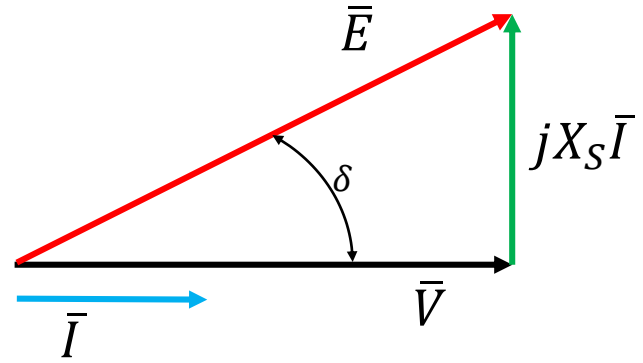


Operation of an isolated generator





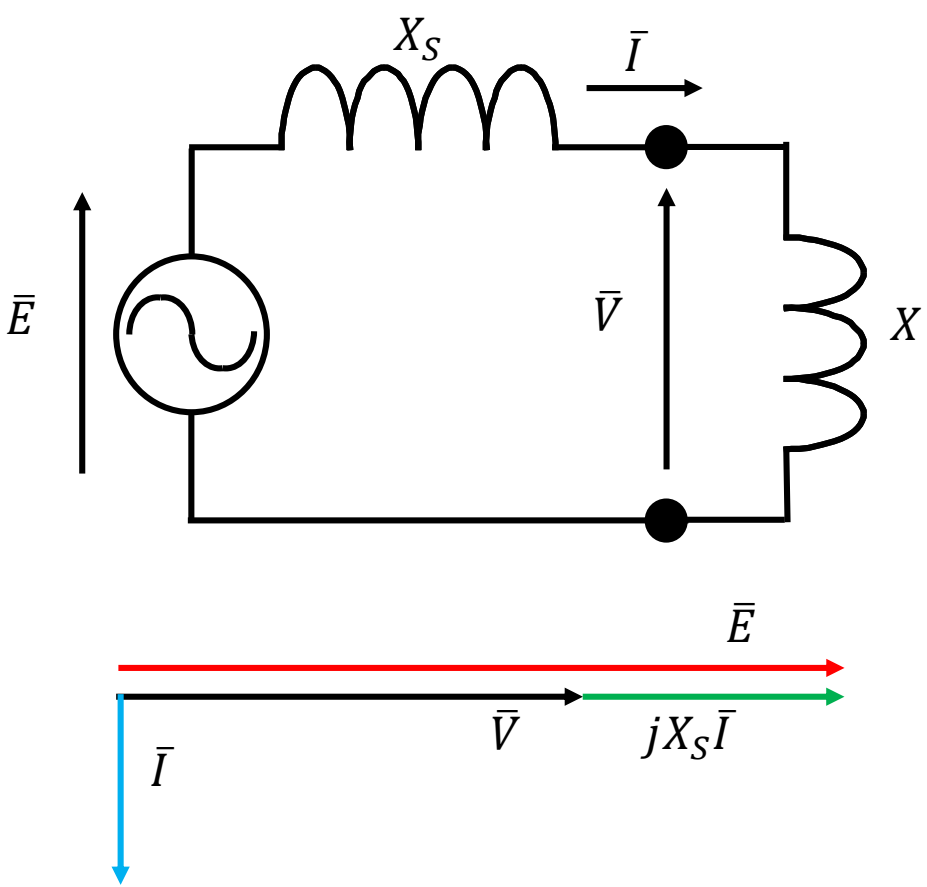




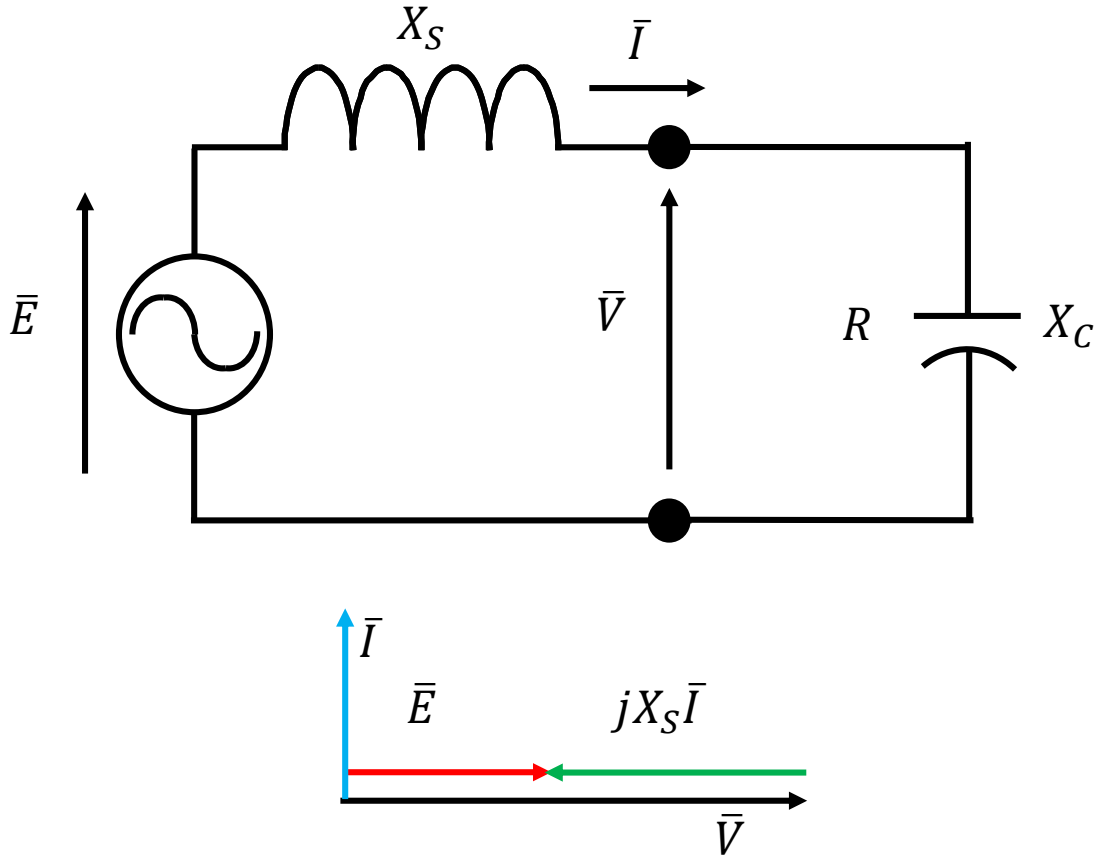
The governor loop keeps the frequency constant

→ mechanical power from prime mover follows the demand for active power

Operation of an isolated generator

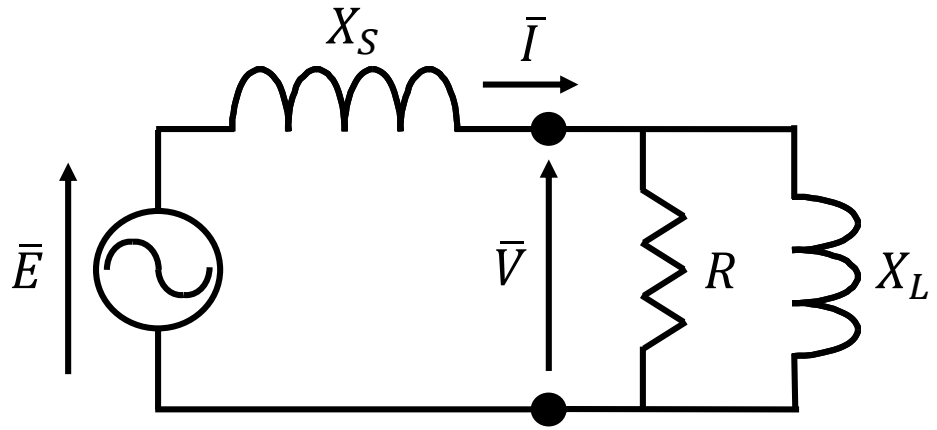


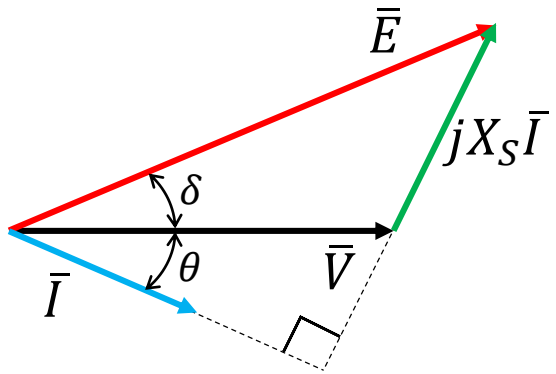
Generator supplies reactive power



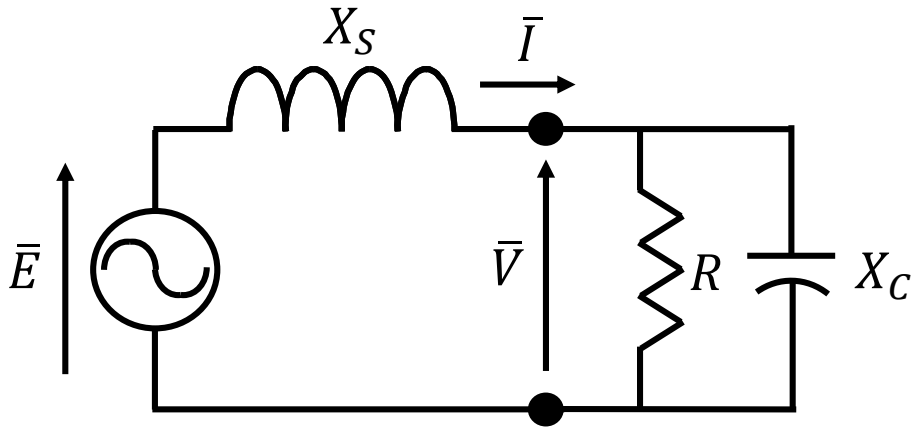
Generator absorbs reactive power

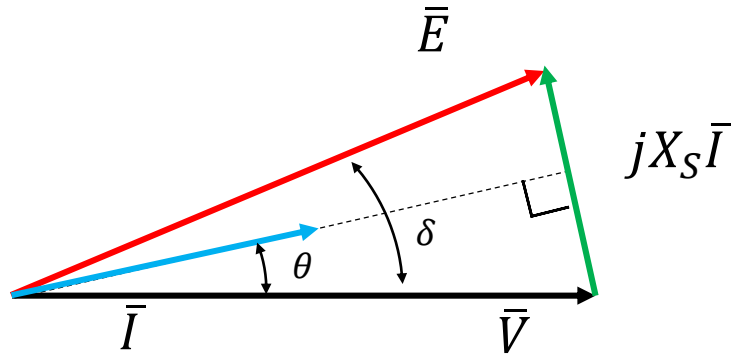
Operation of an isolated generator





Generator supplies active and reactive power



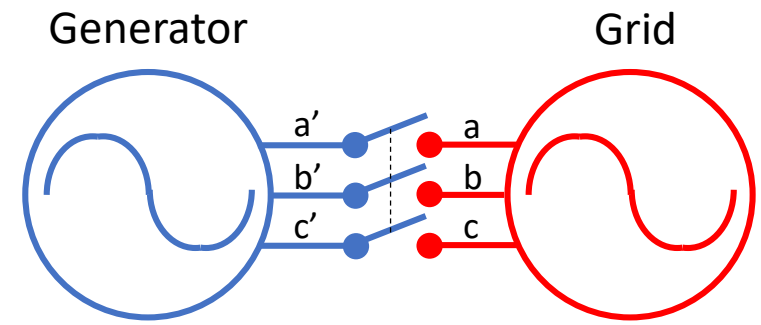


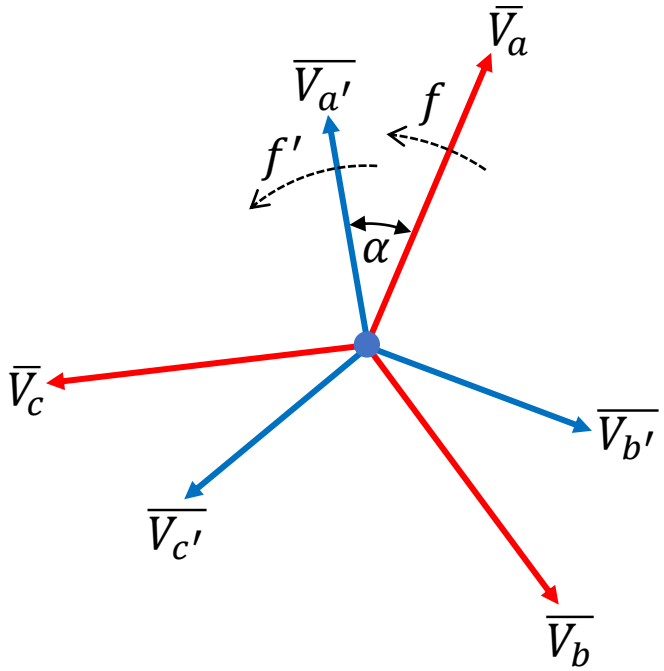
Generator supplies active and absorbs reactive power

θ: power factor angle

δ: power angle

Synchronizing a generator to the grid



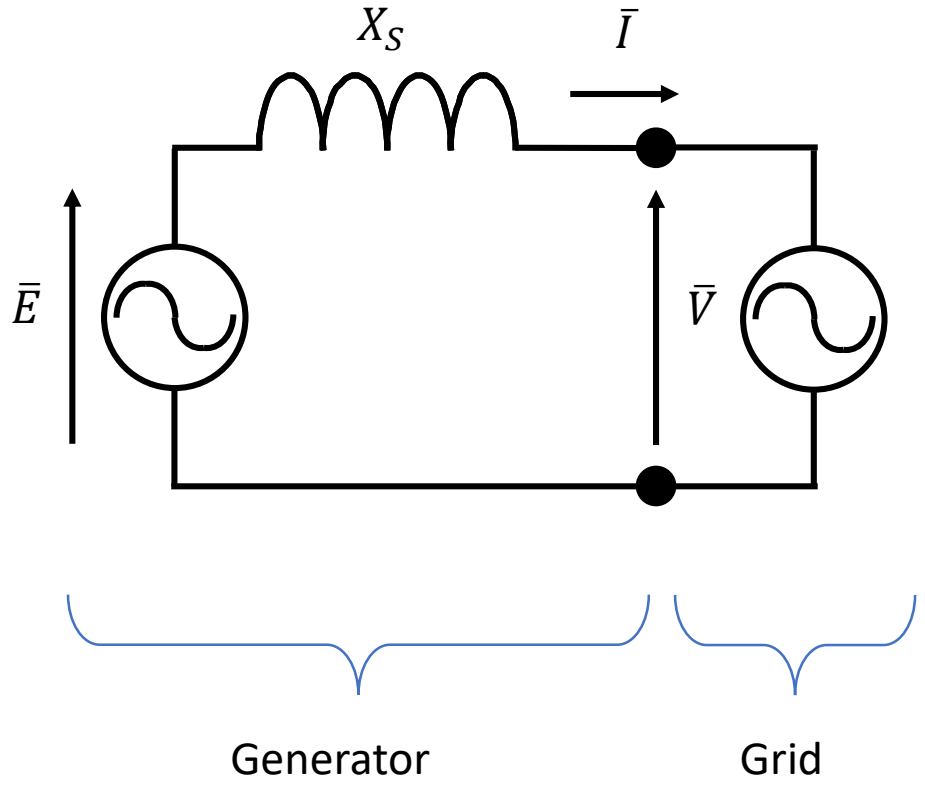


Before closing the switch:

- Same phase sequence
- Same frequency
- Same voltage magnitude
- Minimal phase angle difference (small α)

Operating a small generator synchronized to the grid

- Assume that the rating of the generator is small compared to the rest of the grid
- The grid then sets
  - The frequency (60 Hz)
  - The voltage at the terminal of the generator ($\bar{V}$)
- Model the grid as an ideal voltage source
- We will consider more sophisticated grid models later



Operating a small generator synchronized to the grid

$$\bar{I} = \frac{\bar{E} - \bar{V}}{jX_S}$$

$$\bar{S} = \bar{V}\,\bar{I}^* = \bar{V}\,\frac{\bar{E}^* - \bar{V}^*}{-jX_S}$$

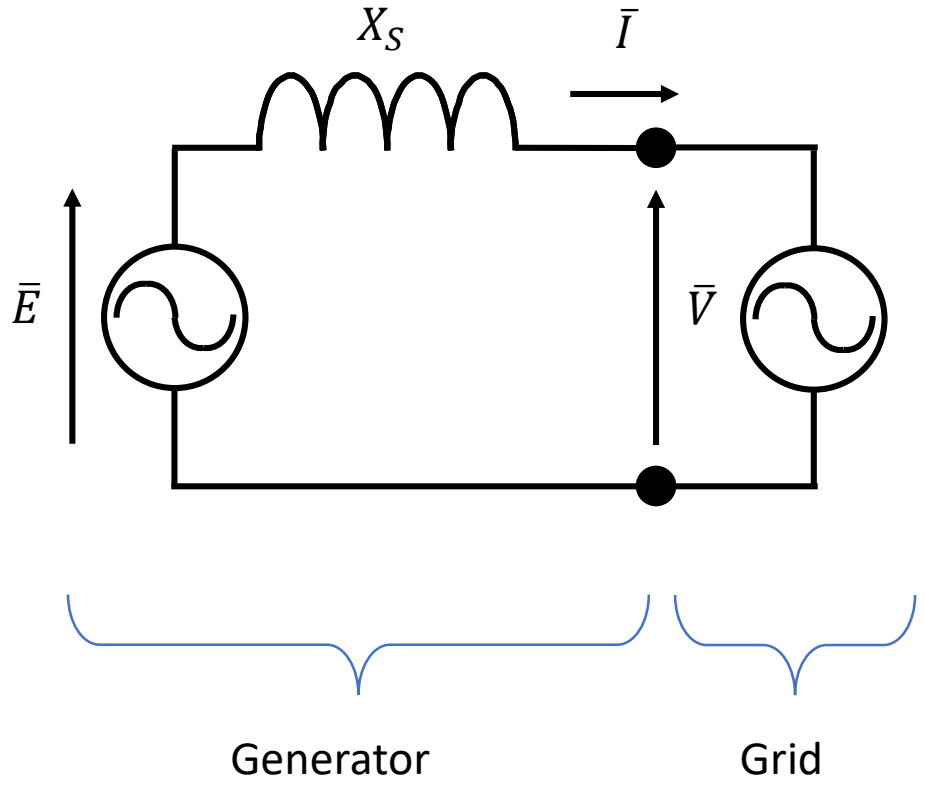
$$\bar{S} = \frac{V(E\cos\delta - jE\sin\delta) - V^2}{-jX_S}$$

$$\bar{S} = P + jQ = \frac{EV\sin\delta}{X_S} + j\frac{VE\cos\delta - V^2}{X_S}$$

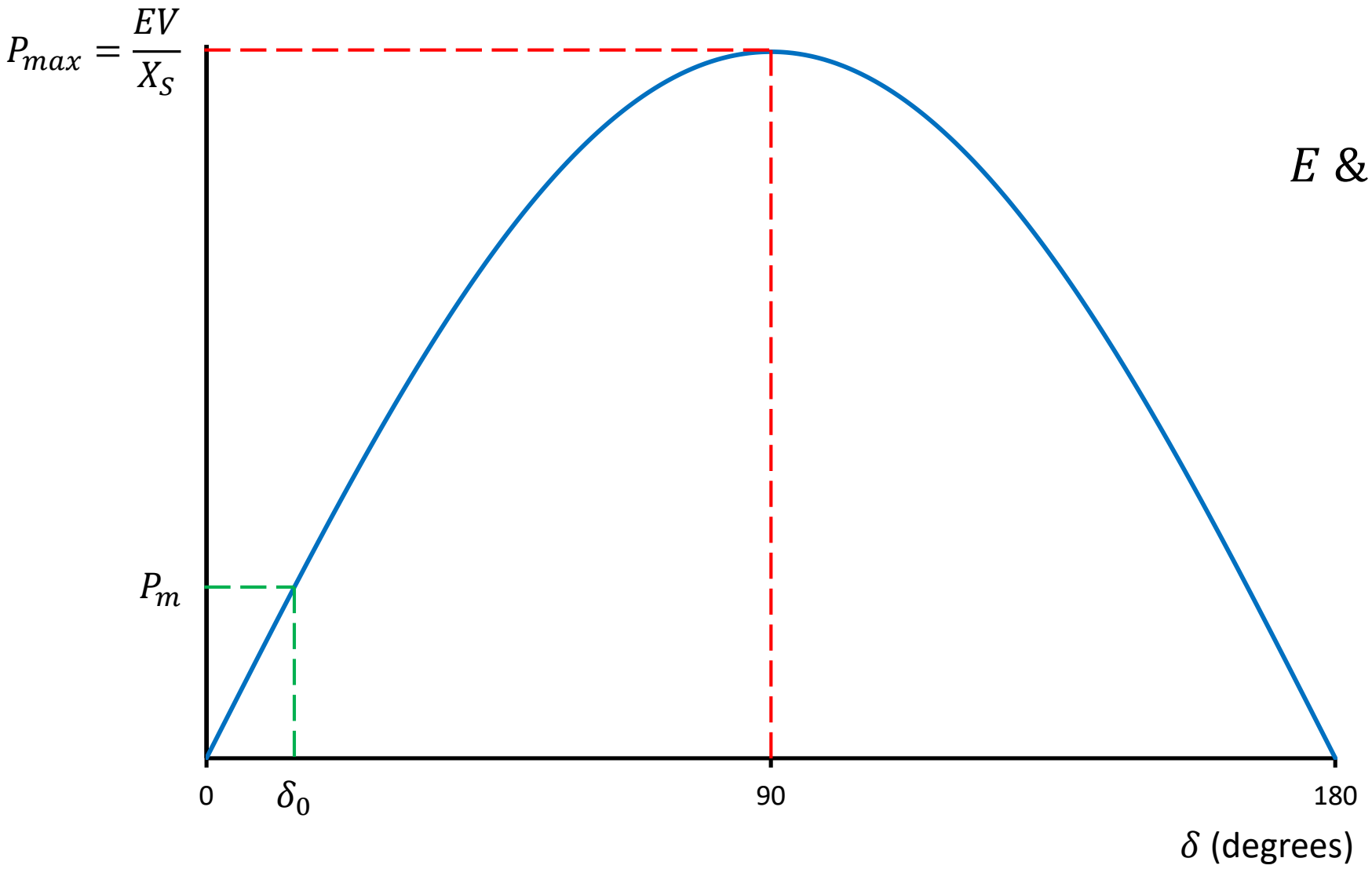
$$P = \frac{EV}{X_S}\sin\delta$$

$$Q = \frac{EV\cos\delta - V^2}{X_S}$$

$$\bar{V} = V\angle 0° = V$$
$$\bar{E} = E\angle\delta = E\cos\delta + jE\sin\delta$$



Power angle curve



$$P = \frac{EV}{X_S} \sin \delta$$

$$E \,\&\, V \; constant \Longrightarrow P = f(\delta)$$

$$P_m = P = \frac{EV}{X_S} \sin \delta_0$$

Mechanical power supplied by the prime mover

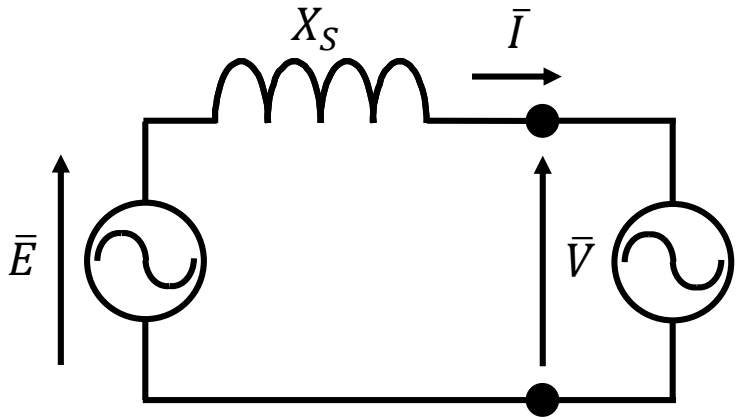
Reactive power

$$Q = \frac{EV\cos\delta_0 - V^2}{X_S}$$

δ_0 is usually small → $\cos\delta_0 \approx 1$ → $Q \sim (E - V)$

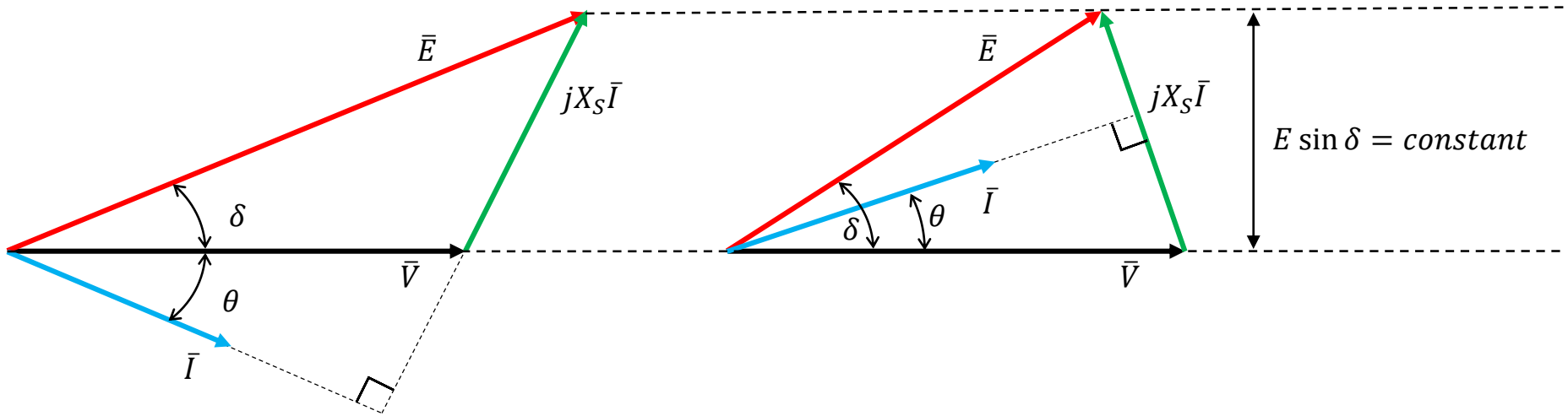
The difference in magnitude between E and V determines the reactive power injection

Reactive power



$$P = P_m = \frac{EV}{X_S}\sin\delta = constant$$

$$V = constant$$



Larger $|E|$
- Injects a lagging current
- Injects reactive power in the grid
- "Overexcited"

Smaller $|E|$
- Injects a leading current
- Absorbs reactive power from the grid
- Same active power injection
- "Underexcited"

Example 4.9: Steady state generator operation

The prime mover of a generator provides 0.25 pu of power. Its synchronous reactance has a value of 0.8 pu and it operates at its nominal voltage. The value of the field current is such that the magnitude of the internal emf is 1.15 pu. Calculate the power angle, the reactive power produced by this generator and the power factor.

$$P_m = 0.25, V = 1.0, E = 1.15, X_S = 0.8$$

$$P = P_m = \frac{EV}{X_S}\sin\delta \rightarrow \sin\delta = \frac{0.25 \times 0.8}{1.0 \times 1.15} \rightarrow \delta = 10.01°$$

$$Q = \frac{[1.15 \times 1.0 \times \cos(10.01°)] - 1.0^2}{0.8} = 0.166\ pu$$

$$pf = \cos\theta = \cos[tan^{-1}(\frac{Q}{P})]\cos[tan^{-1}(0.662)] = 0.834$$

$$I = \frac{S}{V} = \frac{\sqrt{P^2+Q^2}}{V} = \frac{0.3}{1.0} = 0.3\ pu$$

$$\bar{I} = 0.3\angle -33.5°\ pu$$

The current lags the voltage because this generator supplies reactive power.

Example 4.10: Effect of the field current

Repeat the calculation of the previous example for the case where the value of the field current is such that the magnitude of the internal emf is 0.95 pu.

$P_m = 0.25, V = 1.0, E = 0.95, X_S = 0.8$

$P = P_m = \frac{EV}{X_S}\sin\delta \rightarrow \sin\delta = \frac{0.25 \times 0.8}{1.0 \times 0.95} \rightarrow \delta = 12.15°$

$Q = \frac{[0.95 \times 1.0 \times \cos(12.15°)] - 1.0^2}{0.8} = -0.089 \text{ pu}$

$pf = \cos\theta = \cos[tan^{-1}(\frac{Q}{P})]\cos[tan^{-1}(0.356)] = 0.942$

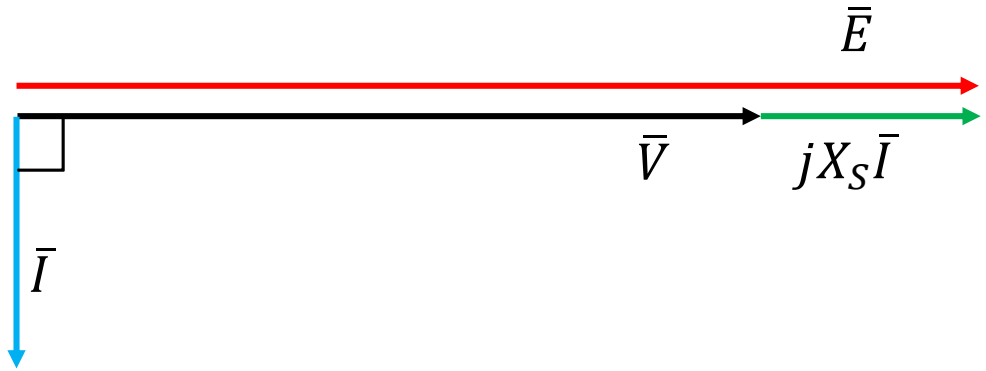
$I = \frac{S}{V} = \frac{\sqrt{P^2+Q^2}}{V} = \frac{0.265}{1.0} = 0.265\ pu$

$\bar{I} = 0.265\angle 19.6°\ pu$

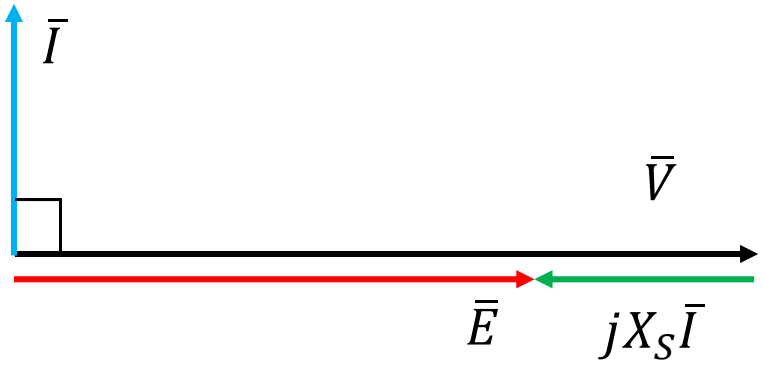
The current leads the voltage because the generator absorbs reactive power.

Synchronous condenser

- Synchronous generator not driven by a prime mover
- Losses provided by the grid
- Produces or absorbs only reactive power



Injects reactive power in the grid



Absorbs reactive power from the grid

Generator limits

- Maximum active power: $P \leq P_{max}$
  - Determined by prime mover
- Minimum active power: $P \geq P_{min}$
  - Flame stability in the boiler
- Stator current limit: $I \leq I_{max}$
  - Avoid damaging conductor insulation due to overheating
- Field current limit: $I_f \leq I_{f,max}$
  - Avoid damaging conductor insulation due to overheating

How to express as a function of P and Q?

$I \leq I_{max}$ as a function of P and Q

$$S = \sqrt{P^2 + Q^2} = VI$$

$$I = \frac{\sqrt{P^2+Q^2}}{V} \leq I_{max}$$

$$\frac{P^2+Q^2}{V^2} \leq I_{max}^2$$

$P^2 + Q^2 \leq V^2 I_{max}^2$ → Circle in the P, Q plane

$x^2 + y^2 = r^2$

$I_f \leq I_{f,max}$ as a function of P and Q

$$I_f \leq I_{f,max} \Longrightarrow E \leq E_{max}$$

$$P = \frac{EV}{X_S}\sin\delta \quad \rightarrow \quad P = \frac{E_{max}\,V}{X_S}\sin\delta$$

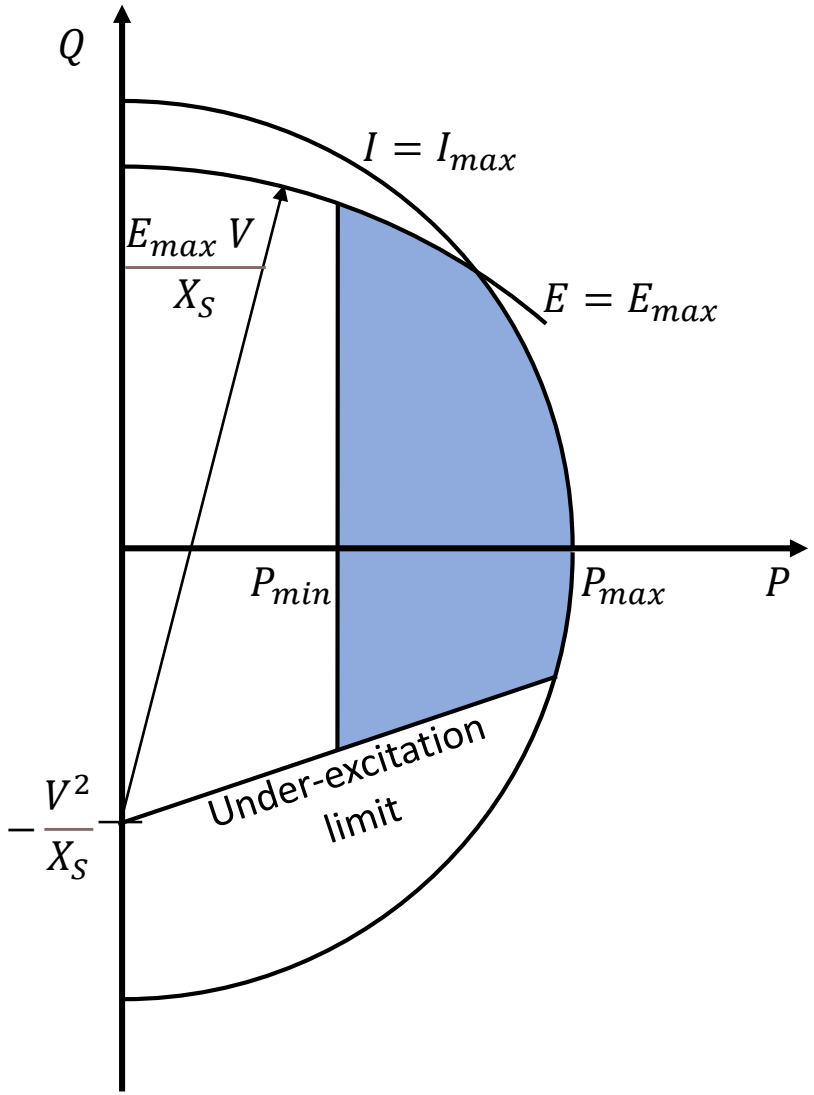
$$Q = \frac{EV\cos\delta \; - V^2}{X_S} \quad \rightarrow \; Q + \frac{V^2}{X_S} = \frac{E_{max}\,V}{X_S}\cos\delta$$

$$P^2 + \left(Q + \frac{V^2}{X_S}\right)^2 = \left(\frac{E_{max}\,V}{X_S}\right)^2$$

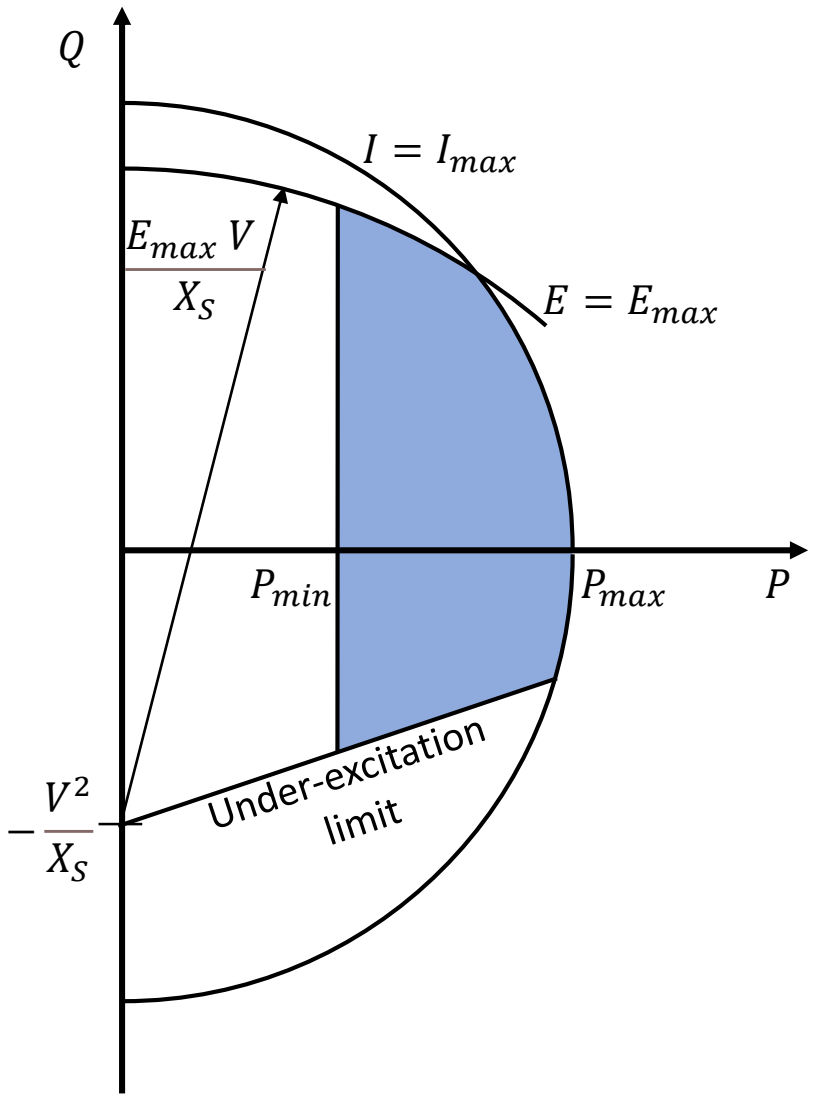
Circle or radius $\frac{E_{max}\,V}{X_S}$

centered at $\left(0, -\frac{V^2}{X_S}\right)$ in the $P - Q$ plane

Loading capability diagram of a generator



Simplified generator limits



$$P_{min} \leq P \leq P_{max}$$

$$Q_{min} \leq \mathrm{Q} \leq Q_{max}$$

