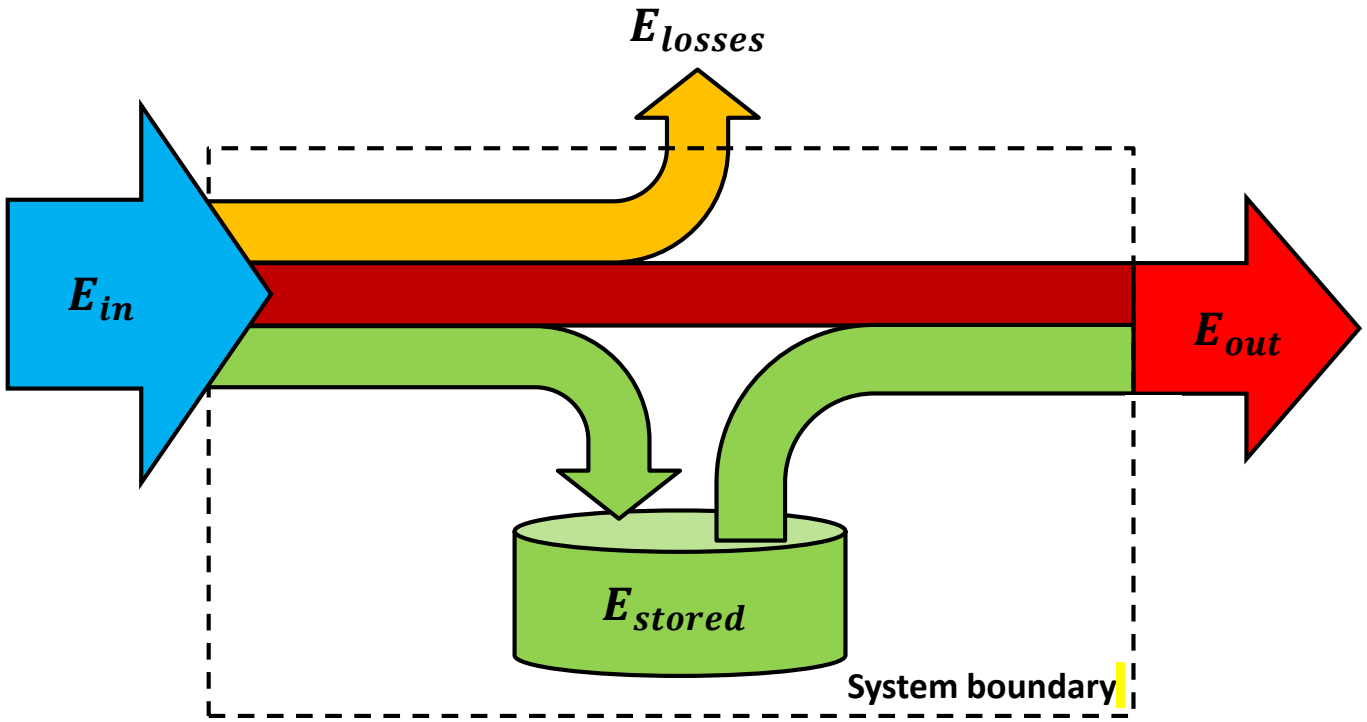
Conservation of energy



$$E_{in} = E_{out} + E_{stored} + E_{losses}$$

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Power balance

$$E_{in} = E_{out} + E_{stored} + E_{losses}$$

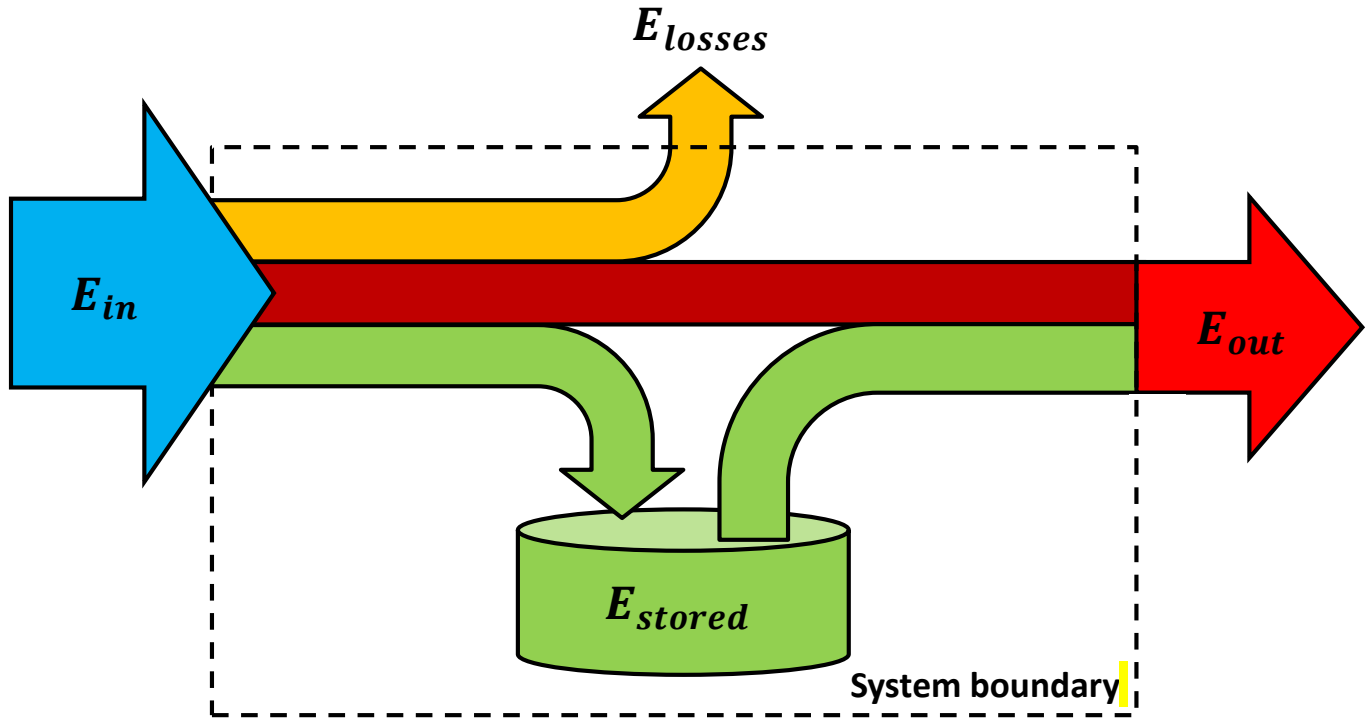
$$\frac{dE_{in}}{dt} = \frac{dE_{out}}{dt} + \frac{dE_{stored}}{dt} + \frac{dE_{losses}}{dt}$$

$$P_{in} = P_{out} + P_{stored} + P_{losses}$$

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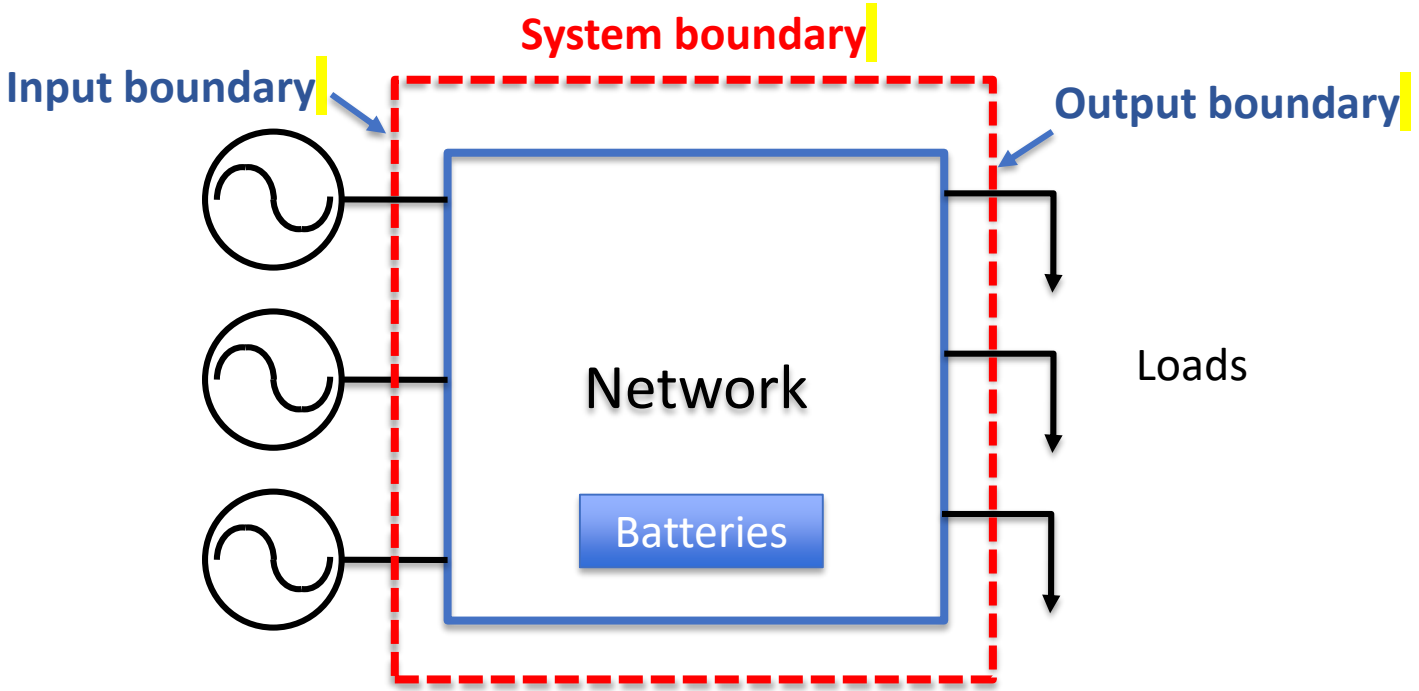
Drawing the boundaries of the system

- $P_{in} = P_{out} + P_{stored} + P_{losses}$ is always valid
- Meaning of the terms depends on how we draw the boundaries of the system
- Particularly important when considering the energy stored



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Input boundary at the terminals of the generators

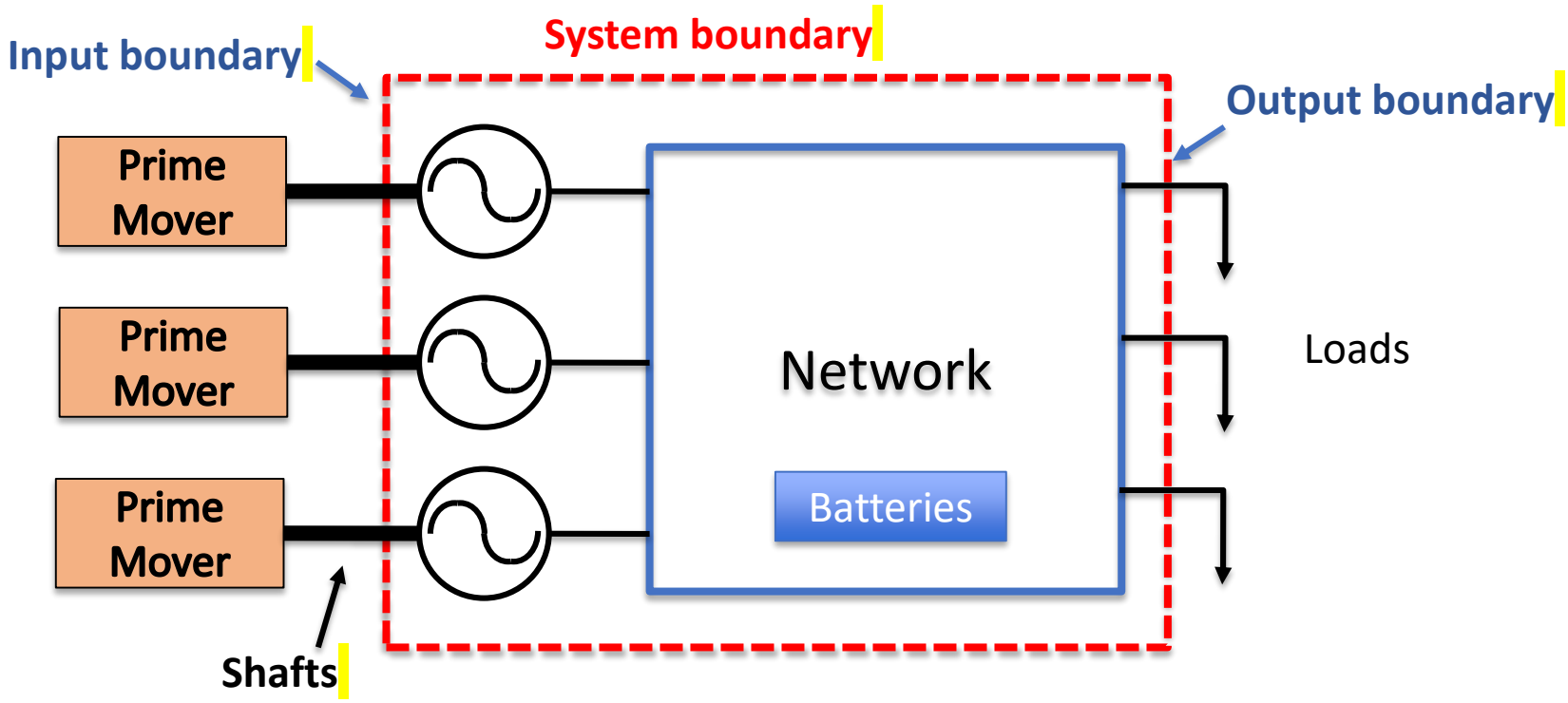


- P_{out}: electrical loads
- P_{in}: electrical power injected by these generators
- P_{losses}: losses in the network
- P_{stored}: batteries in the network (> 0: charging < 0: discharging)

(Energy stored in inductances and capacitances assumed essentially constant and negligible)

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Input boundary at the shafts of the generators



- P_{out}: electrical loads
- P_{in}: mechanical power provided by the prime mover
- P_{losses}: losses in the network + losses in the generator
- P_{stored}: batteries in the network + kinetic energy stored in the rotating generators

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Kinetic energy stored in the rotating generators



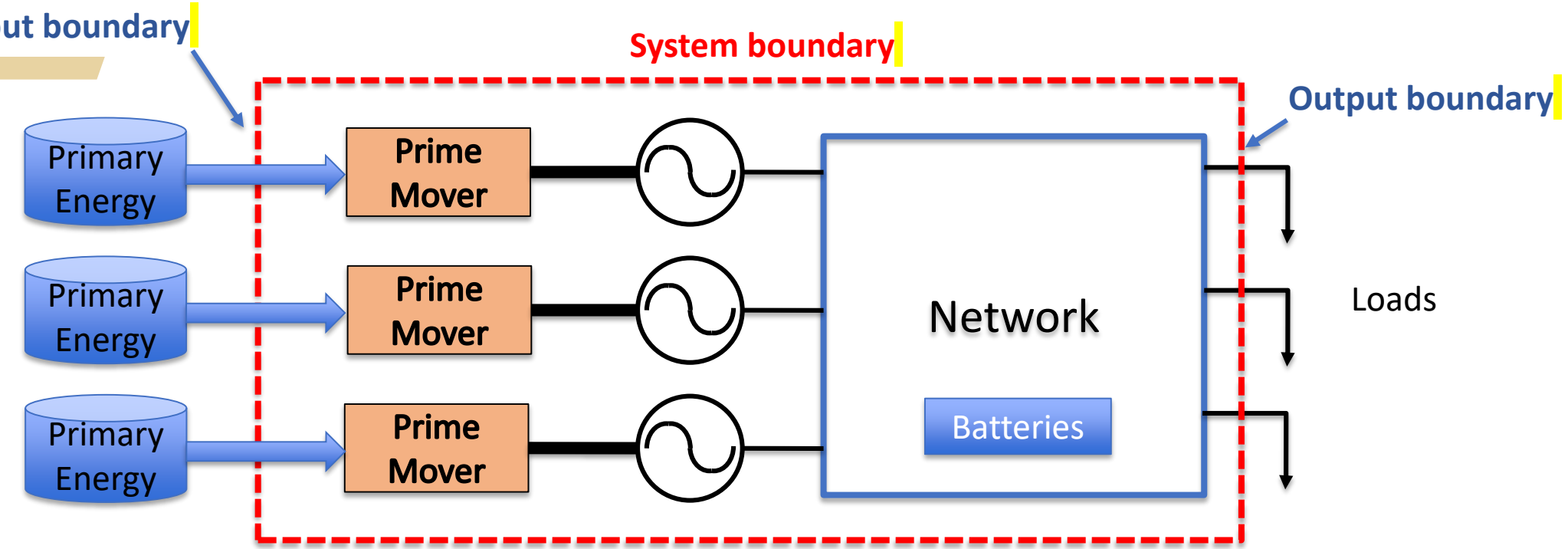
$$E_{kinetic} = \frac{1}{2} \sum_{g \in G} J_g \, \omega_{mg}^2$$

- G: set of all synchronous generators
- J_g moment of inertia of generator g
- ω_{mg}^2 square of the mechanical rotational speed of generator g
- ω_{mg} is proportional to the grid frequency
- A change in frequency results in a change in stored energy:

$$\frac{dE_{kinetic}}{dt} = \sum_{g \in G} J_g \, \omega_{mg} \, \frac{d\omega_{mg}}{dt}$$

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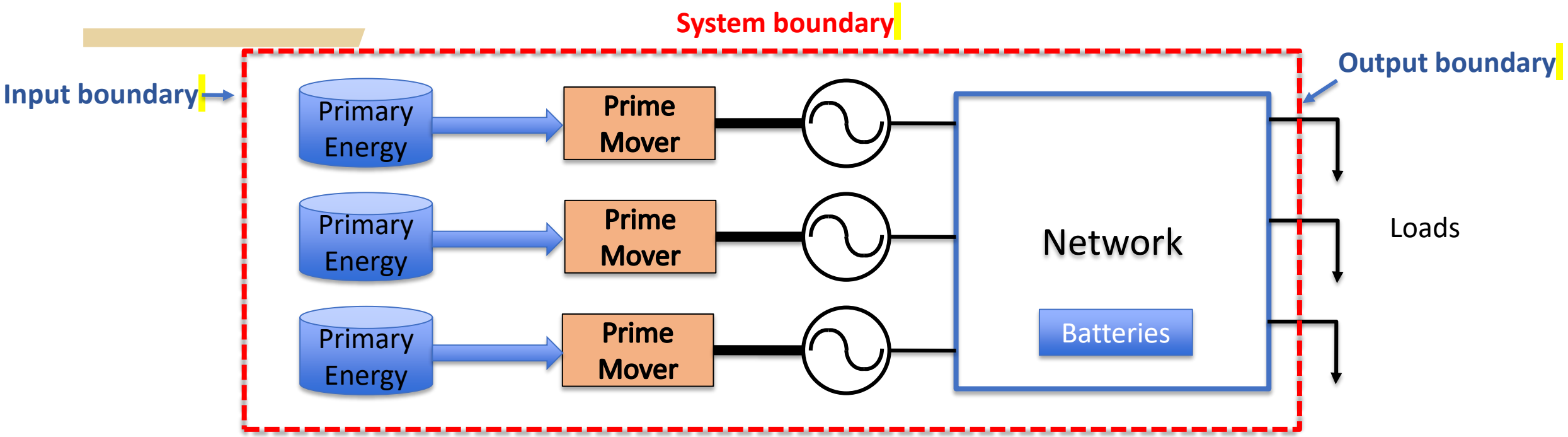
Input boundary at the primary energy input to the prime mover



- P_{out}: Electrical loads
- P_{in}: Primary energy input
- P_{losses}: losses in the network + losses in the generator + losses in the prime mover
- P_{stored}: batteries in the network + kinetic energy stored in the rotating generators + thermodynamic energy in the prime mover

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Input boundary at the storage of primary energy



- P_{losses}: losses in the network + losses in the generator + losses in the prime mover + losses in primary energy
- P_{stored}: batteries in the network + kinetic energy stored in the rotating generators + thermodynamic energy in the prime mover + primary energy storage

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Steady state

- $P_{in} = constant$
- $P_{out} = constant$
- $P_{stored} = 0$ (neither charging nor discharging)
- $P_{in} = P_{out} + P_{stored} + P_{losses} \Longrightarrow P_{in} = P_{out} + P_{losses}$

- Steady state assumption is very useful when planning the operation of the system
- Example:
  - Forecast that the load + losses tomorrow at noon will be 1000 MW
  - Must be prepared to generate 1000 MW
  - $P_{stored} = 0$

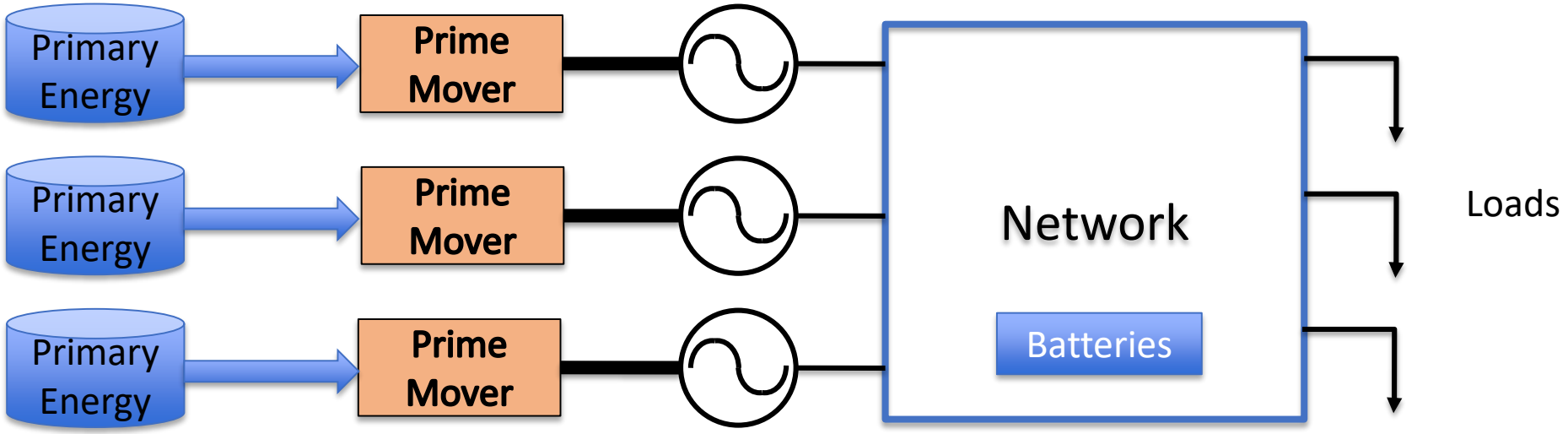
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Perturbation

- Steady state:
  - $P_{in} = P_{out} + P_{losses}$
  - $P_{stored} = 0$
- Sudden increase in load: $P_{out} \rightarrow P'_{out} > P_{out}$
  - Generators cannot increase their output instantaneously
  - We must still have: $P_{in} = P'_{out} + P_{stored} + P_{losses}$
  - Therefore, we must have $P_{stored} < 0$ until we have:$P'_{in} = P'_{out} + P_{losses}$
  - The amount of energy stored in the system decreases
- Where this stored energy comes from depends on the time frame that we consider

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Time frames



- $P_{out} \rightarrow P'_{out} > P_{out}$
- First draw power from kinetic energy stored in generator-prime movers
- Then draw power from thermodynamic energy stored in turbines
- Then increase power input to prime mover (increase P_{in})

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Single generator

- Mechanical power & torque

$$P_m = \omega_m T_m$$

- Electrical power & reaction torque

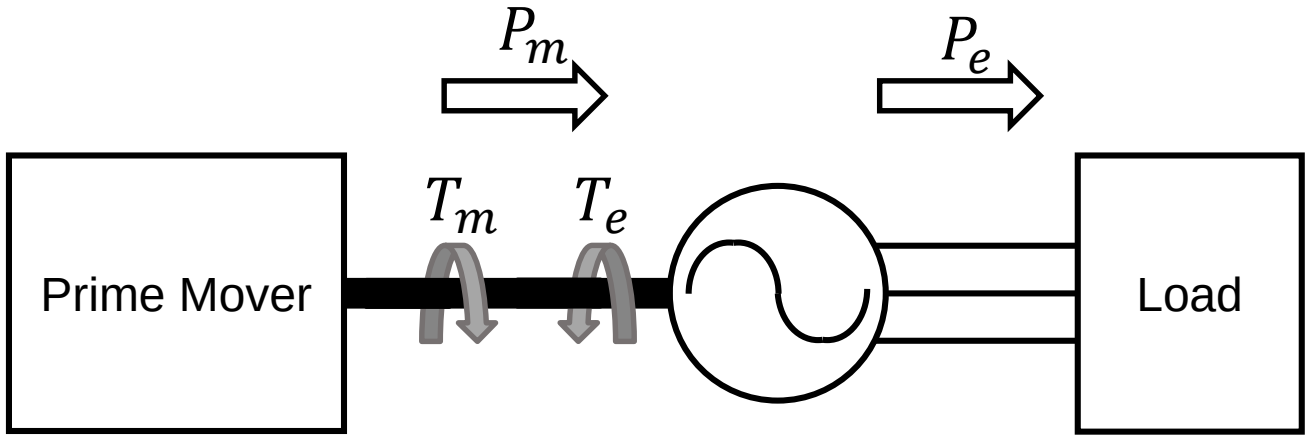
$$P_e = \omega_m T_e$$

- Newton's law of motion

$$J\frac{d\omega_m}{dt} = T_m - T_e$$

J: combined moment of inertia of the prime mover and the generator

$$J\omega_m\frac{d\omega_m}{dt} = P_m - P_e$$



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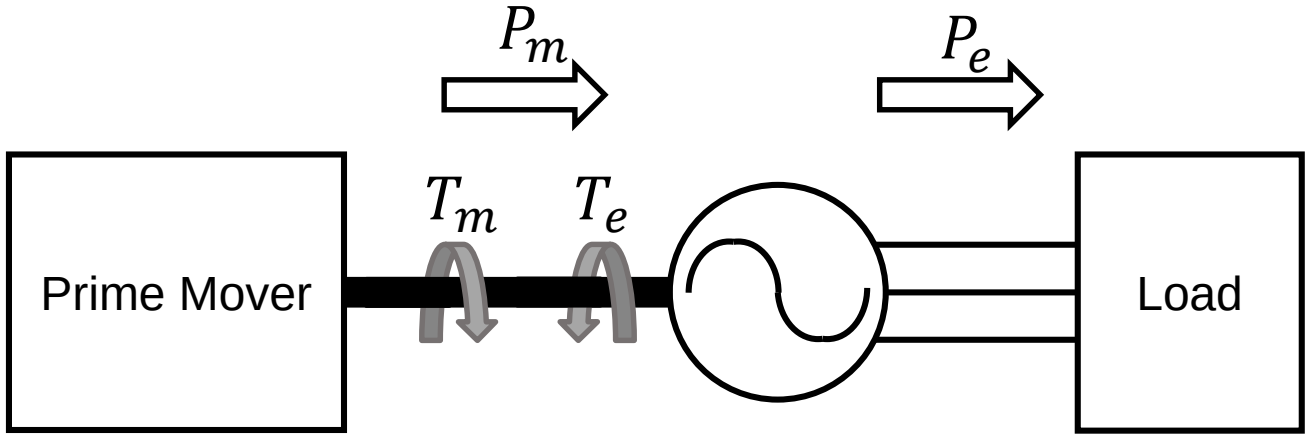
Single generator

$$J\omega_m \frac{d\omega_m}{dt} = P_m - P_e$$

- Steady state:

$$P_m = P_e \rightarrow J\omega_m \frac{d\omega_m}{dt} = 0$$

$\rightarrow \frac{d\omega_m}{dt} = 0 \rightarrow$ frequency is constant



- Load suddenly increase from $P_e \rightarrow P'_e$

$$J\omega_m \frac{d\omega_m}{dt} = P_m - P'_e < 0$$

$\rightarrow \frac{d\omega_m}{dt} < 0 \rightarrow$ frequency decreases

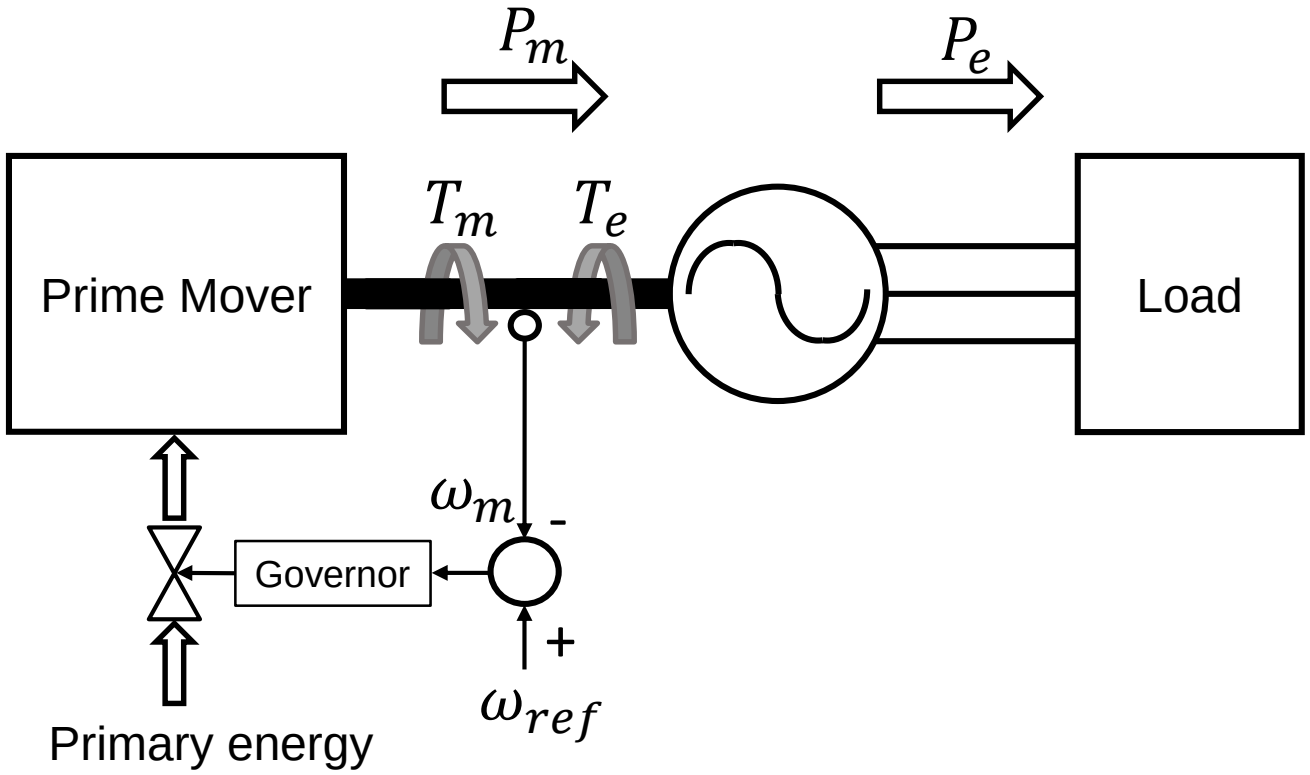
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Governor control

- Detects positive or negative difference between actual speed and reference speed
- Adjusts the amount of primary energy injected in the prime mover
- Restores the equilibrium:

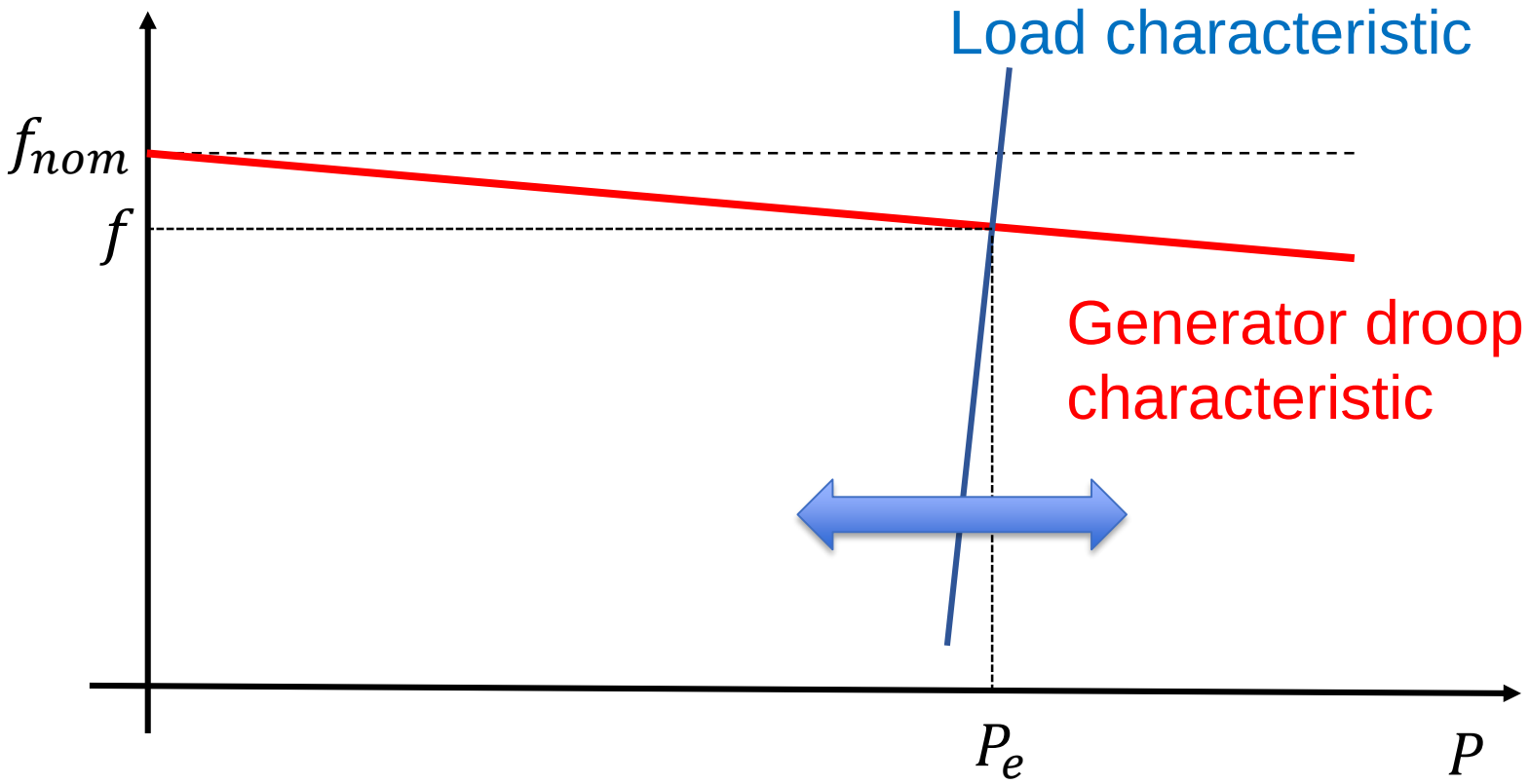
$$P'_m = P'_e \rightarrow J\omega_m \frac{d\omega_m}{dt} = 0$$

- Frequency is stabilized
- *Primary frequency control*



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Droop characteristic



- Primary frequency control does not maintain the frequency at its nominal value as the load changes

$$\frac{f}{f^{nom}} = 1 - \rho \frac{P}{P^{max}}$$

$$P = \frac{P^{max}}{\rho}\left(\frac{f^{nom} - f}{f^{nom}}\right)$$

$$0.04 \leq \rho \leq 0.09$$

- Aggregated load has a slight dependence on frequency

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Example 6.1

The governor of a 50kW, 60 Hz generator has a droop of 4%. Calculate the frequency provided by this generator when it supplies a 25kW load. Neglect the frequency dependence of the load.

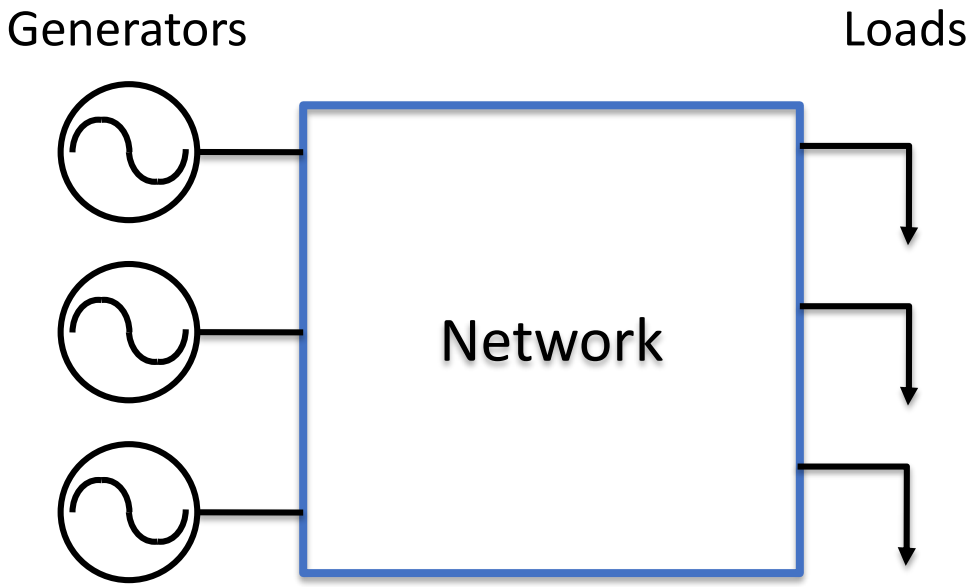
$$f = 60\left(1 - 0.04\frac{25}{50}\right) = 58.8\text{Hz}$$

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Multiple generators

In the steady state:

$$\sum_{g \in G} P_{m,g} = \sum_{g \in G} P_{e,g} = \sum_{l \in L} P_l + P_{losses}$$



G: set of generators

$P_{m,g}$: mechanical power of the prime mover of generator g

$P_{e,g}$: electrical power produced by the generator g

L: set of loads

P_l: power consumed by load l

P_{losses}: power losses in the network and the generators

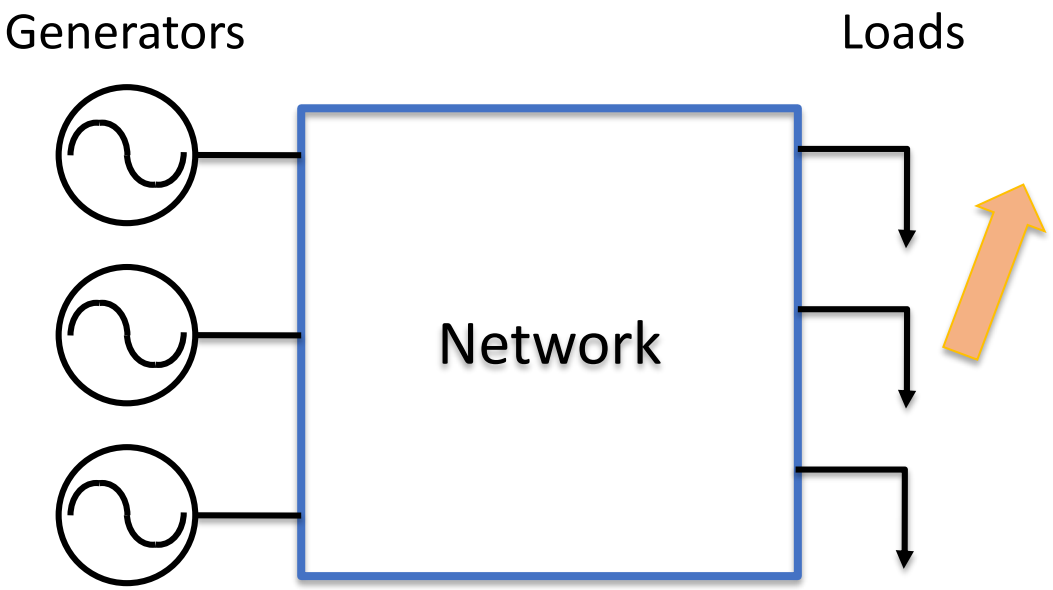
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Multiple generators

Sudden increase in load:

$$J_G \omega_m \frac{d\omega_m}{dt} = \sum_{g \in G} P_{m,g} - \sum_{l \in L} P_l + P_{losses} < 0$$

J_G: combined inertia of all the generators and their prime movers



→ All the generators decelerate together

→ System frequency decreases

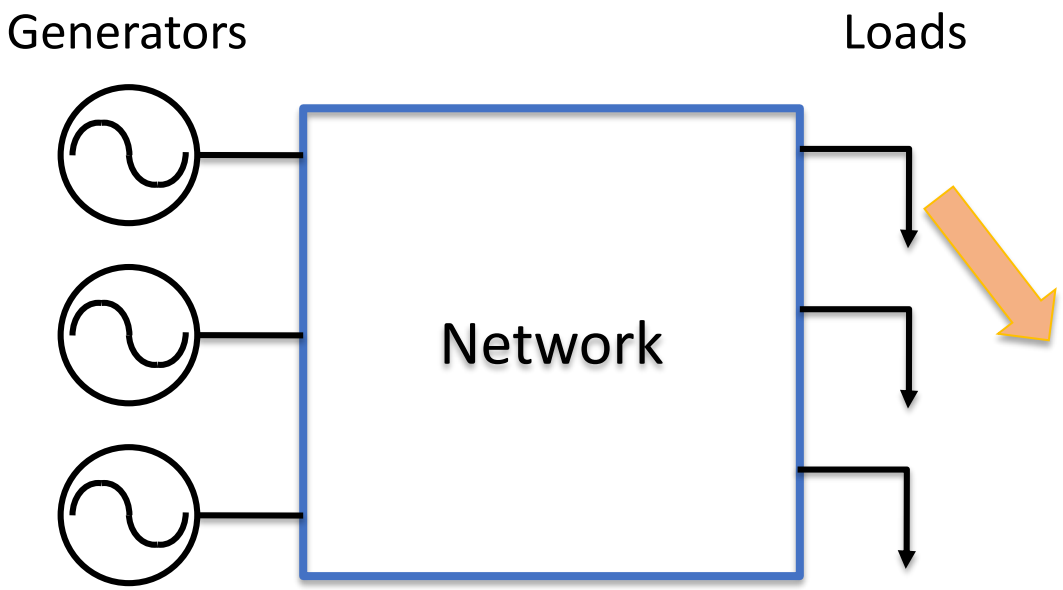
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Multiple generators

Sudden decrease in load:

$$J_G \omega_m \frac{d\omega_m}{dt} = \sum_{g \in G} P_{m,g} - \sum_{l \in L} P_l + P_{losses} > 0$$

J_G: combined inertia of all the generators and their prime movers



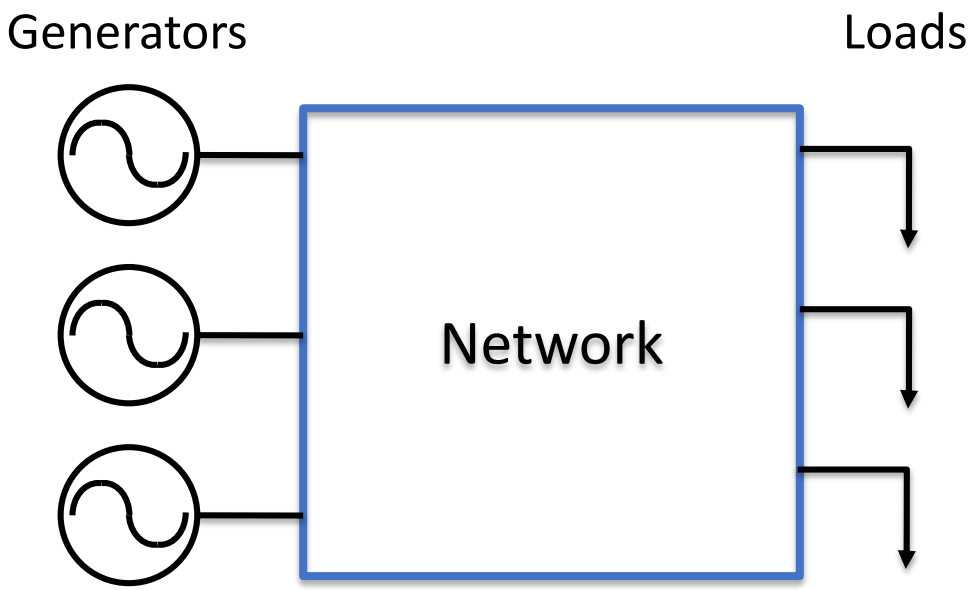
→ All the generators accelerate together

→ System frequency increases

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Multiple generators

$$J_G \omega_m \frac{d\omega_m}{dt} = \sum_{g \in G} P_{m,g} - \sum_{l \in L} P_l + P_{losses}$$



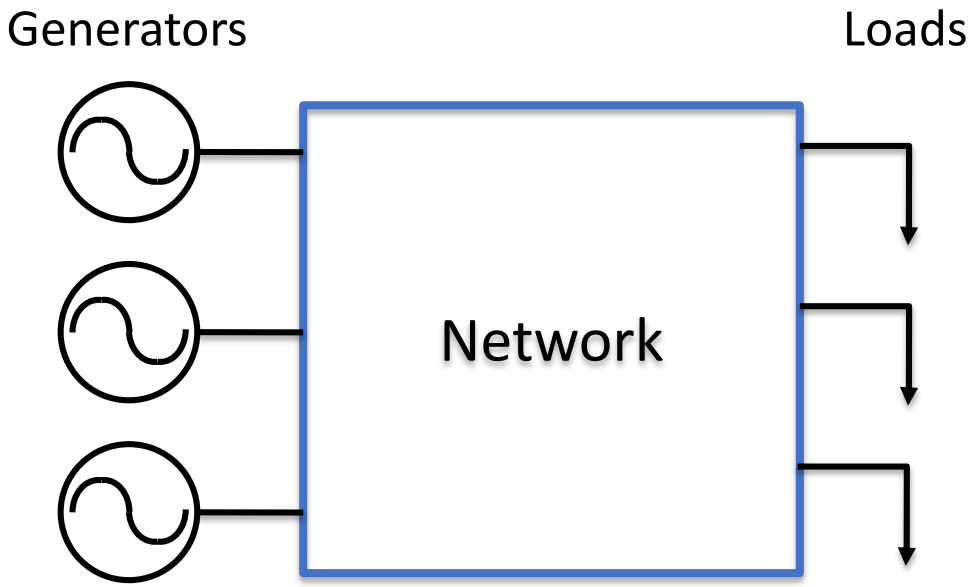
J_G: **combined inertia** of all the generators and their prime movers

→ A sudden change in load has a much smaller effect in large systems than in small systems

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Multiple generators

$$J_G \omega_m \frac{d\omega_m}{dt} = \sum_{g \in G} P_{m,g} - \sum_{l \in L} P_l + P_{losses}$$

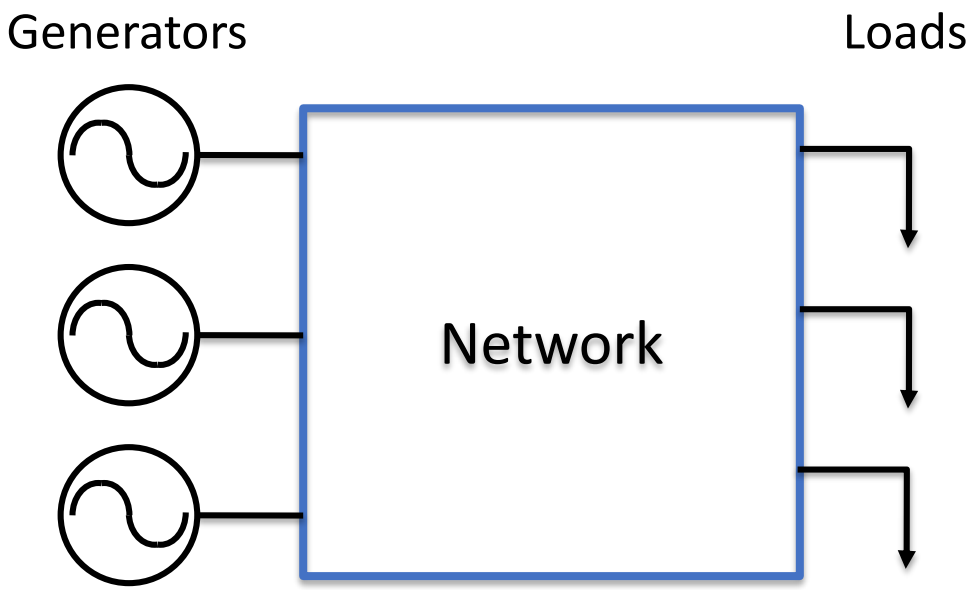


Changes in loads are usually gradual. Large changes in generation can be sudden because of generator failures

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Multiple generators: Governor control

- Any imbalance between generation and load causes the frequency to drop or rise
- The governor of each generator reacts to this frequency change according to its droop characteristic



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Load sharing between generators

$$P_1 = P_1^{max}\left(\frac{f^{nom}-f}{\rho f^{nom}}\right)$$

$$P_2 = P_2^{max}\left(\frac{f^{nom}-f}{\rho f^{nom}}\right)$$

Two generators with same droop coefficient ρ

$$P_T = P_1 + P_2 = (P_1^{max} + P_2^{max})\left(\frac{f^{nom}-f}{\rho f^{nom}}\right)$$

These two generators share the load and operate at the same frequency

$$f = f^{nom}\left[1 - \rho\frac{P_T}{(P_1^{max}+P_2^{max})}\right]$$

Frequency determined by the combined droop characteristic

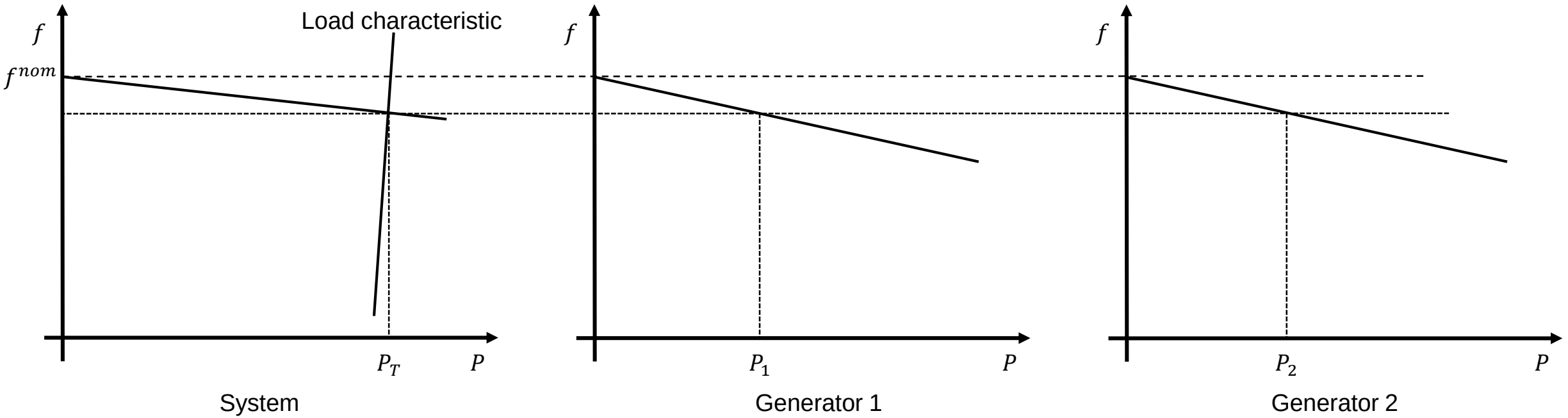
$$P_1 = \frac{P_1^{max}}{(P_1^{max}+P_2^{max})}P_T$$

$$P_2 = \frac{P_2^{max}}{(P_1^{max}+P_2^{max})}P_T$$

Generators share the load in proportion to their capacity

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System droop characteristic



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Example 6.2

Two 60 Hz generators supply a load of 20 kW. The first generator is rated at 20 kW and the second at 30 kW. They have the same droop coefficient of 0.04. Calculate the power produced by each generator and the operating frequency.

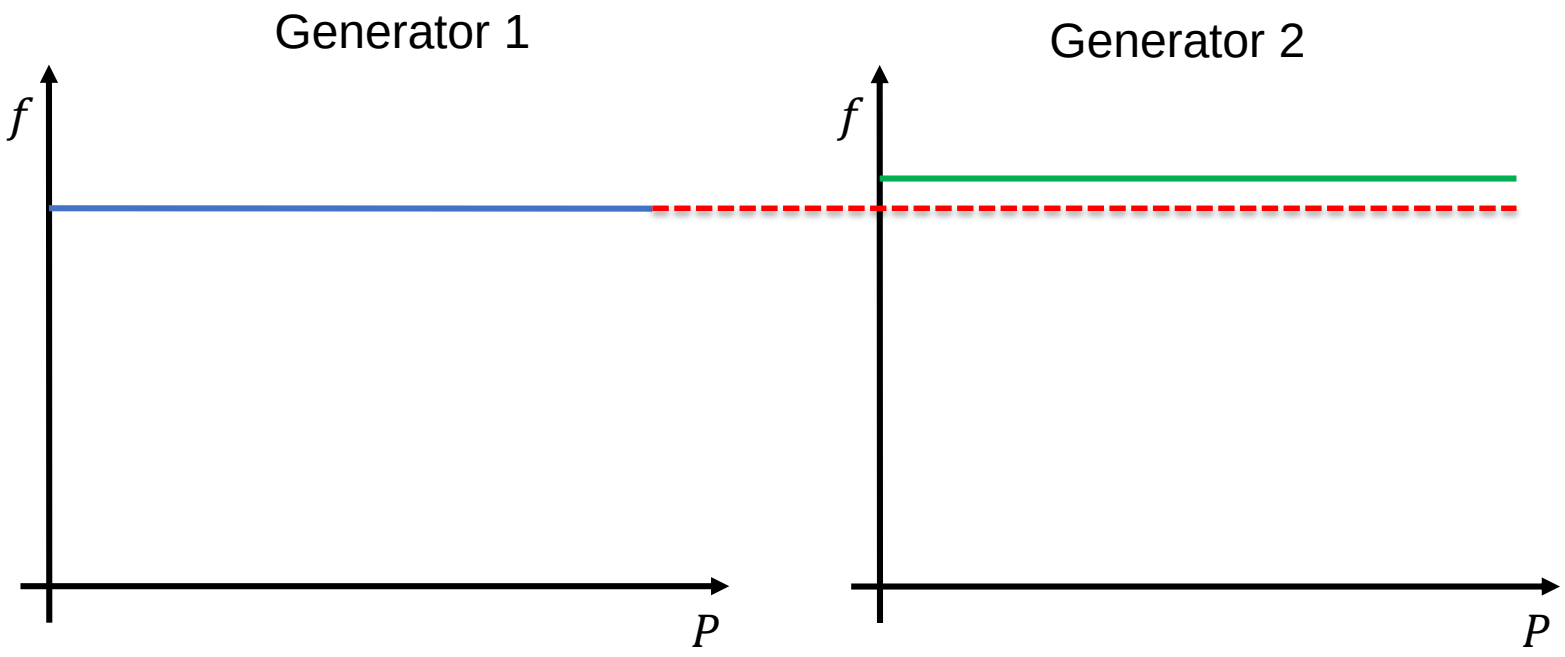
$$P_1 = \frac{20}{20+30} \times 20 = 8\text{kW}$$

$$P_2 = \frac{30}{20+30} \times 20 = 12\text{kW}$$

$$f = 60\left(1 - 0.04 \times \frac{20}{20+30}\right) = 59.04\text{Hz}$$

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Why a droop characteristic?



Assume that the two generators try to keep the frequency exactly constant. However, the frequencies that they measure would never be exactly the same. As a result, they would fight, and the system would be unstable.

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Electronically-connected generators





Electronically-connected generators do not naturally contribute to the inertia of the system

This poses a challenge as the proportion of renewable generation increases

Special control systems are being developed so these generators can provide *virtual inertia*

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Secondary frequency control

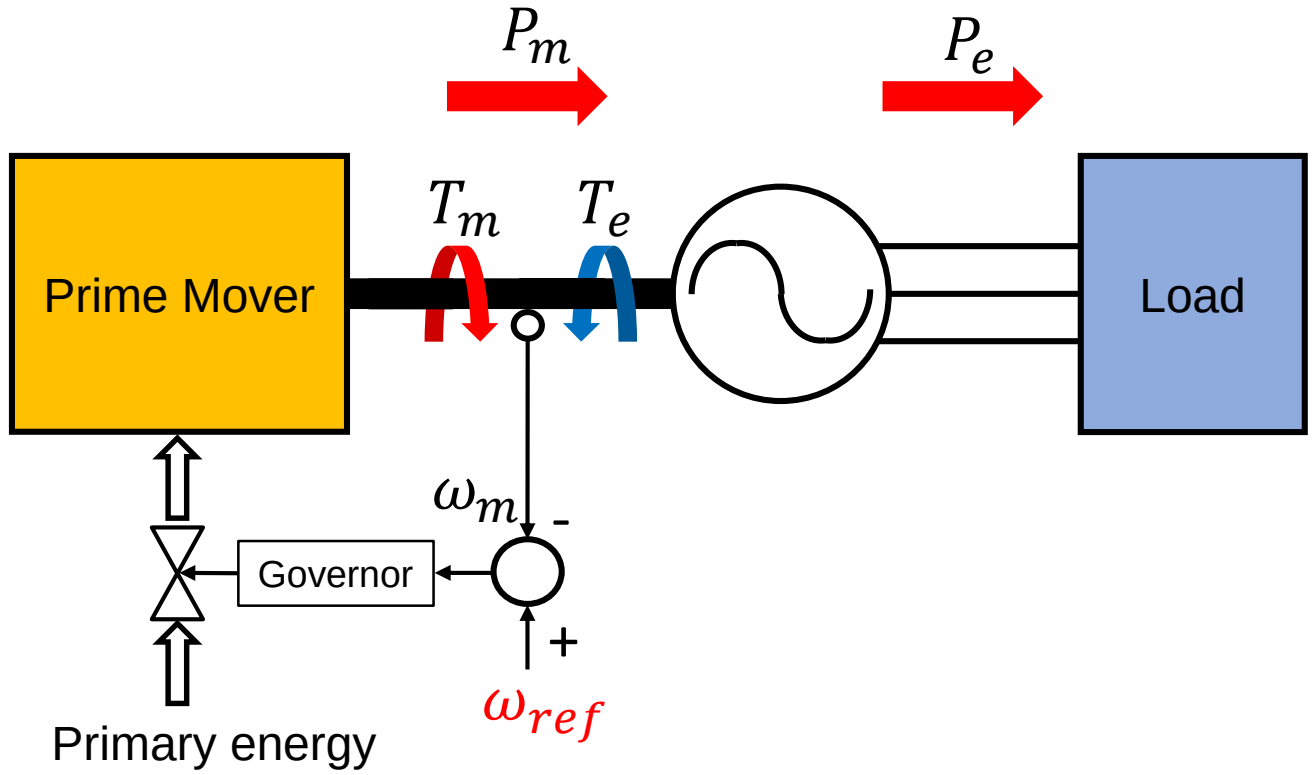
- Primary frequency control stops frequency from declining/increasing when there is an imbalance
- What's good about it?
  - Fast
  - Simple
  - Distributed (no need for communication)
  - Allocates imbalances in proportion to generator rating
- What are its limitations?
  - Does not restore frequency to its nominal value
  - Does not allocate generation economically

} Need for secondary frequency control

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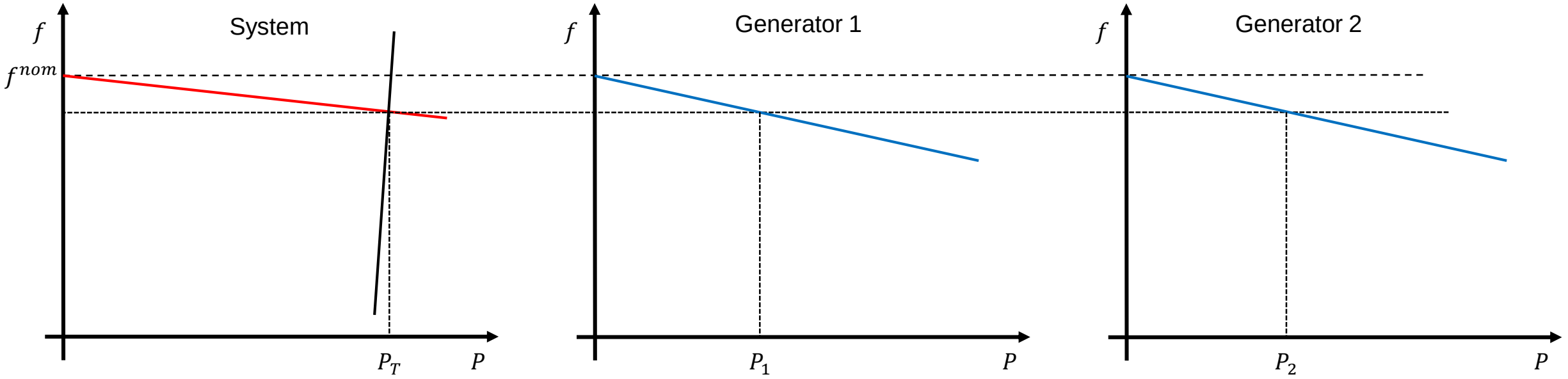
Secondary frequency control

- Adjusts ω_{ref} up or down
- Shifts the whole droop characteristic up or down
- Restores the frequency to its nominal value
- Can be used to re-allocate load among the generators

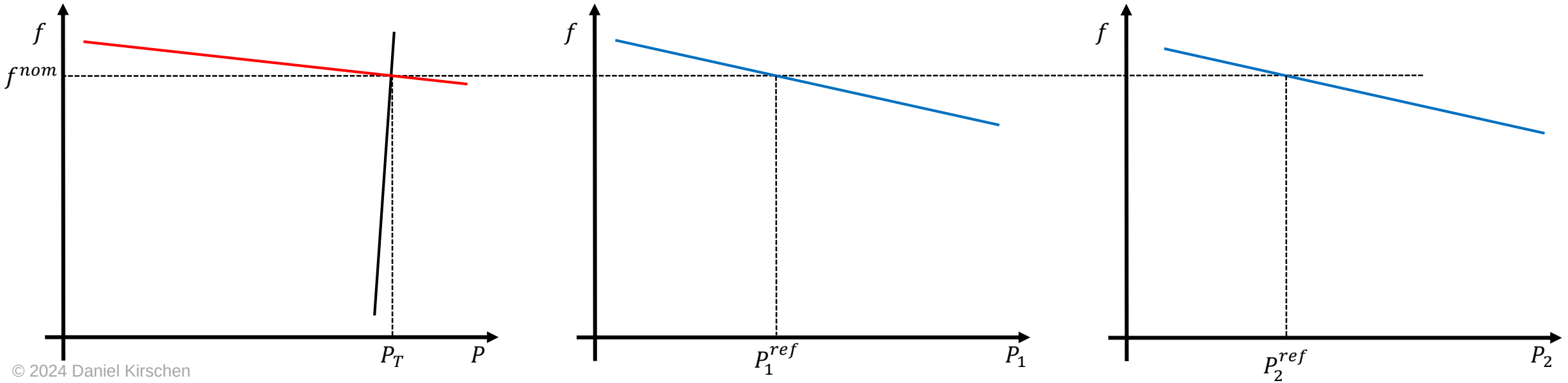


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Droop characteristics without secondary frequency control



Droop characteristics with secondary frequency control



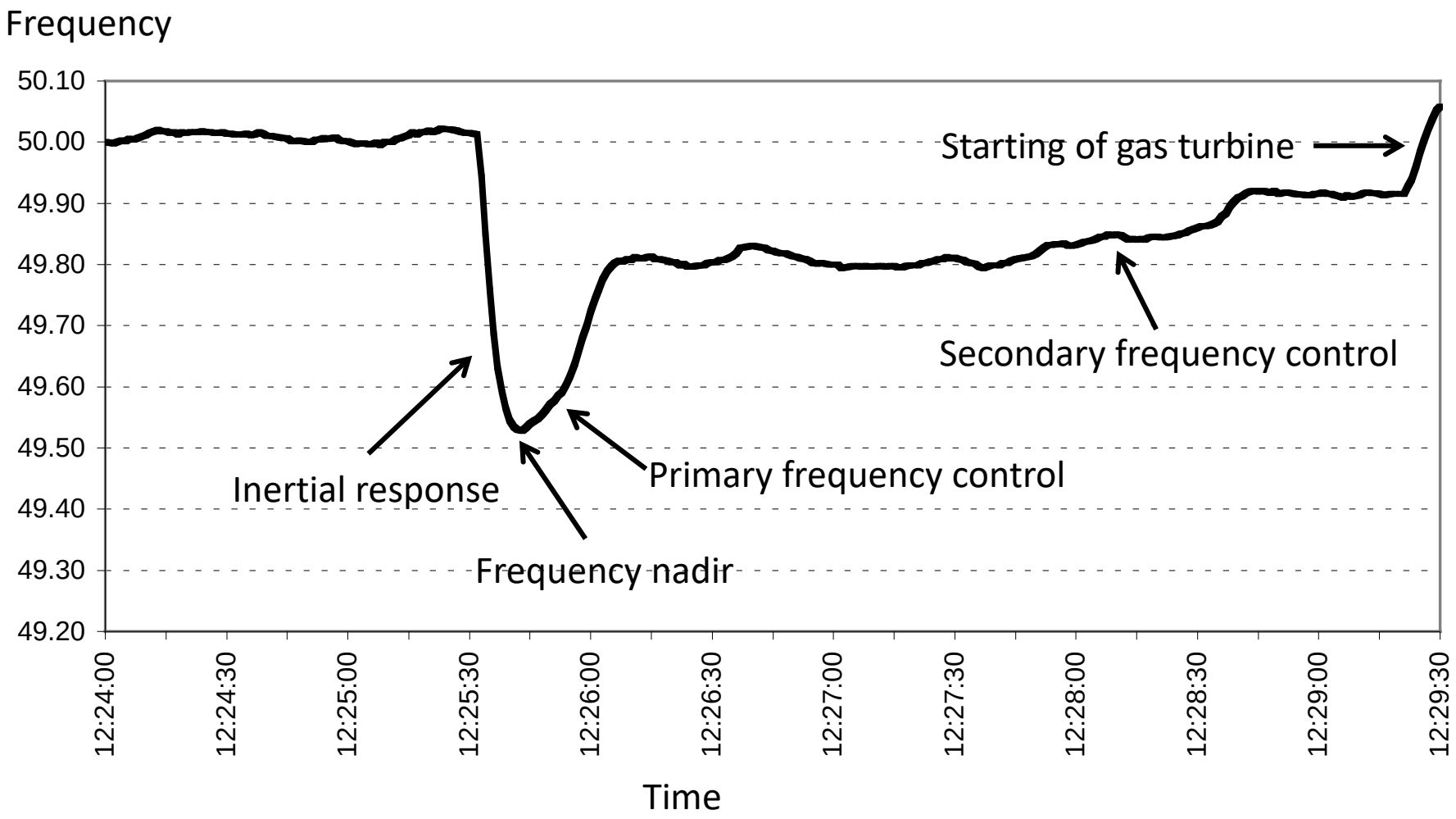
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Secondary frequency control

- Centralized control
- Control center sends reference signal P_i^{ref} to each generator
- Adjusts the P_i^{ref} to restore the frequency at its nominal value (e.g., 60 Hz)
- At that point we have $\sum_{i \in G} P_i^{ref} \sim P_T$
- Since we don't know the value of the load, we control the frequency
- Primary frequency control reacts fast to keep the frequency from changing uncontrollably
- Secondary frequency control reacts more slowly to restore the frequency to its nominal value
- We'll see later how secondary frequency control implements economic dispatch

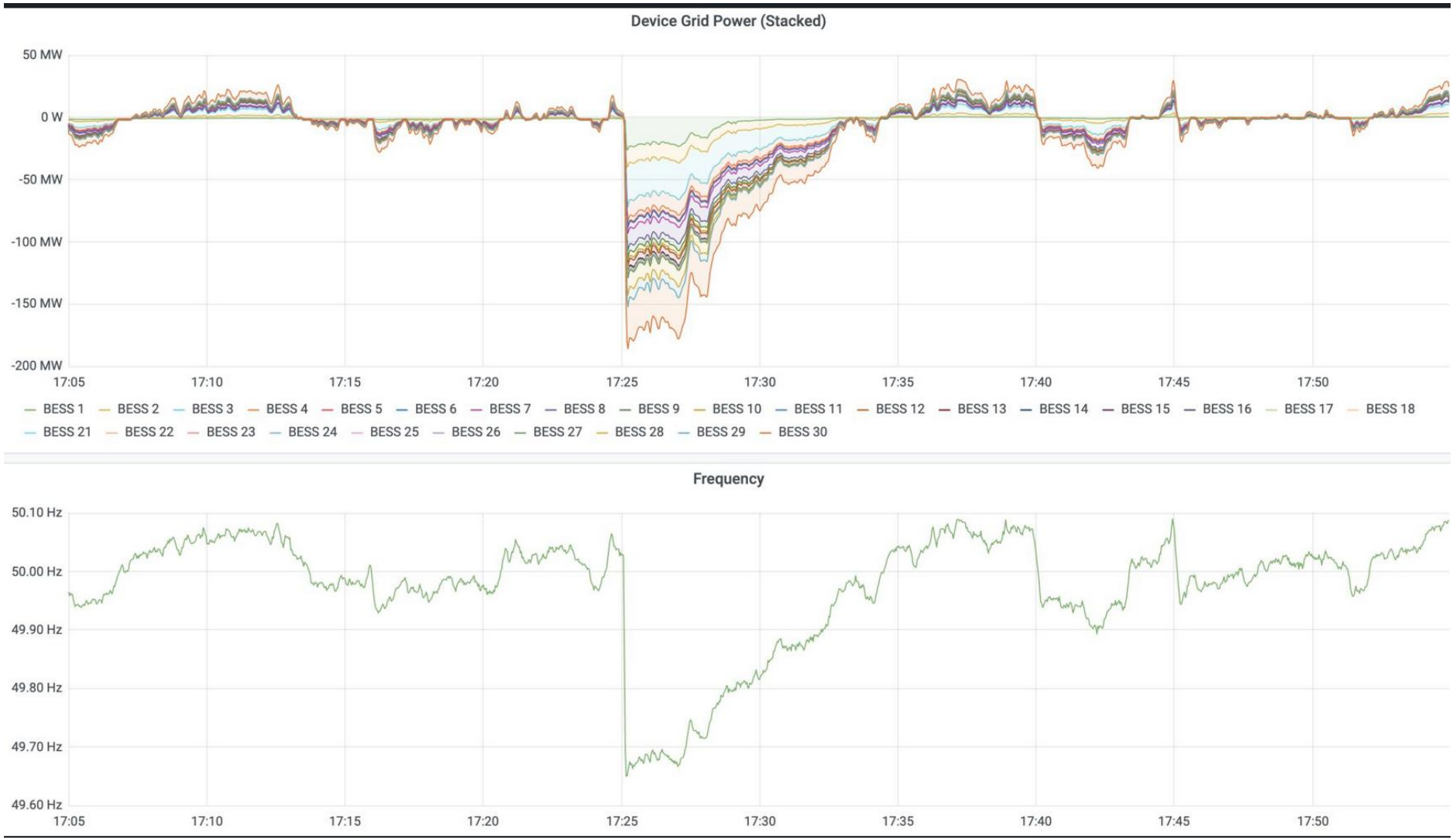
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Response to a large disturbance



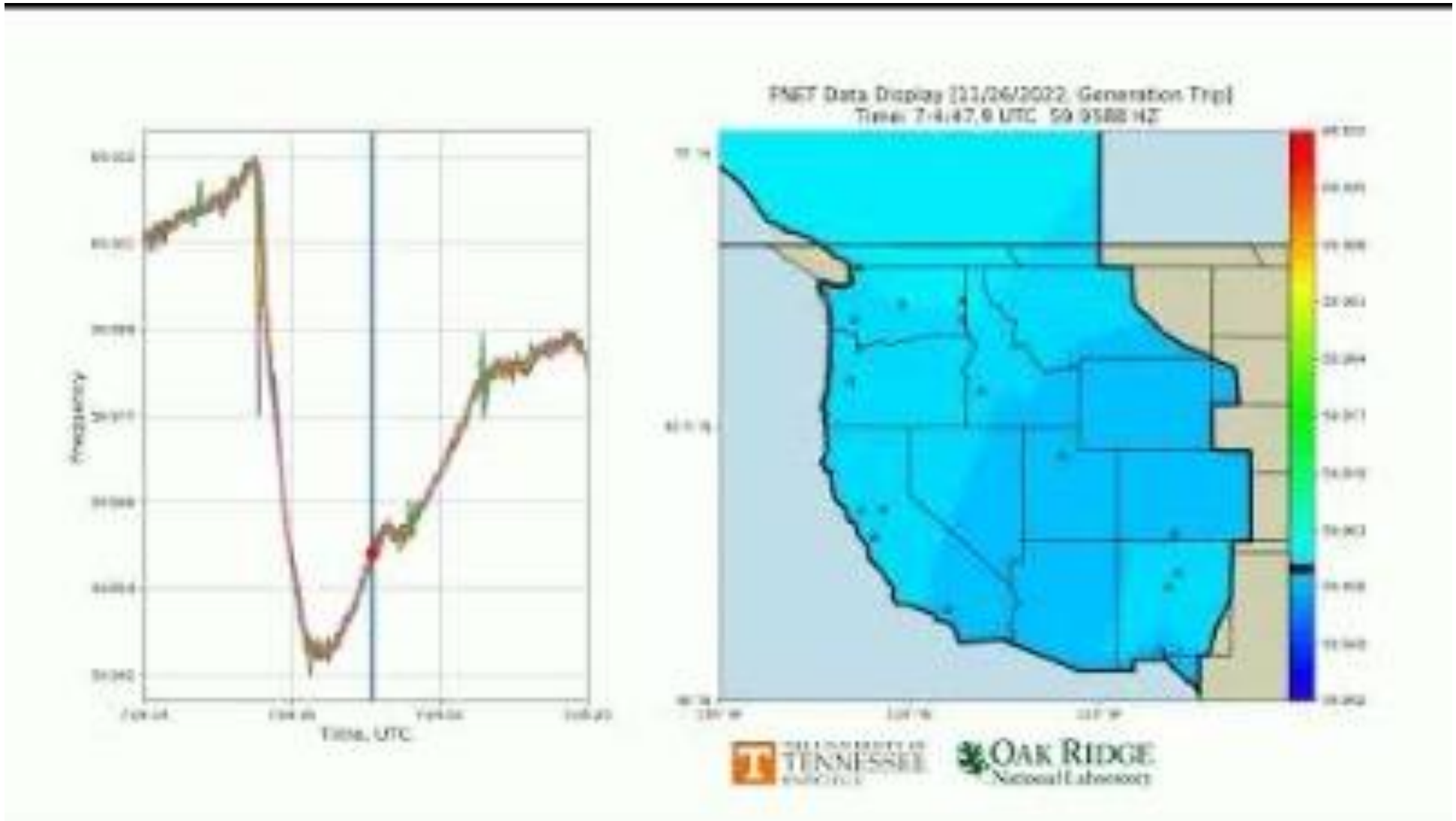
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Loss of 1200 MW in the GB system on 4/18/2022



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Loss of generation event in the Western US 11/26/2022



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Major incidents

- What if generators do not react fast enough to a large imbalance?
- Frequency continues to decrease
- Operating generators at too low a speed will damage them
  - Automatic disconnection
- Last line of defense: under-frequency load shedding

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Underfrequency load shedding in WECC

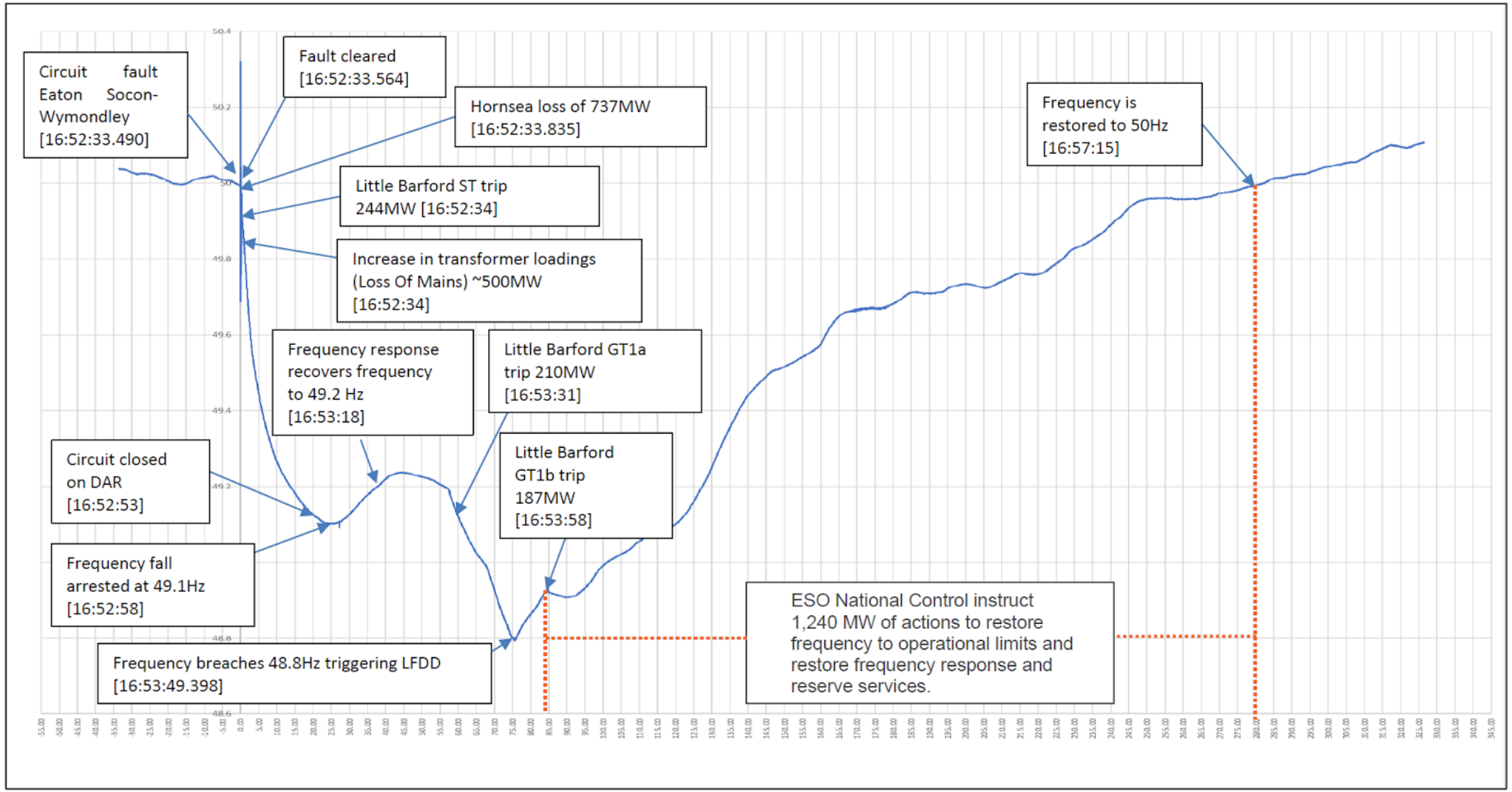
Block	% of Load	Frequency
1	5.3	59.1
2	5.9	58.9
3	6.5	58.7
4	6.7	58.5
5	6.7	58.3

Each block of load must be shed within 14 cycles of the frequency dropping below the corresponding threshold

WECC: Western US interconnection

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Great Britain, 9 August 2019



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Consequences



Passengers travelling on Thameslink were delayed



People walked in complete darkness at Clapham Junction station in London during a power cut



Passengers on a train near Kentish Town station got off and began walking along the tracks

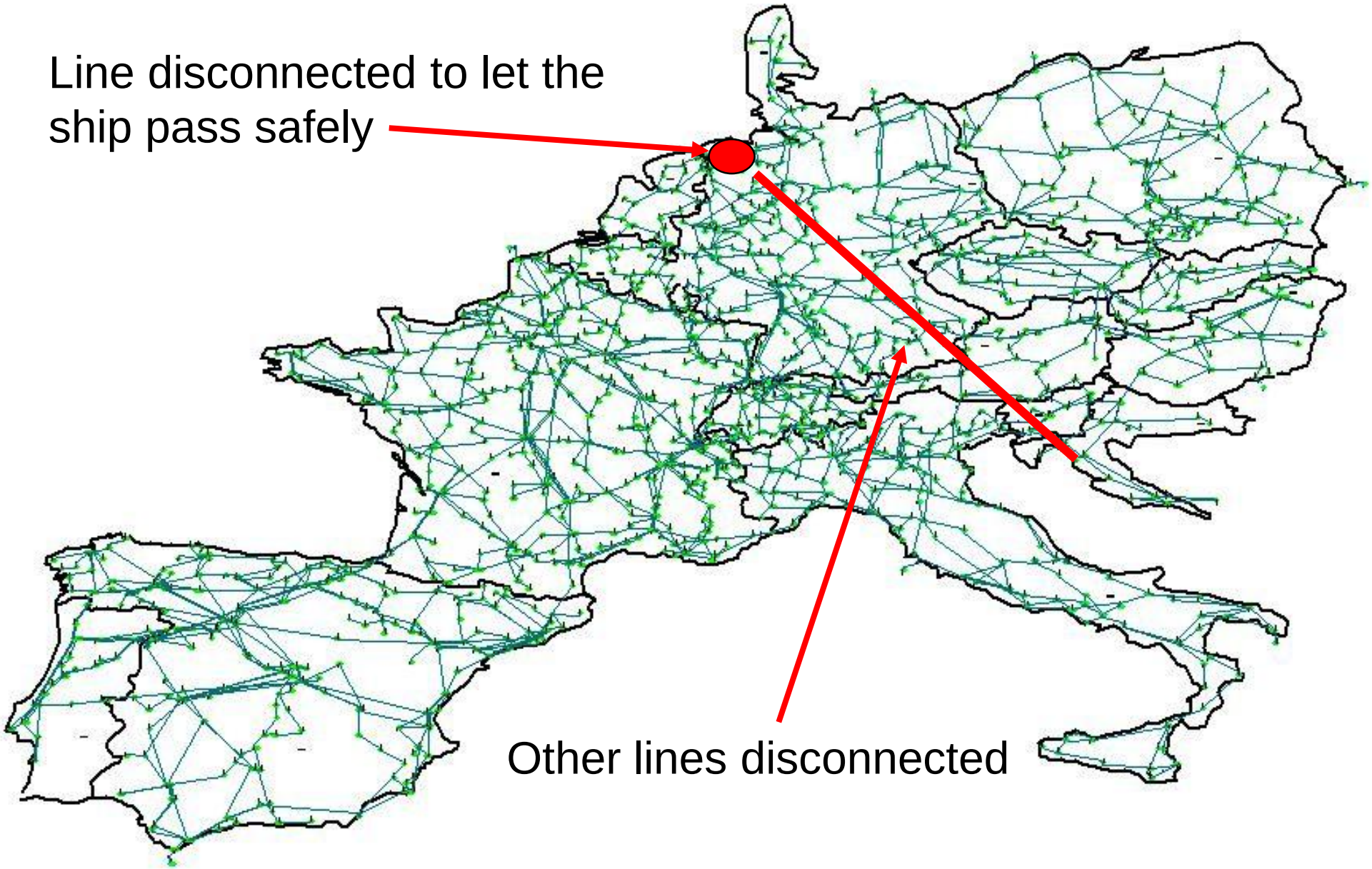
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European incident of 4 November 2006



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European incident of 4 November 2006



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European incident of 4 November 2006

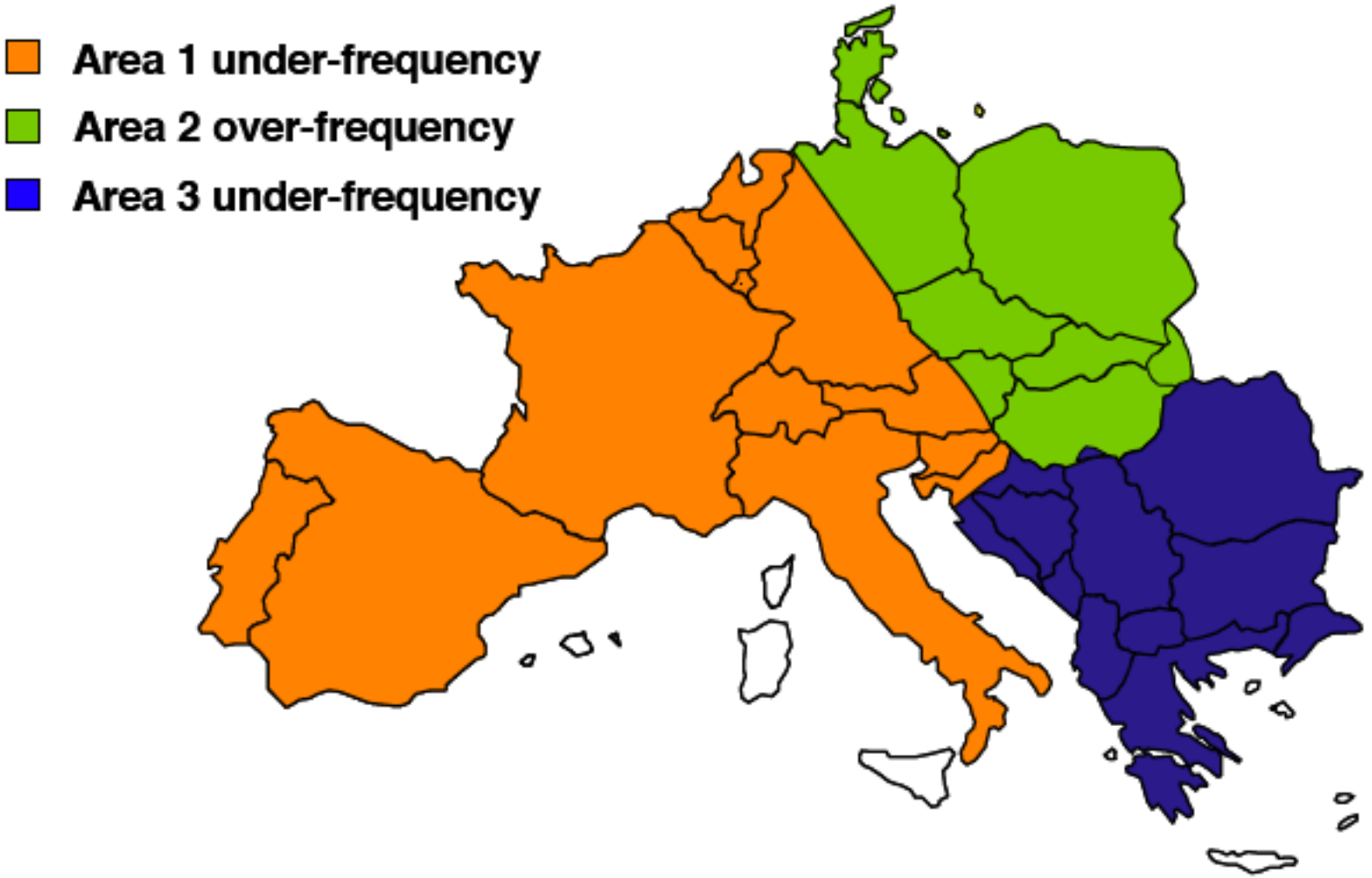
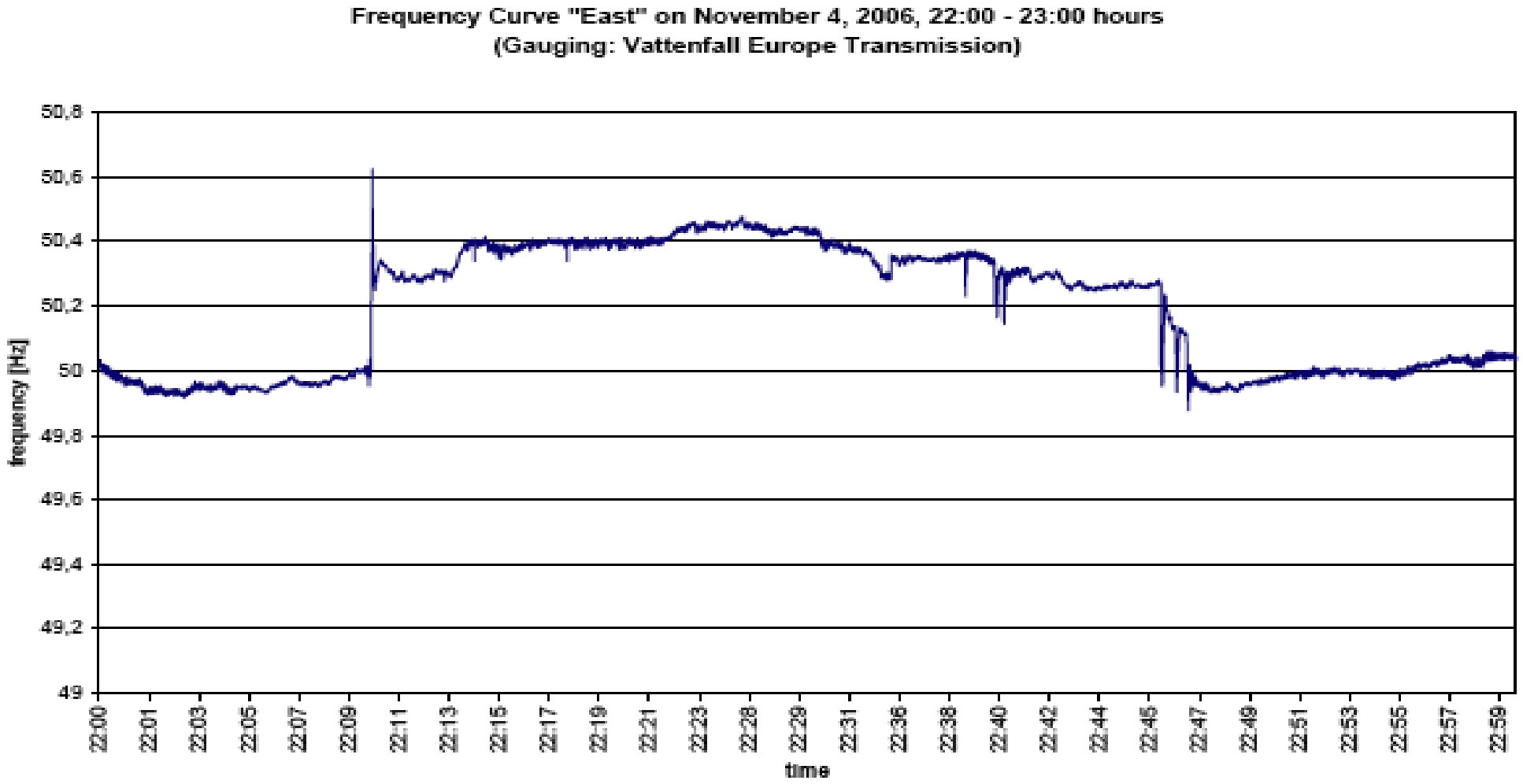


Figure 4: Schematic map of UCTE area split into three areas

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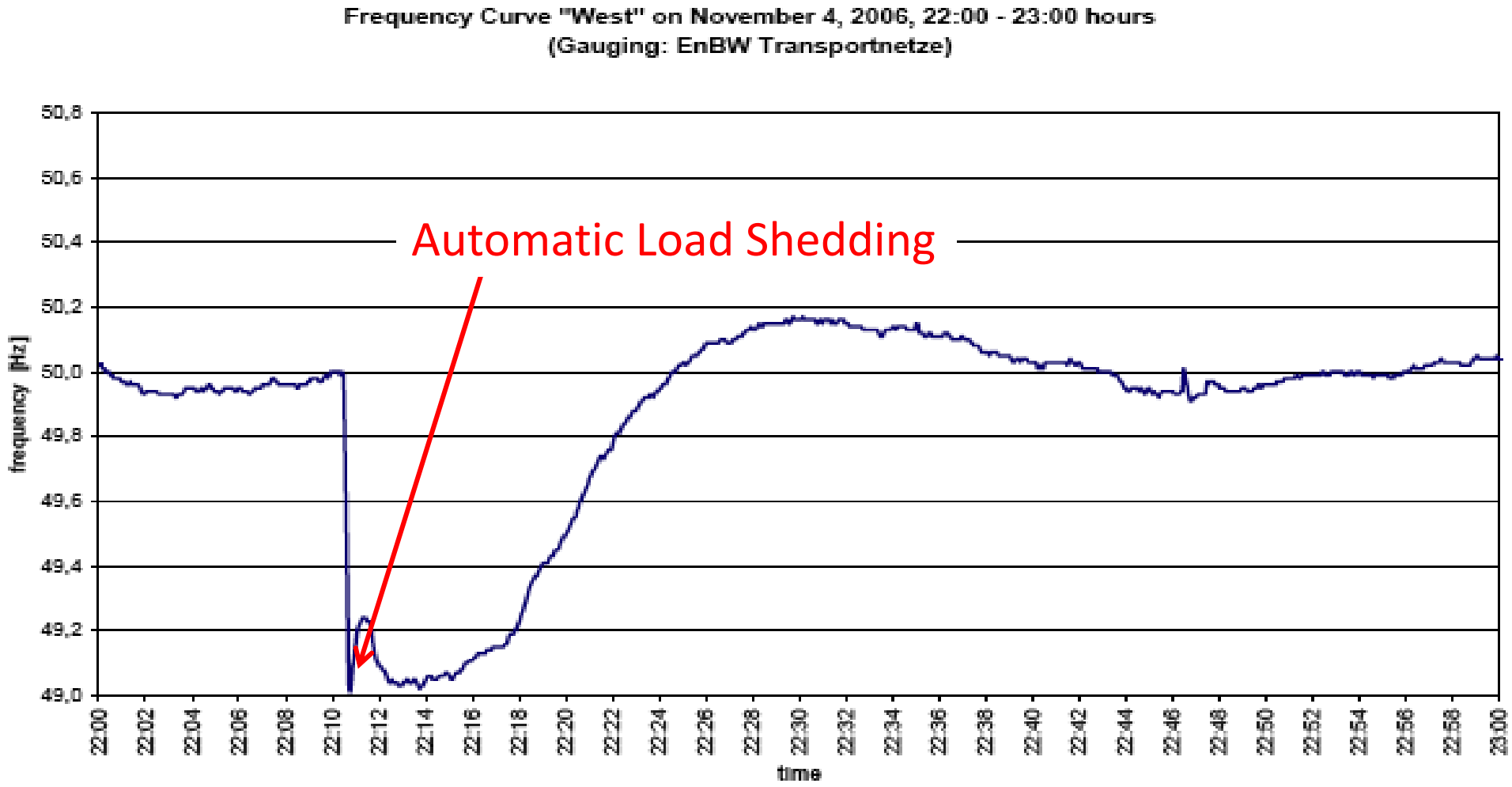
European incident of 4 November 2006



In the Eastern part of the system, because there was more generation than load, the frequency increased suddenly. This was corrected by reducing the output of the generators. After 45 minutes, the system was reconnected.

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European incident of 4 November 2006



In the Western part of the system, there was much more load than generation. The frequency dropped very fast. To prevent the system from collapsing, 15 million customers were disconnected automatically. Generation was increased to restore frequency to its normal value.

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European incident of 4 November 2006

Country	Load shed
Austria	1500 MW
Belgium	800 MW
France	5200 MW
Germany/E.ON Netz	400 MW
Germany/RWE TSO	2000 MW
Italy	1500 MW
Netherlands	400 MW
Portugal	500 MW
Spain	1050 MW

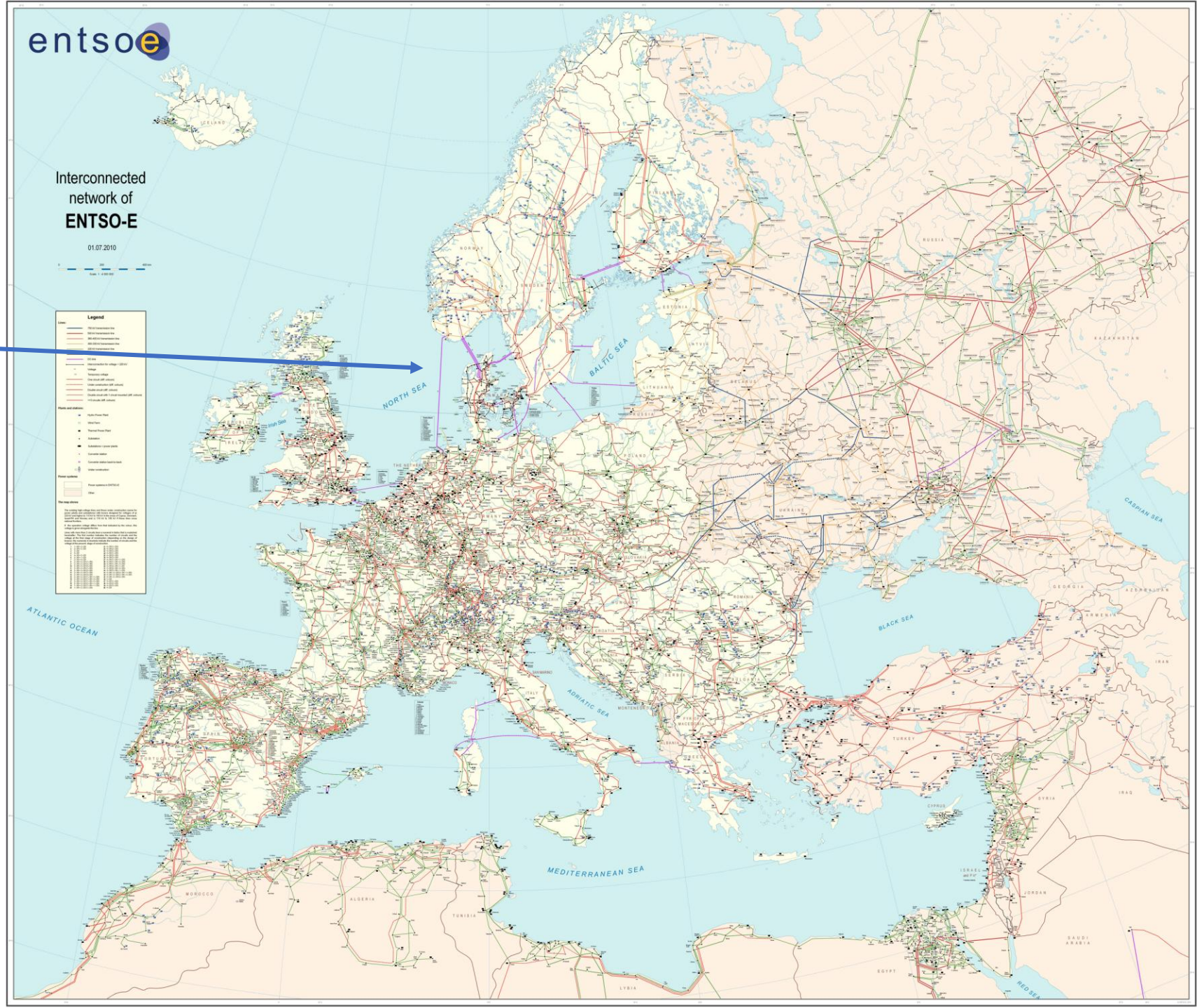
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Incident on DC interconnector between Norway and Germany

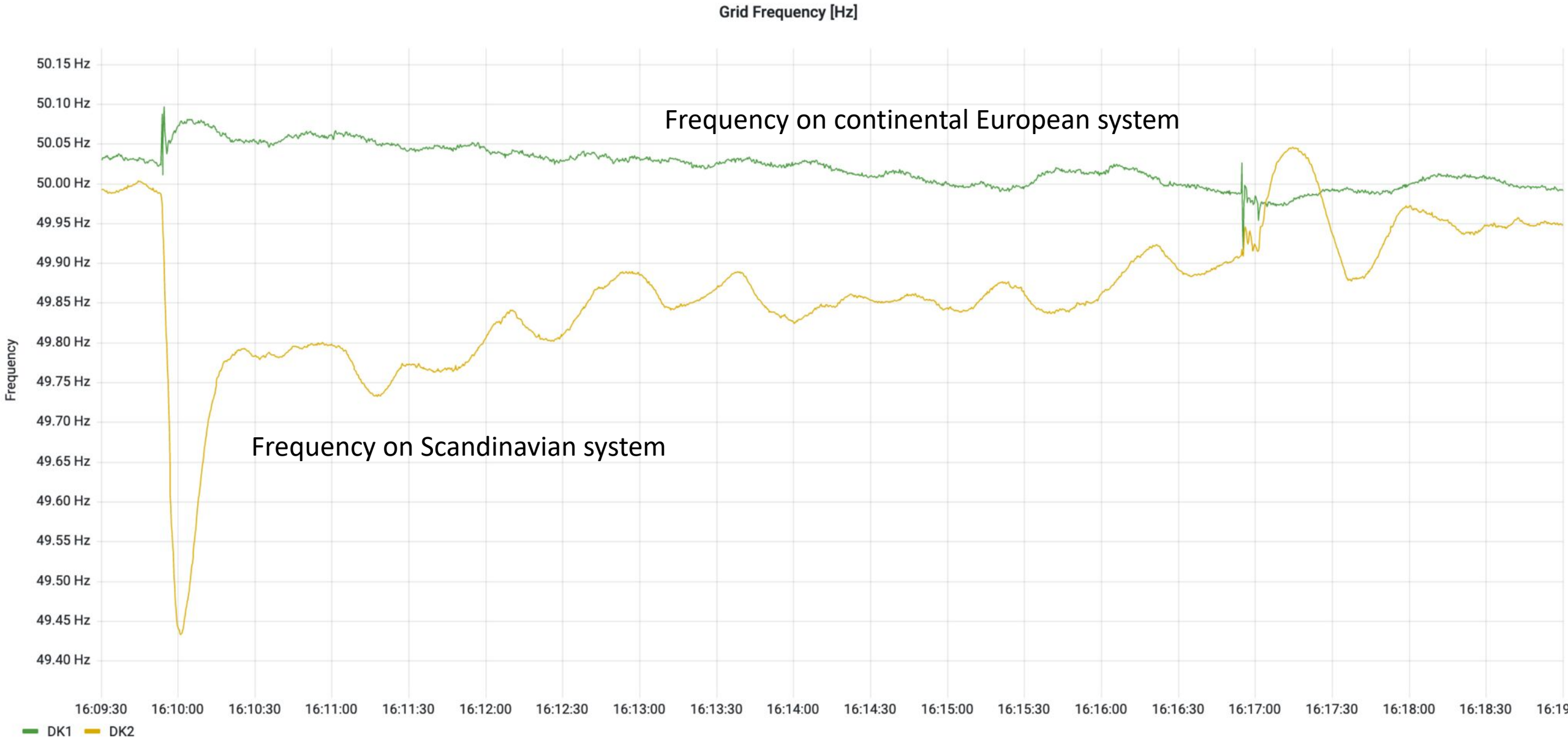
Sudden reversal of power flow

Norway, Sweden, Finland went from importing 1200 MW to exporting 600 MW

This created a large load/generation imbalance



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Economic dispatch



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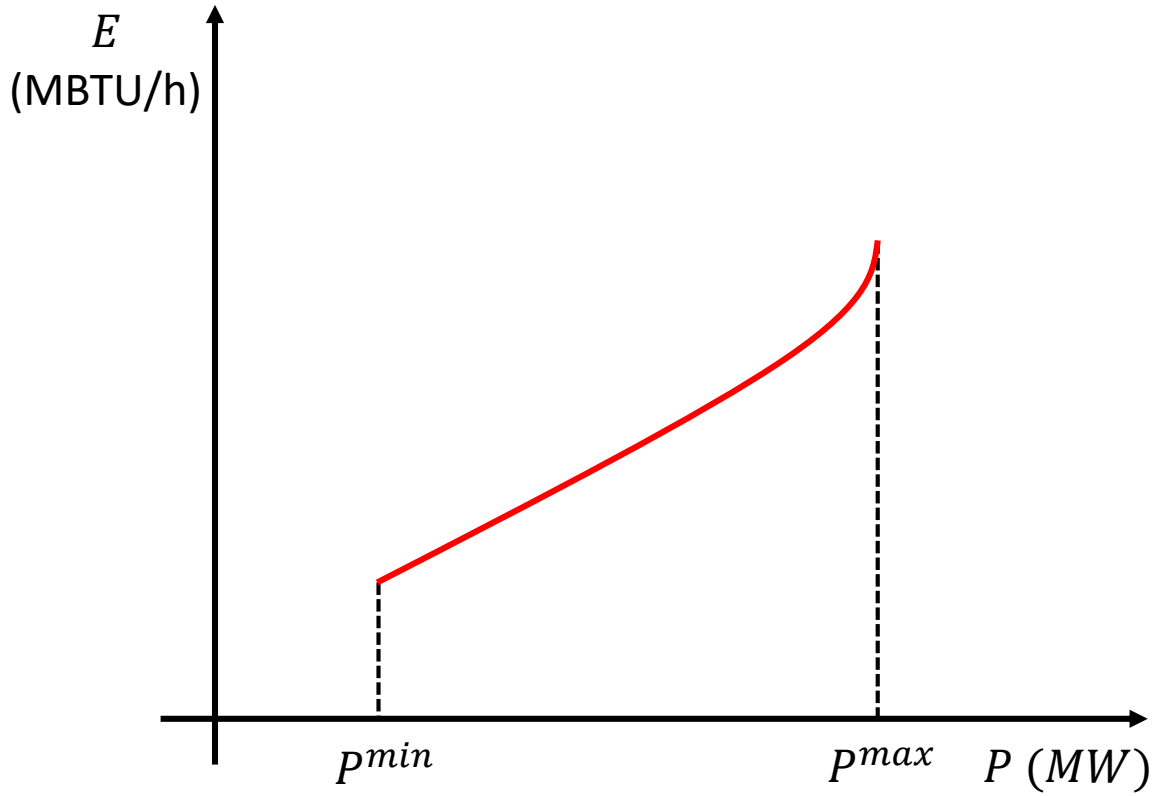
Goal of the economic dispatch problem

- Given set of generating units are synchronized to the power system
- How much power should each of these units produce to supply the load at minimum cost?
- Minimize the cost of the fuel needed to produce this power
- For wind and solar generation, this fuel cost is zero
- Wind and solar generation are taken as given in the economic dispatch problem

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Heat rate curve of a thermal generating unit

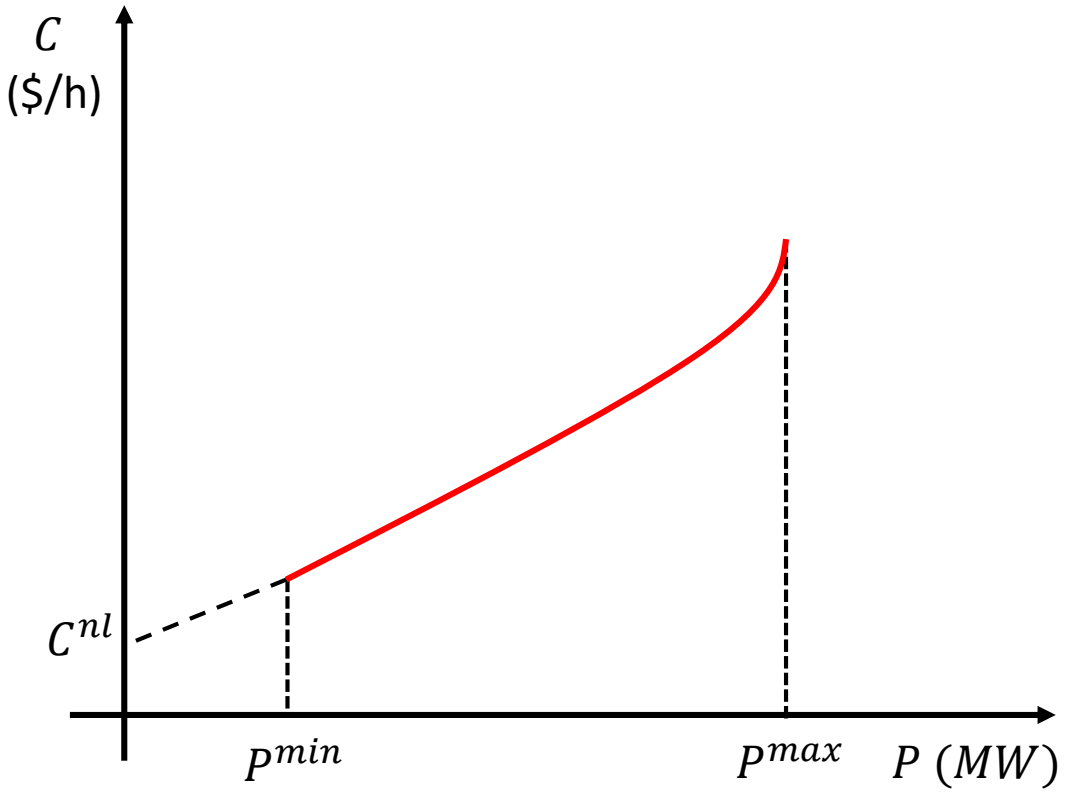
- Shows how much fuel must be burned each hour for the unit to produce a certain amount of power
- Measured in the energy content of the fuel
- MBTU: Million British Thermal Unit
- Note the typical shape of the curve
- Note the physical limits on the power production



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Cost curve of a thermal generating unit

- Multiply hourly fuel input by cost of fuel in $/MBTU
- Extrapolation of the curve to $P = 0$ usually leads to a value called the no-load cost



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Mathematical formulation

- Economic dispatch is a *constrained optimization problem*
- Must define:
  - Objective function
  - Equality constraint

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Objective function

- Minimize the cost of producing power with all the generating units

$$C^{total} = \sum_{i \in G} C_i(P_i)$$

- G: set of generating units
- P_i: amount of power produced by unit i
- $C_i(P_i)$: hourly cost of producing P_i with unit i

- The P_i are the decision variables

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Equality constraint

- Load/generation balance:

$$\sum_{i \in G} P_i = L$$

- L: load to be supplied by these generating units

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Solving this constrained optimization problem

- Build the Lagrangian function of the problem:

$$\mathcal{L}(P_i, \lambda) = \sum_{i \in G} C_i(P_i) + \lambda \left(L - \sum_{i \in G} P_i \right)$$

Objective function

Equality constraint

Lagrange multiplier

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Optimality conditions

$$\mathcal{L}(P_i, \lambda) = \sum_{i \in G} C_i(P_i) + \lambda \left(L - \sum_{i \in G} P_i \right)$$

- Set the partial derivatives of the Lagrangian function to zero
- Partial derivatives with respect to the decision variables:

$$\frac{\partial \mathcal{L}(P_i, \lambda)}{\partial P_i} \equiv \frac{dC_i(P_i)}{dP_i} - \lambda = 0 \quad \forall\, i \in G$$

- Derivative with respect to the Lagrange multiplier:

$$\frac{\partial \mathcal{L}(P_i, \lambda)}{\partial \lambda} \equiv L - \sum_{i \in G} P_i = 0$$

- Solve these equations for the P_i and λ

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Physical interpretation of the solution

$$\frac{\partial \mathcal{L}(P_i,\lambda)}{\partial P_i} \equiv \frac{dC_i(P_i)}{dP_i} - \lambda = 0 \quad \forall\, i \in G$$

$$\frac{dC_1(P_1)}{dP_1} = \frac{dC_2(P_2)}{dP_2} = \cdots = \lambda$$

At the optimum, all units operate at the same marginal cost

$$\frac{dC_i(P_i)}{dP_i} \approx \frac{\Delta C_i}{\Delta P_i} \rightarrow \Delta C_i = \frac{dC_i(P_i)}{dP_i}\Delta P_i$$

$\frac{dC_i(P_i)}{dP_i}$: cost of an extra MW from unit i

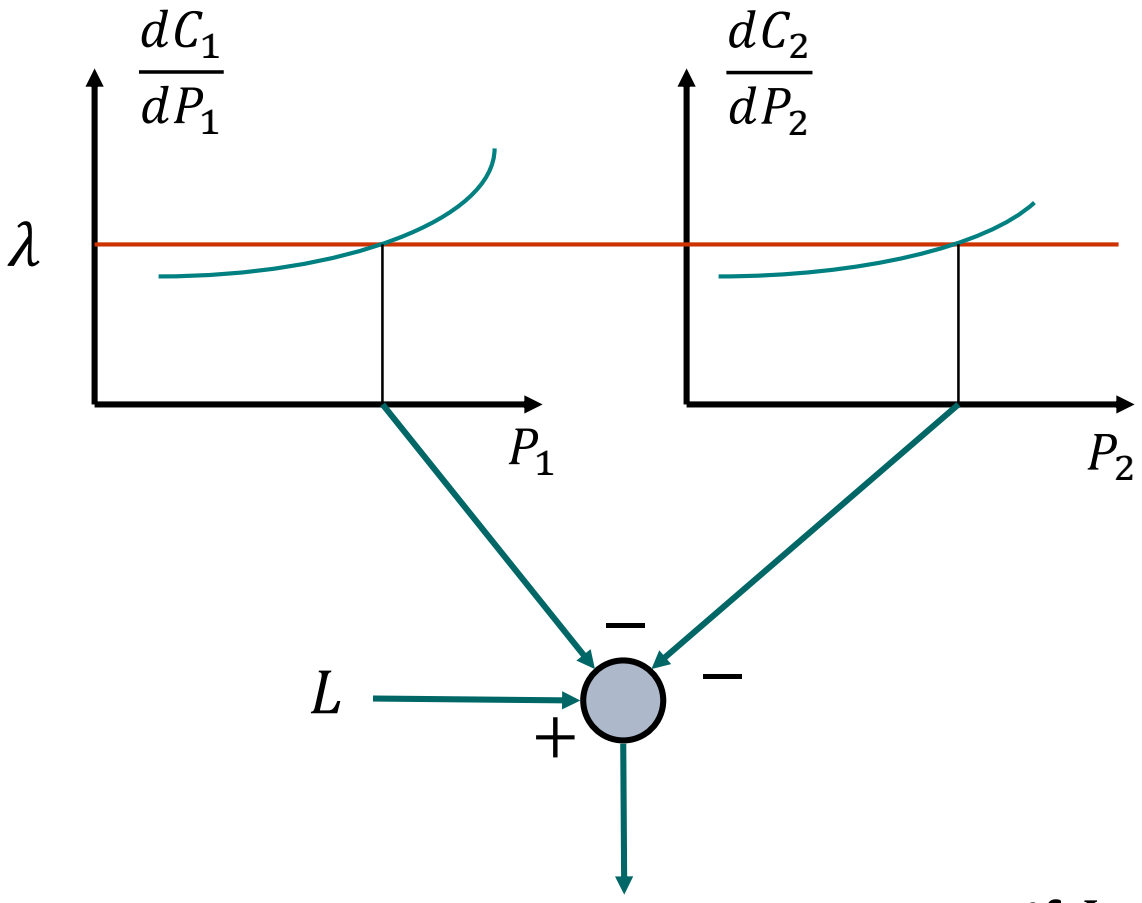
At the optimum, a MW has the same cost from all generating units

Lagrange multiplier λ: cost of an extra MW at the optimum dispatch

λ: system marginal cost

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Visualisation of economic dispatch



$L - P_1 - P_2$ ⇒ If $L - P_1 - P_2 < 0$, reduce λ
If $L - P_1 - P_2 < 0$, increase λ

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Example 6.3: Economic dispatch

- Cost functions of three generating units:

$C_1(P_1) = 2100 + 8P_1 + 0.1\ P_1^2$

$C_2(P_2) = 7200 + 7P_2 + 0.06\ P_2^2$

$C_3(P_3) = 6250 + 9P_3 + 0.07\ P_3^2$

- Load to be supplied at minimum cost by these three units:

$L = 800\text{MW}$

- Lagrangian function:

$\mathcal{L}(P_1, P_2, P_3, \lambda) = 2100 + 8P_1 + 0.1\ P_1^2 + 7200 + 7P_2 + 0.06\ P_2^2 + 6250 + 9P_3 + 0.07\ P_3^2 + \lambda(L - P_1 - P_2 - P_3)$

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Example 6.3: Optimality conditions

- Lagrangian function:

$$\mathcal{L}(P_1, P_2, P_3, \lambda) = 2100 + 8P_1 + 0.1\,P_1^2 + 7200 + 7P_2 + 0.06\,P_2^2 + 6250 + 9P_3 + 0.07\,P_3^2 + \lambda(L - P_1 - P_2 - P_3)$$

- Partial derivatives:

$$\frac{\partial \mathcal{L}(P_1,P_2,P_3,\lambda)}{\partial P_1} \equiv \frac{dC_1(P_1)}{dP_1} - \lambda = 8 + 0.2P_1 - \lambda = 0$$

$$\frac{\partial \mathcal{L}(P_1,P_2,P_3,\lambda)}{\partial P_2} \equiv \frac{dC_2(P_2)}{dP_2} - \lambda = 7 + 0.12P_2 - \lambda = 0$$

$$\frac{\partial \mathcal{L}(P_1,P_2,P_3,\lambda)}{\partial P_3} \equiv \frac{dC_3(P_3)}{dP_3} - \lambda = 9 + 0.14P_3 - \lambda = 0$$

$$\frac{\partial \mathcal{L}(P_1,P_2,P_3,\lambda)}{\partial \lambda} \equiv L - P_1 - P_2 - P_3 = 0$$

$$\rightarrow P_1 = 5.00\lambda - 40.00$$

$$\rightarrow P_2 = 8.33\lambda - 58.33$$

$$\rightarrow P_3 = 7.14\lambda - 64.29$$

$$\lambda = \frac{L + 162.619}{20.476}$$

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Example 6.3: Solution for $L = 800\text{MW}$

- $\lambda = \frac{L+162.619}{20.476}$
- $L = 800\text{MW}$

} → $\lambda = 47.012$ \$/MWh

- $P_1 = 5.00\lambda - 40.00$ → $P_1 = 195.06$ MW
- $P_2 = 8.33\lambda - 58.33$ → $P_2 = 333.43$ MW
- $P_3 = 7.14\lambda - 64.29$ → $P_3 = 271.51$ MW
- $\frac{dC_1(P_1)}{dP_1} = \frac{dC_2(P_2)}{dP_2} = \frac{dC_3(P_3)}{dP_3} = \lambda = 47.012$ \$/MWh
- $C = C_1(P_1) + C_2(P_2) + C_3(P_3) = 37{,}523.69$ \$/h

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Example 6.3: Solution for $L = 1200\text{MW}$

- $\lambda = \frac{L+162.619}{20.476}$
- $L = 1200\text{MW}$

} → $\lambda = 66.547$ \$/MWh → increases with the load

- $P_1 = 5.00\lambda - 40.00$ → $P_1 = 292.73$ MW
- $P_2 = 8.33\lambda - 58.33$ → $P_2 = 496.22$ MW
- $P_3 = 7.14\lambda - 64.29$ → $P_3 = 411.05$ MW
- $\frac{dC_1(P_1)}{dP_1} = \frac{dC_2(P_2)}{dP_2} = \frac{dC_3(P_3)}{dP_3} = \lambda = 66.547$ \$/MWh
- $C = C_1(P_1) + C_2(P_2) + C_3(P_3) = 60{,}235.32$ \$/h

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Example 6.4: Economic dispatch and renewable generation

- Same three thermal generating units:

$C_1(P_1) = 2100 + 8P_1 + 0.1\ P_1^2$

$C_2(P_2) = 7200 + 7P_2 + 0.06\ P_2^2$

$C_3(P_3) = 6250 + 9P_3 + 0.07\ P_3^2$

- Generation from a renewable source: 200 MW
- Load $L = 800\text{MW}$
- Net load to be supplied by the three thermal units: $L_{net} = 800 - 200 = 600\text{MW}$.
- Intermediate result of previous example: $\lambda = \frac{L_{net}+162.619}{20.476}$
- $\lambda = 37.244$ \$/MWh
- $P_1 = 146.22$ MW; $P_2 = 252.03$ MW; $P_3 = 201.74$ MW
- $C = C_1(P_1) + C_2(P_2) + C_3(P_3) = 29{,}098.11$ \$/h

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Limits on generating units

- Must compare the calculated P_i against the upper and lower limit of each unit:

$$P_i \geq P_i^{min} \quad \forall\, i \in G$$

$$P_i \leq P_i^{max} \quad \forall\, i \in G$$

- If all limits are satisfied → done

- If a limit is violated → fix the corresponding P_i at the limit and repeat the solution

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Example 6.5: Effect of generator limits on economic dispatch

- Same thermal generating units as in Example 6.3
- Limits on these units:
  $200 \leq P_1 \leq 500$ MW
  $200 \leq P_2 \leq 450$ MW
  $150 \leq P_3 \leq 500$ MW
- For $L = 1200\text{MW}$, we calculated:
- $P_1 = 292.73$ MW; $P_2 = 496.22$ MW; $P_3 = 411.05$MW; $\lambda = 66.547$ \$/MWh
- This is not a feasible solution
- Set $P_2 = 450$ MW
- $L_{net} = 1200 - 450 = 750\text{MW}$ to be supplied by units 1 and 3

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Example 6.5: Effect of generator limits on economic dispatch

- Optimally dispatch 750MW with units 1 and 3

  $\lambda = 70.353$ \$/MWh

  $P_1 = 311.76$ MW

  $P_3 = 438.24$ MW

  $P_2 = 450\text{MW}$

  $C =$ \$60,451.47

- Calculate the marginal cost of each unit at this optimum;

  $$\frac{dC_1(P_1)}{dP_1} = \frac{dC_3(P_3)}{dP_3} = \lambda = 70.353 \ \ \$/\text{MWh}$$

  $$\frac{dC_2(P_2)}{dP_2} = 61.00 \ \$/\text{MWh}$$

- All the units no longer operate at the same marginal cost
- Producing more power with unit 2 would reduce the total cost but is not possible

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Example 6.6: Effect of generator limits on economic dispatch

- Same thermal and renewable generating units as in Example 6.4
- Limits on the thermal units:
  $200 \leq P_1 \leq 500$ MW
  $200 \leq P_2 \leq 450$ MW
  $150 \leq P_3 \leq 500$ MW
- In Example 6.4, for $L_{net} = 800 - 200 = 600\text{MW}$, we found:
- $P_1 =$ 146.22 MW; $P_2 =$ 252.03 MW; $P_3 =$ 201.74 MW
- Infeasible solution
- Set $P_1 = 200$ MW
- $L_{net} = 800 - 200 - 200 = 400\text{MW}$ to be supplied by units 2 and 3

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Example 6.6: Effect of generator limits on economic dispatch

- Optimally dispatch 400MW with units 2 and 3

  $\lambda = 33.769$ \$/MWh

  $P_1 = 200$ MW

  $P_2 = 223.08$ MW

  $P_3 = 176.92$ MW

- Calculate the marginal cost of each unit at this optimum;

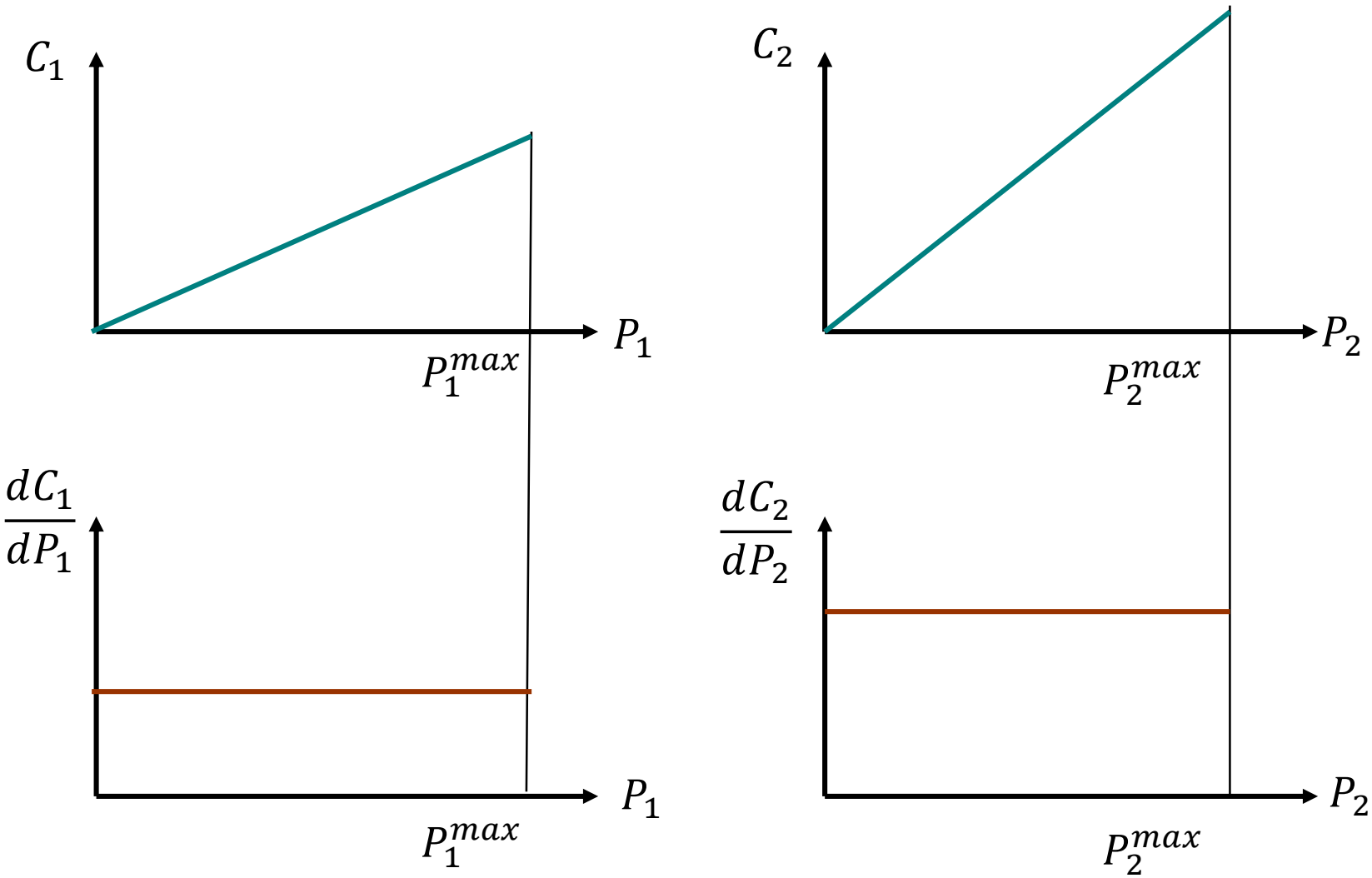
$$\frac{dC_2(P_2)}{dP_2} = \frac{dC_3(P_3)}{dP_3} = \lambda = 33.769 \ \ \$/\text{MWh}$$

$$\frac{dC_1(P_1)}{dP_1} = 48.00 \ \$/\text{MWh}$$

- The marginal cost of unit 1 is higher than the system marginal cost because it is at its lower limit and cannot produce any less

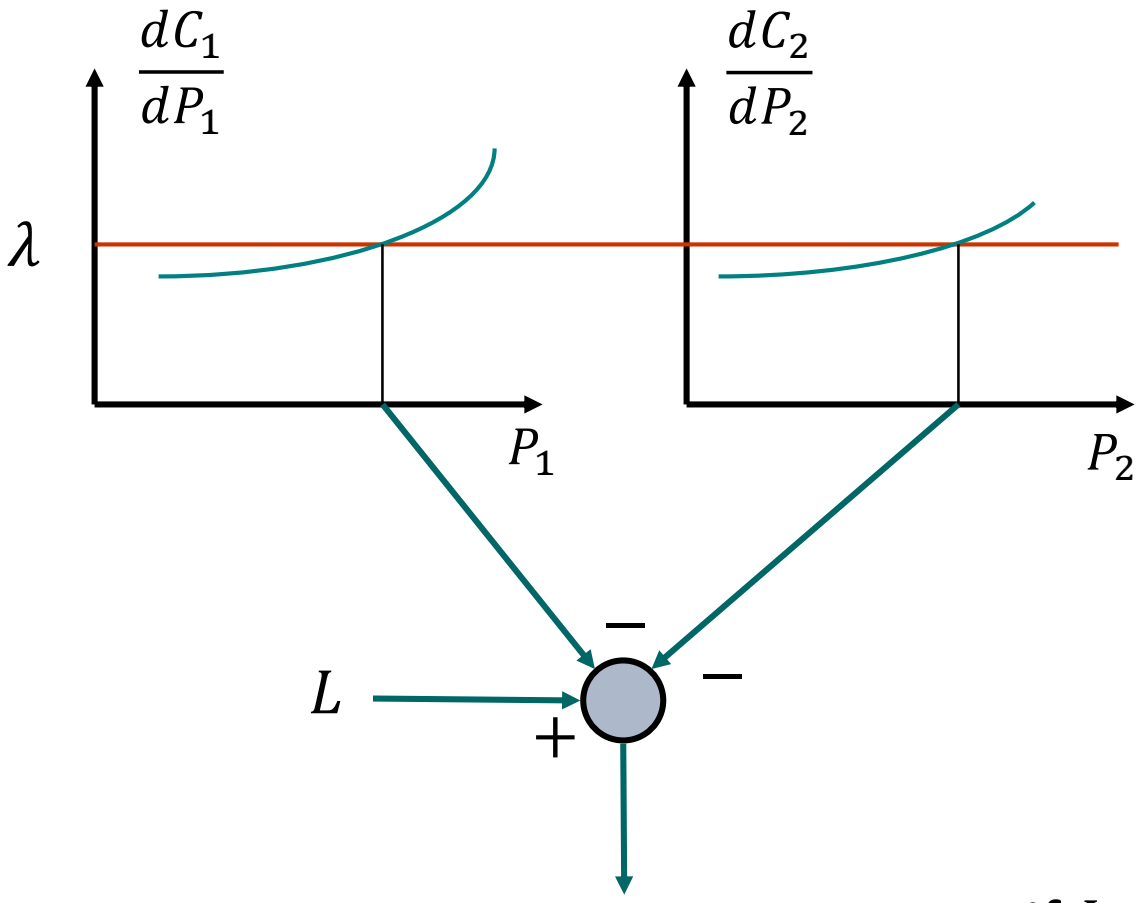
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Economic dispatch with linear cost curves



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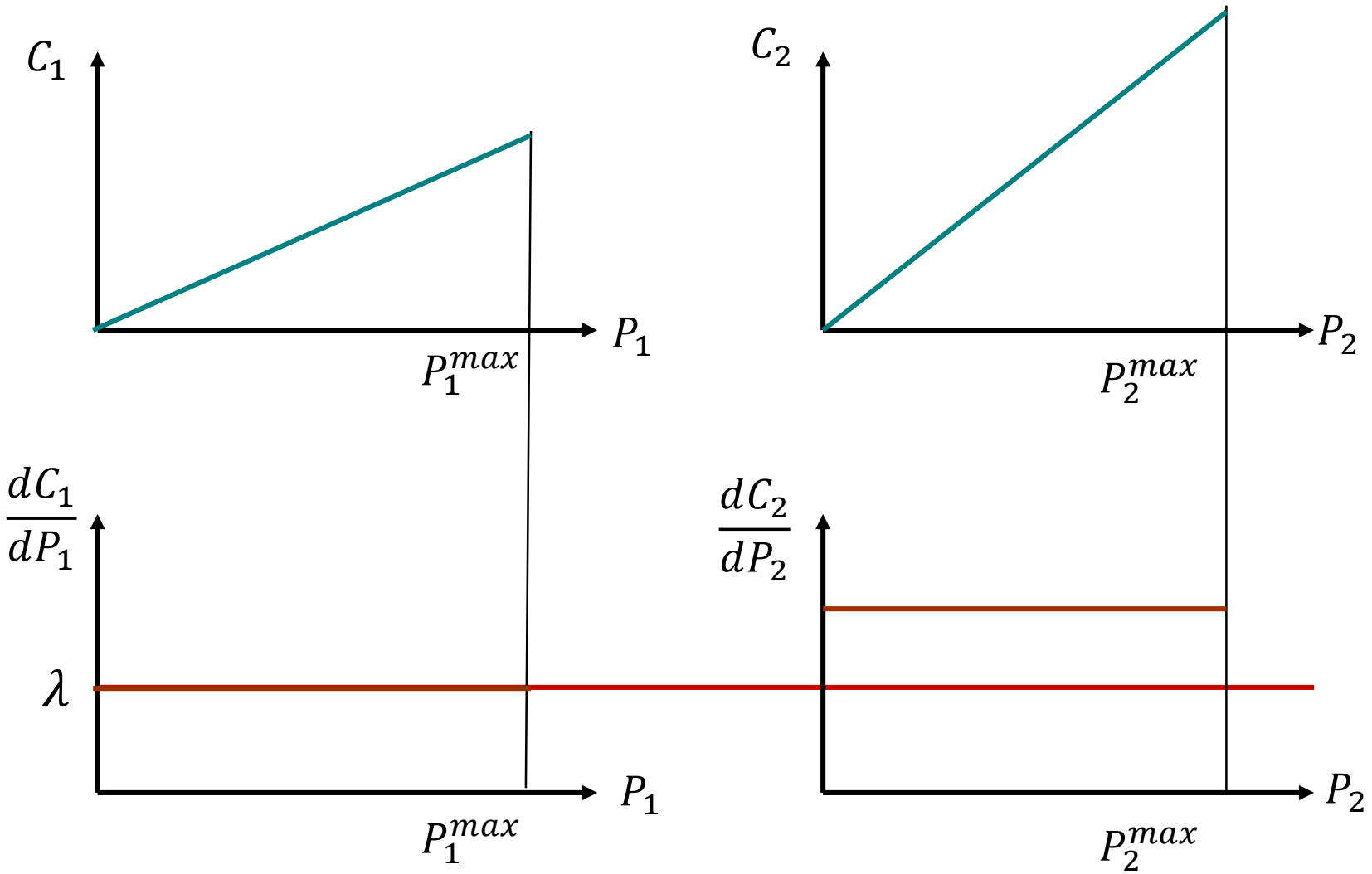
Visualisation of economic dispatch



$L - P_1 - P_2$ ⇒ If $L - P_1 - P_2 < 0$, reduce λ
If $L - P_1 - P_2 < 0$, increase λ

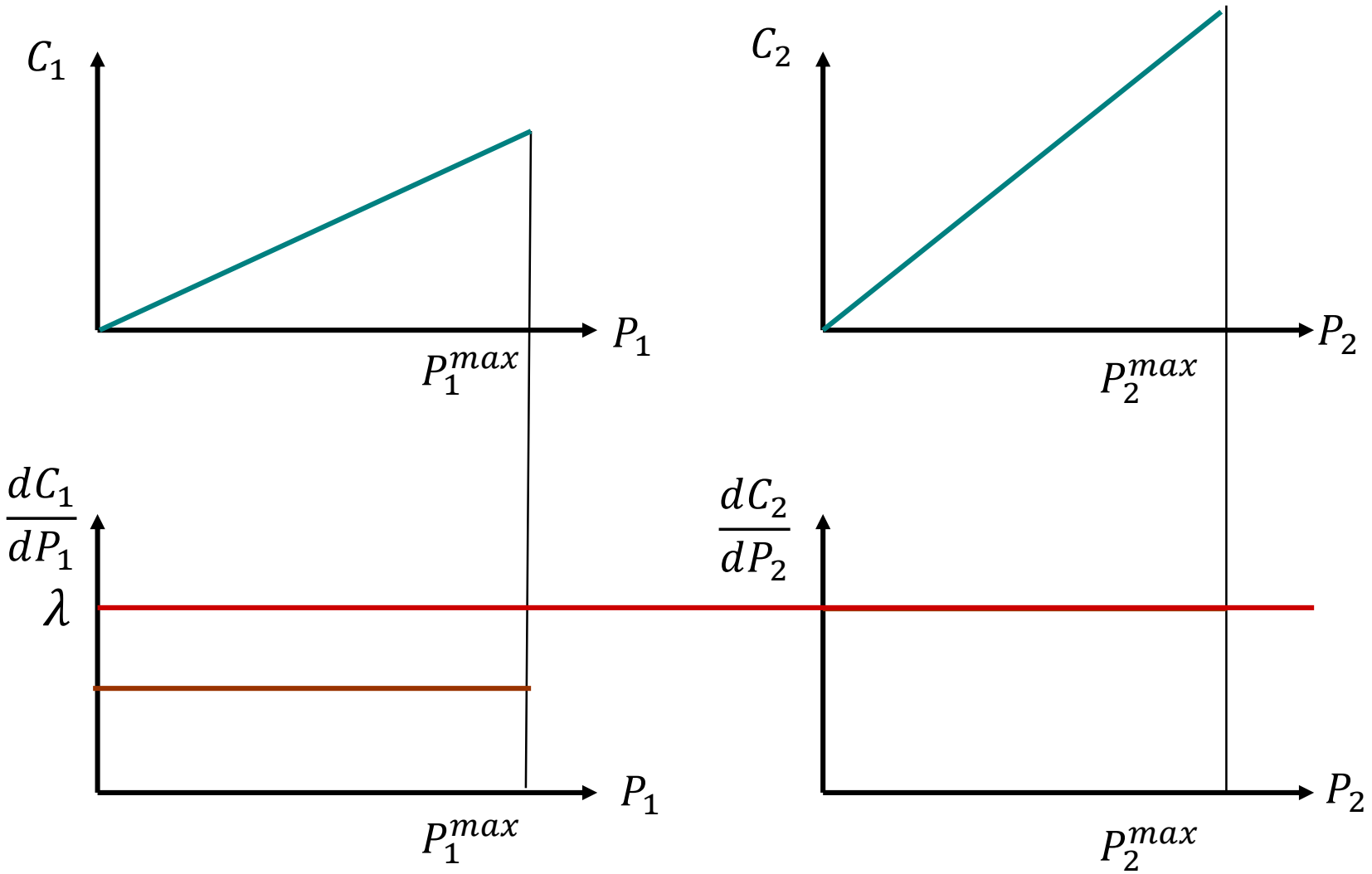
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Economic dispatch with linear cost curves



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Economic dispatch with linear cost curves



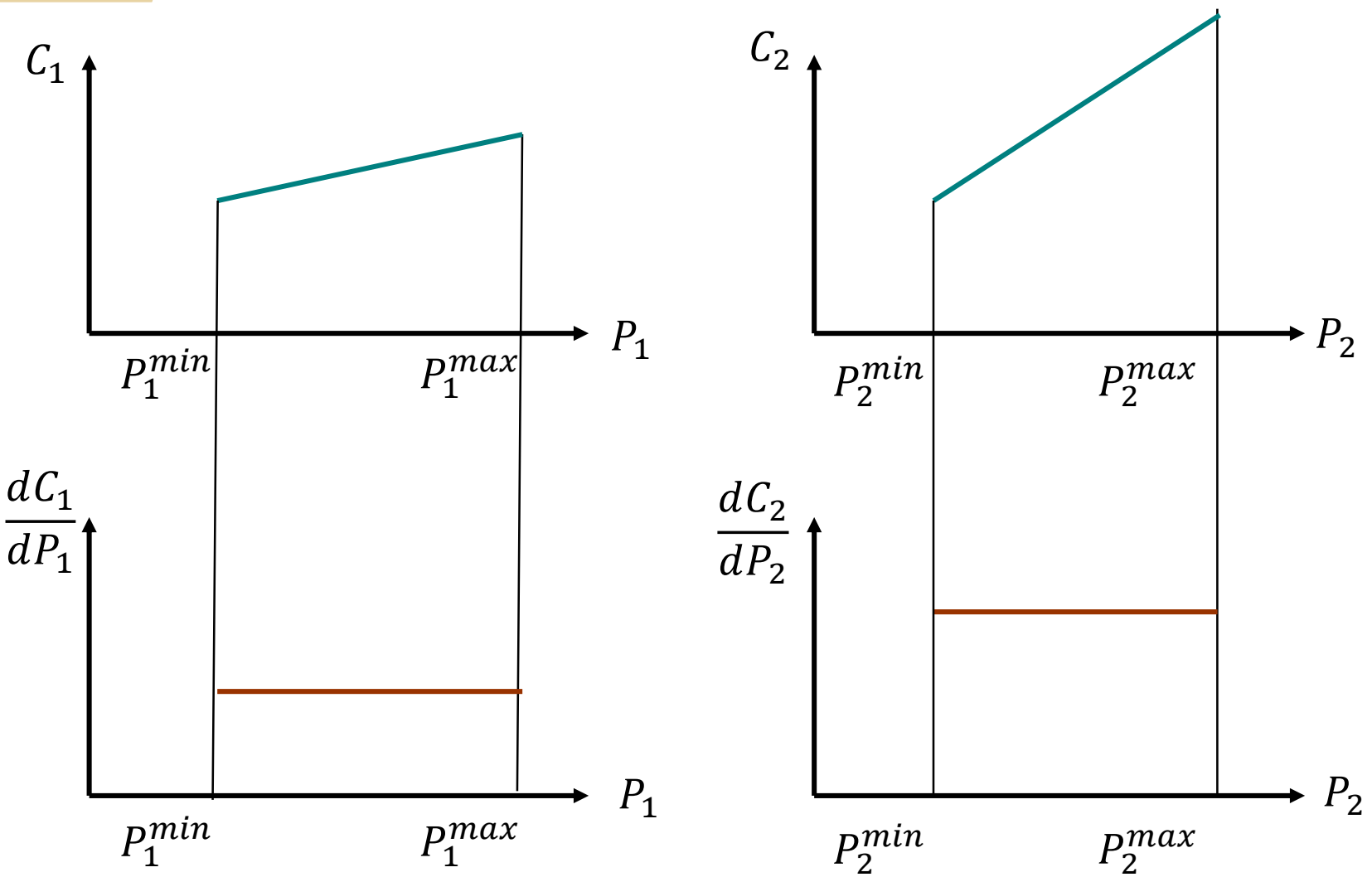
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Linear Cost Curves

- The output of the unit with the lowest incremental cost (in this case unit A) is increased first (it is “loaded” first)
- The output of the other units is not increased until unit A is loaded up to its maximum
- The output of the unit with second lowest incremental cost is then increased (i.e. the “second cheapest” unit)
- And so on until the generation meets the load

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Linear cost curves with minimum generation



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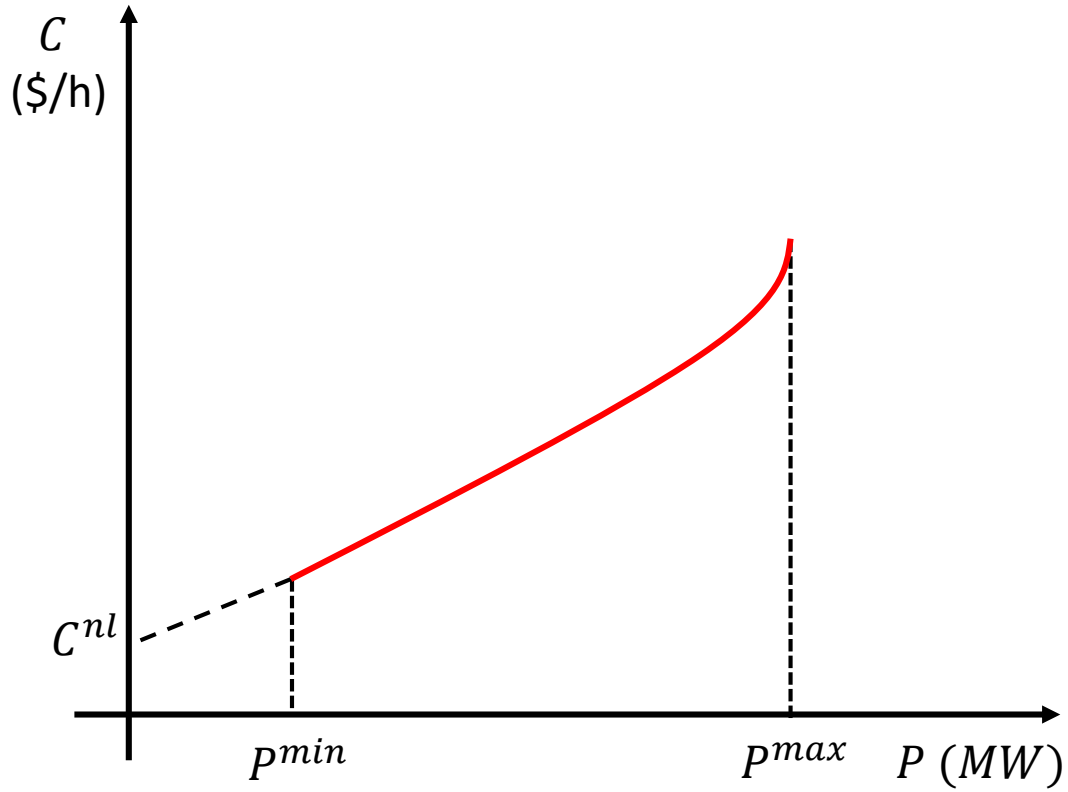
Linear cost curves with minimum generation

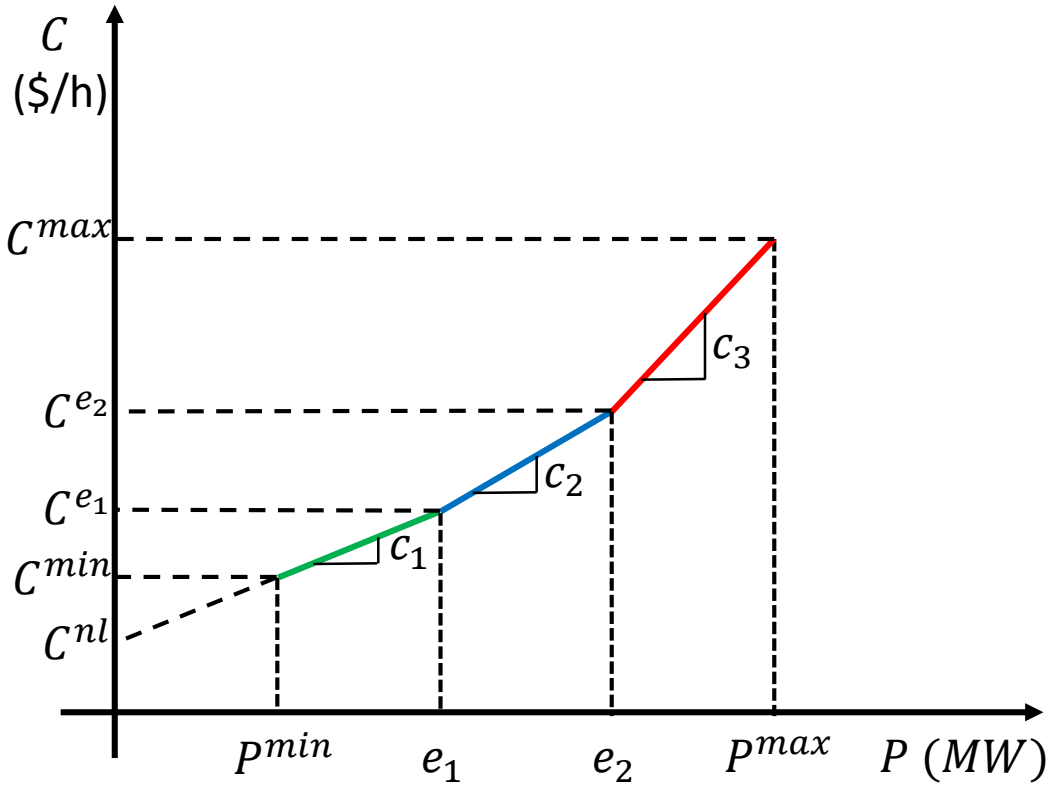
- All generating units are first loaded up to their minimum generation
- The generating units are then loaded in order of incremental cost

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Piecewise linear cost curves

- Quadratic cost curves are convenient when solving ED problems by hand
- Practical applications rely on piecewise linear cost curves





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Piecewise linear cost curves

$$C = C^{nl} + c^1 P \quad \forall P \subset \left[P^{min}, e_1\right]$$
$$C = C^{e_1} + c^2(P - e_1) \ \forall P \subset [e_1, e_2]$$
$$C = C^{e_2} + c^3(P - e_2) \ \forall P \subset [e_2, P^{max}]$$

Where:

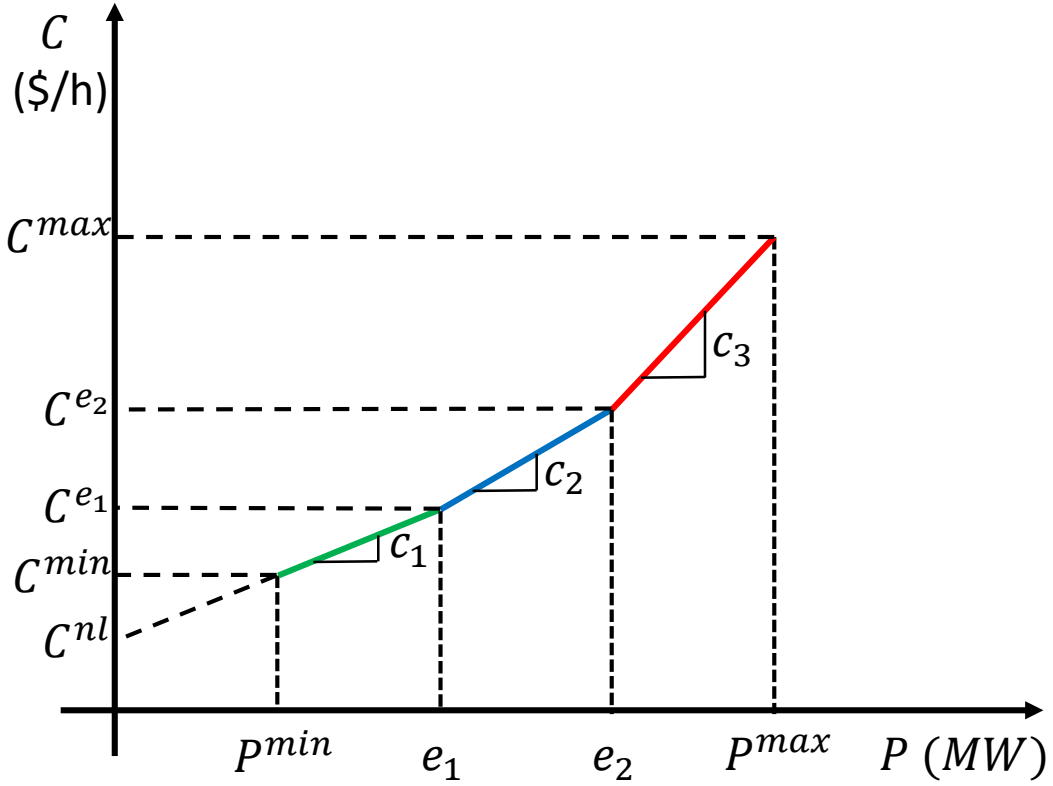
c^1: slope of the first segment

c^2 : slope of the second segment

c^3 : slope of the third segment

$$C^{e_1} = C^{nl} + c^1 e_1$$

$$C^{e_2} = C^{e_1} + c^2(e_2 - e_1)$$



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Example 6.7: Piecewise linear cost curves

$$C_1(P_1) = 2100 + 8P_1 + 0.1\ P_1^2$$
$$C_2(P_2) = 7200 + 7P_2 + 0.06\ P_2^2$$
$$C_3(P_3) = 6250 + 9P_3 + 0.07\ P_3^2$$

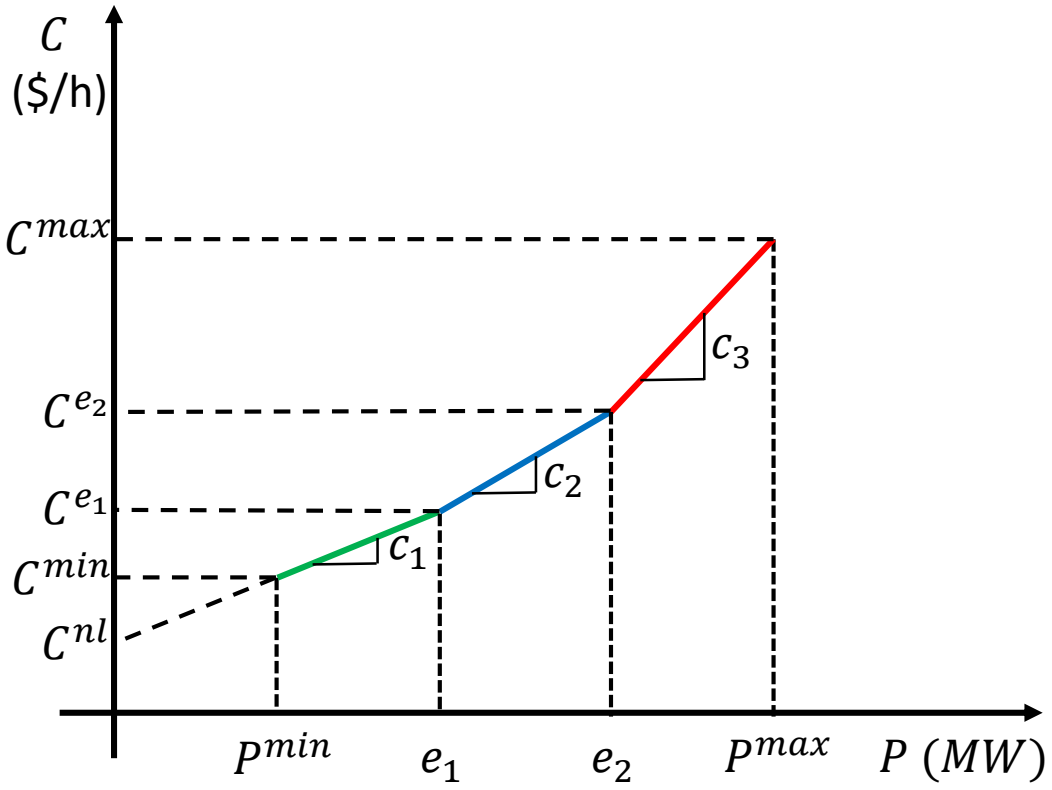
$$200 \leq P_1 \leq 500 \text{ MW}$$
$$200 \leq P_2 \leq 450 \text{ MW}$$
$$150 \leq P_3 \leq 500 \text{ MW}$$

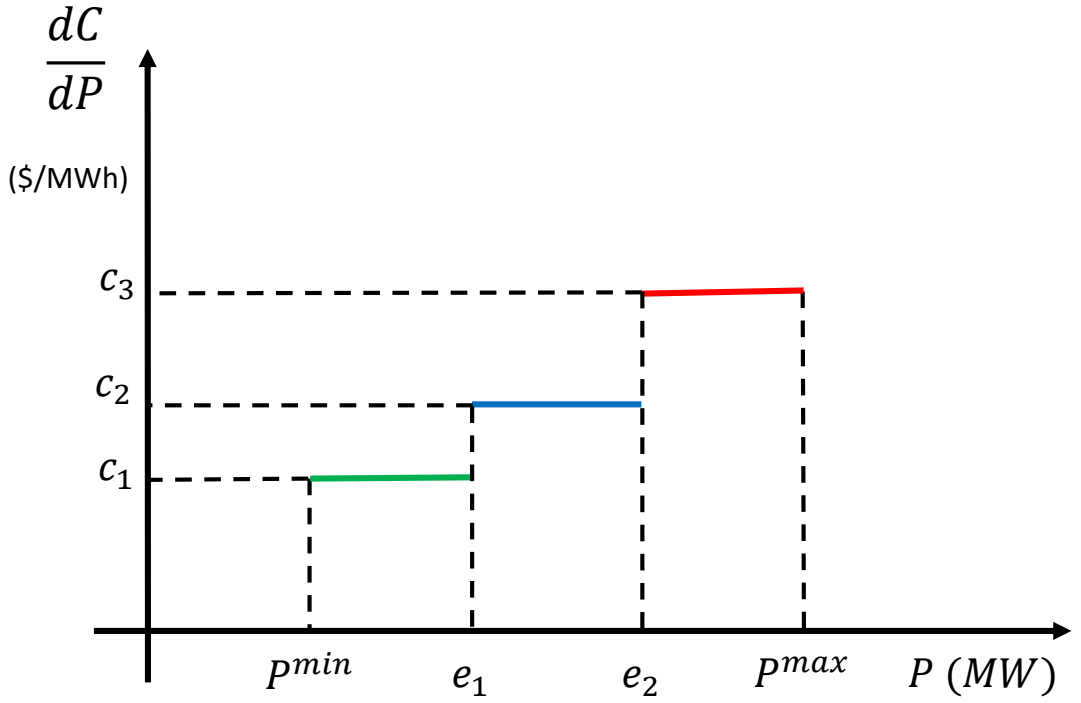
Arbitrary choice of elbow points e_1, e_2, e_3

	Unit 1	Unit 2	Unit 3
P^{min} (MW)	200	200	200
e_1 (MW)	300	300	300
e_2 (MW)	400	420	420
P^{max} (MW)	500	450	500
C^{min} ($/h)	7700	11000	10850
C^{e_1} ($/h)	13500	14700	15250
C^{e_2} ($/h)	21300	20724	22378
C^{max} ($/h)	31100	22500	28250
c^1($/MWh)	58.00	37.00	44.00
c^2 ($/MWh)	78.00	50.20	59.40
c^3 ($/MWh)	98.00	59.20	73.40

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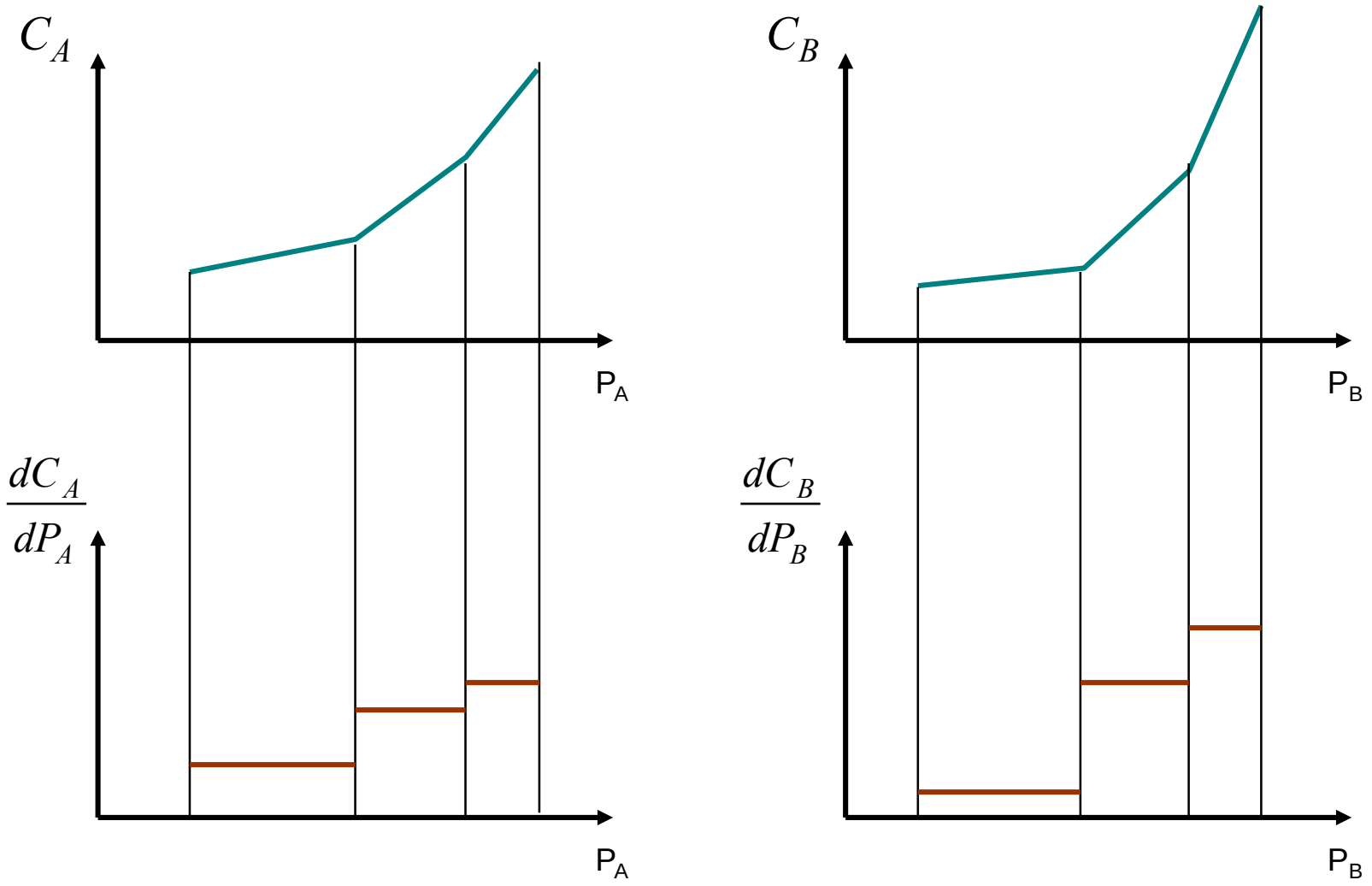
Piecewise constant marginal cost curves





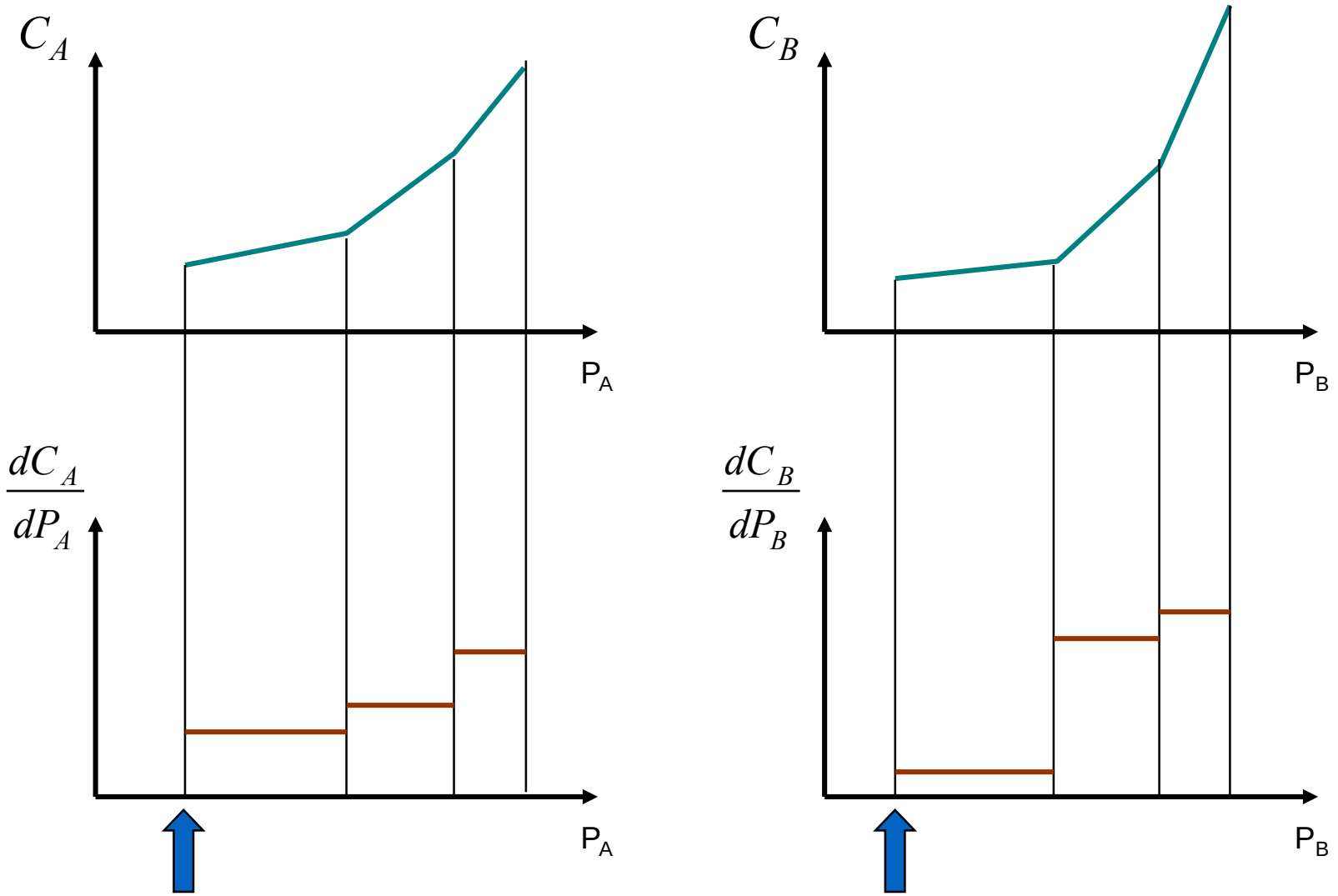
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Economic Dispatch with Piecewise Linear Cost Curves



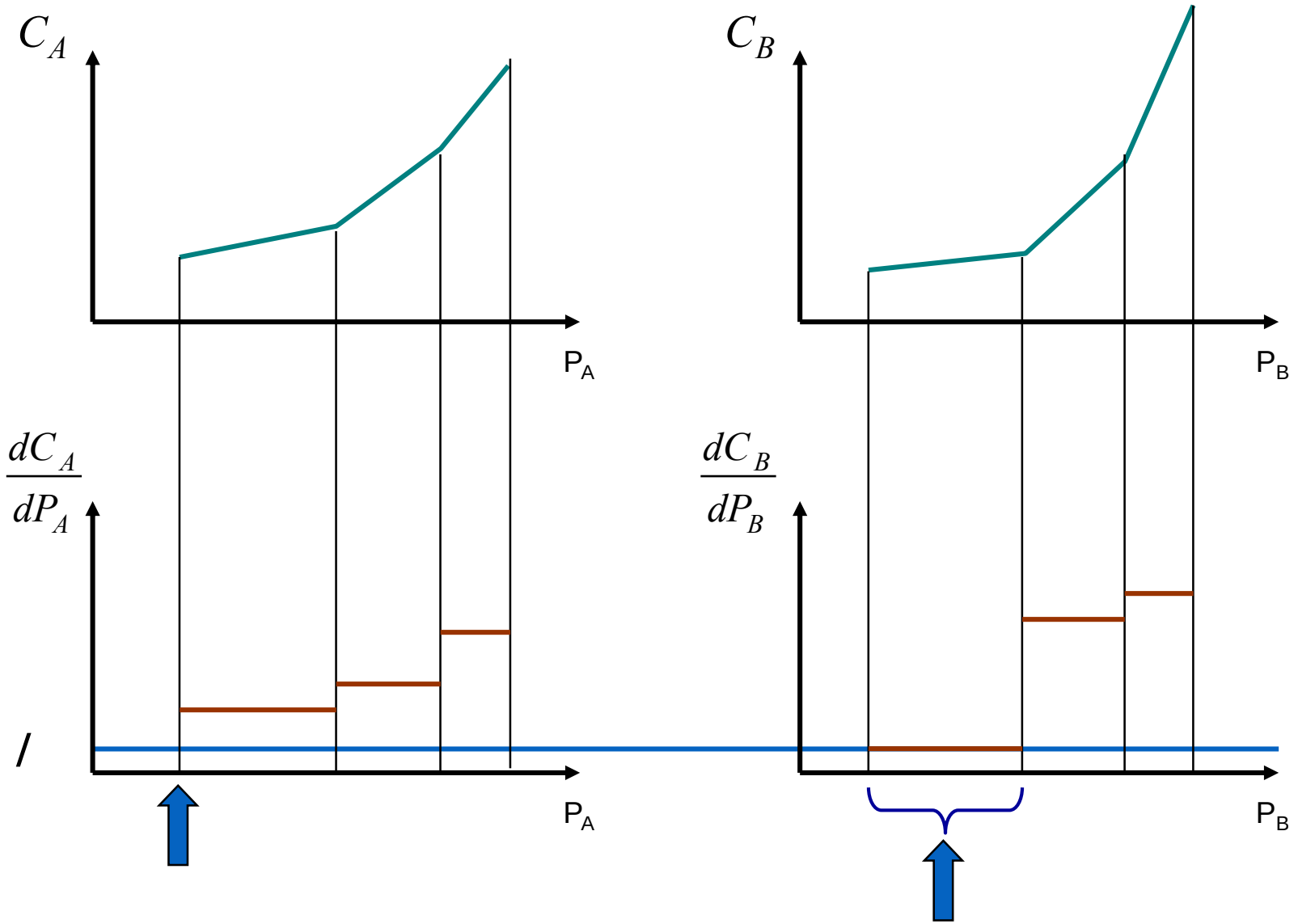
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Economic Dispatch with Piecewise Linear Cost Curves



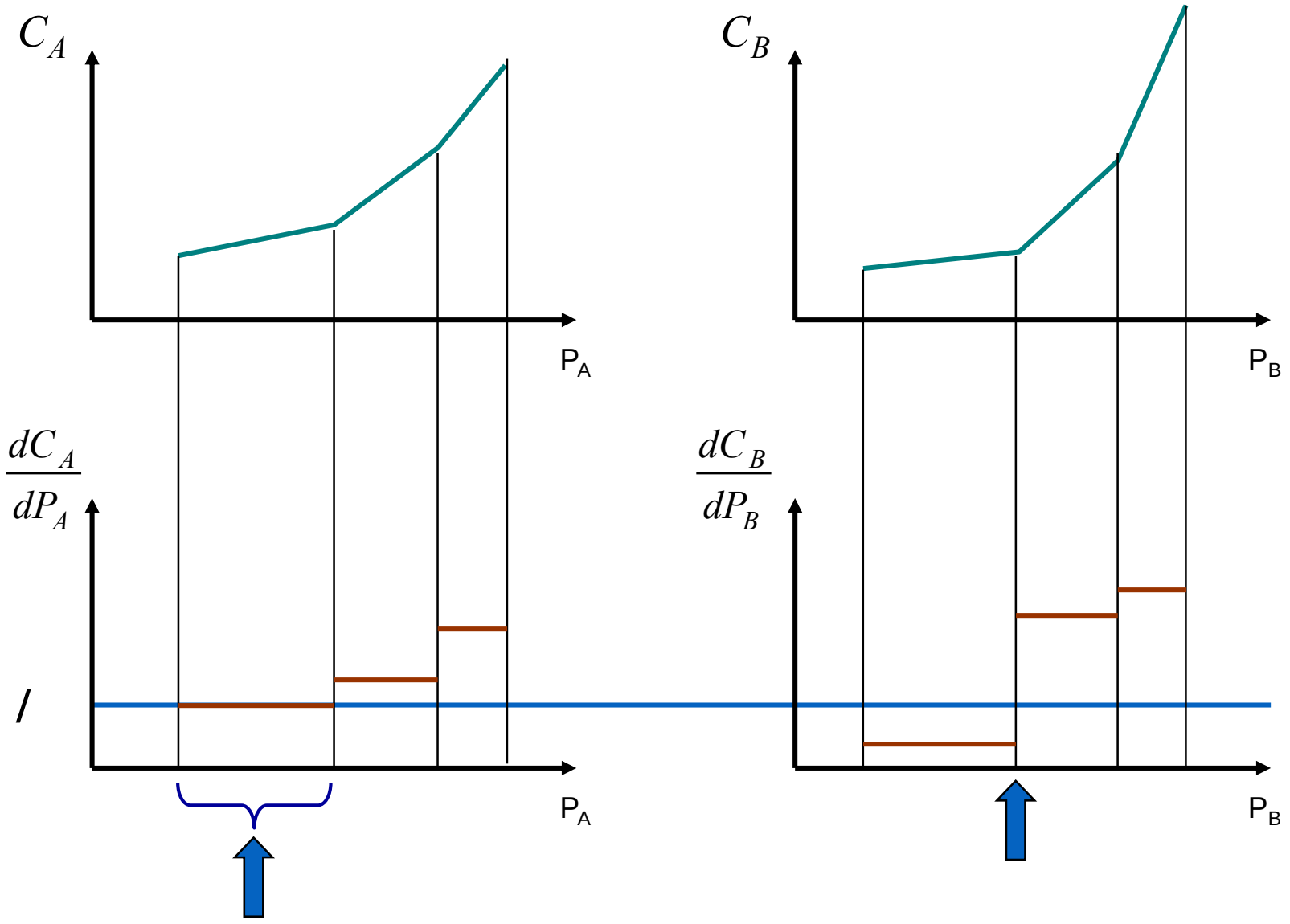
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Economic Dispatch with Piecewise Linear Cost Curves



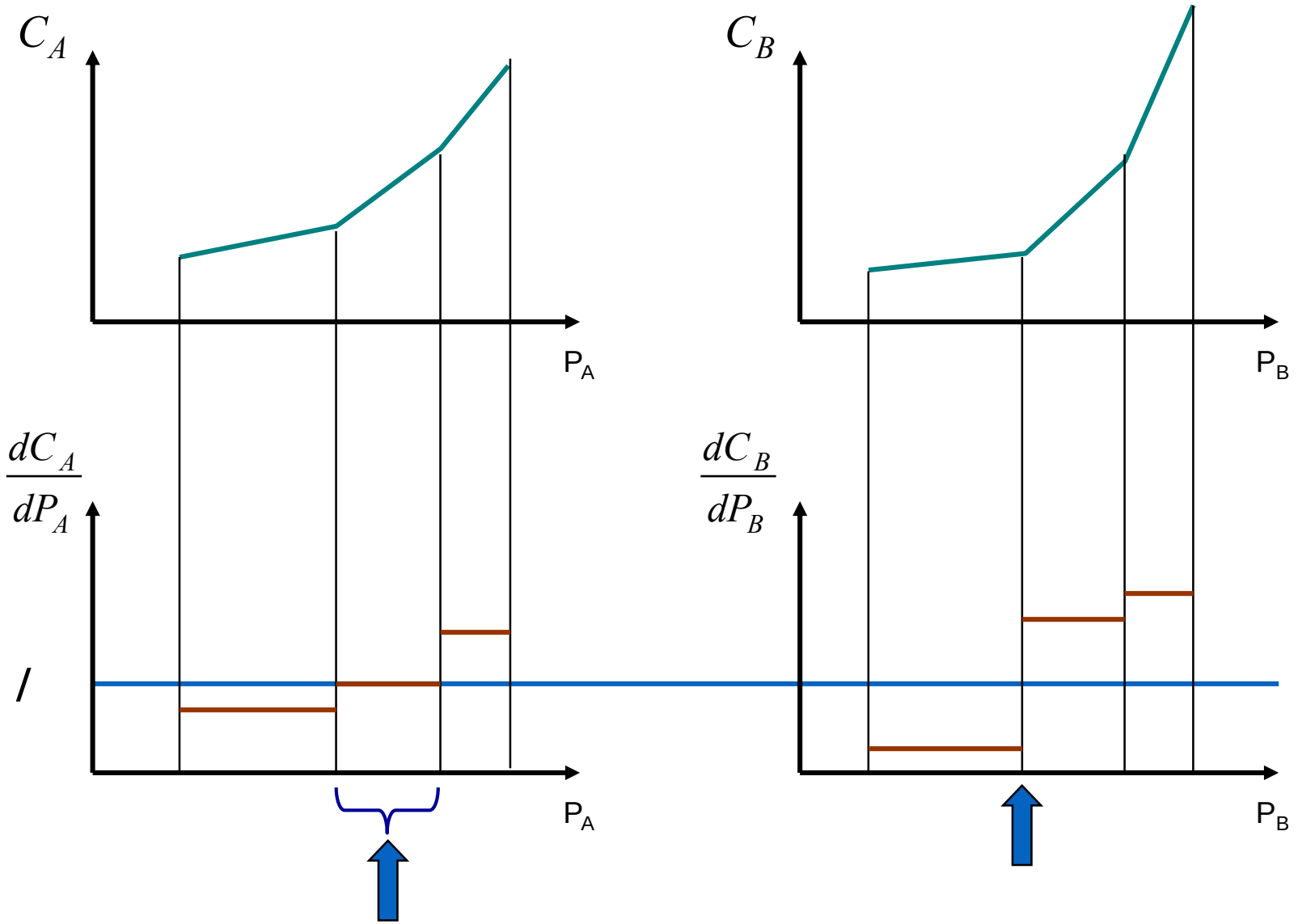
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Economic Dispatch with Piecewise Linear Cost Curves



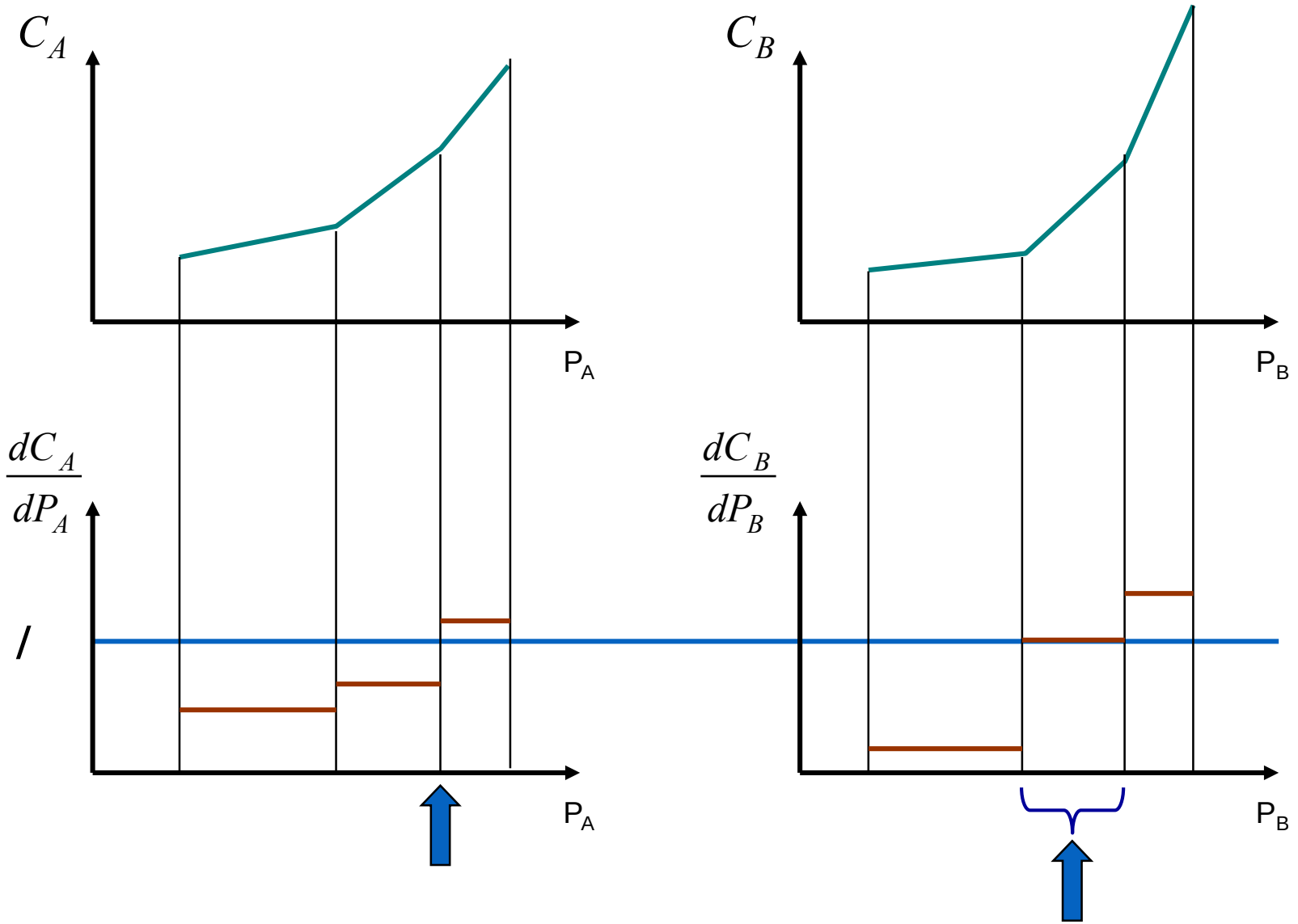
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Economic Dispatch with Piecewise Linear Cost Curves



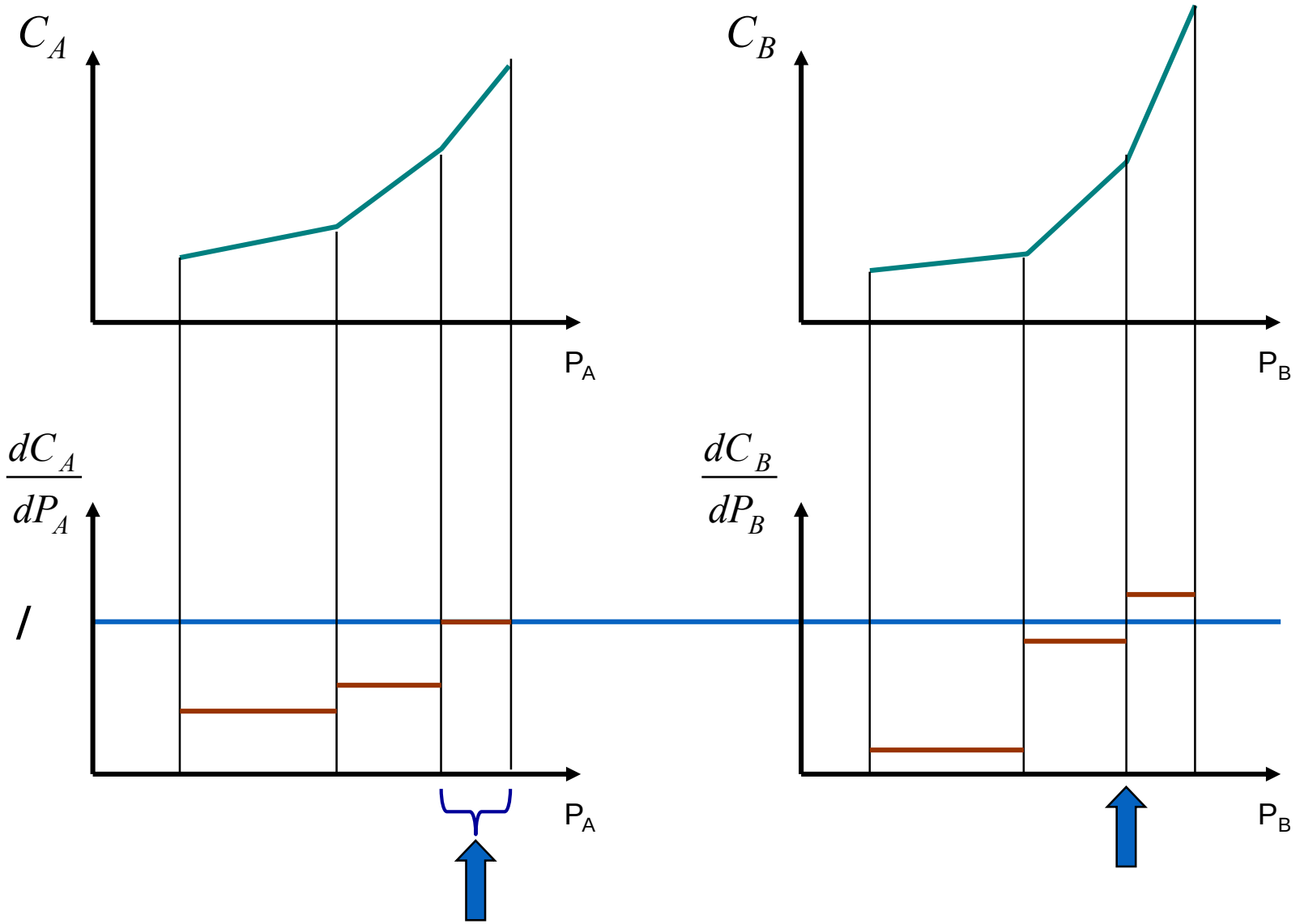
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Economic Dispatch with Piecewise Linear Cost Curves



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Economic Dispatch with Piecewise Linear Cost Curves



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ED algorithm using piecewise linear cost curves

1. Input the cost curve data of all the generating units
2. Create a list of all the segments of the cost curves of all the generators
3. Order this list in increasing order of marginal cost
4. Set the power output of all the generators at their respective minimum value
5. Set the residual load at the actual load minus the sum of these minimum generations
6. Take the first or next segment in the list
7. If the length of this segment is less than the residual load:
    a. Increase the output of the corresponding generator by the length of this segment
    b. Decrease the residual load by the length of this segment
    c. Go to step 6
8. Else
    a. Increase the output of the corresponding generator by the residual load
    b. Exit.

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ED algorithm using piecewise linear cost curves

ED with this algorithm leads to a solution where:

- All generators, except one, will be set at
  - P^{min} , or
  - P^{max}, or
  - one of the elbow points
- One generator operates somewhere along one of its segments
- This generator would provide the next MW of load
- This generator is called the marginal generator
- The slope of this segment is the system marginal cost.

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Example 6.8: ED using piecewise linear cost curves

Table with ordered segments for the units of Example 6.7

Segment number	1	2	3	4	5	6	7	8	9
Unit	2	3	2	1	2	3	3	1	1
Segment length	100	100	120	100	30	120	80	100	100
Marginal cost	37.0	44.0	50.2	58.0	59.2	59.4	73.4	78.0	98.0

Economic dispatch for a load of 800 MW

$$P_1^{min} + P_2^{min} + P_3^{min} = 600 \text{ MW}$$

Residual load is thus 200 MW → covered by the first two segments in the list

Economic dispatch: $P_1 = 200.00$ MW; $P_2 = 300.00$ MW; $P_3 = 300.00$ MW

Next MW of load would be provided by Unit 2 on its second segment
→ the system marginal cost is 50.2 $/MWh

$$C_{800} = C_1^{min} + C_2^{e_1} + C_3^{e_1} = 37{,}650 \text{ \$/h}$$

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Example 6.8: ED using piecewise linear cost curves

Table with ordered segments for the units of Example 6.7

Segment number	1	2	3	4	5	6	7	8	9
Unit	2	3	2	1	2	3	3	1	1
Segment length	100	100	120	100	30	120	80	100	100
Marginal cost	37.0	44.0	50.2	58.0	59.2	59.4	73.4	78.0	98.0

Economic dispatch for a load of 1200 MW

$$P_1^{min} + P_2^{min} + P_3^{min} = 600 \text{ MW}$$

Residual load is thus 600 MW → first six segments + 30 MW of seventh segment

Economic dispatch: $P_1 = 300.00$ MW; $P_2 = 450.00$ MW; $P_3 = 450.00$ MW

System operates on the seventh segment → the system marginal cost is 73.4 $/MWh

Unit 3 is the marginal unit

$$C_{1200} = C_1^{e_1} + C_2^{max} + C_3^{e_2} + c_3^3 \times 30 = \text{ 60,580 \$/h}$$

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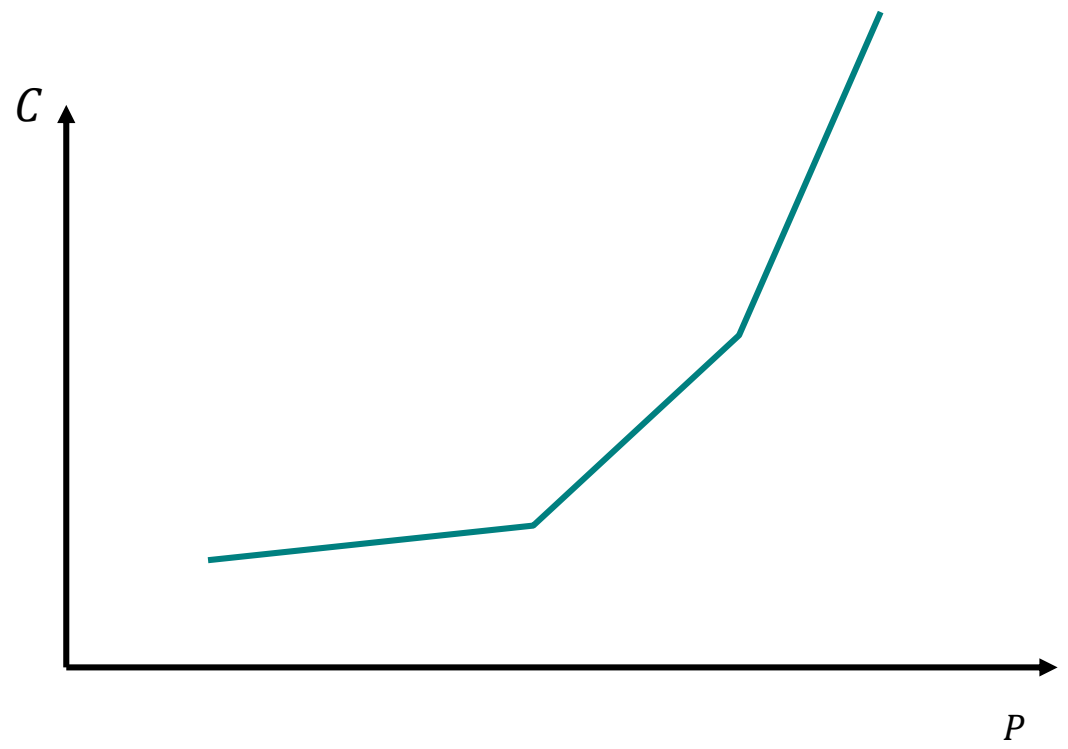
Load flexibility and storage

- Conventional ED:
  - Load is given
  - Minimize cost by adjusting the output of the generators
- Observation:
  - Marginal cost increases significantly with the load
  - A MW of load costs significantly more to produce when the load is high
- Could reduce the overall cost by shifting loads from period of high loads to period of light loads
- Use load flexibility and storage to achieve this

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Load flexibility

- Due to the nature of the cost curves, the total cost increases more than linearly with the load
- Shifting loads from periods of high demand to periods of low demand would thus reduce the total cost



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Example 6.9: Load shifting

- What is the effect of shifting 50 MW from the 1200 MW load period to the 800 MW load period with the units of Example 6.8?

- ED for an 850 MW load:

  $P_1 = 200.00$ MW; $P_2 = 350.00$ MW; $P_3 = 300.00$ MW

  $C_{850} = C_1^{min} + C_2^{e_1} + c_2^2 \times 50 + C_3^{e_1} = 40{,}160$ \$/h

- ED for an 1150 MW load:

  $P_1 = 300.00$ MW; $P_2 = 450.00$ MW; $P_3 = 400.00$ MW

  $C_{1150} = C_1^{e_1} + C_2^{max} + C_3^{e_1} + c_3^2 \times 100 = 57{,}190$ \$/h

- $C_{850} + C_{1150} = \$97{,}350 < C_{800} + C_{1200} = \$98{,}230$

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Example 6.10: Storage

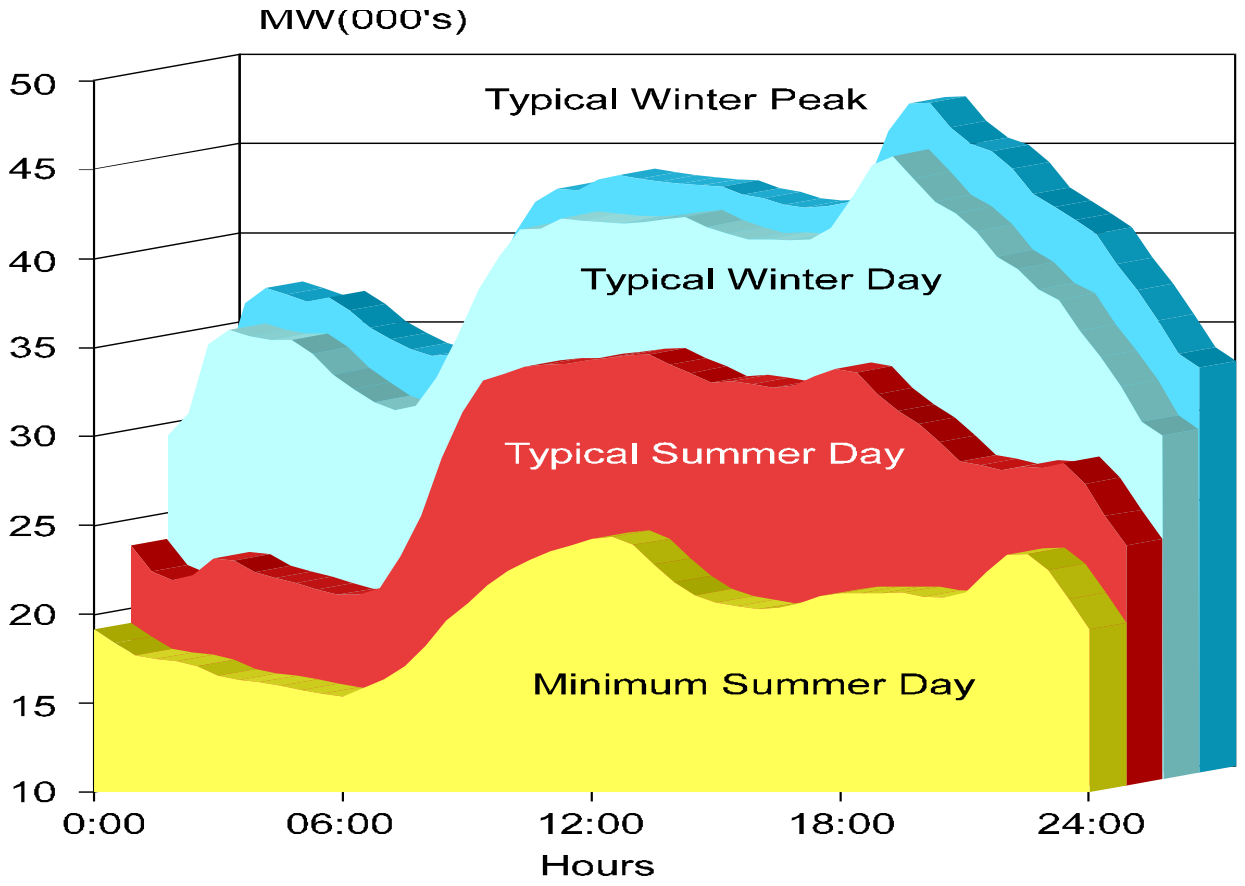
- What is the effect of storing 50 MWh during the low load period and releasing this energy during the high load period
- Use the units of Example 6.8 and assume a storage roundtrip efficiency of 80%
- Net load during the low load period: 800 + 50 = 850 MW
- From Example 6.9: $C_{850} = 40{,}160$ \$/h
- Available energy from the battery: $E_{battery} = 0.8 \times 50 = 40$ MWh
- Net load during the high load period: 1200 - 40 = 1160 MW
- ED for 1160MW: $P_1 = 300.00$ MW; $P_2 = 450.00$ MW; $P_3 = 410$ MW
- $C_{1160} = C_1^{e_1} + C_2^{max} + C_3^{e_1} + c_3^2 \times 110 = 57{,}784\$/\text{h}$
- $C_{850} + C_{1160} = \$97{,}944 < C_{800} + C_{1200} = \$98{,}230$
- Due to the efficiency of the storage, this is marginally profitable

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Unit Commitment (UC)

Unit commitment

- Periods of high load
  → need more generating units
- Periods of light load
  → need fewer generating units



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Why unit commitment?

- Questions:
  - How many generating units do we need at any given time?
  - Which set of units would be optimal?
  - When should each unit be synchronized to the grid?
  - When should each unit be de-synchronized from the grid?
- Approach:
  - Develop an intuitive understanding of the problem
  - Formulate the unit commitment problem as a mathematical optimization problem

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Example 6.11: How many units?

- Consider the generating units of Example 6.7
- Perform an ED for each of the combinations of these units and a load of 475MW

Unit 1 (MW)	Unit 2 (MW)	Unit 3 (MW)	Total Cost ($)	Marginal Cost ($/MWh)	
-	-	475	26,415.00	73.4	
-	X	-	Infeasible	-	← Infeasible because load is larger than P_2^{max}
475	-	-	28,650.00	98.0	
-	275	200	24,625.00	37.0	
200	275	-	21,475.00	37.0	← Lowest cost
200	-	275	21,850.00	44.0	
X	X	X	Infeasible	-	← Impossible because $P_1^{min} + P_2^{min} + P_3^{min} > 475\ MW$

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Marginal cost vs. average cost

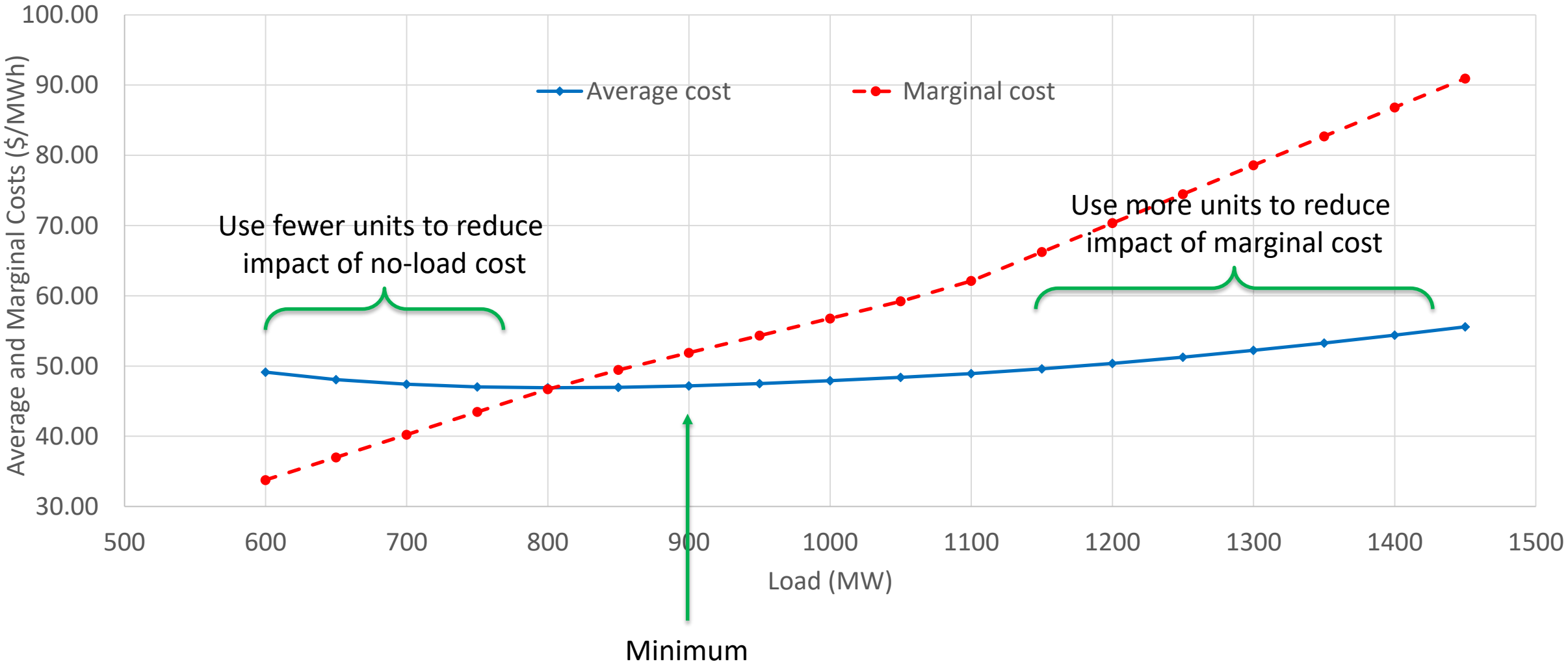
$$C_1(P_1) = 2100 + 8P_1 + 0.1\,P_1^2$$
$$C_2(P_2) = 7200 + 7P_2 + 0.06\,P_2^2$$
$$C_3(P_3) = \underbrace{6250}_{\text{No-load cost}} + \underbrace{9P_3 + 0.07\,P_3^2}_{\text{Variable cost}}$$

Marginal cost: $\frac{dC_1(P_1)}{dP_1} = \frac{dC_2(P_2)}{dP_2} = \frac{dC_3(P_3)}{dP_3} = \lambda$ ← Does not involve the no-load cost

Average cost: $\frac{C_1(P_1)+C_2(P_2)+C_3(P_3)}{L}$ ← Does involve the no-load cost

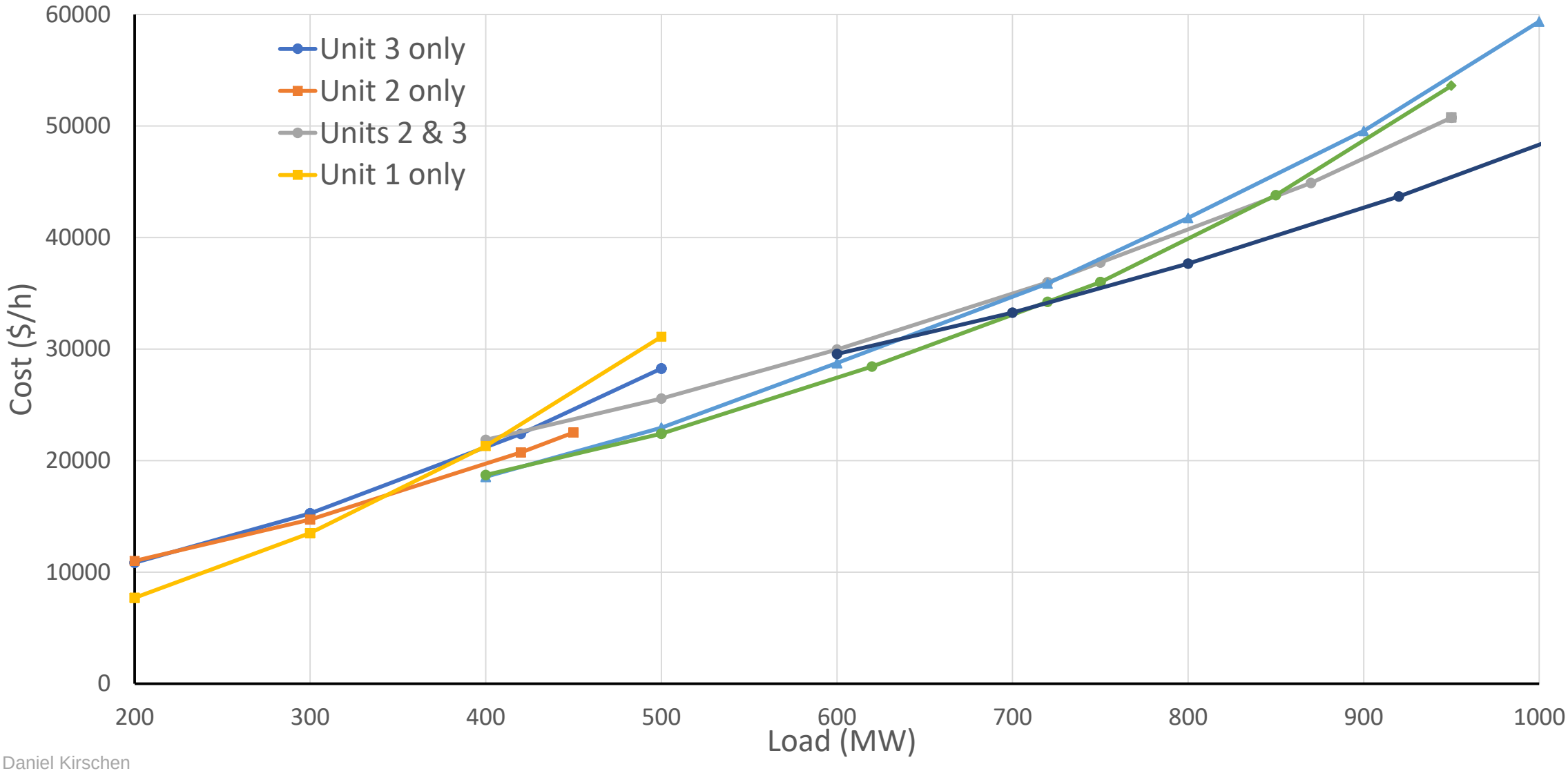
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Optimal marginal and average costs as a function of the load



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Total operating cost for different combinations of units



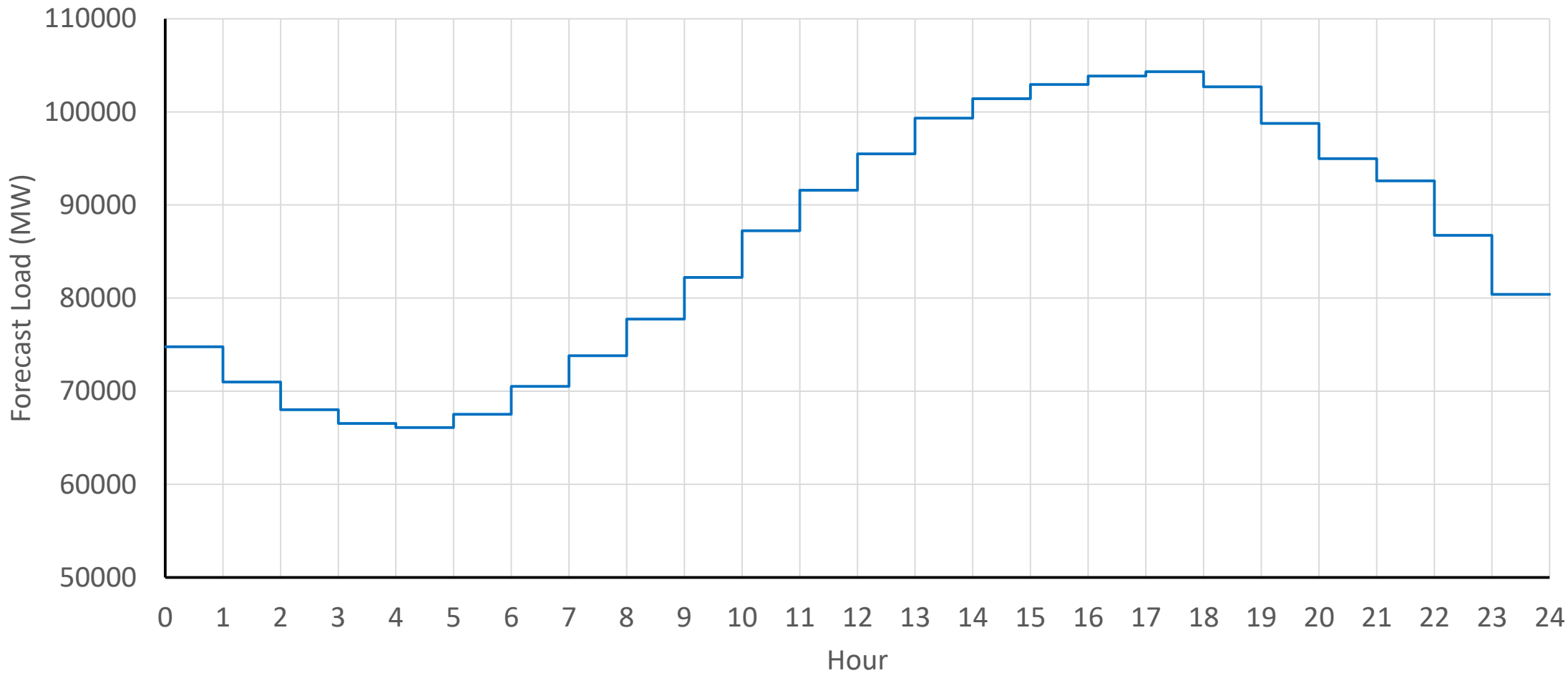
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Setting up the unit commitment

- Done daily for the next day
- Forecast the load profile for the next day
- Subtract from this forecast the expected generation from renewable sources
- Discretize this net load forecast profile in hourly time intervals
- Schedule the production of the conventional generating units to meet this profile at minimum cost

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Example of a 24-hour hourly net load forecast



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Startup cost of thermal generating units

- Thermal generating units must be “warmed up” before they can be synchronized
- A certain amount of fuel must be burned to do this
- Burning up this fuel costs money

→ Startup cost C^{st}

- C^{st} is incurred only when a unit is started
- C^{st} is not a function of the power produced
- Production cost $C_i(P_i)$ is a function of the power produced
- Production cost $C_i(P_i)$ includes the no-load cost C_i^{nl}
- C_i^{nl} is incurred every hour unit i is running
- C^{st} can be viewed as an investment in future power production

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Balancing startup and production costs

- Thermal unit with large C_i^{st} and low $C_i(P_i)$
  - Example: large coal or nuclear unit

  → Bring this unit online only if C_i^{st} can be amortized over the production of a lot of power

- Thermal unit with large C_i^{st} and low $C_i(P_i)$
  - Example: OCGT or diesel generator

  →Bring this unit online if a small amount of power is needed for a short time

→ Scheduling cannot be done for each period separately

→ Scheduling must be done over a time horizon, typically one day

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Formulating UC as an optimization problem

Objective function:

$$\min_{u_i(t),P_i(t)} \sum_{i\in G}\sum_{t\in T}\left[C_i^{st}(t) + u_i(t)\,.\,C_i(P_i(t))\right]$$

- G: Set of available thermal generating units
- T: Set of hourly time intervals included in the horizon
- $u_i(t)$: Status of unit i
  - $u_i(t) = 1$ if unit i is generating (i.e. "on") during period t
  - $u_i(t) = 0$ otherwise (i.e., "off")
- $P_i(t)$: Power produced by unit i during period t
- $C_i^{st}(t)$: Startup cost of unit i.
  - Non-zero only if unit i is turned on at time t i.e., $u_i(t) = 1$ and $u_i(t-1) = 0$
- $C_i(P_i(t))$: Cost of producing P_i for an hour with unit i,

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Formulating UC as an optimization problem

Load/generation balance constraint:

$$\sum_{i \in G} u_i(t) \,.\, P_i(t) = L(t) \quad \forall t \in T$$

Constraints on the output of generating units:

$$P_i^{min} \leq P_i(t) \leq P_i^{max} \quad \forall\, i \in G, \forall t \in T$$

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Formulating UC as an optimization problem

Ramp rate limits

To avoid potentially damaging temperature gradients in the turbine, changes in the power output must be limited between subsequent time intervals:

$$P_i(t) - P_i(t-1) \leq \Delta \mathrm{P}_i^{up} \quad \forall\, i \in G, \forall t \in T$$

$$P_i(t-1) - P_i(t) \leq \Delta \mathrm{P}_i^{down} \; \forall\, i \in G, \forall t \in T$$

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Formulating UC as an optimization problem

Minimum up- and down-time constraints

For similar reasons, if a thermal generating units is turned on, it should remain on for a minimum time T_{up}^{min}, and if it is turned off, it should remain off for a minimum number time T_{down}^{min} :

$$\text{If } \left\{u_i(t-1) = 1 \text{ and } \exists\ \tau\ > t - T_{up}^{min} \text{ such that } u_i(\tau) = 0\right\} \Rightarrow u_i(t) = 1$$

$$\text{If } \{u_i(t-1) = 0 \text{ and } \exists\ \tau > t - T_{down}^{min} \text{ such that } u_i(\tau) = 1\} \Rightarrow u_i(t) = 0$$

The ramp rate and minimum up- and down-time constraints create a linkage between the optimization at different time intervals.

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Decision variables

- Binary decision variables $u_i(t)$
  - Status of unit i (on or off) at every time interval t
- Continuous decision variables $P_i(t)$
  - Power produced by unit i at every time interval t

- Specify $u_i(t)$ for all units and time interval → perform ED to find the $P_i(t)$
- Theoretical optimum → must consider all combinations of $u_i(t)$
- N units → 2^N possible combinations of unit status at each time interval t
- T time intervals → $(2^N)^T$ combinations

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Example 6.12: How many combinations?

- $N = 30$
- $T = 24$
- $(2^N)^T = (2^{30})^{24} = 5.5 \times 10^{216}$
- Assessing one billion combinations per second would require 1.7×10^{200} years

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Solving the unit commitment problem

- Assessing ALL combinations to find the optimal one is clearly impossible
- UC is a mixed integer linear programming (MILP) problem
  - $u_i(t)$ are integer variables
  - $P_i(t)$ are continuous variables
- Solve the ED using piecewise linear cost curves → linear programming
- Use commercial-grade optimization software that implement ”branch and cut” algorithm to reduce the number of combinations that are considered
- Provides a solution that is close enough to the optimal solution

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Example 6.13: Unit commitment

- Example inspired by the documentation of the pypsa software tool
- Load profile:

Hour	1	2	3	4
Load (MW)	4000	800	5000	3000

- Parameters of generating units:

Unit Name	P^{min} (MW)	P^{max} (MW)	Marginal cost ($/MWh)
Hansel	3,000	10,000	20
Gretel	500	5,000	70

- Single segment cost curves
- Start by neglecting the no-load and startup costs

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Example 6.13: Unit commitment

- Base case solution:

Hour	1	2	3	4
P_{Hansel}	4000	0	5000	3000
P_{Gretel}	0	800	0	0

- Since Hansel is much cheaper and has enough capacity, it carries the entire load during hours 1, 3 and 4.
- However, the load at hour 2 is smaller than P^{min} of Hansel
  - Hansel must be shut down
  - Gretel must be brought online
  - At hour 3, Hansel is brought back online and Gretel is shutdown

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Example 6.13: Unit commitment

- Effect of a minimum up-time of three hours for Gretel:

Hour	1	2	3	4
P_{Hansel}	3500	0	4500	3000
P_{Gretel}	500	800	500	0

- Effect of a minimum down-time of two hours for Hansel:

Hour	1	2	3	4
P_{Hansel}	4000	800	500	0
P_{Gretel}	0	0	4500	3000

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Example 6.13: Unit commitment

- Adding constraints:
  - Changes the solution
  - Increases the operating cost

	Cost ($)
Base Case	296000
Add min uptime	346000
Add min down time	521000

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Example 6.13: Unit commitment

- Add a startup cost of $375,000 to to Hansel:

Hour	1	2	3	4
P_{Hansel}	0	0	0	0
P_{Gretel}	4000	800	5000	3000

- While Hansel is cheaper to run, it is not worth starting for such a short time

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Handling uncertainty

- Sources of uncertainty:
  - Load forecasting errors
    - Typical error less than 2%
    - Much larger errors can occur during unusual weather conditions
  - Renewable generation
    - Wind and solar generation can change rapidly with a weather front
  - “Behind the meter” solar generation is not monitored by operators
  - Large conventional generators can fail at any time
- Operators need resources to handle uncertainty
- Have generation capacity in reserve to handle load/generation imbalances

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Unit commitment with reserve

- Headroom:

$$P_i^{head}(t) = P_i^{max} - P_i(t)$$

- Total headroom must be larger than the reserve requirement:

$$\sum_{i \in G} u_i(t)\, P_i^{head}(t) = \sum_{i \in G} u_i(t)\, P_i^{max} - \sum_{i \in G} u_i(t)\, P_i(t) \geq R(t) \quad \forall t \in T$$

- Combine with load generation balance:

$$\sum_{i \in G} u_i(t)\, .P_i(t) = L(t) \quad \forall t \in T$$

- Reserve constraint:

$$\sum_{i \in G} u_i(t)\, P_i^{max} \geq L(t) + R(t) \quad \forall t \in T$$

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Reserve requirement $R(t)$

- Must be able to handle the largest credible contingency
- In a small system:
  - Loss of largest committed unit

  → $R(t)$ = capacity of this unit
- In a large system:
  - Nearly simultaneous loss of multiple units or large decrease in renewable generation

  → $R(t)$ determined using probabilistic calculations
- Committed generation must also be able to ramp up or down sufficiently fast
- Including these constraints in UC often forces the commitment of additional units
  - Increase in operating cost
- Can also use voluntary load shedding or battery energy storage for reserve

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Example 6.14: Unit commitment with reserve

Unit Name	P^{min} (MW)	P^{max} (MW)	Marginal cost ($/MWh)	Startup cost ($)	Initial state
Alpha	10	20	10	100	ON
Beta	5	30	12	100	ON
Gamma	5	50	13	100	OFF
Delta	10	20	20	0	OFF

Hour	1	2	3	4
Load (MW)	30	40	60	50

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Example 6.14: Unit commitment with reserve

UC solution without reserve constraint:

Hour	1	2	3	4
P_{Alpha}	20	20	20	20
P_{Beta}	10	20	30	30
P_{Gamma}	0	0	0	0
P_{Delta}	0	0	10	0

Total headroom:

Hour	1	2	3	4
Headroom	20	10	10	0

Not enough headroom to meet a reserve requirement of 20 MW at hours 2, 3, and 4

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Example 6.14: Unit commitment with reserve

UC solution without 20 MW reserve constraint:

Hour	1	2	3	4
P_{Alpha}	20	20	20	20
P_{Beta}	10	10	30	25
P_{Gamma}	0	10	10	5
P_{Delta}	0	0	0	0

Bring Gamma online to meet the reserve requirement. No longer need Delta.

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Example 6.14: Unit commitment with reserve

Enforce a maximum ramp up rate ΔP_i^{up} of 20 MW/h on Alpha, Beta, and Delta, and of 5 MW/h on Gamma:

Hour	1	2	3	4
P_{Alpha}	15	20	20	20
P_{Beta}	10	10	15	25
P_{Gamma}	5	10	15	5
P_{Delta}	0	0	10	0

Adding constraints increases the cost of the schedule:

	Cost ($)
Base Case	2080
Add reserve constraint	2125
Add max ramp rate constraint	2225

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