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G H RAISONI INSTITUTE OF ENGINEERING & TECHNOLOGY, NAGPUR

Department of Mechanical Engineering

PPT On

"Heat Transfer"

For B.E & B. Tech in Mechanical Engineering

By

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Assistant Professor
Mechanical Engineering Department
GHRIET, Nagpur (India)

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Syllabus- Heat Transfer

BEME504T: HEAT TRANSFER (Theory)

CREDITS: 04

Teaching Scheme
Lectures: 3 Hours/Week
Tutorial: 1 Hour/Week

Examination Scheme
Duration of Paper: 03 Hours
University Assessment: 80 Marks
College Assessment: 20 Marks

Course Objectives and Expected Outcomes:

- This course is designed to learn the various modes of heat transfer and laws associated with it.
- During this course, students can distinguish between steady state and unsteady state heat transfer; will be able to apply their knowledge of Dimensional Analysis to forced and free convection.
- Students will also be able to analyse radiation with and without radiation shield. Apart from this, students will also be able to analyse & design heat exchangers.

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Syllabus- Heat Transfer

UNIT – I [8 Hrs.]

Introduction to basic modes of heat transfer, conduction, convection & radiation. Laws of heat transfer & conservation of energy requirement. General heat conduction equation in cartesian, cylindrical and spherical coordinates. One dimensional steady state heat conduction equation for the plane wall, cylinder and sphere, overall heat transfer coefficient. Thermal resistance of composite structure, contact resistance, variable thermal conductivity, critical thickness of insulation.

UNIT – II [8 Hrs.]

Conduction with internal heat generation for plane wall, cylinder and sphere. Extended surface, types of fins. Fins of uniform cross section area, temperature distribution and heat transfer rate, fin efficiency & effectiveness. Error in temperature measurement. Unsteady state heat transfer, lumped heat capacity analysis, Heisler's charts. Biot Number, Fourier's Number & its significance.

UNIT – III [8 Hrs.]

Forced convection, physical significance of non-dimensional parameter. Flow of high, moderate & low Prandtl fluid flow over a flat plate. Concept of hydrodynamics & thermal boundary layer thickness, local and heat transfer coefficient. Empirical co-relations for external, internal flows, laminar & turbulent flow conduits. Dimensional analysis applied to forced convection.

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Syllabus- Heat Transfer

UNIT – IV [8 Hrs.]

Free or natural convection. Grashoff's number, Rayleigh number, flow over horizontal and vertical plate, Empirical Co-relations for cylinders and spheres, heat transfer with phase change, pool boiling curve & regimes of pool boiling, Film & Drop wise condensation, laminar film condensation on vertical surface, on horizontal tubes, effect of super heated & non-condensable gases on condensation heat transfer, Dimensional analysis applied to free or Natural convection.

UNIT – V [8 Hrs.]

Radiation, spectrum of radiation, black body radiation, radiation intensity, laws of radiation-Kirchoffs, Plancks, Weins displacement law, Stefan Boltzmann & Lamberts Co-sine law. Emissivity, Absorbtivity, Transmissivity, Reflectivity, Radiosity, Emissive power, Irradiation. Radiation network, radiation exchange between parallel plate cylinder & sphere, shape factor & its laws, radiation between parallel plates, cylinder & spheres. Radiation shields.

UNIT – VI [8 Hrs.]

Heat exchanger : Classification, overall heat transfer coefficient, fouling factor, LMTD & effectiveness, NTU method
:changer analysis for parallel, counter flow & cross flow arrangement, design aspect of heat exchangers,
on to compact heat exchanger, Heat Pipe, Introduction to mass transfer.

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Syllabus- Heat Transfer

TEXT BOOKS:

1. Heat Transfer, J.P. Holman, McGraw Hill Book Company, New York.
2. Fundamentals of Heat and Mass Transfer, K. N. Seetharam & T.R. Seetharam, Willey.
3. A Text Book of Heat Transfer, S.P. Sukhatme, University Press.

REFERENCE BOOKS:

1. Fundamentals of Heat and Mass Transfer, Venkanna B.K., PHI Publication.
2. Principles of Heat Transfer, Frank Kreith, Harper and Row Publishers, New York.
3. Heat Transfer - A Practical Approach, Yunus A. Cengel, Tata McGraw Hill Publishing Company Ltd., New Delhi.
4. Heat & Mass Transfer, M.N. Ozisik, Tata McGraw Hill Publishing Company Ltd., New Delhi.
5. Heat & Mass Transfer, R.K. Rajput, Laxmi Publication.

DATA BOOK:

Heat and Mass Transfer Data Book_6th edition_by C.P. Kothandaraman & S. Subramanyan.
Heat & Mass Transfer, Domkundwar, Dhanapat Rai & Sons Publication

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Course Outcomes [COs]- Heat Transfer

After the completion of course student will be able to

1. **Understand and solve conduction, convection and radiation problems**
2. **Analyze steady state and unsteady state heat transfer and internal heat generation**
3. **Apply their knowledge of Dimensional Analysis to forced convection**
4. **Apply their knowledge of Dimensional Analysis to free convection**

Analyze radiation heat transfer for with & without radiation shield

Analyze, Solve & Design heat exchangers

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Unit No.1

Introduction of Heat Transfer, Conduction, Convection and Radiation

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What is Heat & Heat Transfer

"*The energy in transit is termed* **heat**"

- Heat always moves from a warmer place to a cooler place.
- Hot objects in a cooler room will cool to room temperature.
- Cold objects in a warmer room will heat up to room temperature.

What is Temperature ?

Temperature is a measure of how hot or cold something is; specifically, a measure of the average kinetic energy of the particles in an object, which is a type of energy associated with motion.

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Differentiate between Thermodynamics and Heat Transfer

- ✓ "**Thermodynamics**" deals with the amount of energy in form of **heat** or work during a process and only considers the end states in equilibrium.

- ✓ "**Heat Transfer**" deals with the rate of energy **transfer** thus, it gives idea of how long a **heat transfer** will occur? **Heat transfer** deals with time and non equilibrium phenomena.



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The areas *covered* under the discipline of *heat transfer* are :

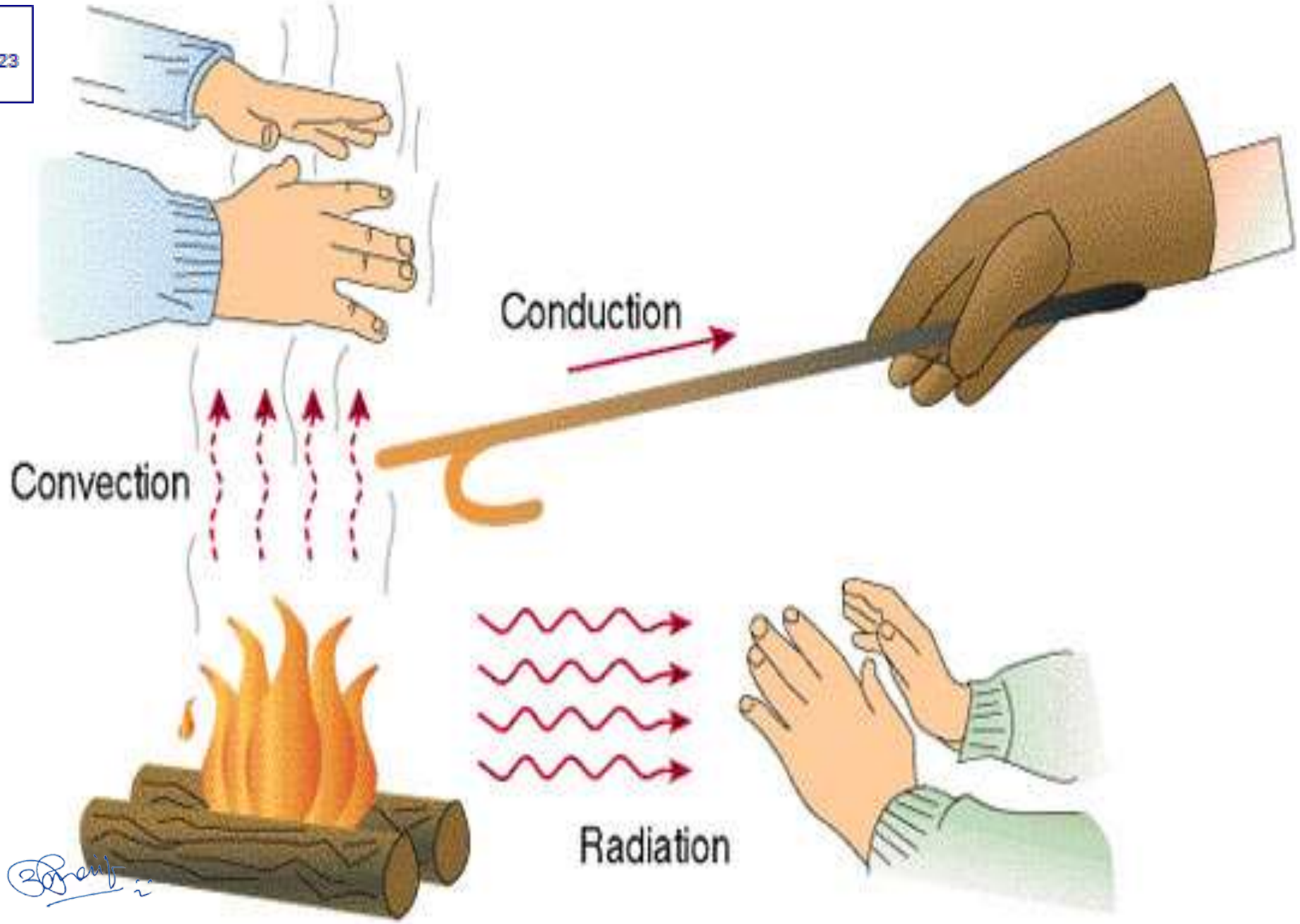
- Design of thermal and nuclear power plants including heat engines, steam generators, condensers and other heat exchange equipments, catalytic converters, heat shields for space vehicles, furnaces, electronic equipments etc.
- Internal combustion engines.
- Refrigeration and air conditioning units.
- Design of cooling systems for electric motors, generators and transformers.
- Heating and cooling of fluids etc. in chemical operations.
- Construction of dams and structures; minimisation of building-heat losses using improved insulation techniques.
- ermal control of space vehicles.
- at treatment of metals.
- spersion of atmospheric pollutants.

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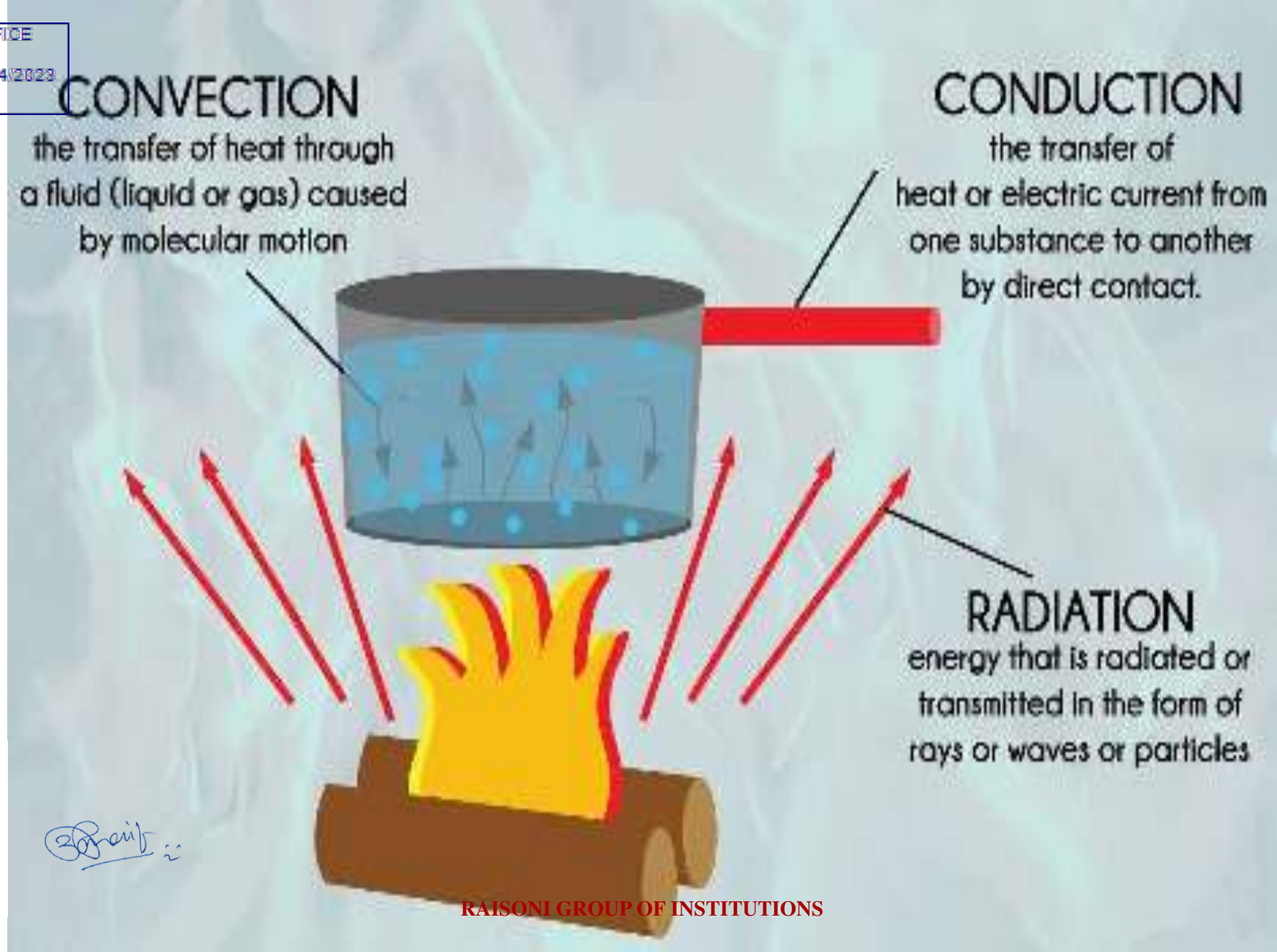
Heat Transfer Methods

- Heat transfers in three ways:
  - Conduction
  - Convection
  - Radiation

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Question

- If a cup of coffee and a Ice Cube were left on the table in this room what would happen to them? Why?

✓ The cup of coffee will cool until it reaches room temperature.

✓ The Ice Cube will melt and then the liquid will warm to room temperature.

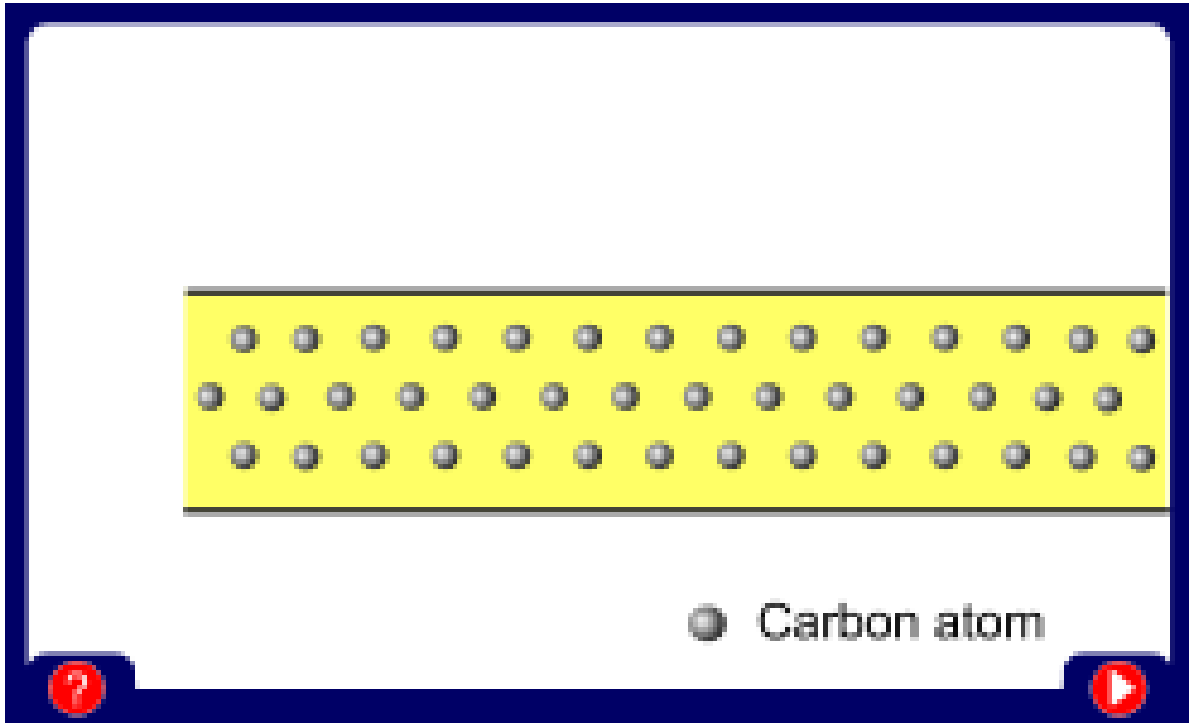
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Conduction Heat Transfer



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When you heat a metal strip at one end, the heat travels to the other end.



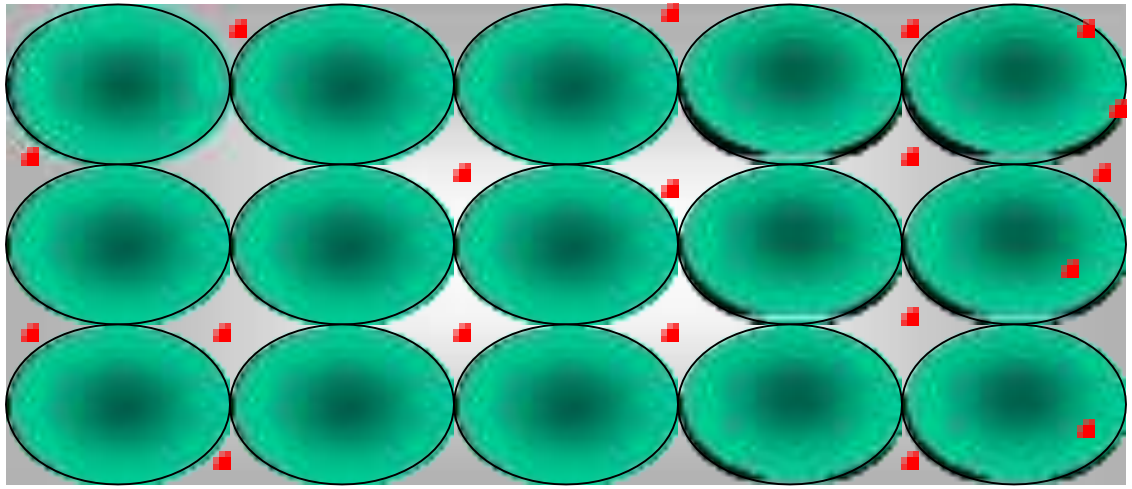
As you heat the metal, the particles vibrate, these vibrations make the
acent particles vibrate, and so on and so on, the vibrations are passed
ıg the metal and
so is the heat. We call this? **Conduction**

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Metals are different

The outer electrons of metal atoms drift, and are free to move.

When the metal is heated, this 'sea of electrons' gain kinetic energy and transfer it throughout the metal.

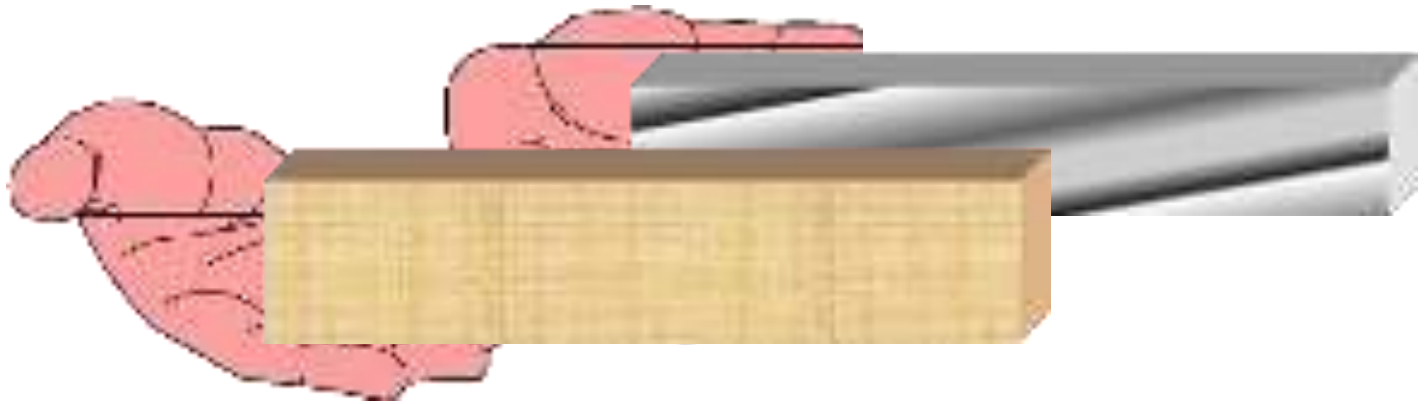


Insulators, such as wood and plastic , do not have this 'sea of electrons' which is why they do not conduct heat as well as metals.

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Why does metal feel colder than wood, if they are both at the same temperature?

Metal is a conductor, wood is an insulator. Metal conducts the heat away from your hands. Wood does not conduct the heat away from your hands as well as the metal, so the wood feels warmer than the metal.



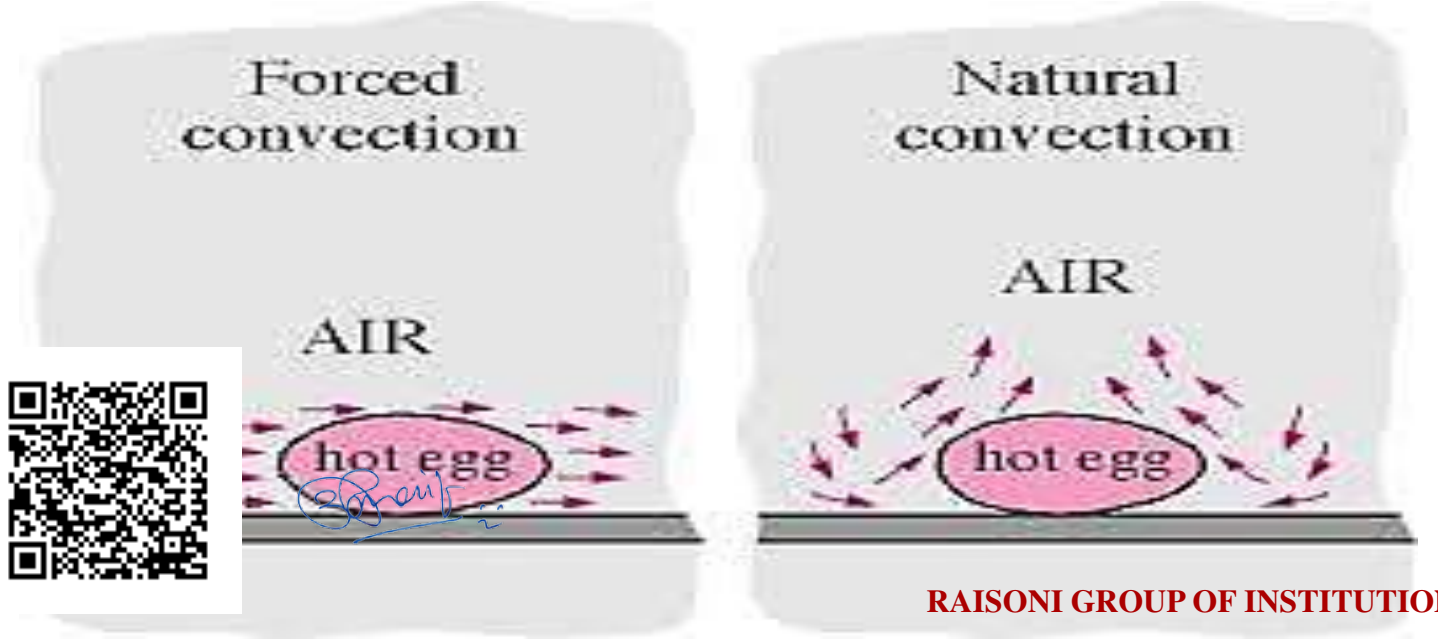
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Convection

What is Convection Heat Transfer

Convection is **heat transfer** by mass motion of a fluid such as air or water when the heated fluid is caused to move away from the source of **heat**, carrying **energy** with it. **Convection** above a hot surface occurs because hot air expands, becomes less dense, and rises (As per Ideal Gas Law).

Type of Convection Heat Transfer



Understanding of Convection

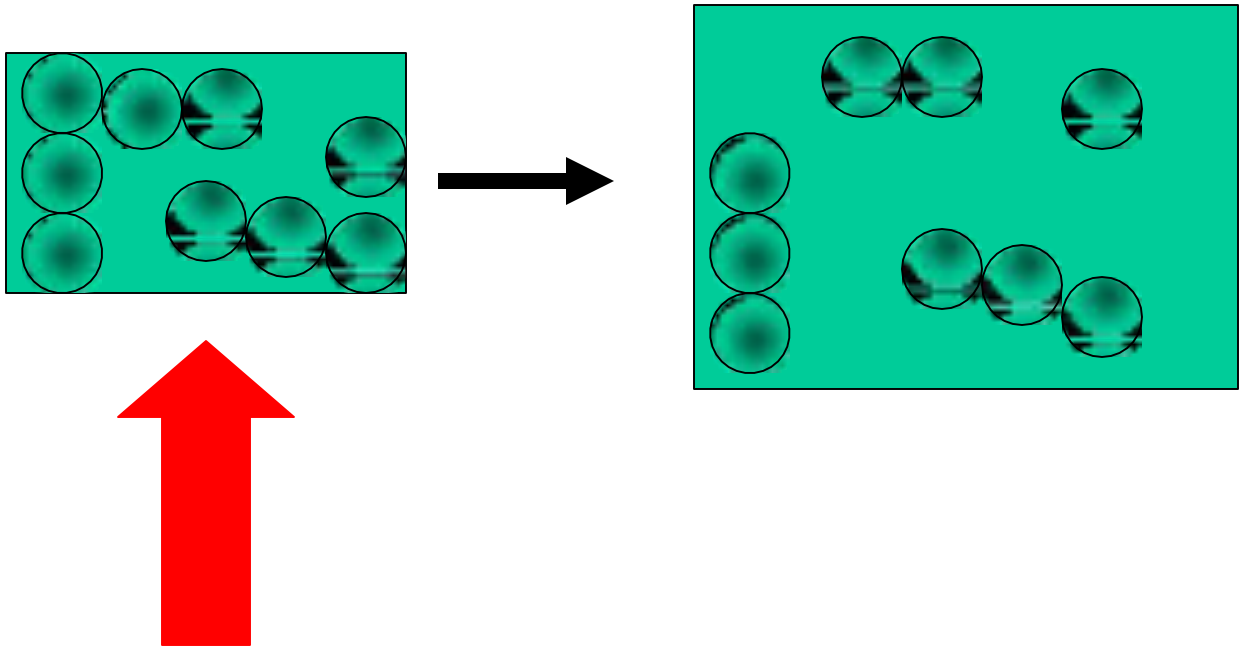


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Convection

What happens to the particles in a liquid or a gas when you heat them?

The particles spread out and become less dense.



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Water movement

Cools at the surface

Convection current

Cooler water sinks



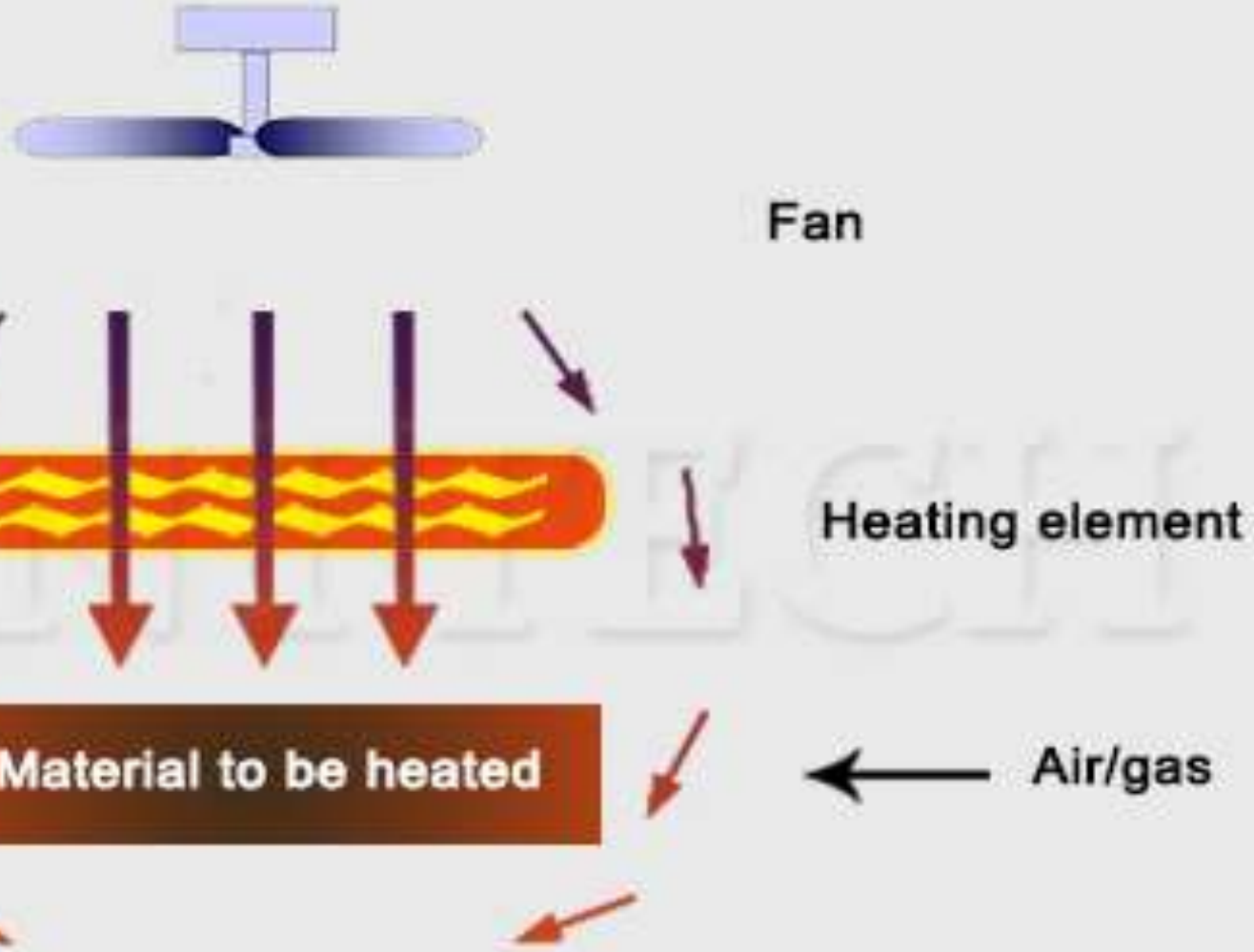
Hot water rises

Why does hot air rise and cold air sink?

Cool air is more dense than warm air, so the cool air 'falls through' the warm air.

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Heat Transfer by Convection



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Examples Of Convection In Everyday Life

- ✓ Boiling Water. Convection comes into play while boiling water. ...
- ✓ Blood Circulation in Warm-Blooded Mammals. ...
- ✓ Air-Conditioner. ...
- ✓ Radiator. ...
- ✓ Refrigerator. ...
- ✓ Hot Air Popper. ...
- ✓ Hot Air Balloon. etc……..

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Newton's law of cooling states that the rate of heat loss of a body is directly proportional to the difference in the temperatures between the body and its surroundings.

$$Q = hA\,(t_s - t_f)$$

where,

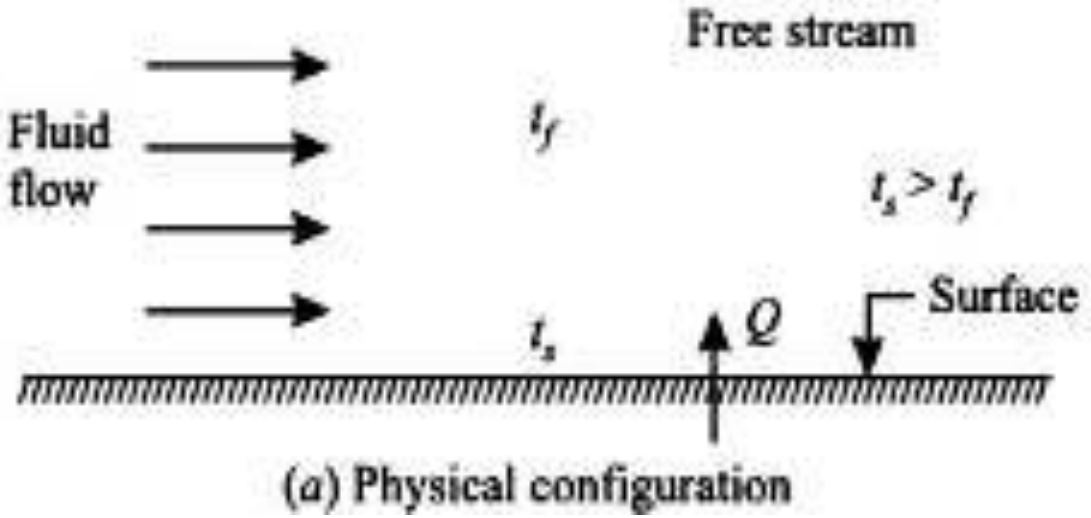
Q = Rate of conductive heat transfer,

A = Area exposed to heat transfer,

t_s = Surface temperature,

t_f = Fluid temperature, and

h = Co-efficient of convective heat transfer.



(a) Physical configuration

are,

$$h = \frac{Q}{A\,(t_s - t_f)} = \frac{W}{m^2\,°C} \quad \text{or} \quad W/m^2\,°C$$

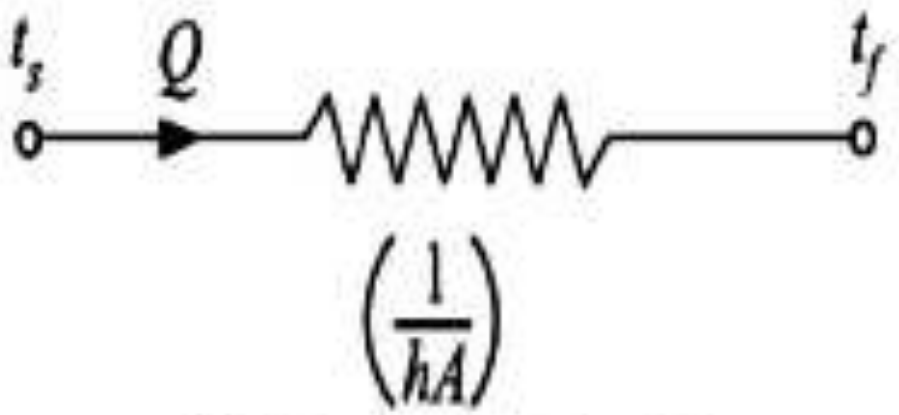
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What is Heat Transfer Coefficient ?

The **heat transfer coefficient is the heat** transferred per unit area per kelvin. Thus area is included in the equation as it represents the area over which the **transfer** of **heat** takes place. The areas for each flow will be different as they represent the contact area for each fluid side.

The value of '*h*' depends on the following factors :

(*i*) Thermodynamic and transport properties (*e.g.* viscosity, density, specific heat etc.).
(*ii*) Nature of fluid flow.
(*iii*) Geometry of the surface.
(*iv*) Prevailing thermal conditions.



(*b*) Equivalent circuit



...Eqn (1.6)

called **convection thermal resistance** $[(R_{th})_{conv}]$ to heat flow.

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Example 1.4. *A hot plate 1m × 1.5 m is maintained at 300°C. Air at 20°C blows over the plate. If the convective heat transfer coefficient is 20W/m²°C, calculate the rate of heat transfer.*

Solution. Area of the plate exposed to heat transfer, $A = 1 \times 1.5 = 1.5\ m^2$

Plate surface temperature, $t_s = 300°C$

Temperature of air (fluid), $t_f = 20°C$

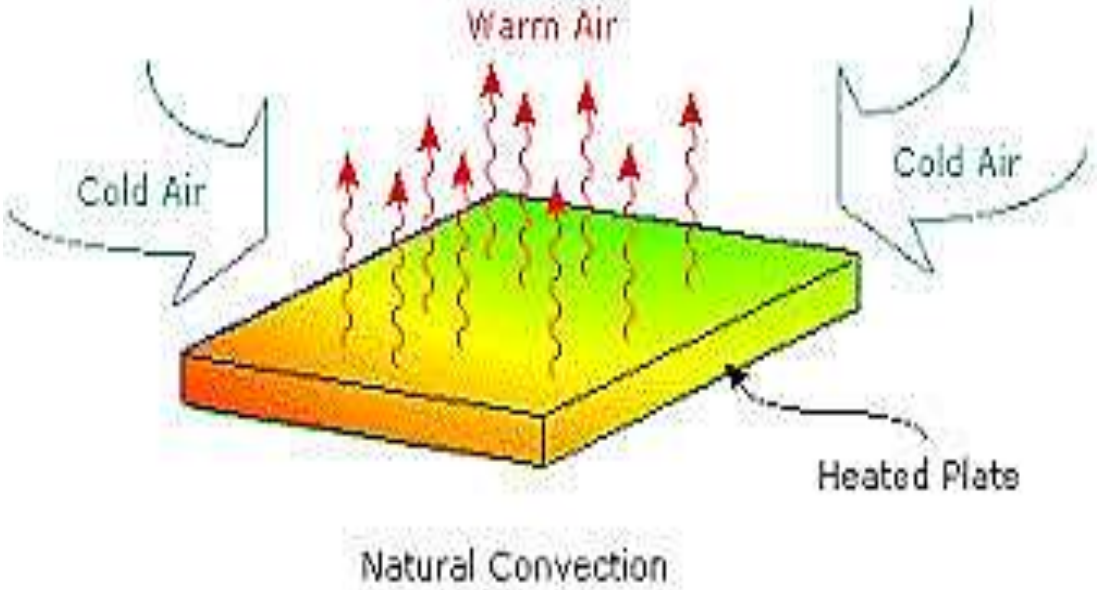
Connvective heat-transfer coefficient, $h = 20\ W/m^2\,°C$

Rate of heat transfer, Q :

From Newton's law of cooling,

$$Q = hA\,(t_s - t_f)$$

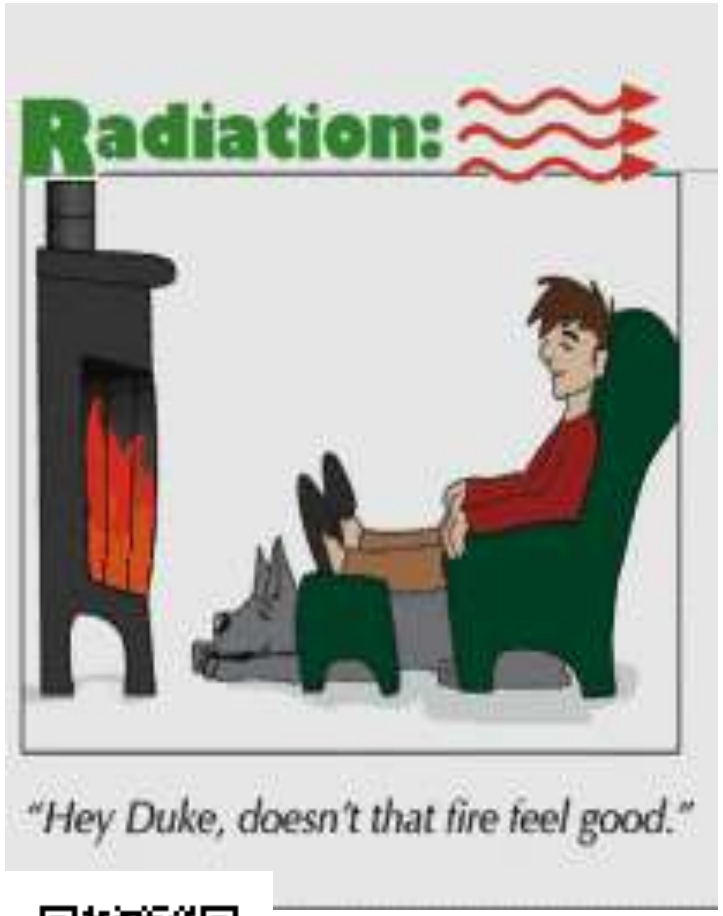
$$20 \times 1.5\,(300 - 20) = 8400\ W \text{ or } \mathbf{8.4\ kW}$$



Natural Convection

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RADIATION







"Radiation" is the transfer of heat through space or matter by means other than conduction or convection.

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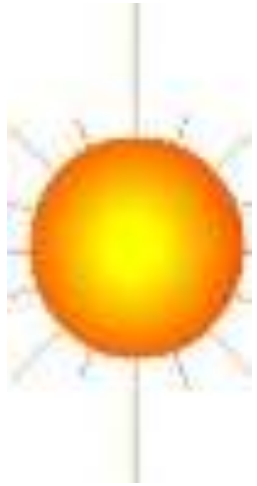
Radiation: The energy emitted by matter in the form of *electromagnetic waves* (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules.

- Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an *intervening medium*.
- In fact, heat transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum.
- This is how the energy of the sun reaches the earth.



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How does heat energy get from the Sun to the Earth?



?



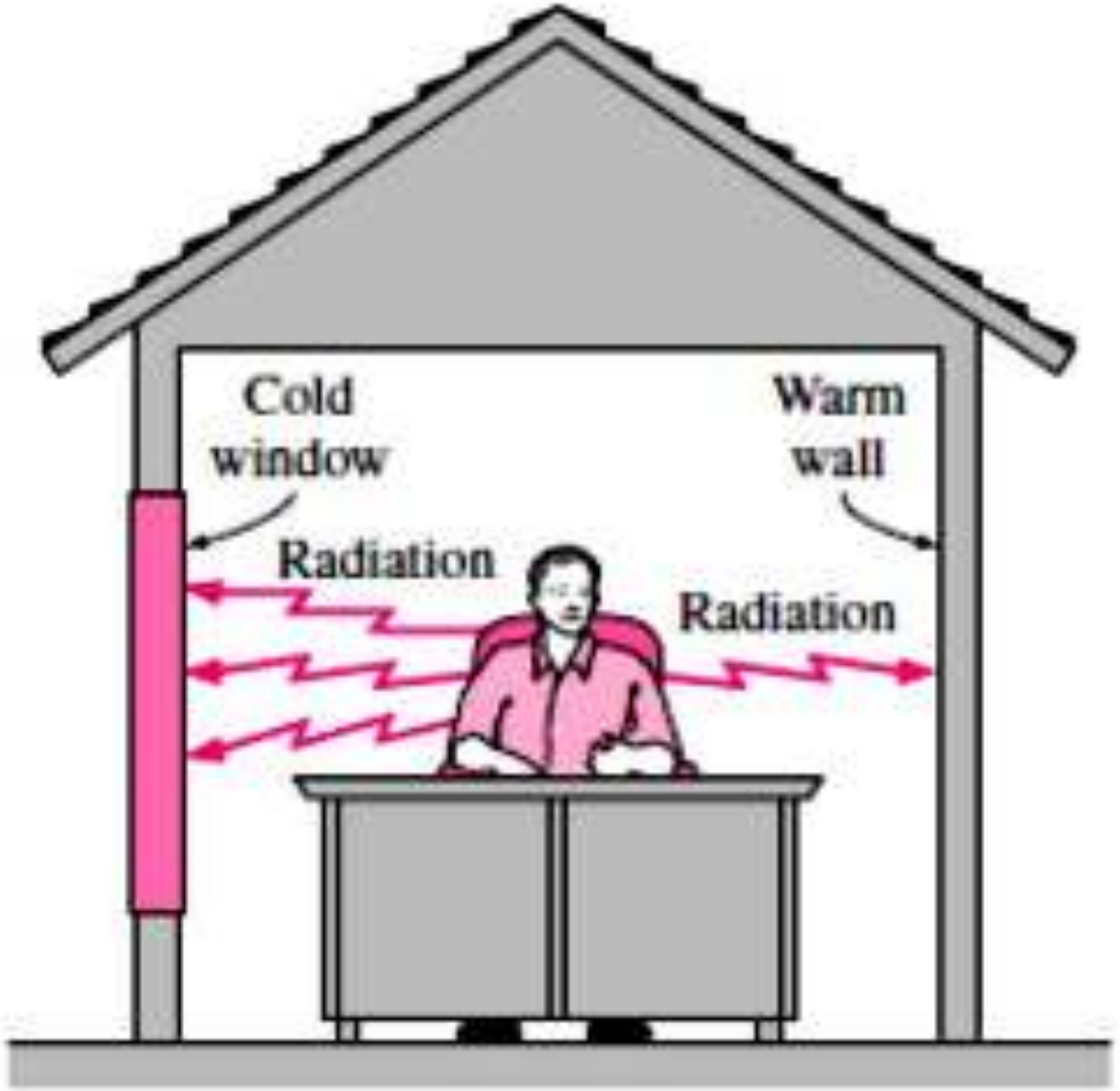
There are no particles between the Sun and the Earth so it CANNOT travel by conduction or by convection.

RADIATION

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Example of Radiation Heat Transfer ……

- ✓ **Cold Surface cause Excessive heat loss from the body by radiation and**
- ✓ **Thus discomfort on that side of the body**



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Radiation

Radiation travels in straight lines

True/~~False~~

Radiation can travel through a vacuum ~~True~~/False

Radiation requires particles to travel

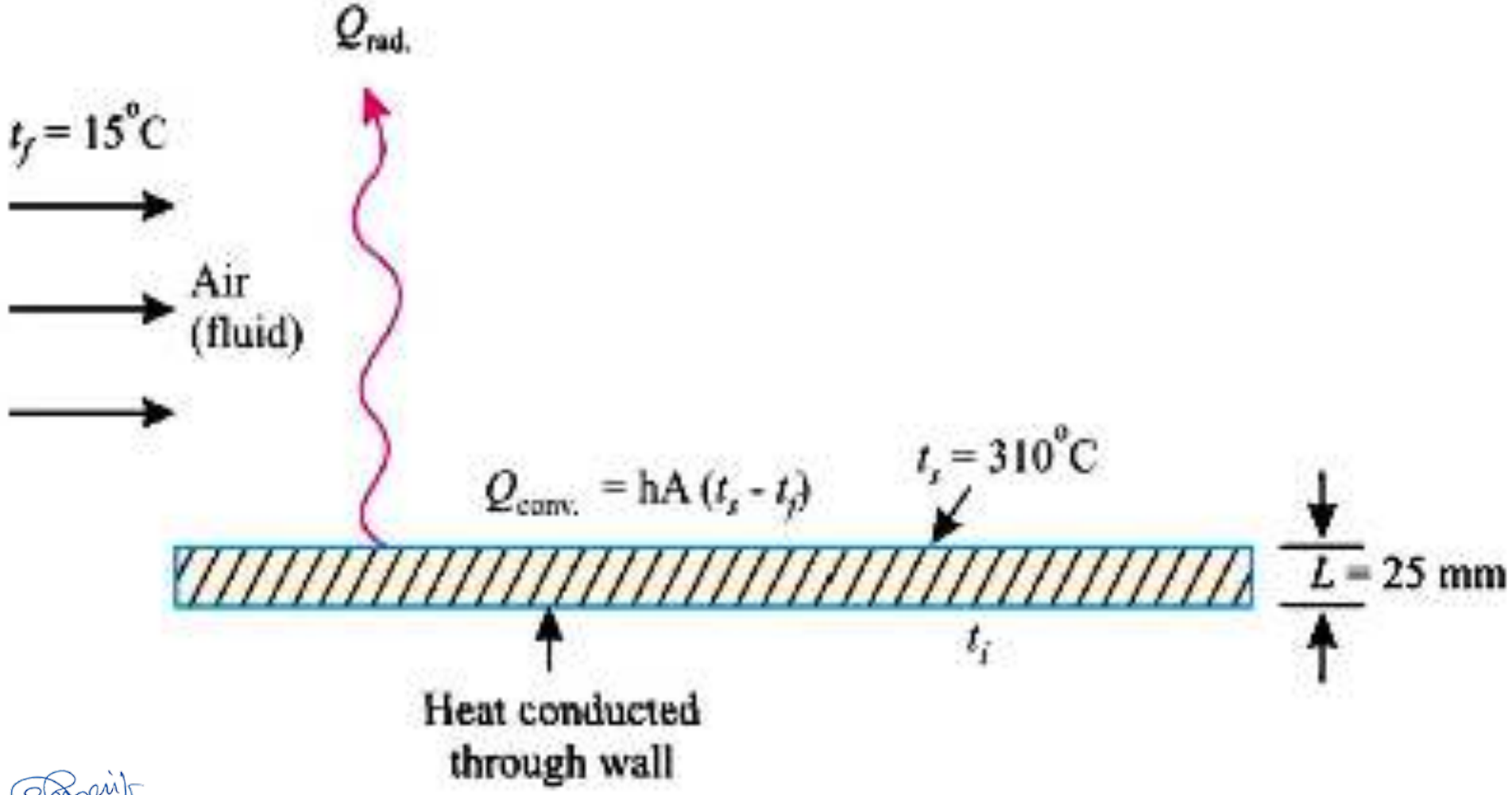
~~True~~/False

Radiation travels at the speed of light

True/~~False~~

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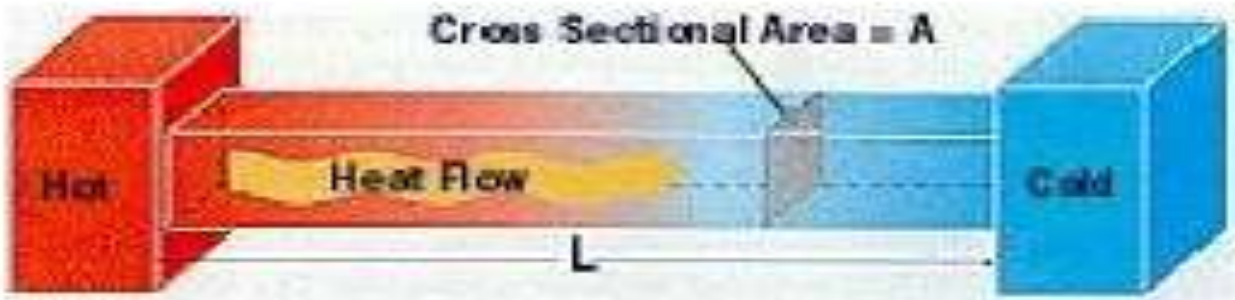
Combination of conduction, convection and radiation heat transfer.



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HEAT TRANSFER BY CONDUCTION

- Heat transfer through a solid medium via direct contact
- Expressed by Fourier's Law



$$Q = k\,A\left(\frac{\Delta T}{L}\right)t$$

Q = heat transferred
k = thermal conductivity
A = cross sectional area
ΔT = temperature difference between two ends
L = length
t = duration of heat transfer

∴ What is the unit of k?

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FOURIER'S LAWS OF HEAT CONDUCTION

Fourier's law of heat conduction is an emperical law based on observation and states as follows :

"The rate of flow of heat through a simple homogeneous solid is directly proportional to the area of the section at right angles to the direction of heat flow, and to change of temperature with respect to the length of the path of the heat flow".

Mathematically, it can be represented by the equation :

$$Q \propto A \cdot \frac{dt}{dx}$$

where, Q = Heat flow through a body per unit time (in watts), W,

A = Surface area of heat flow (*perpendicular to the direction of flow*), m^2,

dt = Temperature difference of the faces of block (homogeneous solid) of thickness 'dx' through which heat flows, °C or K, and

dx = Thickness of body in the direction of flow, m.

Thus, $$Q = -k \cdot A \frac{dt}{dx}$$ where, k = Constant of proportionality and is known as *thermal conductivity of the body.*

– ve sign of k [eqn. (1.1)] is to take care of the decreasing temperature alongwith the direction asing thickness or the direction of heat flow. The temperature gradient $\frac{dt}{dx}$ is *always negative* ositive x direction and, therefore, the value of Q becomes + ve.

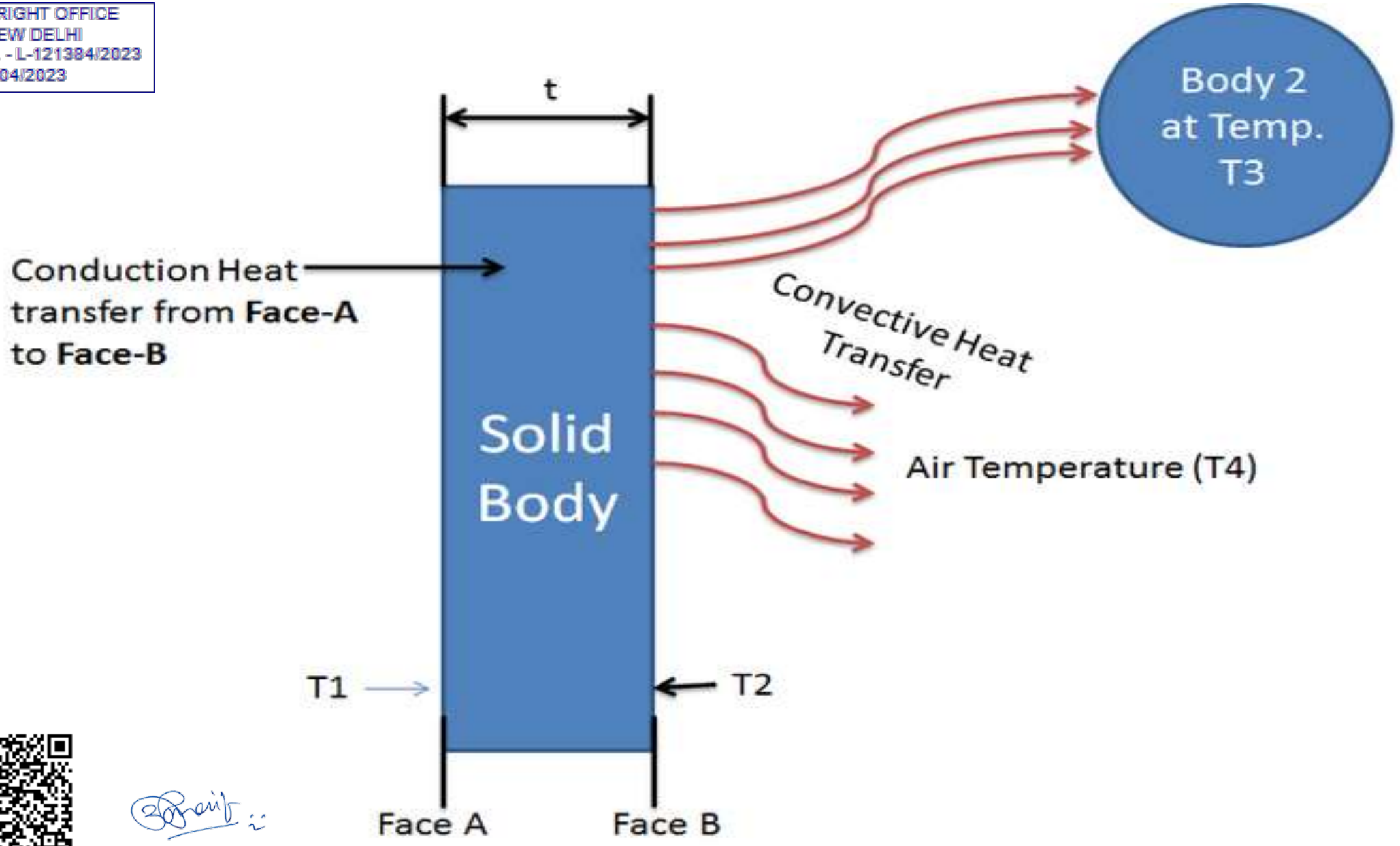
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Assumptions :

The following are the assumptions on which Fourier's law is based :

1. Conduction of heat takes place under *steady state conditions*.
2. The heat flow is unidirectional.
3. The temperatures gradient is *constant* and the temperature profile is *linear*.
4. There is no internal heat generation.
5. The bounding surfaces are isothermal in character.
6. The material is homogeneous and isotropic (*i.e.*, the value of thermal conductivity is *constant in all directions*).

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Example 1.1. *Calculate the rate of heat transfer per unit area through a copper plate 45 mm thick, whose one face is maintained at 350°C and the other face at 50°C. Take thermal conductivity of copper as 370 W/m°C.*

Solution. Temperature difference, $dt\ (= t_2 - t_1) = (50 - 350)$

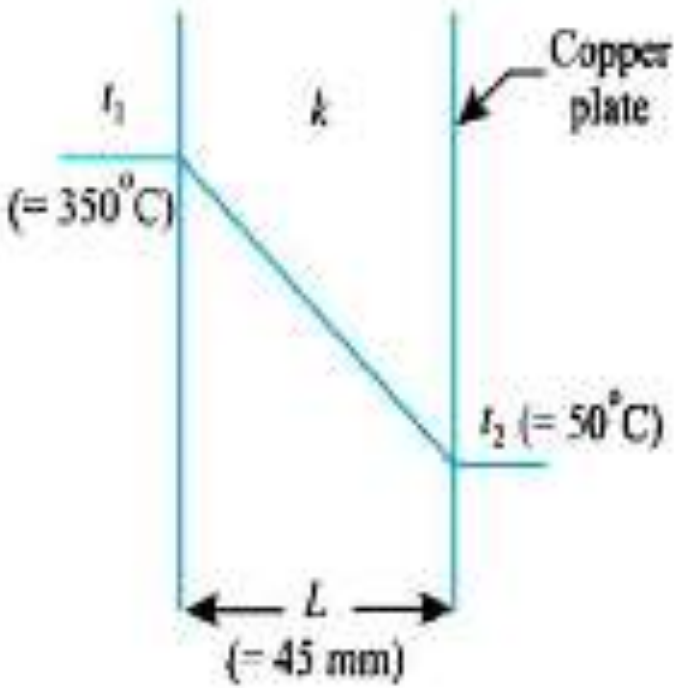
Thickness of copper plate, $L = 45$ mm $= 0.045$ m

Thermal conductivity of copper, $k = 370$ W/m°C

Rate of heat-transfer per unit area, q :

From Fourier's law

$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{L}$$



or,
$$q = \frac{Q}{A} = -k\frac{dt}{dx}$$

$$= -370 \times \frac{(50 - 350)}{0.045}$$

$$= 2.466 \times 10^6 \text{ W/m}^2 \text{ or}$$

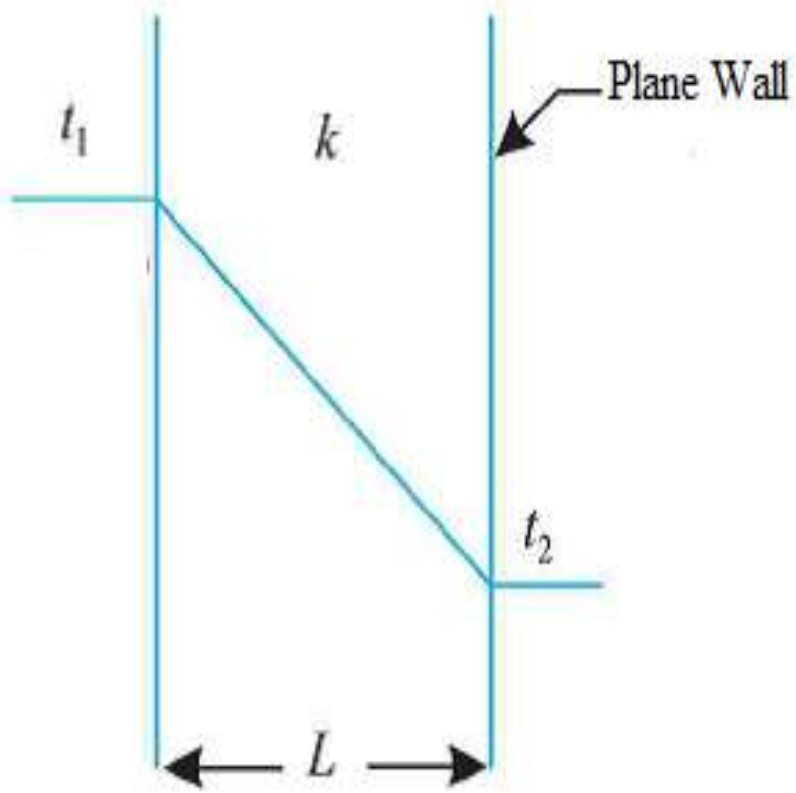
2.466 MW/m² (Ans.)

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Example 1.2. *A plane wall is 150 mm thick and its wall area is 4.5 m^2. If its conductivity is 9.35 W/m°C and surface temperatures are steady at 150°C and 45°C, determine :*

(i) Heat flow across the plane wall;

(ii) Temperature gradient in the flow direction.



Solution. Thickness of the plane wall,

$$L = 150 \text{ mm} = 0.15 \text{ m}$$

Area of the wall, $A = 4.5 \text{ m}^2$

Temperature difference, $dt = t_2 - t_1 = 45 - 150 = -105°C$

Thermal conductivity of wall material,

$$k = 9.35 \text{ W/m°C}$$

(i) Heat flow across the plane wall, Q :

As per Fourier's law,

$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{L}$$

$$= -9.35 \times 4.5 \times \frac{(-105)}{0.15} = \mathbf{29452.5\ W}$$

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(ii) Temperature gradient, $\frac{dt}{dx}$:

From Fourier's law, we have

$$\frac{dt}{dx} = -\frac{Q}{kA}$$

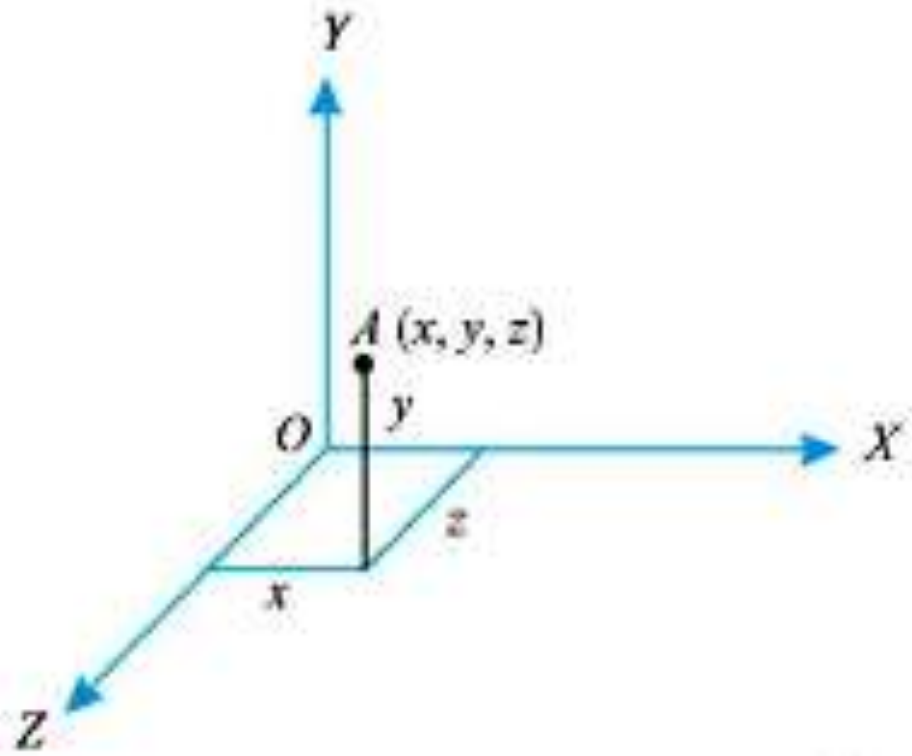
$$= \frac{29452.5}{9.35 \times 4.5} = -700°C/m$$



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Hear Transfer Dimensional Study

- 1 - Dimensional
- 2 - Dimensional
- 3 - Dimensional



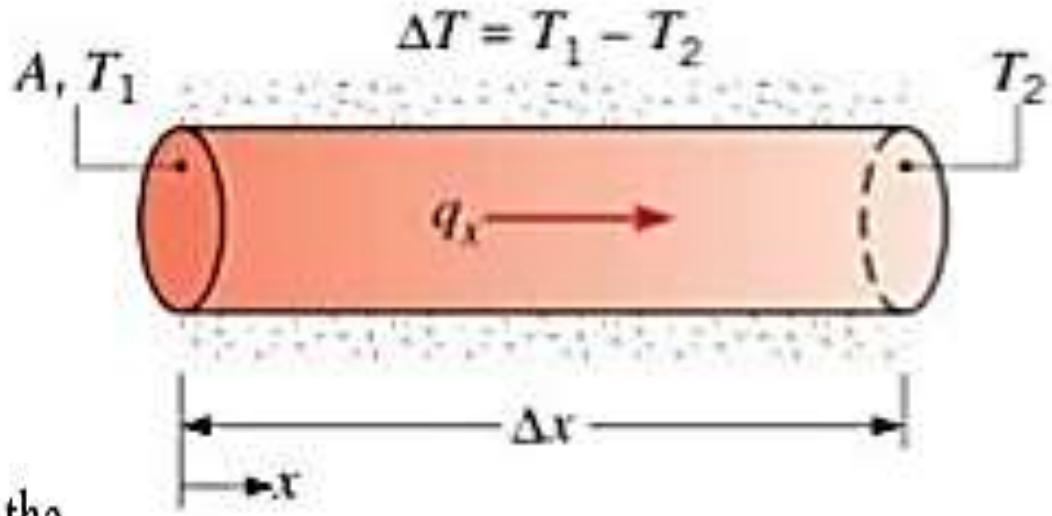
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ONE DIMENSIONAL STEADY STATE CONDUCTION

The Conduction Rate Equation

For example, consider the steady-state conduction experiment.

- A cylindrical rod of known material is insulated on its lateral surface, while its end faces are maintained at different, with $T_1 > T_2$
- The temperature difference causes conduction heat transfer in the positive x- direction.
- How q_x depends on the following variables: ΔT, *the temperature difference;* Δx, the rod length; and A, *the cross-sectional area.*



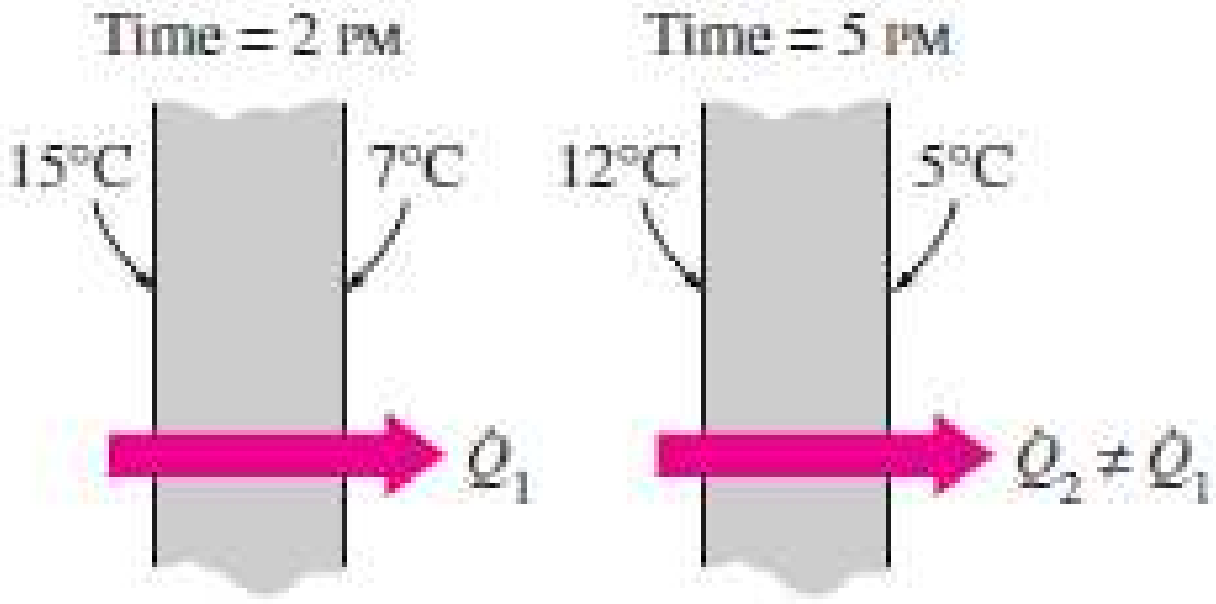
(ΔT, Δx, A) using a chain rule differentiate on by taking one constant and two variable

$$q_x \propto A \frac{\Delta T}{\Delta x}$$

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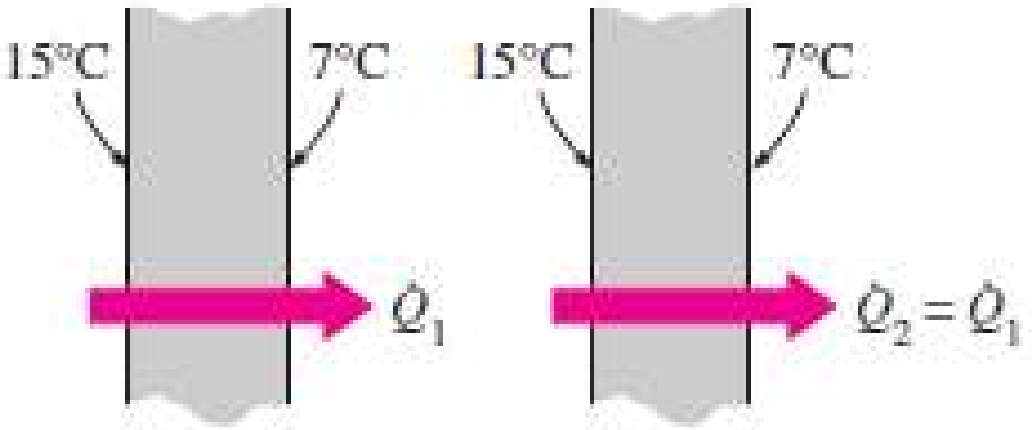
1 - Dimensional Conduction Heat Transfer

(d) Transient



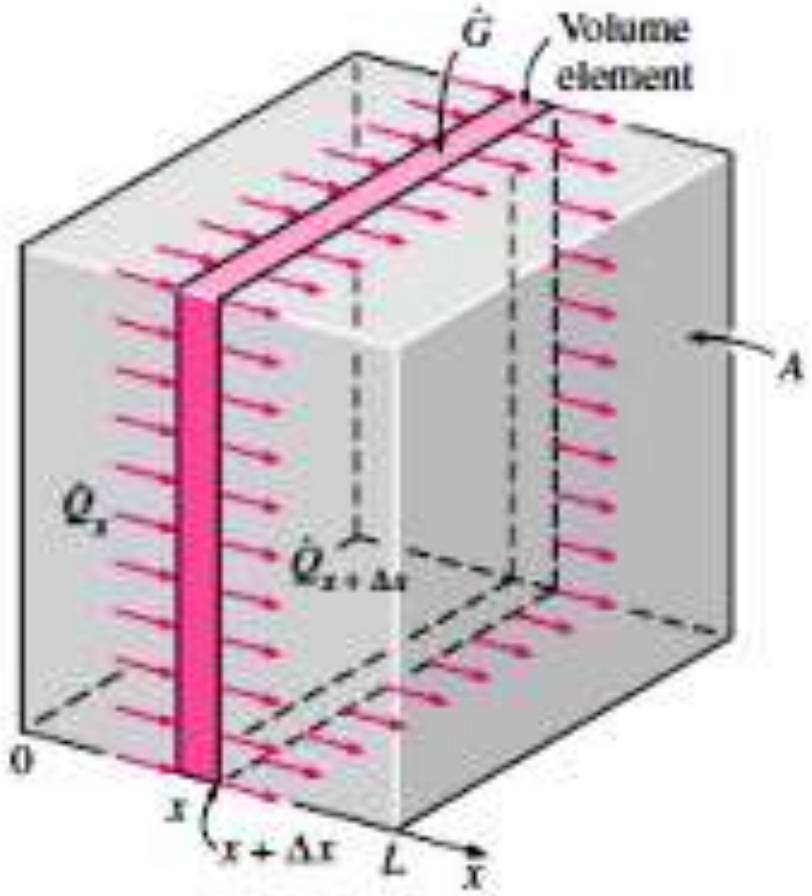
Steady and transient heat conduction in a plane wall.

(b) Steady-state



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1 - Dimensional Conduction Heat Transfer



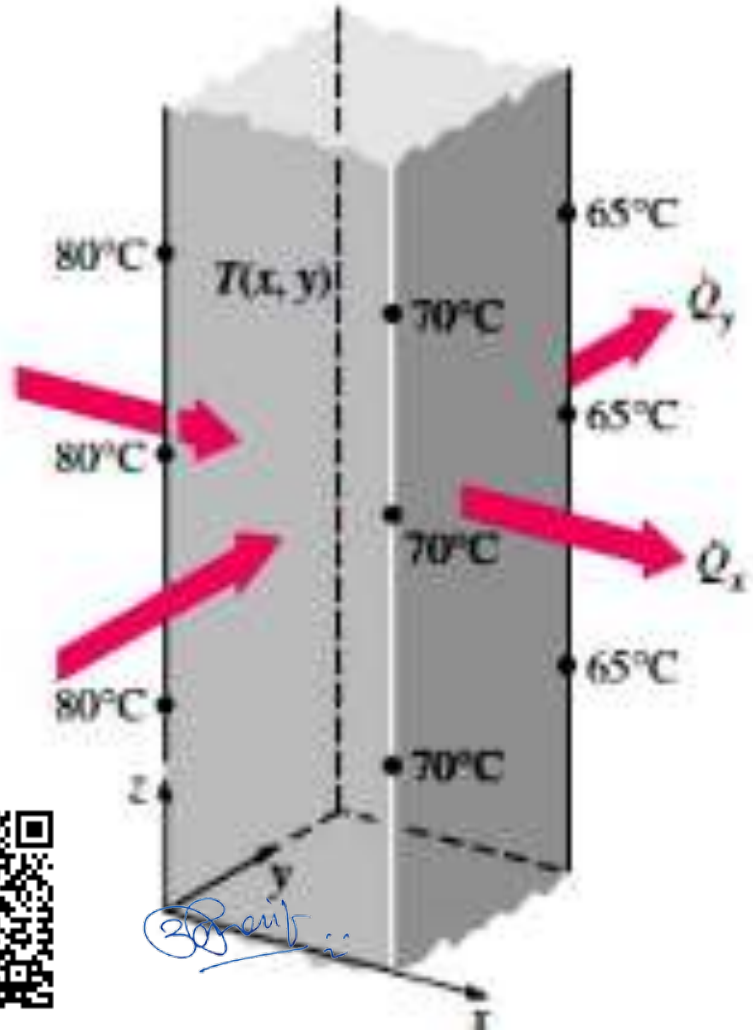
$$\left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x\end{array}\right) - \left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x\end{array}\right) + \left(\begin{array}{c}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{array}\right) = \left(\begin{array}{c}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{array}\right)$$

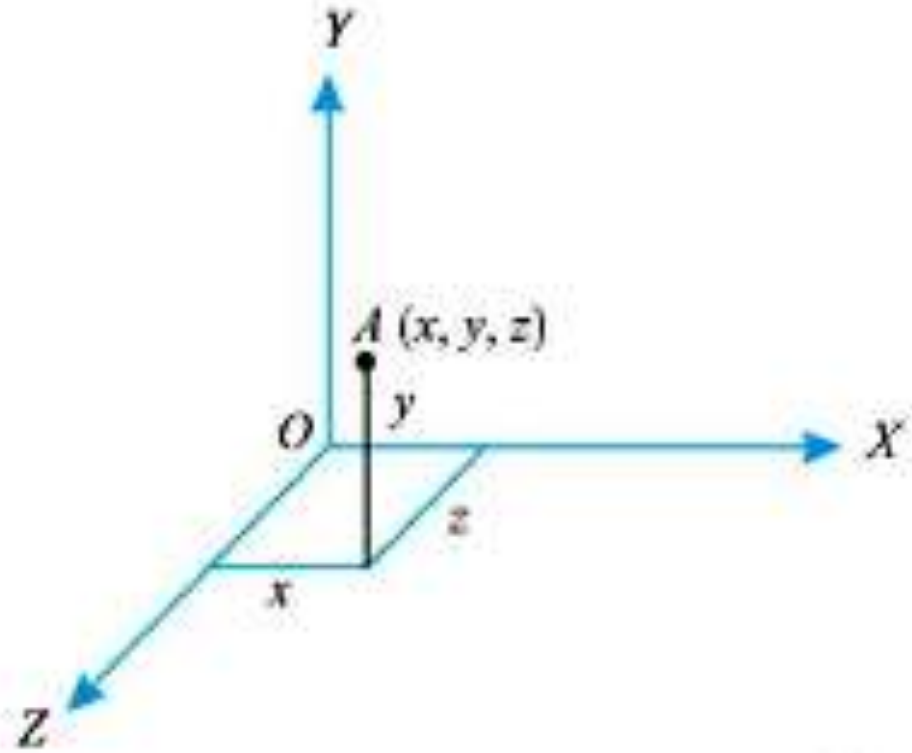
$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

ensional heat conduction
a volume element
plane wall.

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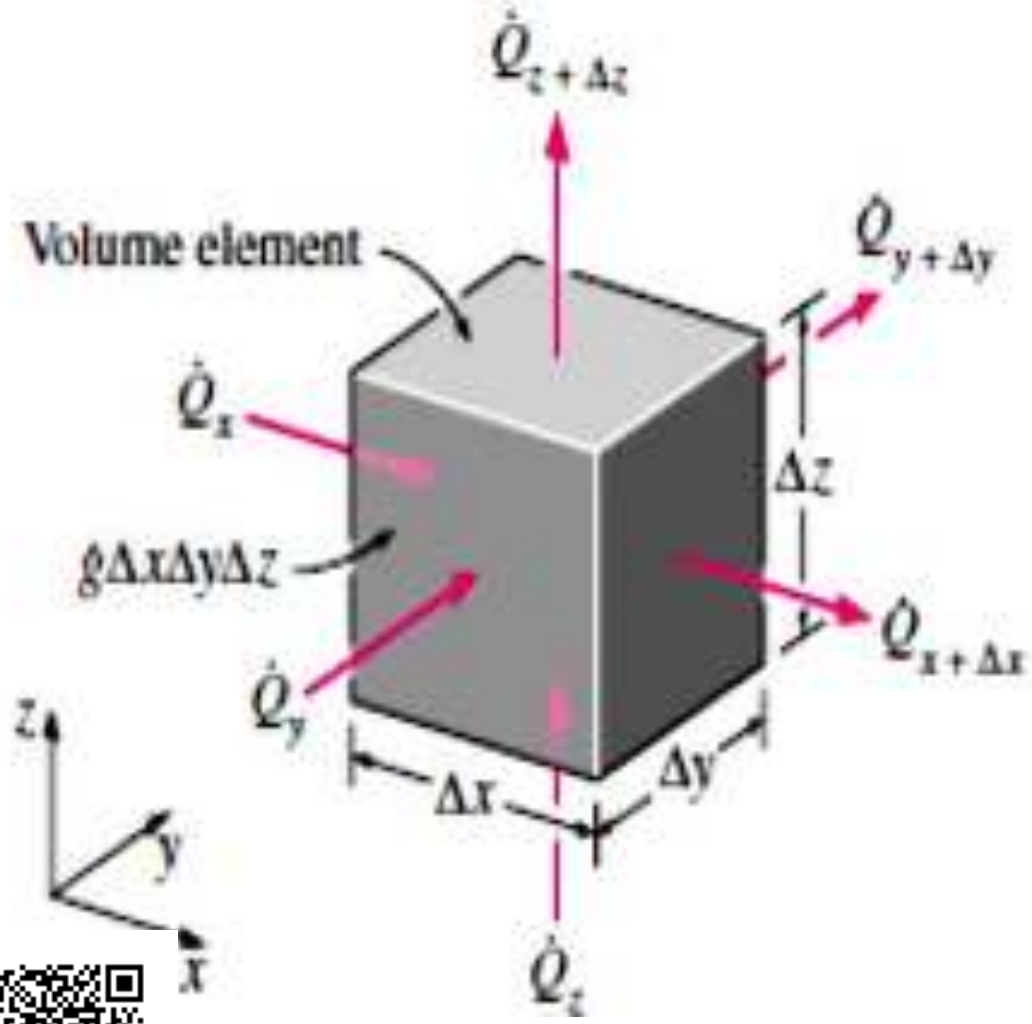
2 - Dimensional Conduction Heat Transfer





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3 - Dimensional Conduction Heat Transfer



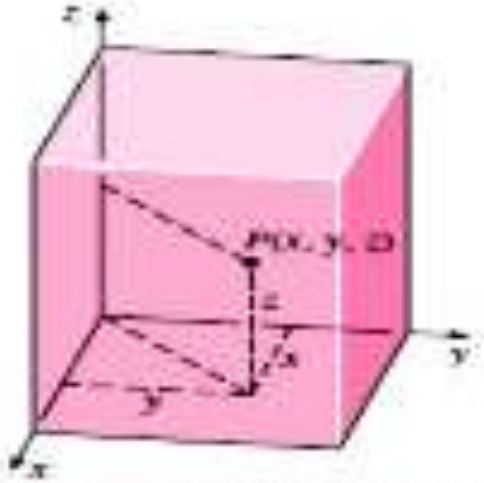
$$\left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction at}\\ x, y, \text{and } z\end{array}\right) - \left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x,\\ y+\Delta y, \text{and } z+\Delta z\end{array}\right) + \left(\begin{array}{c}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{array}\right) = \left(\begin{array}{c}\text{Rate of change}\\ \text{of the energy}\\ \text{content of}\\ \text{the element}\end{array}\right)$$

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

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General Heat Conduction Equation

- Heat transfer problems are also classified as being *one-dimensional, two dimensional, or three-dimensional, depending on the relative magnitudes of* heat transfer rates in different directions and the level of accuracy desired.
- In the most general case, heat transfer through a medium **3D.**
- That is, the temperature varies along all three primary directions within the **medium** during the heat transfer process.



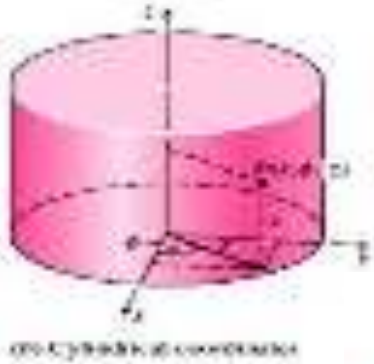
(a) Rectangular coordinates

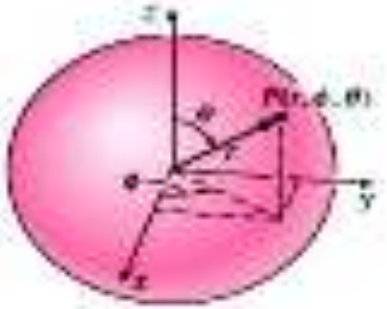
- The temperature distribution throughout the medium at a specified time as well as the heat transfer rate at any location, can be described by a set of three coordinates such as the

i. Rectangular (or Cartesian) coordinate system: **x, y,** and **z;**

ii. ndrical coordinate system: **r,** ϕ *and* **z**

iii. rical (or polar) coordinate system: **r,** ϕ *and* θ



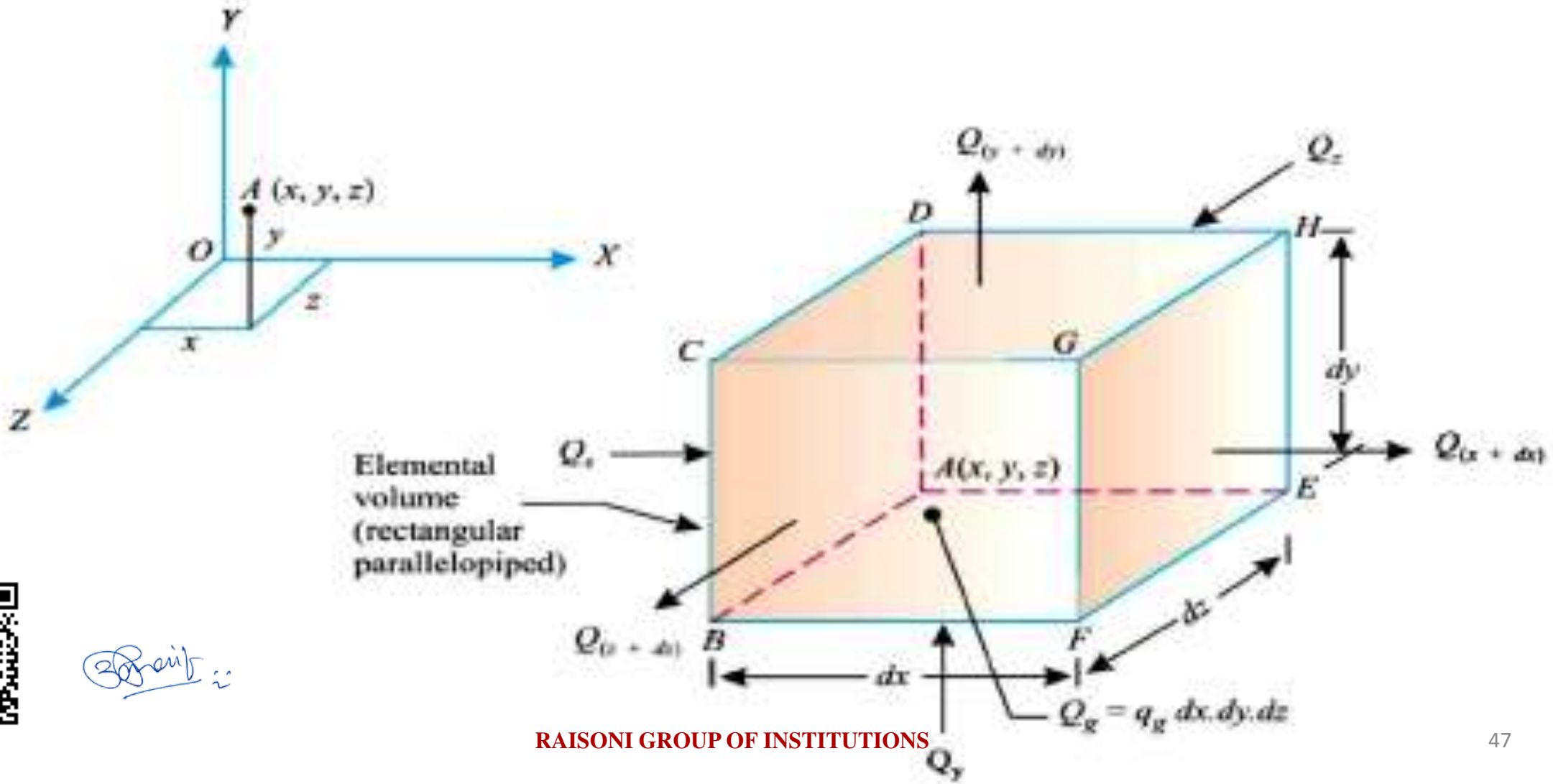


(c) Spherical coordinates

General heat conduction equation in Cartesian Coordinates or Rectangular Block

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Consider an infinitesimal rectangular parallelopiped (volume element) of sides dx, dy and dz parallel, respectively, to the three axes (X, Y, Z) in a medium in which temperature is varying with



General heat conduction equation in Cartesian Coordinates or Rectangular Block

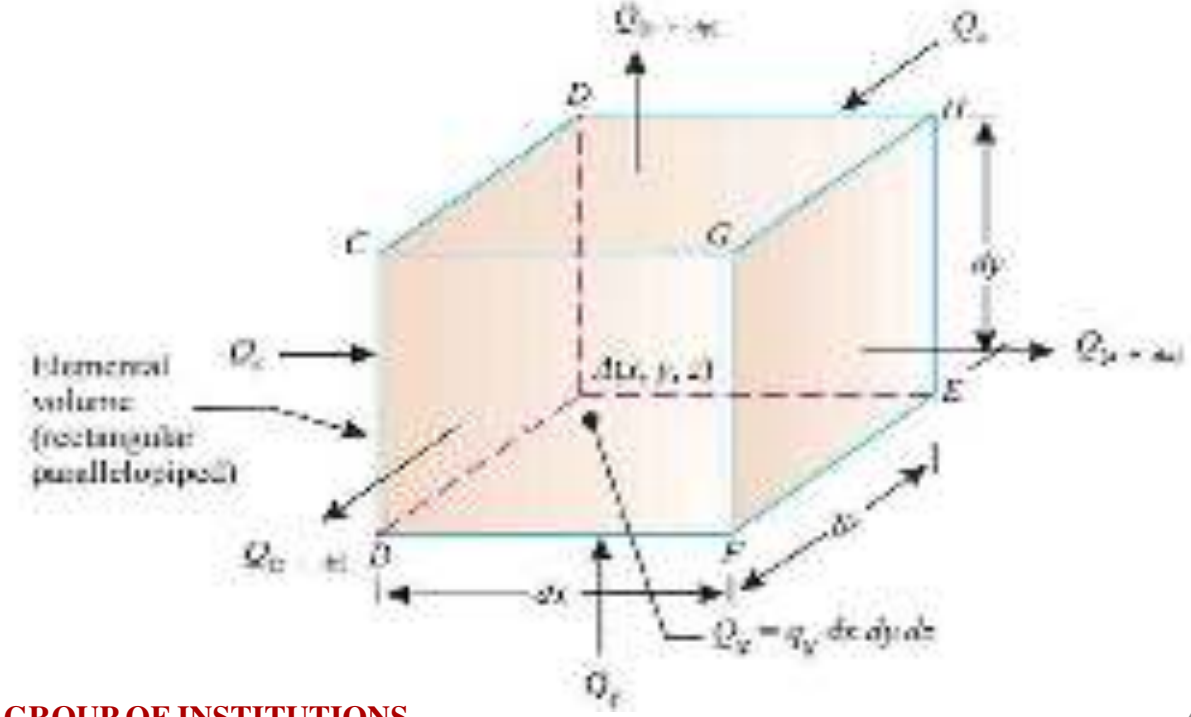
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Rectangular (or Cartesian) coordinate system: **x, y,** and **z**

- Consider a small rectangular element of **length** *dx*, ***width*** d *y*, ***and height*** *dz* *and* an **infinitesimally small (differential) control volume,** d *y*. *dx*.d *z*

Temperature distribution: T(x, y, z)

- In the absence of motion (or with uniform motion), there are no changes in mechanical energy and no work being done on the system.
- Only thermal forms of energy need be considered. Specifically, if there are temperature gradients, conduction heat transfer will occur across each of the control surfaces.
- The conduction heat rates perpendicular the control surfaces at the *x*-, *y*-, and *ate* locations are indicated q_x, q_y,

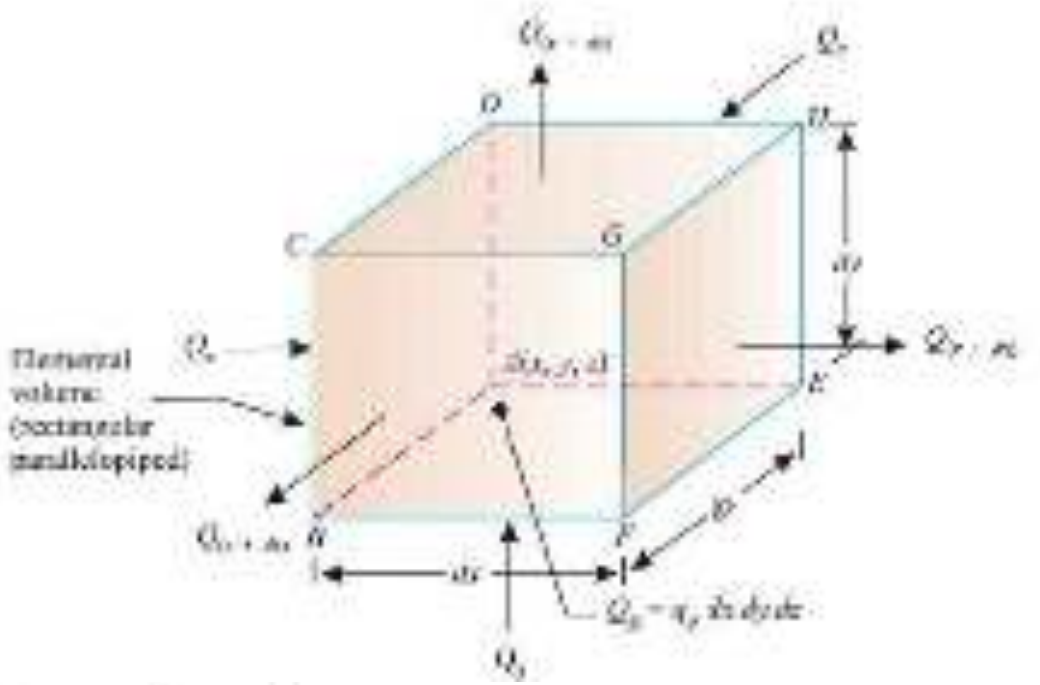


General heat conduction equation in Cartesian Coordinates or Rectangular Block

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Let, t = Temperature at the left face *ABCD*; this temperature may be assumed uniform over the entire surface, since the area of this face can be made arbitrarily *small*, and

$\frac{dt}{dx}$ = Temperature changes and rate of change along *X*-direction.



Then, $\left(\frac{\partial t}{\partial x}\right)dx$ = Change of temperature through distance dx, and

$t + \left(\frac{\partial t}{\partial x}\right)dx$ = temperature on the right face *EFGH* (at a distance dx from the left face *ABCD*).

et, k_x, k_y, k_z = Thermal conductivities (direction characteristics of the material) along *X*, *Y* and *Z* axes.

General heat conduction equation in Cartesian Coordinates or Rectangular Block

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Fourier's law,

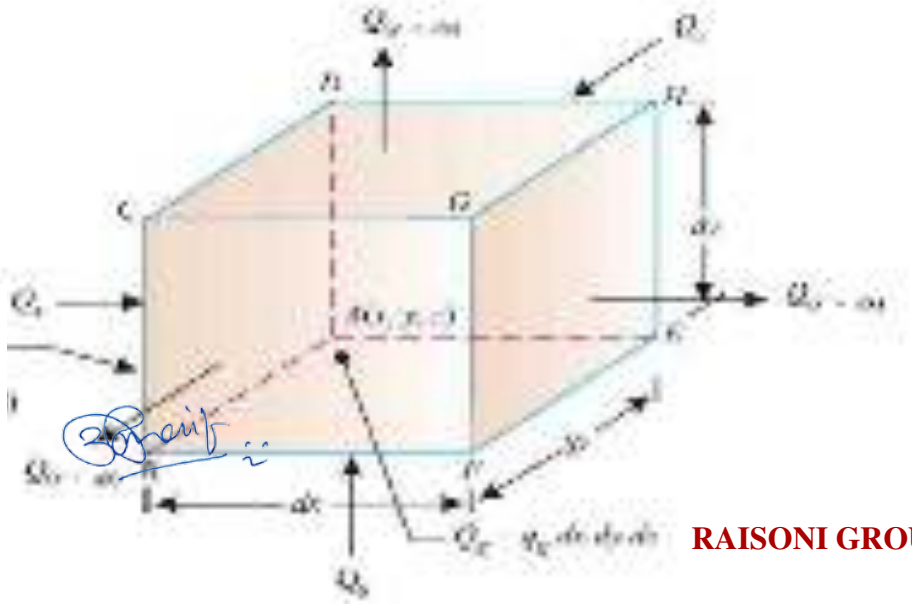
$$q_x = -k\,dy\,dz\frac{\partial T}{\partial x}$$

$$q_y = -k\,dx\,dz\frac{\partial T}{\partial y}$$

$$q_z = -k\,dx\,dy\frac{\partial T}{\partial z}$$

Note: Isotropic material : they have the same properties in all directions. Or, thermal conductivity of a material to be independent of direction.

Fibrous or **composite**, are some **anisontropic** materials



General heat conduction equation in Cartesian Coordinates or Rectangular Block

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Now Considering Face ABCD

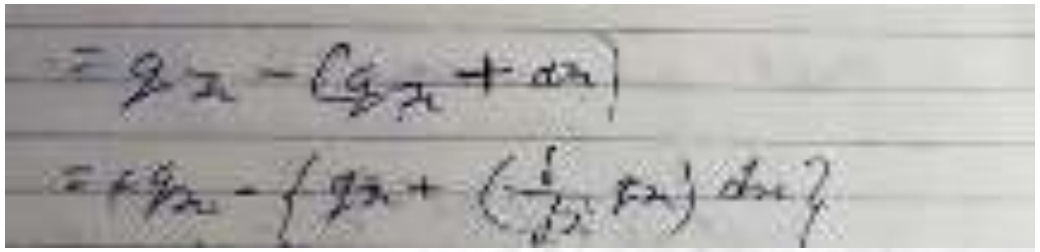
Q = q= Heat in the Face (X direction)

By fourier law $q = -kA\frac{dt}{dx}$

Now Heat Out from the Face EFGH



Now Net Heat Accumulated in X direction

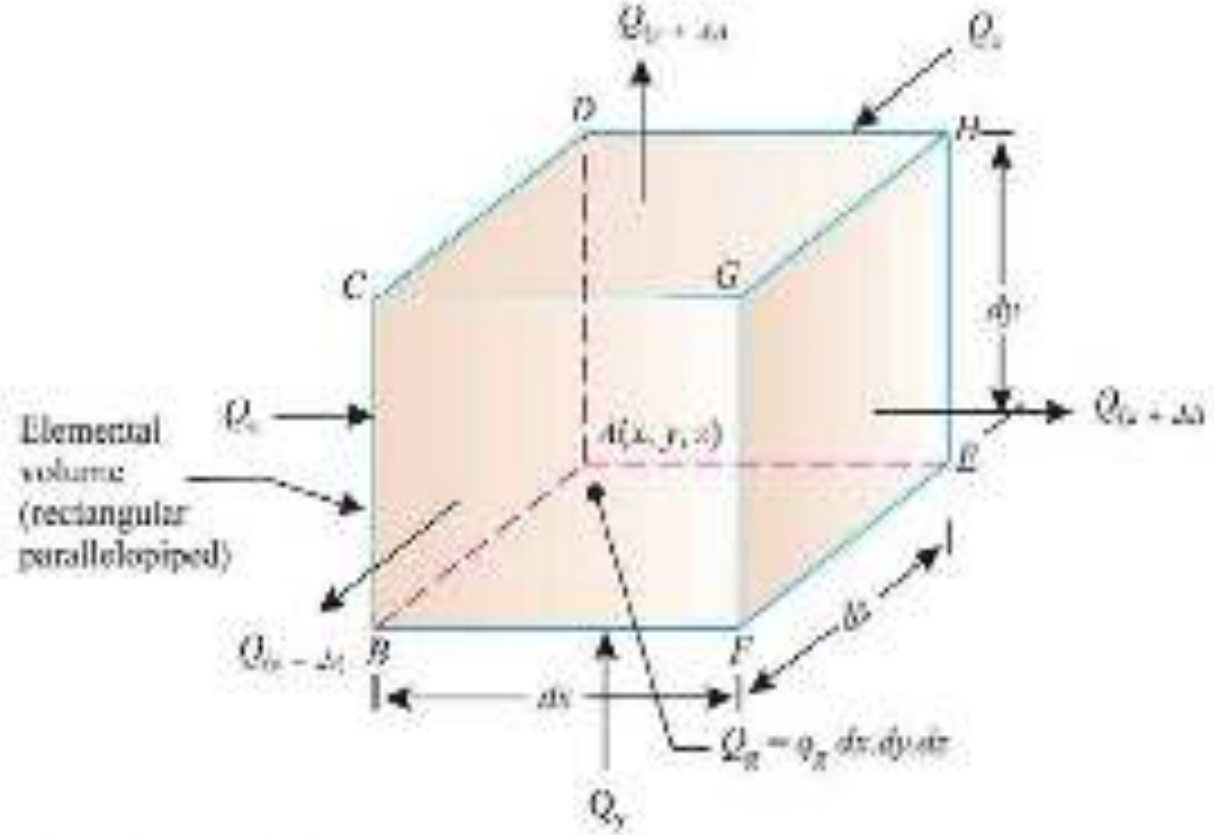


$$\text{Stored} = -\frac{d}{dx}\left(-kA\frac{dt}{dx}\right)dx$$

$$= kA\frac{d^2t}{dx^2}dx$$

$$= k(dy\,dz)\frac{d^2t}{dx^2}dx$$

$$= k\,dx\,dy\,dz\frac{d^2t}{dx^2} \quad \text{---(1)}$$



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Heat accumulated in y direction

$= k\,dx\,dy\,dz\,\frac{d^2t}{dy^2}$ — (2)

in z-direction $= k\,dx\,dy\,dz\,\frac{d^2t}{dz^2}$ — (3)

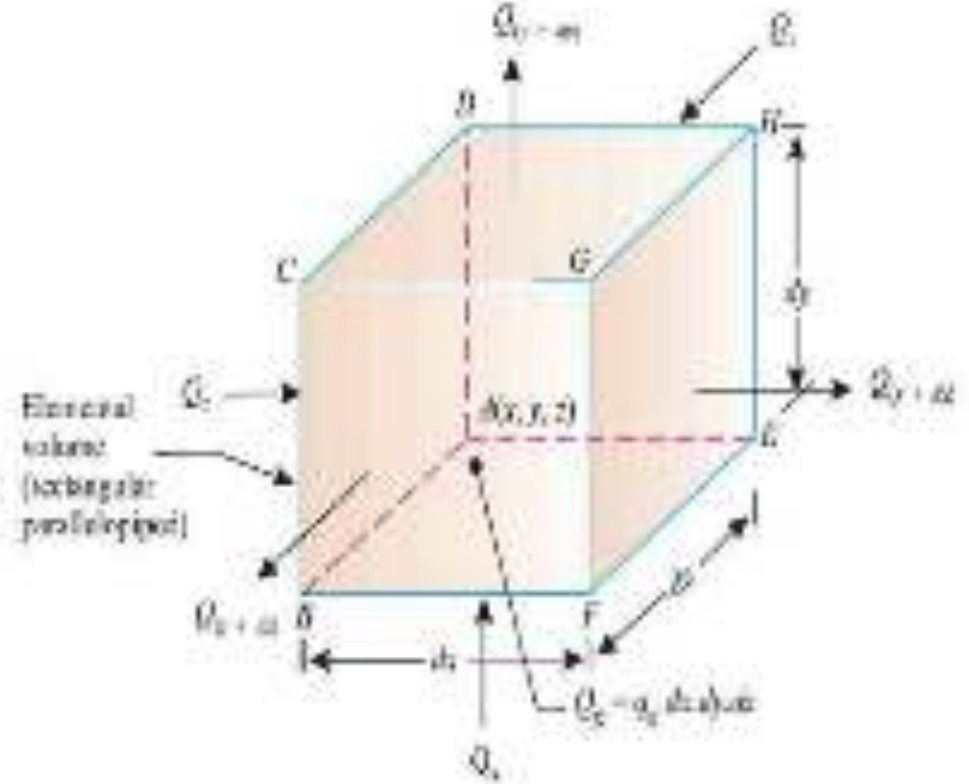
Net heat Accumulated from all dirn.

= eqn (1) + eqn (2) + eqn (3)

Within the medium there may also be an *energy source term associated with the rate of* **thermal energy generation**. This term is represented as

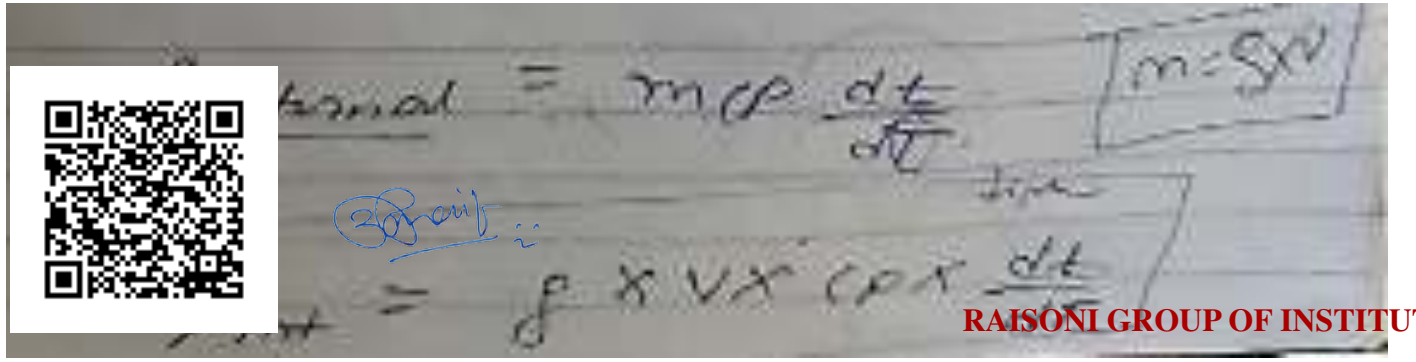
$E_g = q\,dx\,dy\,dz$ q rate at which energy is generated per unit volume (w/m³)

In addition, changes may occur in the amount of the internal thermal energy stored by the material in the control volume.



Now Because of the Heat Accumulated and Heat Generation Internal Energy Changes

i.e.



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$$q_{int} = \rho \times dx \times dy \times dz \times c_p \frac{dt}{dT} \quad -(6)$$

Now by energy balance

$$q_{int} = q_{accumulated} + q_{gen}$$

$$\rho \, dx \, dy \, dz \, c_p \frac{dt}{dT} = K_x \frac{\partial^2 t}{\partial x^2} dx dy dz + K_y \frac{\partial^2 t}{\partial y^2} dx dy dz + K_z \frac{\partial^2 t}{\partial z^2} dx dy dz + q' dx dy dz$$

$$K_x = K_y = K_z = K$$

$$\therefore \rho \, c_p \frac{dt}{dT} = K \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) + q'$$

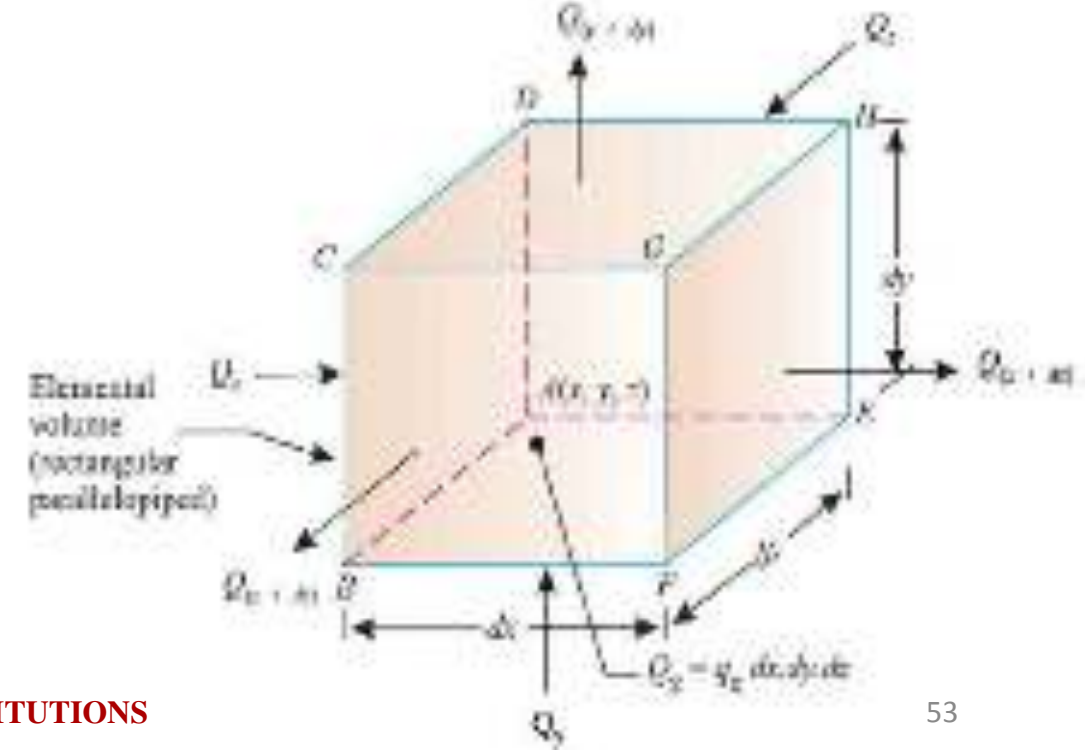
$$\frac{1}{\alpha} \frac{dt}{dT} = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q'}{K}$$

General Cartesian coordinates, of the *heat diffusion equation*.

$$\underbrace{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}_{\text{Net transfer of thermal energy into the control volume (inflow-outflow)}}+\underbrace{\dot{q}}_{\text{Thermal energy generation}}=\underbrace{\rho c_p\frac{\partial T}{\partial t}}_{\text{Change in thermal energy storage}}$$

In the case of constant thermal conductivity, it reduces to

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$



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Thermal Diffusivity (α)

- In heat transfer analysis, the ratio of the thermal conductivity to the heat capacity is an important property termed the *thermal diffusivity* α, *which is*

$$\alpha = \frac{k}{\rho c_p} \quad m^2/s.$$

This value describes how quickly a material reacts to a change in temperature.

Proof its SI units and what is the similar property in fluid flow?

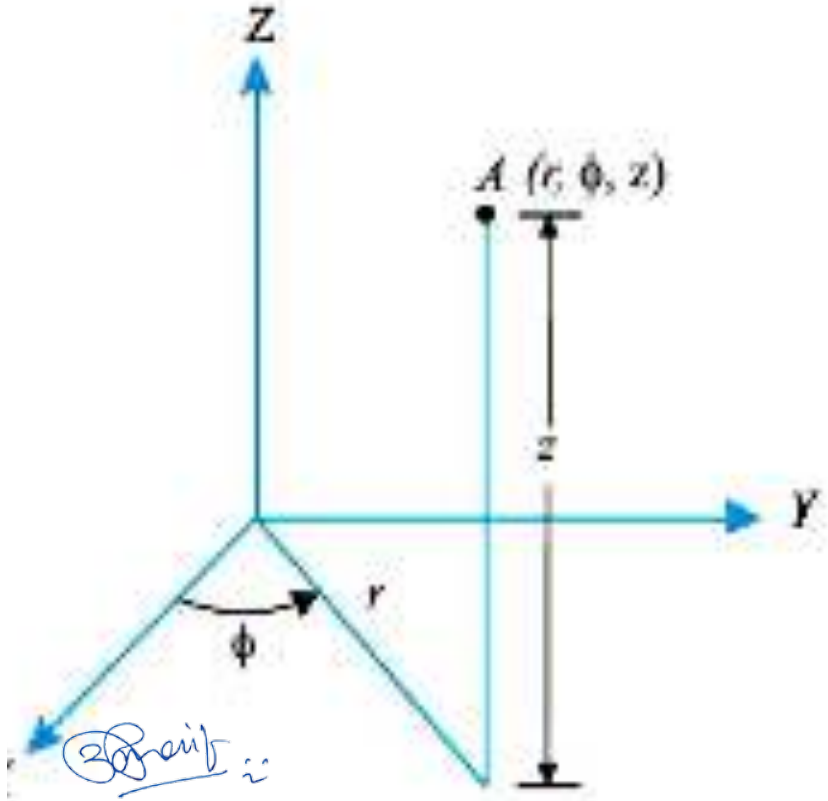
- It measures the ability of a material to **conduct thermal energy** relative to its ability to **store thermal energy**.
- So what does it mean when materials have **large** and **small** value of α?
- For a large α *will respond quickly to changes in their thermal environment, and* ,For a small α will respond more sluggishly, taking longer to reach a new equilibrium condition.

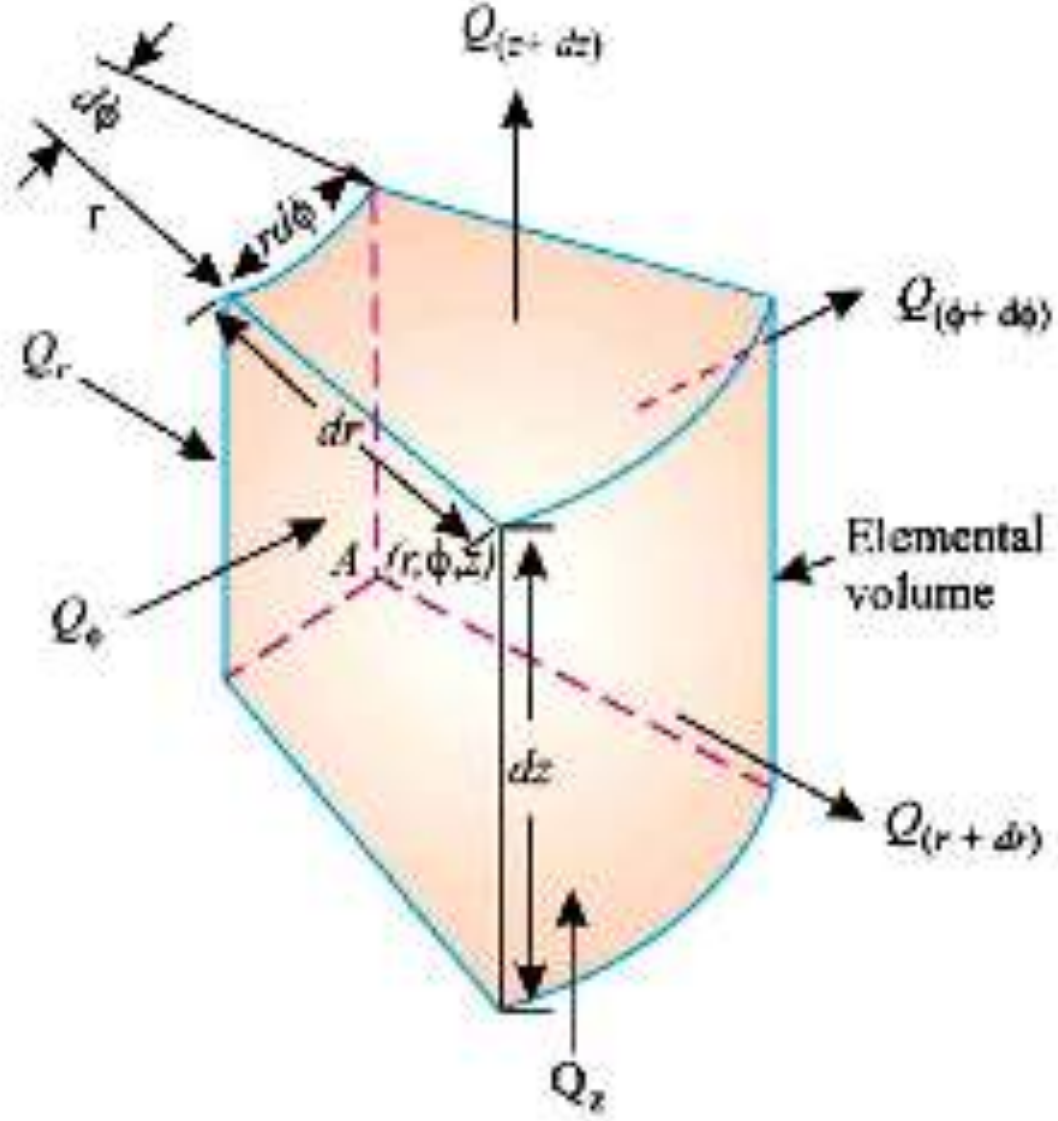
Example: WAX

In order to predict cooling processes or to simulate temperature fields, the thermal diffusivity must be known; it is a requisite for solving the Fourier Differential Equation for unsteady heat conduction.

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GENERAL HEAT CONDUCTION EQUATION IN CYLINDRICAL COORDINATES





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GENERAL HEAT CONDUCTION EQUATION IN CYLINDRICAL COORDINATES

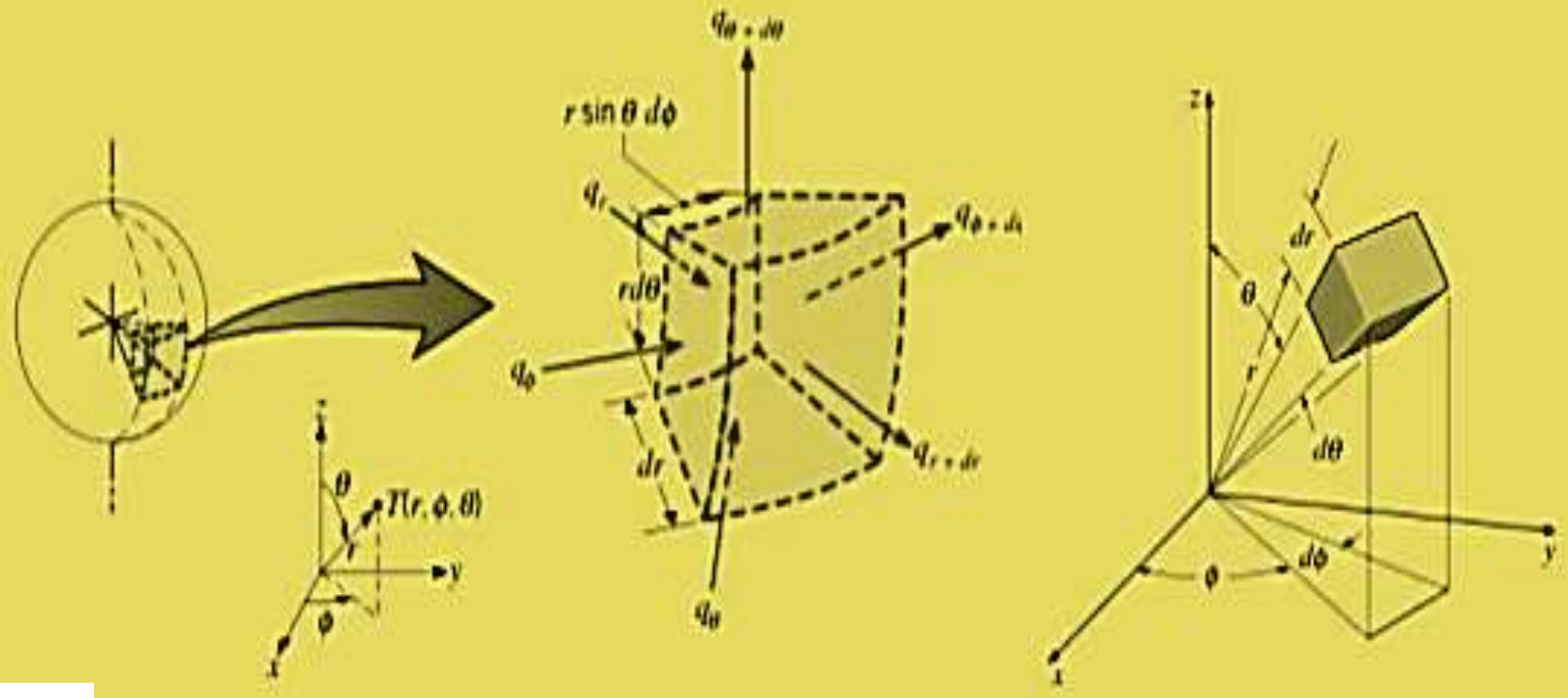
Differential control volume, $dr * rd\phi * dz$, for conduction analysis in cylinderical coordinates(r, ϕ, z).

After lengthy manipulations, we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(kr\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{g}=\rho C\frac{\partial T}{\partial t}$$

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Spherical (or polar) coordinate system: r, ϕ and θ

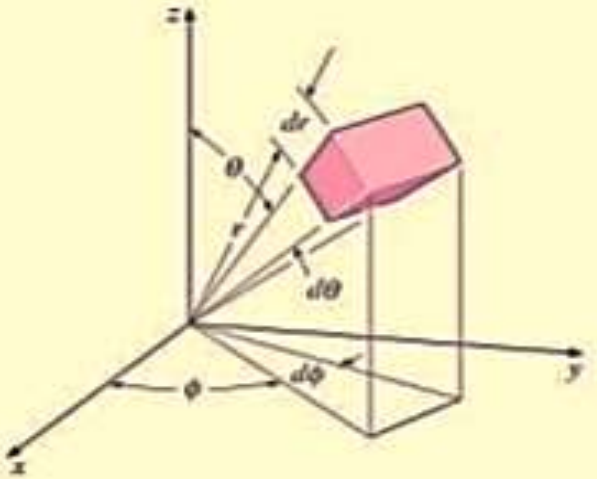


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$x = r\cos\phi\sin\theta$, $y = r\sin\phi\sin\theta$, and $z = \cos\theta$

Differential control volume, $dr * r\sin\theta d\phi * rd\theta$, for conduction analysis in spherical coordinates (r, ϕ, θ).

Again after lengthy manipulations, we obtain



$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(k\frac{\partial T}{\partial\phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(k\sin\theta\frac{\partial T}{\partial\theta}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

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Thermal Resistance





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STEADY HEAT CONDUCTION IN PLANE WALLS

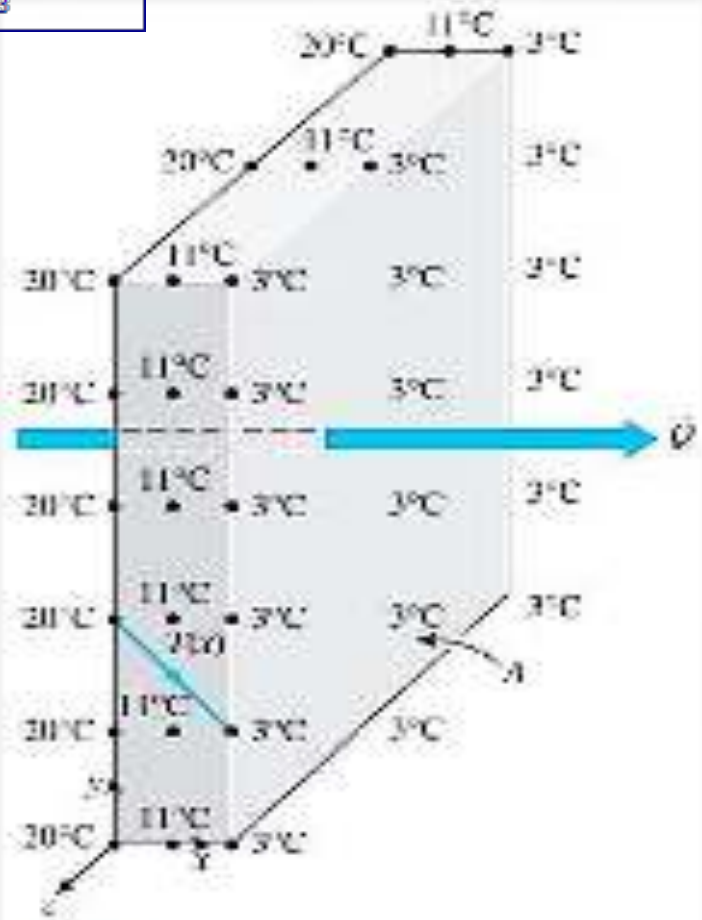


FIGURE
Heat transfer through a wall is one-dimensional when the temperature of the wall varies in one direction only.

Heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional*.

The temperature of the wall in this case depends on one direction only (say the x-direction) and can be expressed as $T(x)$.

$$\begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt}$$

$$dE_{wall}/dt = 0$$

for *steady* operation

In steady operation, the rate of heat transfer through the wall is constant.

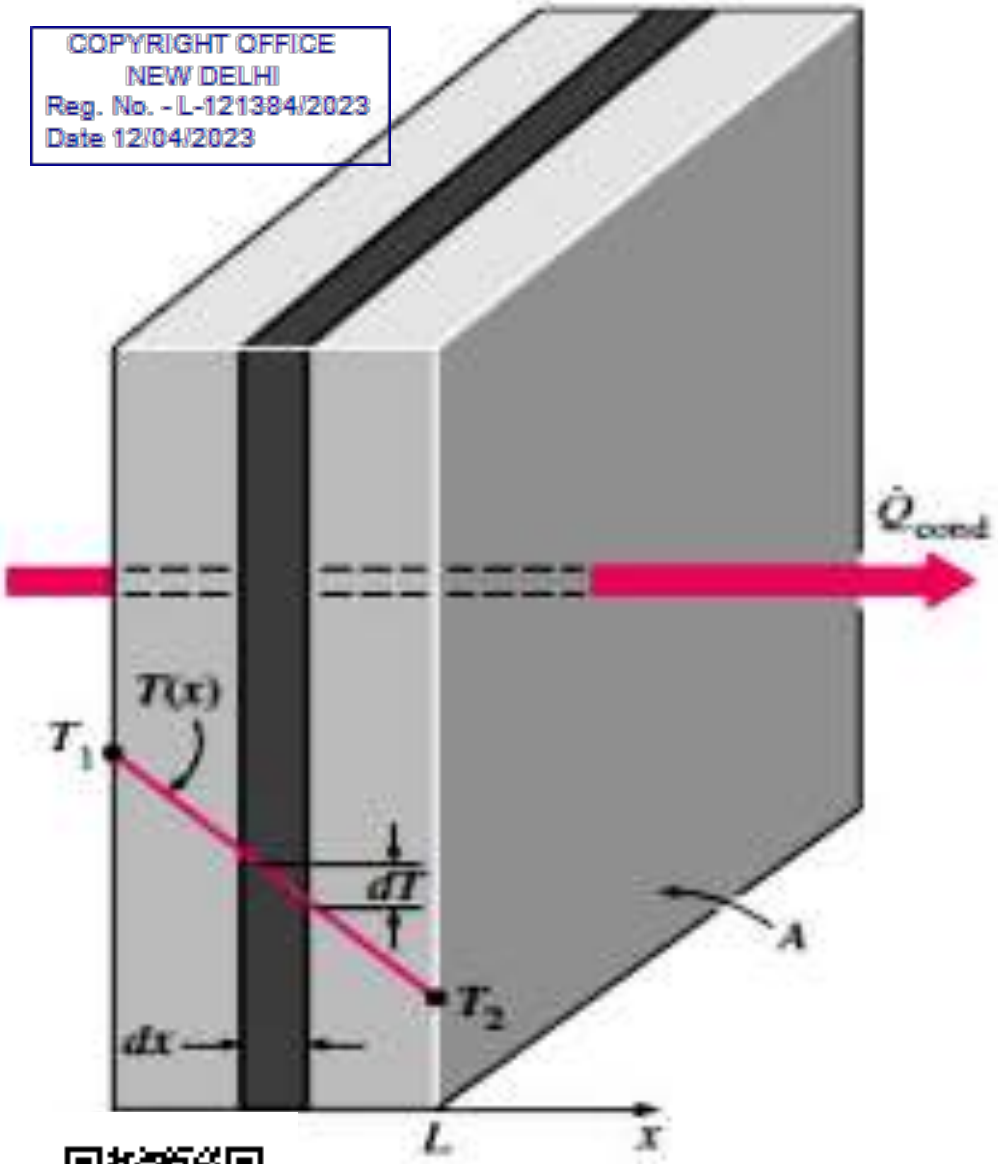
$$\dot{Q}_{cond,\,wall} = -kA\frac{dT}{dx} \quad (W)$$

Fourier's law of heat conduction

3

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STEADY HEAT CONDUCTION



$$Q = \frac{T_1 - T_2}{R}$$





$$R_{th} = \frac{dx}{kA}$$

nder steady conditions.
perature distribution in
e wall is a straight line.

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THERMAL RESITANCE (R_{th})

When two physical systems are described by similar equations and have similar boundary conditions, these are said to be *analogous*. The heat transfer processes may be compared by *analogy* with the flow of electricity in an electrical resistance. As the flow of electric current in the electrical resistance is directly proportional to potential difference (dV); similarly heat flow rate, Q, is directly proportional to temperature difference (dt), the driving force for heat conduction through a medium.

As per Ohm's law (in electric-circuit theory), we have

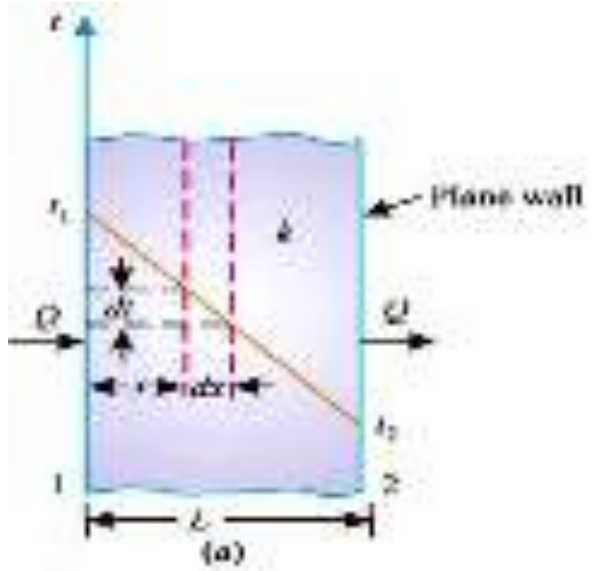
$$\text{Current } (I) = \frac{\text{Potential difference } (dV)}{\text{Electrical resistance } (R)}$$

By analogy, the heat flow equation (Fourier's equation) may be written as

$$\text{Heat flow rate } (Q) = \frac{\text{Temperature difference } (dt)}{\left(\frac{dx}{kA}\right)}$$

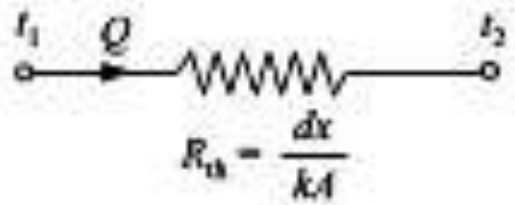
By comparing eqns.

The quantity $\frac{dx}{kA}$ is called



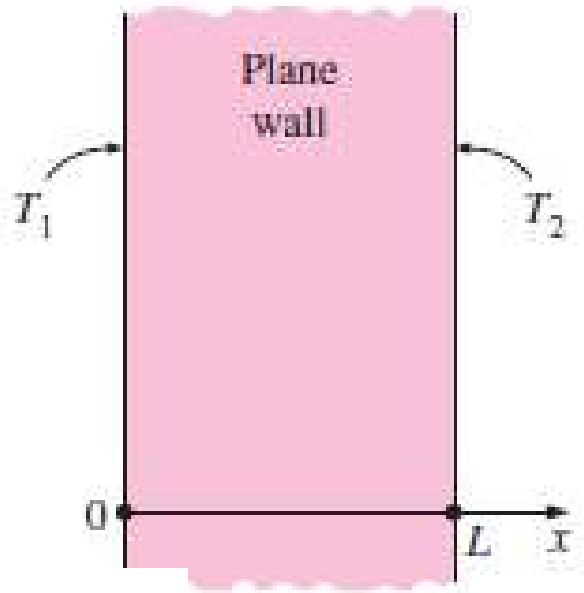
thermal conduction resistance $(R_{th})_{cond.}$ *i.e.*,

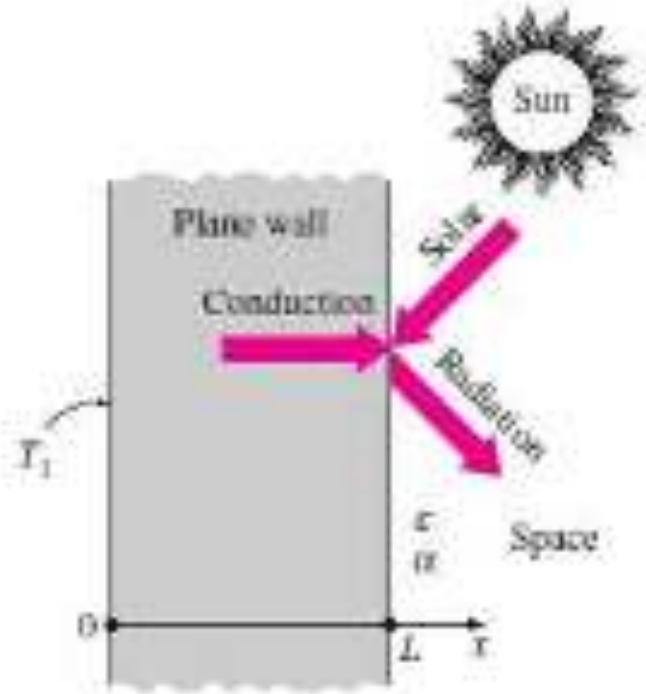
$$(R_{th})_{cond.} = \frac{dx}{kA}$$

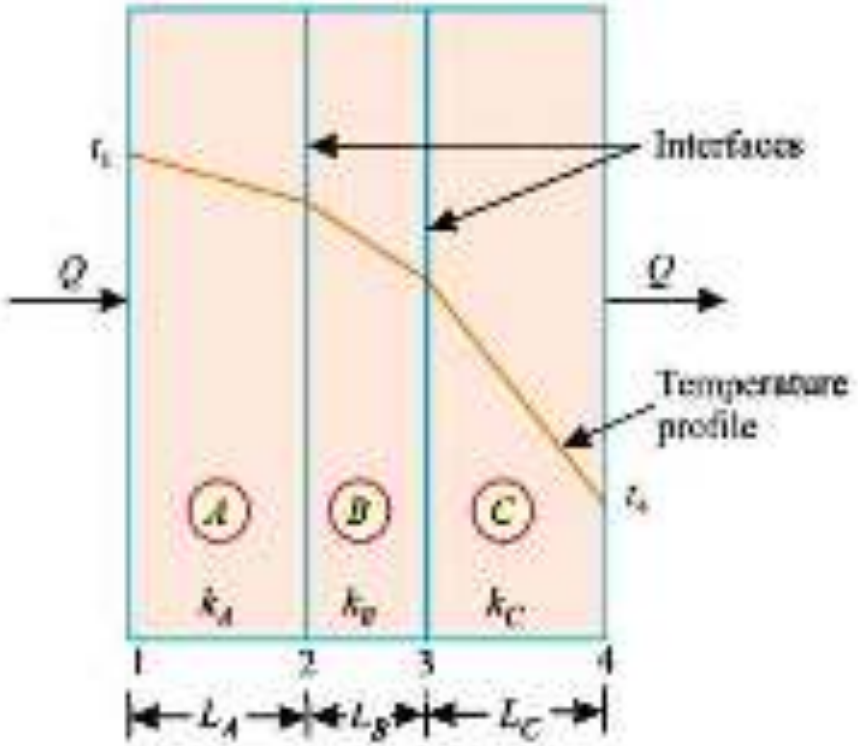


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HEAT CONDUCTION THROUGH PLANE AND COMPOSITE WALLS







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Heat Conduction Through Plane Walls

Consider a plane wall of homogeneous material through which heat is flowing *only in x-direction*.

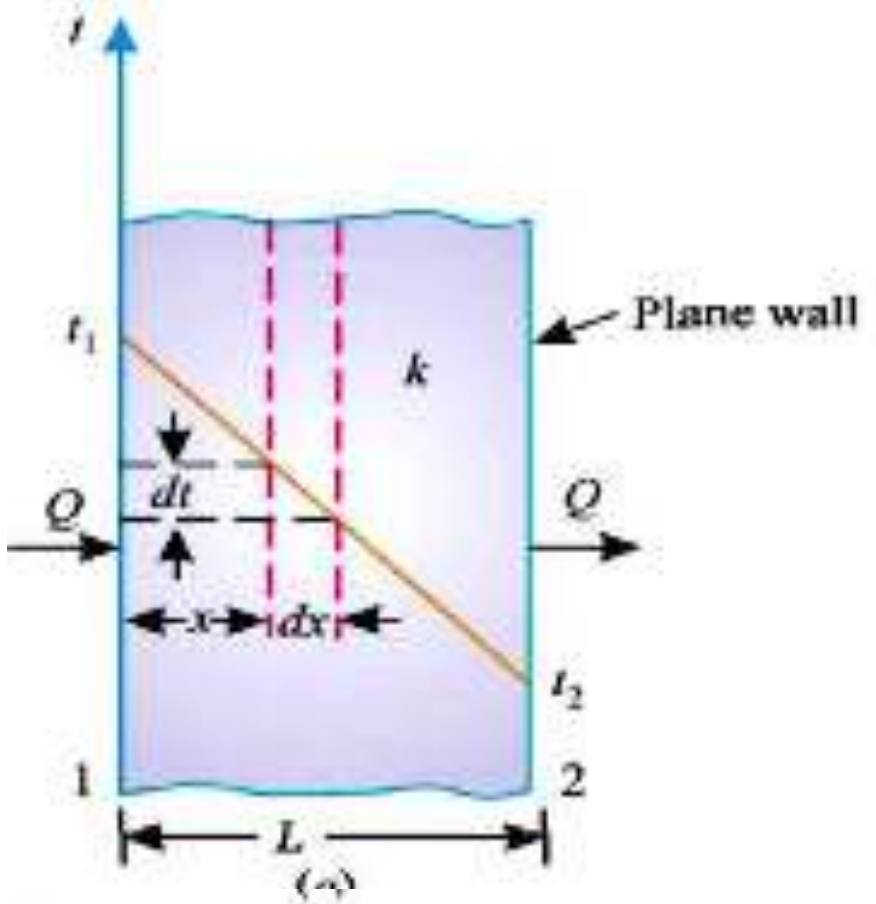
Let,

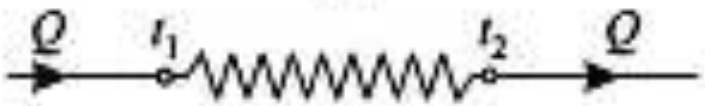
L = Thickness of the plane wall,

A = Cross-sectional area of the wall,

k = Thermal conductivity of the wall material, and

t_1, t_2 = Temperatures maintained at the two faces 1 and 2 of the wall, respectively.





$$(R_{th})_{cond.} = \frac{L}{kA}$$

The general heat conduction equation in cartesian coordinates is given by

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$

$$Q = \frac{(t_1 - t_2)}{(L/kA)} = \frac{(t_1 - t_2)}{(R_{th})_{cond.}}$$

$(R_{th})_{cond.}$ = Thermal resistance to heat conduction.

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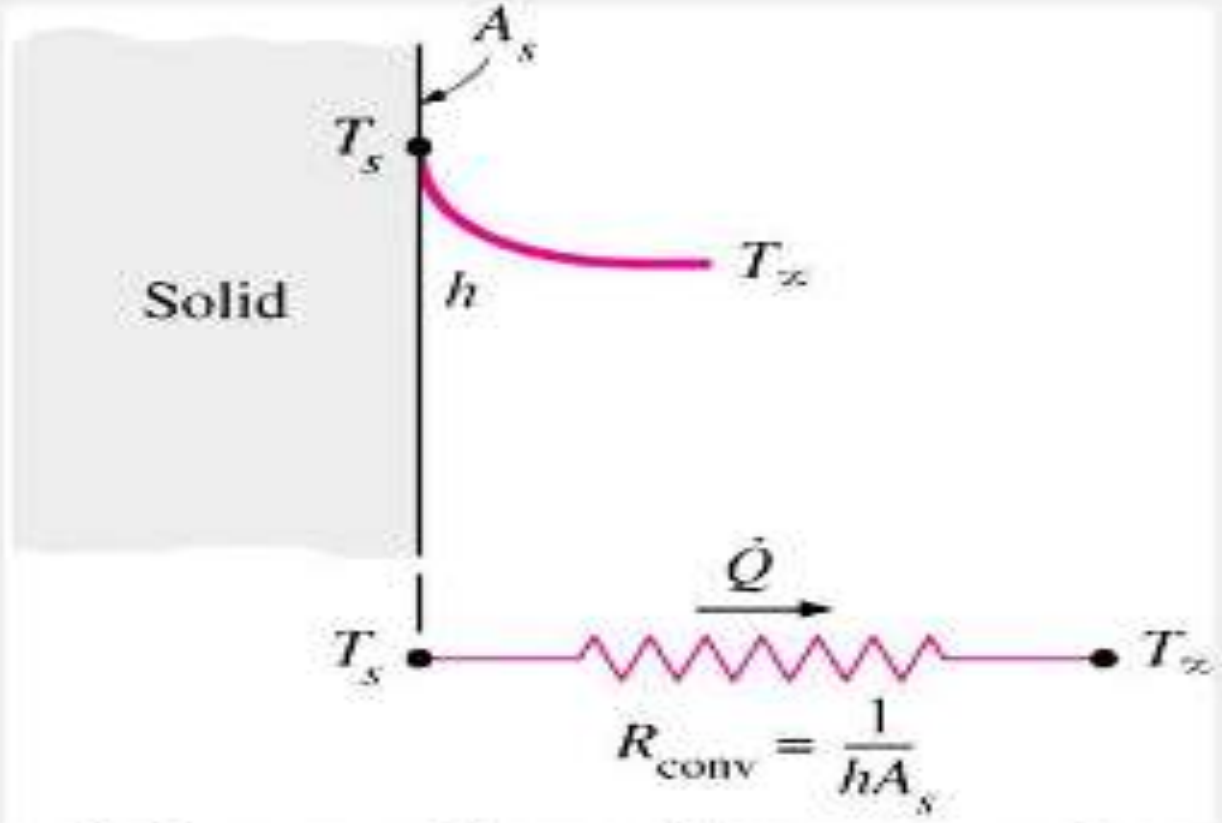
Newton's law of cooling

$$Q_{conv} = hA_s(T_s - T_\infty)$$

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \quad (W)$$

$$R_{conv} = \frac{1}{hA_s} \quad (°C/W)$$

Convection resistance of the surface: *Thermal resistance* of the surface against heat convection.



Schematic for convection resistance at a surface.

When the convection heat transfer coefficient is very large ($h \to \infty$), the convection resistance becomes *zero* and $T_s \approx T_\infty$.

is, the surface offers *no resistance to convection*, and thus it not slow down the heat transfer process.

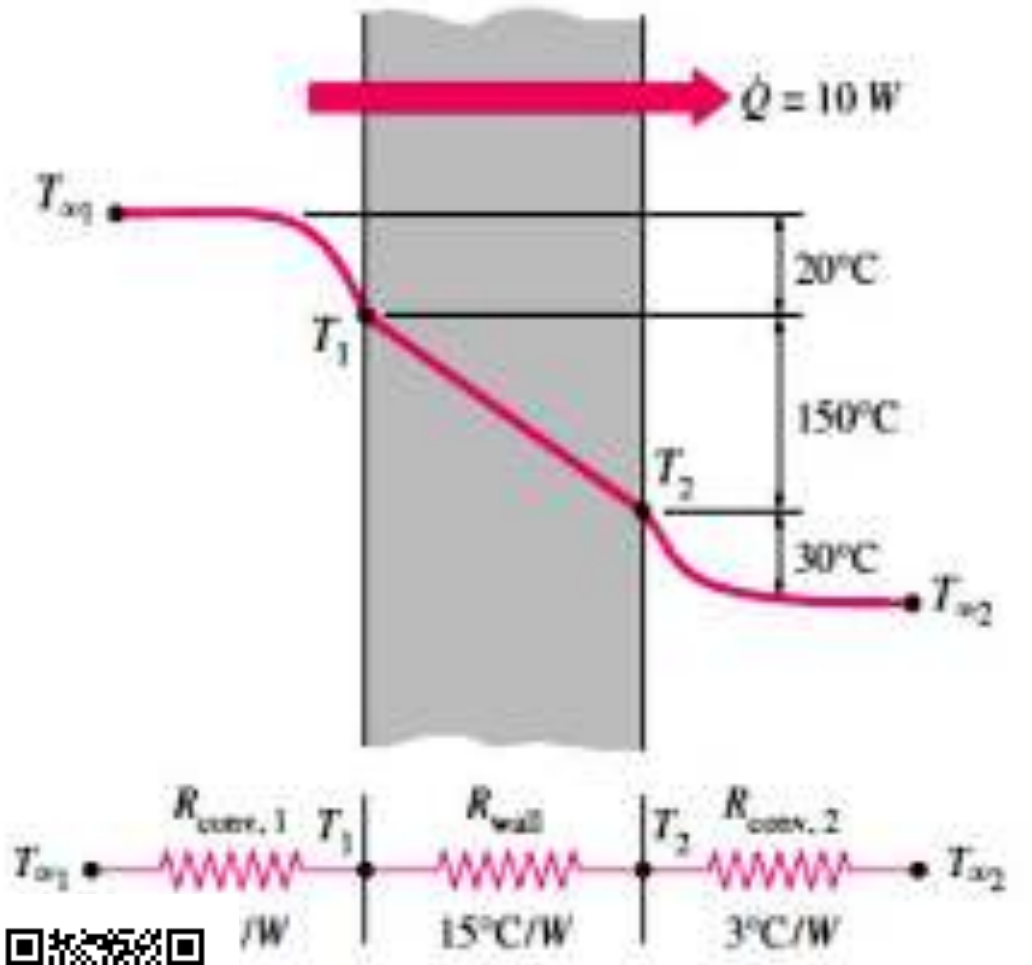
situation is approached in practice at surfaces where boiling condensation occur.

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Thermal Resistance Network

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Heat Conduction Through Plane Walls



$$R_{conv,1} = \frac{1}{h_1 A}$$

$$R_{wall} = \frac{L}{kA}$$

$$R_{conv,2} = \frac{1}{h_2 A}$$

$$\therefore R_{total} = R_{conv,1} + R_{wall} + R_{conv,2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \quad (°C/W)$$

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$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

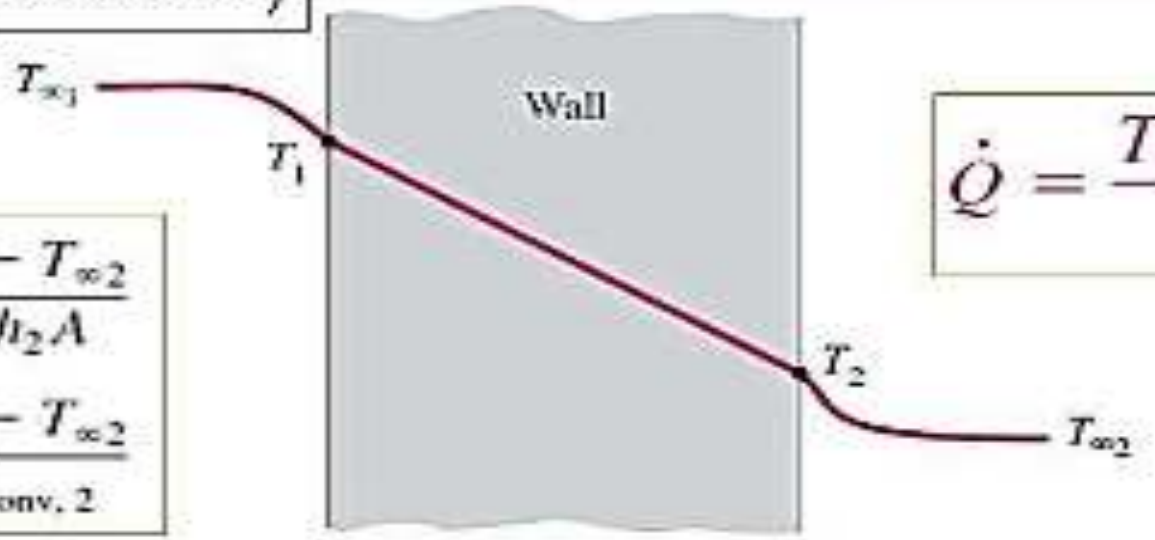
$$= \frac{T_{\infty 1} - T_1}{R_{conv,\,1}} = \frac{T_1 - T_2}{R_{wall}} = \frac{T_2 - T_{\infty 2}}{R_{conv,\,2}}$$

$$\dot{Q} = \frac{T_{\infty} - T_{\infty 2}}{R_{total}} \qquad \text{(W)}$$

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$$\left(\begin{array}{c}\text{Rate of heat convection}\\ \text{into the wall}\end{array}\right) = \left(\begin{array}{c}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{array}\right) = \left(\begin{array}{c}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{array}\right)$$

Thermal Resistance Network



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

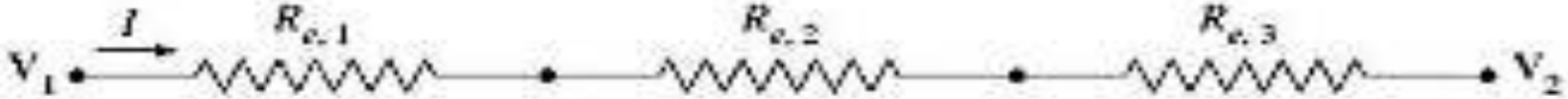
$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv. 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv. 2}}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{conv. 1}} + R_{\text{wall}} + R_{\text{conv. 2}}}$$



Thermal network

$$I = \frac{V_1 - V_2}{R_{e,1} + R_{e,2} + R_{e,3}}$$



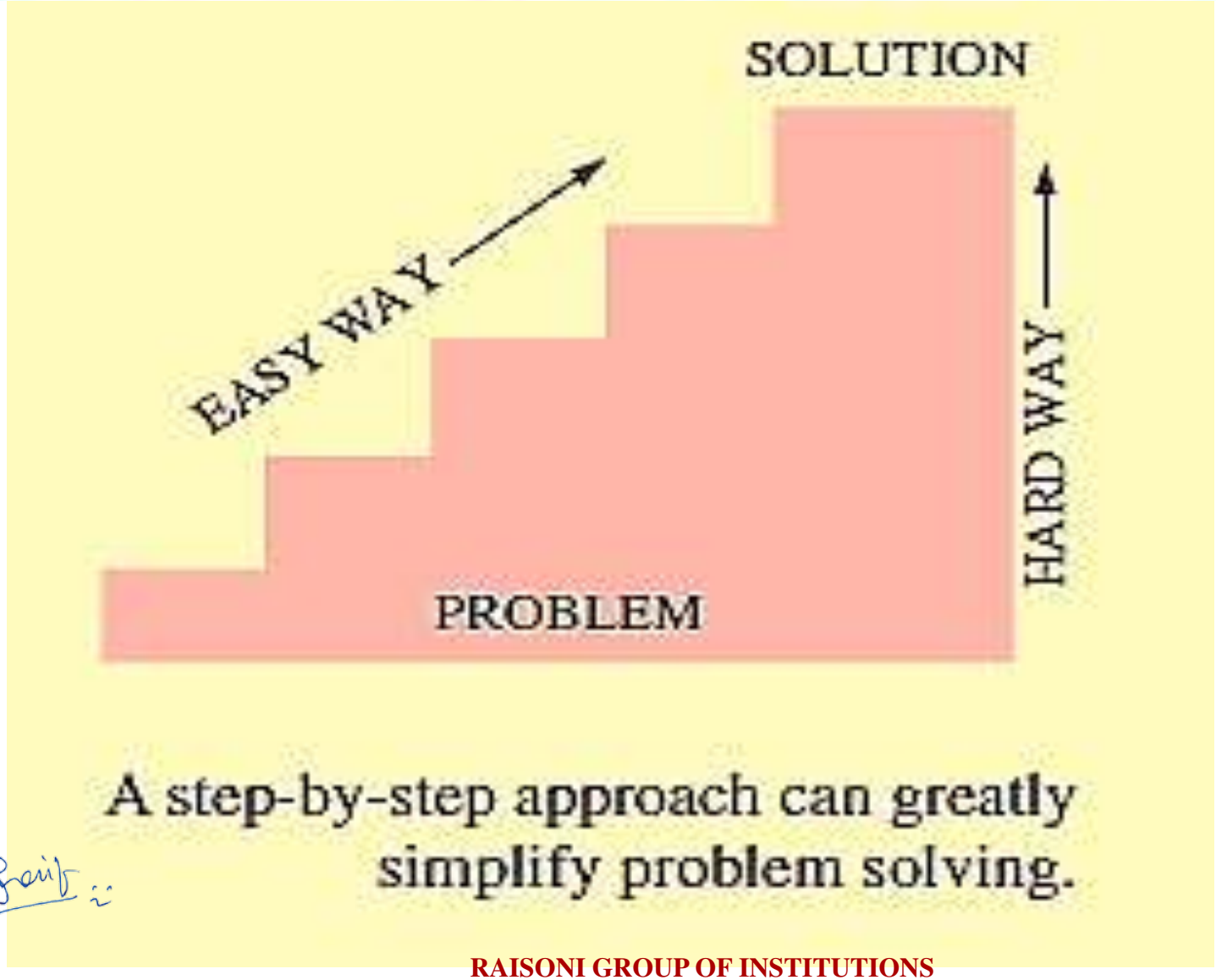
Electrical analogy

rmal resistance network for heat transfer through a plane wall subjected to
on on both sides, and the electrical analogy.

$$R_{\text{total}} = R_{\text{conv. 1}} + R_{\text{wall}} + R_{\text{conv. 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \quad (°C/W)$$

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PROBLEM-SOLVING TECHNIQUE



A step-by-step approach can greatly simplify problem solving.

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Numerical on

Single Layer or Plane Wall



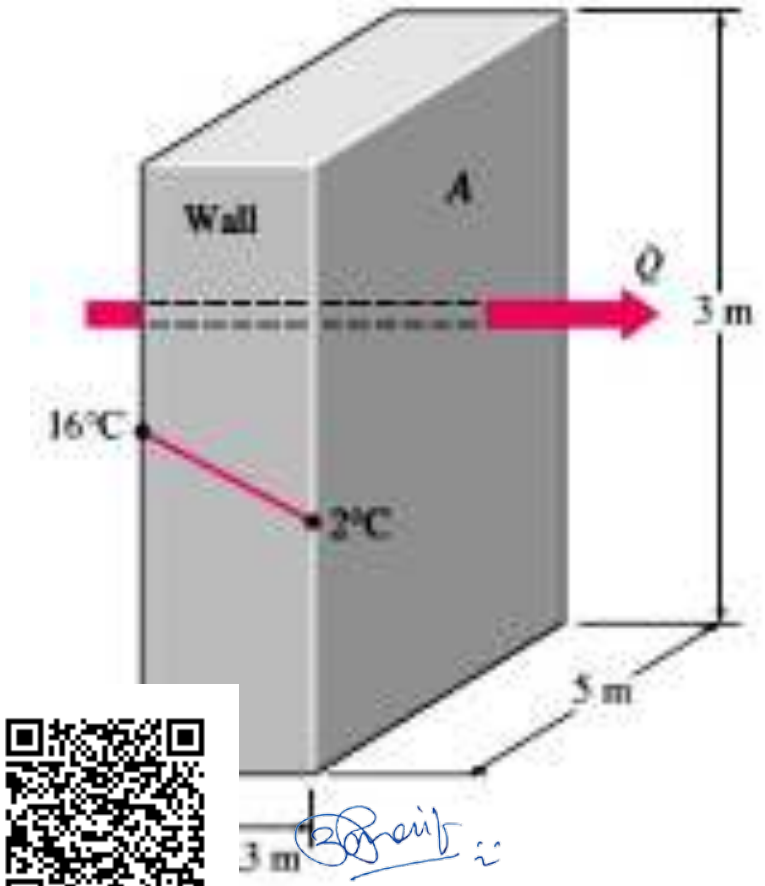
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Numerical on Single Layer or Plane Wall

Example No.1

2. Consider a 3-m-high, 5-m-wide, and 0.3-m-thick wall whose thermal conductivity is k 0.9 W/m ·°C (as shown in Fig.). On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 16°C and 2°C, respectively. Determine the rate of heat loss through the wall on that day.

Solution.



The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

Assumptions

1. Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values.
2. Heat transfer is one dimensional since any significant temperature gradients will exist in the direction
3. from the indoors to the outdoors.
4. Thermal conductivity is constant.

Properties The thermal conductivity is given to be k 0.9 W/m · °C.

Analysis Noting that the heat transfer through the wall is by conduction and the area of the wall is A 3 m 5 m 15 m_2, the steady rate of heat transfer through the wall can be determined from Equation.

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Example No.1

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.9\ \text{W/m} \cdot {}^\circ\text{C})(15\ \text{m}^2)\frac{(16 - 2)^\circ\text{C}}{0.3\ \text{m}} = 630\ \text{W}$$

We could also determine the steady rate of heat transfer through the wall by making use of the thermal resistance concept from

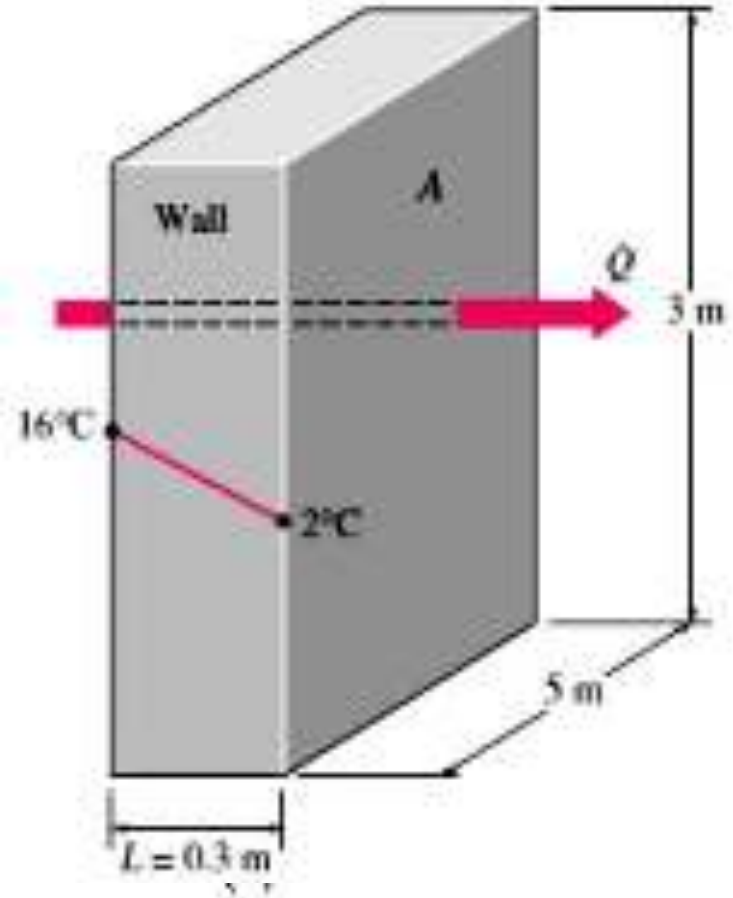
$$\dot{Q} = \frac{\Delta T_{\text{wall}}}{R_{\text{wall}}}$$

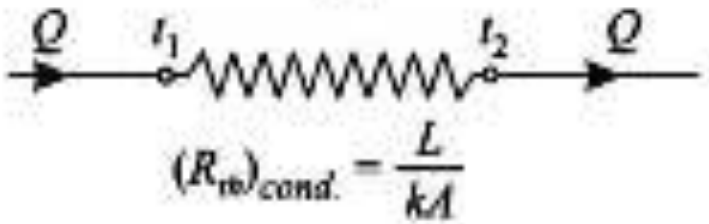
where

$$R_{\text{wall}} = \frac{L}{kA} = \frac{0.3\ \text{m}}{(0.9\ \text{W/m} \cdot {}^\circ\text{C})(15\ \text{m}^2)} = 0.02222^\circ\text{C/W}$$

g. Regret ∴

$$\dot{Q} = \frac{(16 - 2)^\circ\text{C}}{0.02222^\circ\text{C/W}} = 630\ \text{W}$$





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Example No.2

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m · °C. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m² · °C and $h_2 = 40$ W/m² · °C, which includes the effects of radiation.

SOLUTION

Properties The thermal conductivity is given to be $k = 0.78$ W/m · °C.

Analysis This problem involves conduction through the glass window and convection at its surfaces, and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in

12. Noting that the area of the window is $A = 0.8\ \text{m} \times 1.5\ \text{m} = 1.2\ \text{m}^2$, ividual resistances are evaluated from their definitions to be

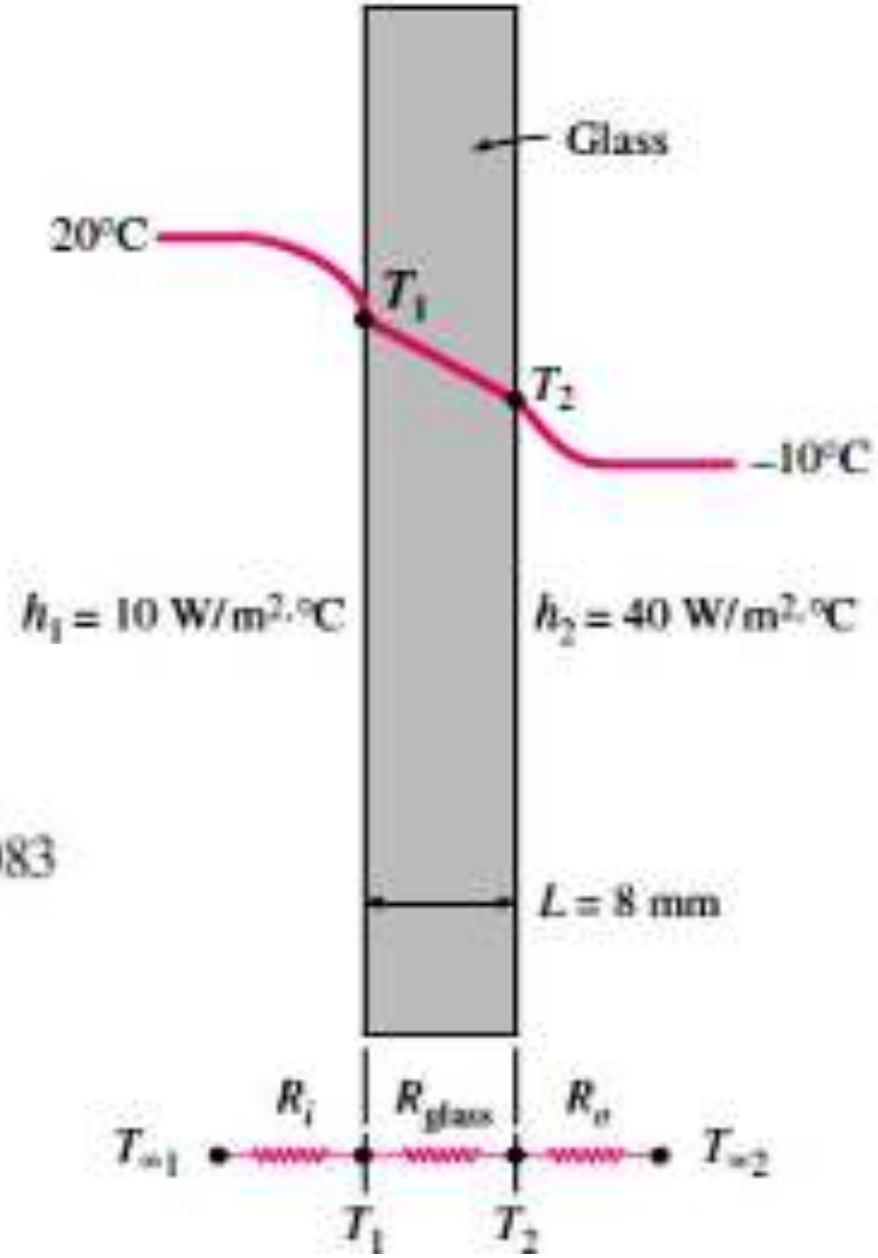
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Example No.2

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_{glass} = \frac{L}{kA} = \frac{0.008\ \text{m}}{(0.78\ \text{W/m} \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.00855^\circ\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(40\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.02083^\circ\text{C/W}$$



Noting that all three resistances are in series, the total resistance is

$$R_{total} = R_{conv,1} + R_{glass} + R_{conv,2} = 0.08333 + 0.00855 + 0.02083$$
$$= 0.1127^\circ\text{C/W}$$

Th dy rate of heat transfer through the window becomes

$$\frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20 - (-10)]^\circ\text{C}}{0.1127^\circ\text{C/W}} = 266\ \text{W}$$

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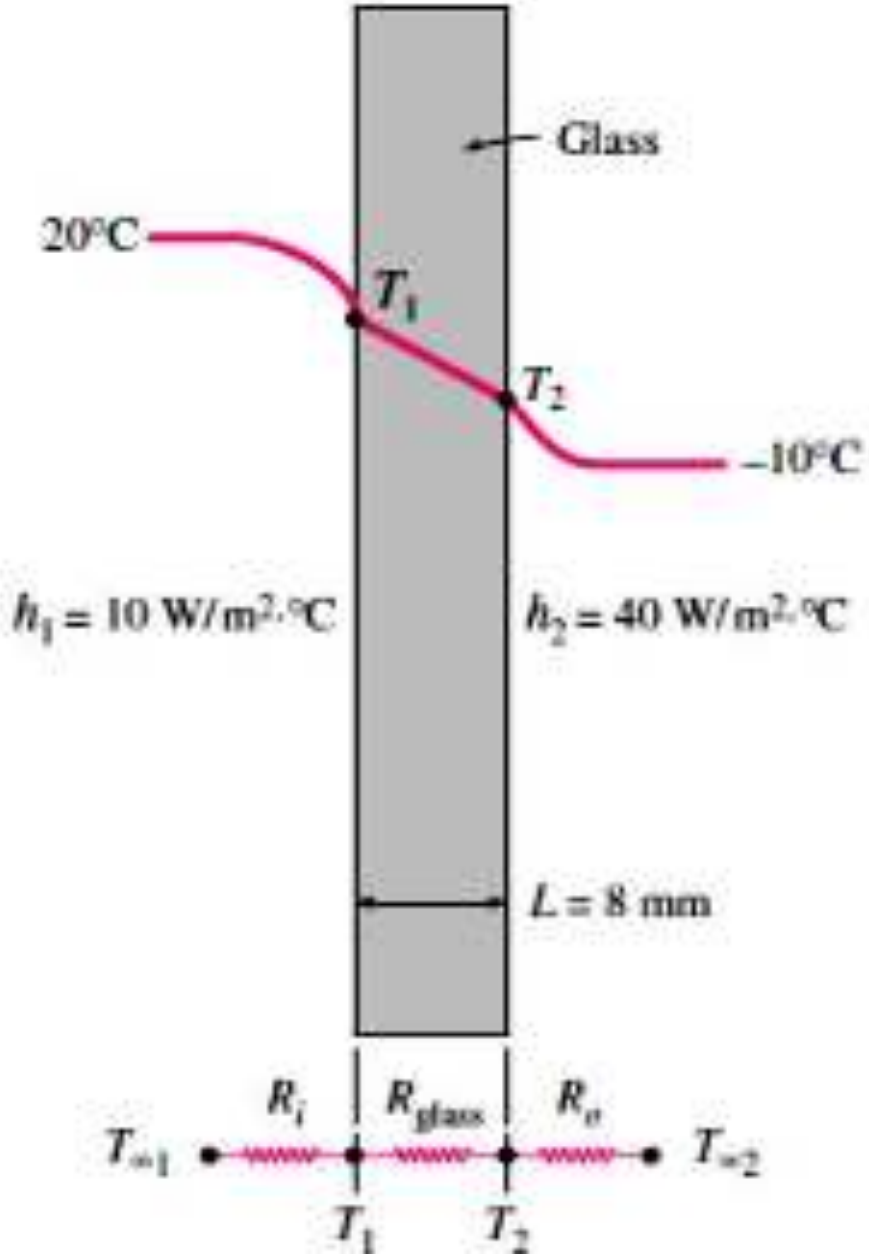
Example No.2

Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1}$$

$$= 20°C - (266\ W)(0.08333°C/W)$$

$$= -2.2°C$$



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HEAT CONDUCTION THROUGH A COMPOSITE WALL

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HEAT CONDUCTION THROUGH A COMPOSITE WALL

Consider the transmission of heat transfer through a composite wall consisting of a number of slabs.

Let, L_A, L_B, L_C = Thicknesses of slabs A, B and C respectively (also called path lengths),

k_A, k_B, k_C = Thermal conductivities of the slabs A, B, and C respectively,

$t_1, t_4\ (t_1 > t_4)$ = Temperatures at the wall surfaces 1 and 4 respectively, and

t_2, t_3 = Temperatures at the interfaces 2 and 3 respectively.

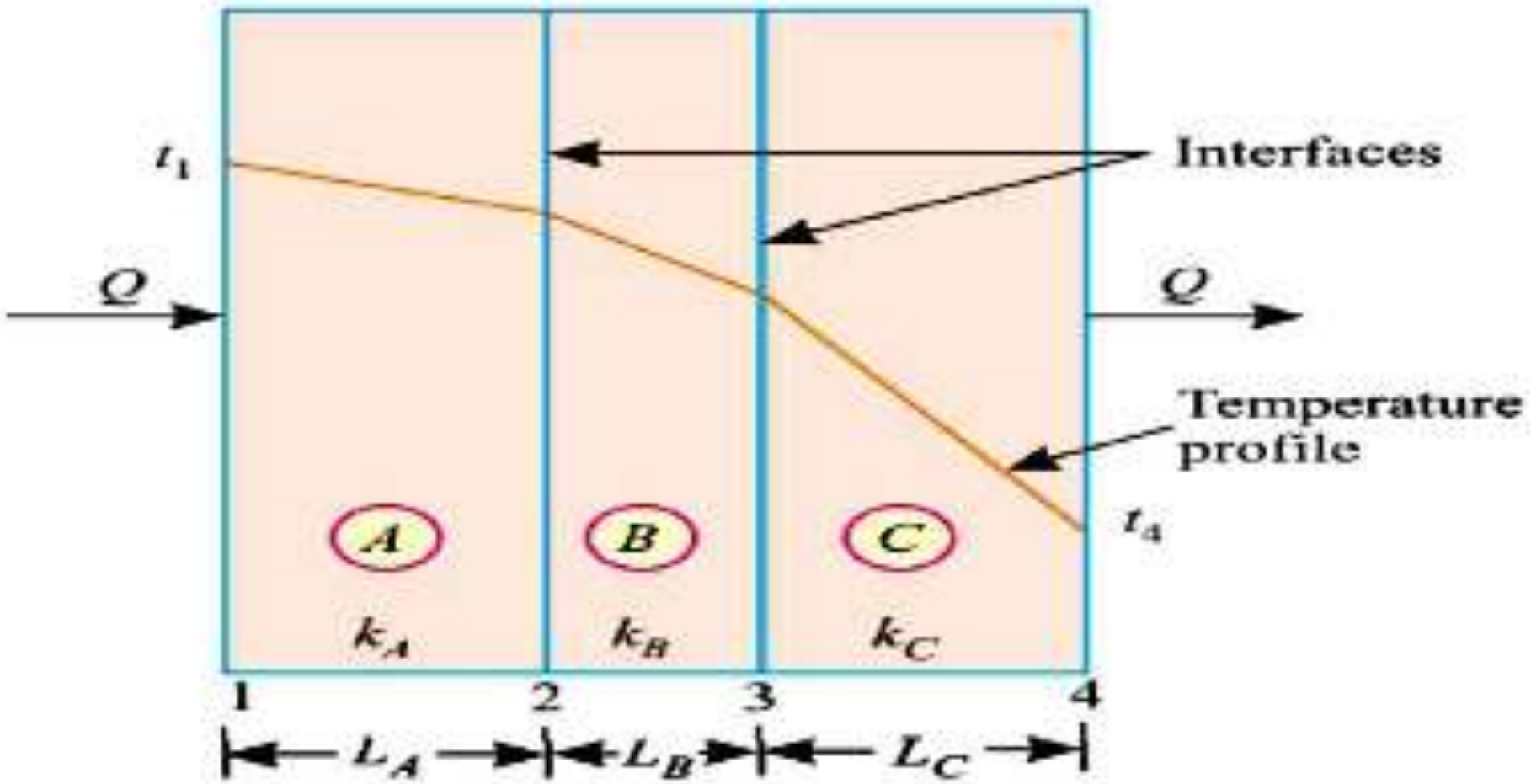


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Cont..

HEAT CONDUCTION THROUGH A COMPOSITE WALL

$$R_{th.A} = \frac{L_A}{k_A . A},$$



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HEAT CONDUCTION THROUGH A COMPOSITE WALL

As per Fourier Law

$$Q = -kA\frac{dt}{dx}$$

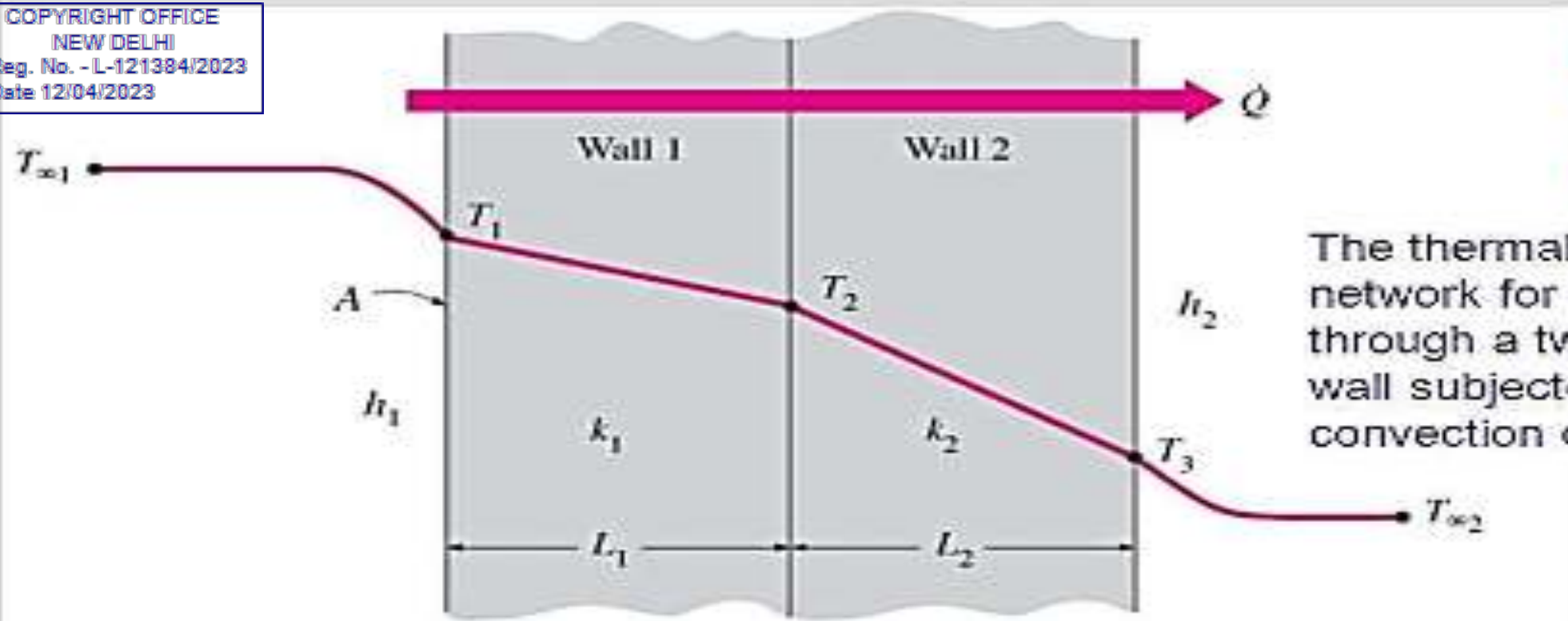
$$Q = \frac{A\,(t_1 - t_4)}{\left[\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}\right]}$$

or

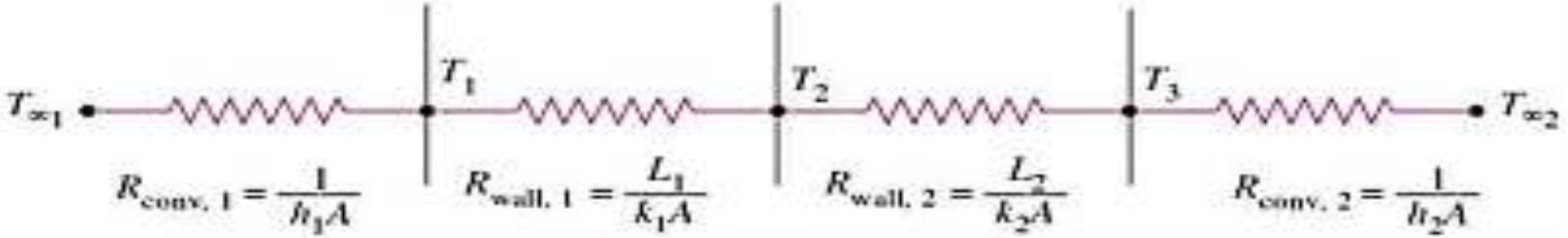
$$Q = \frac{(t_1 - t_4)}{\left[\frac{L_A}{k_A \cdot A} + \frac{L_B}{k_B \cdot A} + \frac{L_C}{k_C \cdot A}\right]} = \frac{(t_1 - t_4)}{[R_{th-A} + R_{th-B} + R_{th-C}]}$$

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Multilayer Plane Walls



The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.



$$\frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall, 1}} + R_{\text{wall, 2}} + R_{\text{conv, 2}}$$

$$= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

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Numerical on Thermal Resistance

Multi-layered Wall



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Example No.1

1. Calculate the rate of heat loss through the vertical walls of a boiler furnace of size 4 m by 3 m by 3 m high. The walls are constructed from an inner fire brick wall 25 cm thick of thermal conductivity 0.4 W/mK, a layer of ceramic blanket insulation of thermal conductivity 0.2 W/mK and 8 cm thick, and a steel protective layer of thermal conductivity 55 W/mK and 2 mm thick. The inside temperature of the fire brick layer was measured at 600° C and the temperature of the outside of the insulation 60° C. Also find the interface temperature of layers.

Solution.

Given:

Composite Wall

l= 4m b= 3m h= 3m

Area of rectangular wall lb = 4x3 = 12m²

L_1 = 25 cm — Fire brick

k_1 = 0.4 W/mK

L_2 =0.002m — Steel

k_2 = 54 W/mK

insulation

mK

T_2 = 60° C

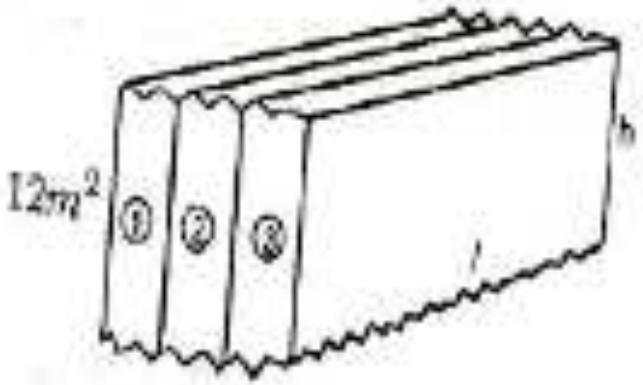


Fig. a

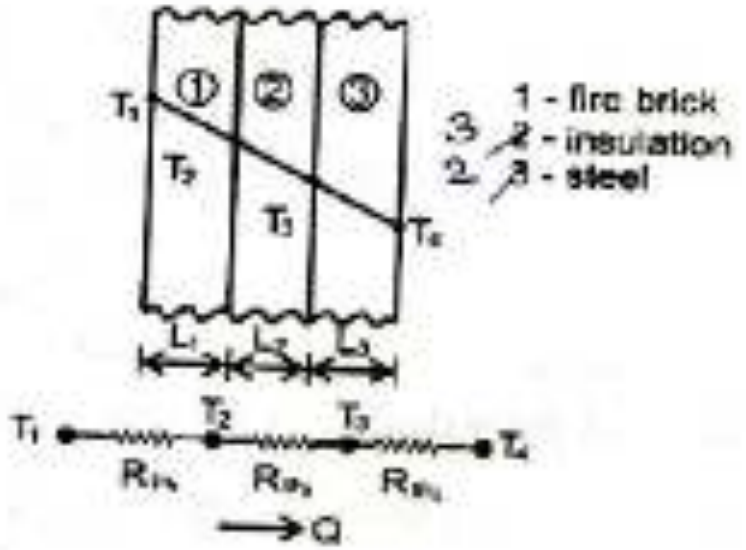


Fig. b

Find

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Example No.1

We know that,

$$Q = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$

Here

(ΔT) overall $= T_1 - T_4$

And $\Sigma R_{th} = R_{th1} + R_{th2} + R_{th3}$

$$R_{th1} = \frac{L_1}{k_1 A} = \frac{0.25}{0.4 \times 12} = 0.0521 \text{ K/W}$$

$$R_{th2} = \frac{L_2}{k_2 A} = \frac{0.08}{0.2 \times 12} = 0.0333 \text{ K/W}$$

$$R_{th3} = \frac{L_3}{k_3 A} = \frac{0.002}{54 \times 12} = 0.0000031 \text{ K/W}$$

$$= \frac{T_1 - T_4}{R_{th1} + R_{th2} + R_{th3}}$$

$$= \frac{600 - 60}{0.0521 + 0.0000031 + 0.0333}$$

$$Q = 6320.96 \text{ W}$$

To find temperature drop across the steel layer $(T_2 - T_3)$

$$Q = \frac{T_2 - T_3}{R_{th3}}$$

$$T_3 - T_4 = Q \times R_{th2}$$

$$= 6320.96 \times 0.0000031$$

$$T_3 - T_4 = 0.0196 \text{ K}.$$

Example No 2

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Example *Calculate the rate of heat flow per m^2 through a furnace wall consisting of 200 mm thick inner layer of chrome brick, a centre layer of kaolin brick 100 mm thick and an outer layer of masonry brick 100 mm thick. The unit surface conductance at the inner surface is 74 $W/m^2°C$ and the outer surface temperature is 70°C. The temperature of the gases inside the furnace is 1670°C. What temperatures prevail at the inner and outer surfaces of the centre layer ?*

Take : $k_{chrome\ brick} = 1.25\ W/m°C$; $k_{kaolin\ brick} = 0.074\ W/m°C$; $k_{masonry\ brick} = 0.555\ W/m°C$

Assume steady heat flow.

Solution.

Thickness of chrome bricks, $L_A = 200\ mm = 0.2\ m$

Thickness of kaolin bricks, $L_B = 100\ mm = 0.1\ m$

Thickness of masonry bricks, $L_C = 100\ mm = 0.1m$

Thermal conductivities : $k_A = 1.25\ W/m°C$;

$k_B = 0.074\ W/m°C$; $k_C = 0.555\ W/m°C$;

The unit surface conductance, $h_{hf} = 74\ W/m^{2o}C$

T f hot fluid, $t_{hf} (= t_g) = 1670°C$

T f the outer surface, $t_4 = 70°C$

Example No. 2

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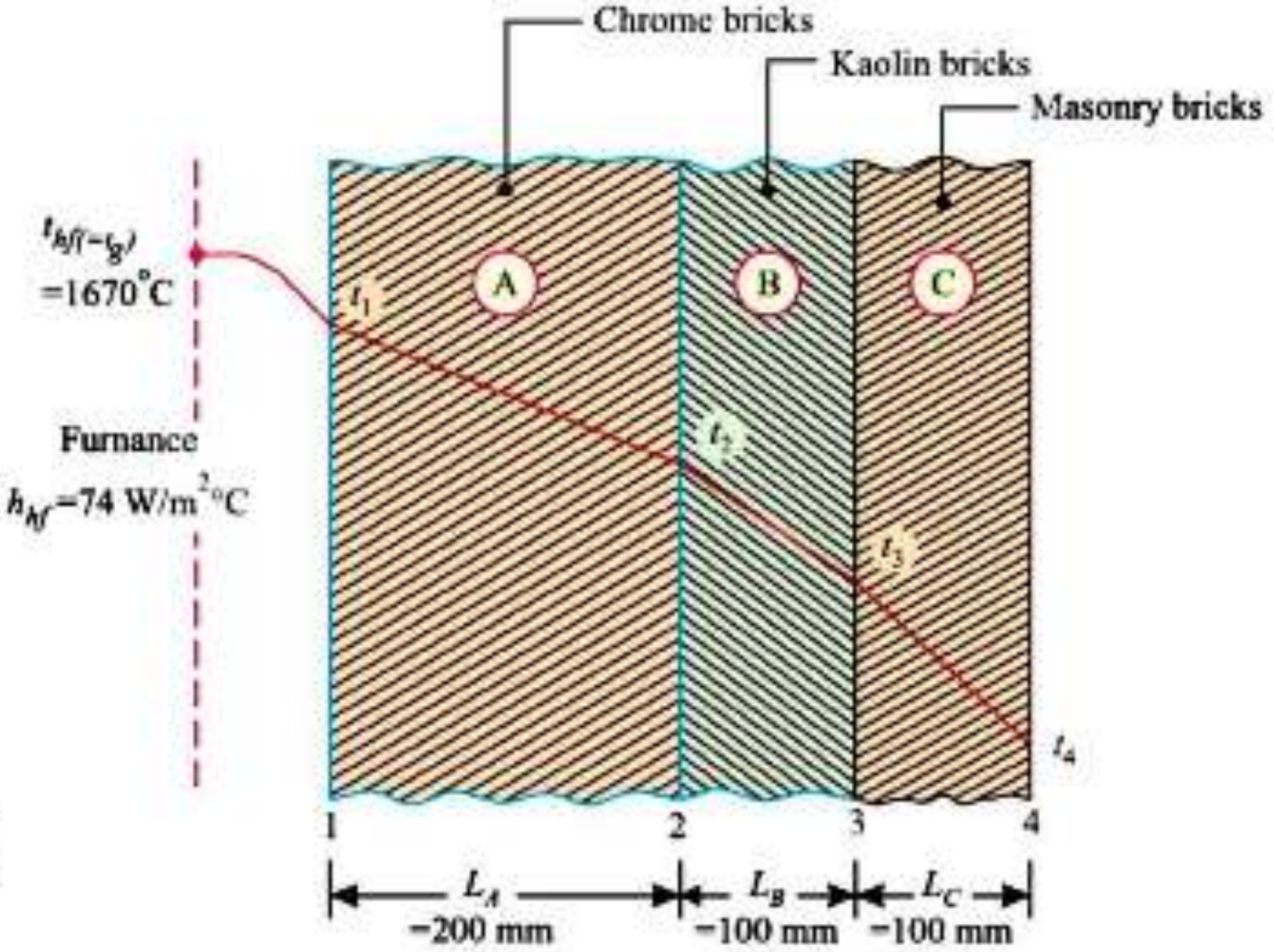
(i) Rate of heat flow per m^2, q :

$$q = \frac{(t_{hf} - t_4)}{\frac{1}{h_{hf}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}}$$

$$= \frac{(1670 - 70)}{\frac{1}{74} + \frac{0.2}{1.25} + \frac{0.1}{0.074} + \frac{0.1}{0.555}}$$

$$= \frac{1600}{\cdot\ 0.16 + 1.351 + 0.1802}$$

W/m² (Ans.)



Example No. 2

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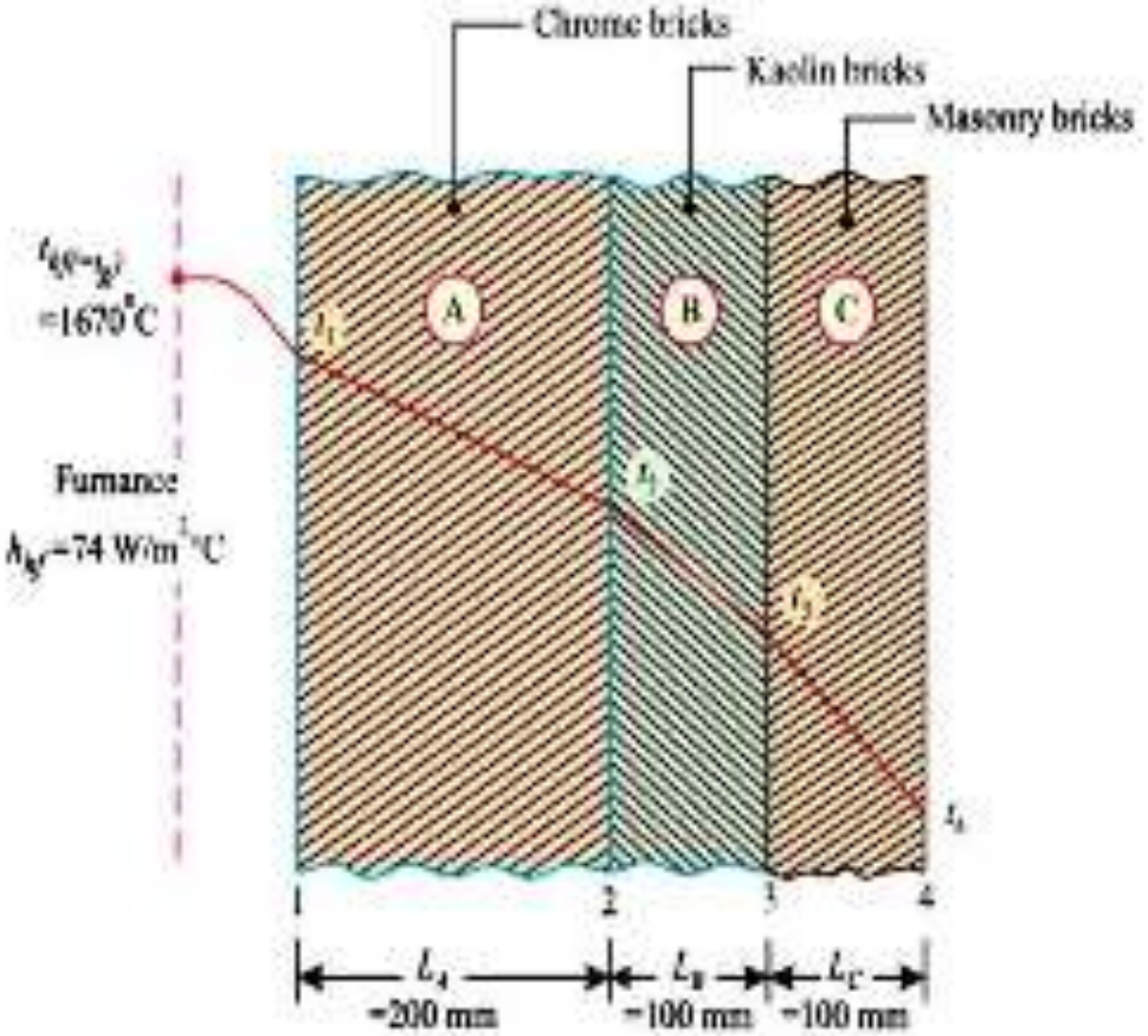
(ii) Temperatures; t_2, t_3 :

The heat flow is given by

$$q = \frac{(t_{hf} - t_1)}{1/h_{hf}} = \frac{(t_1 - t_2)}{L_A/k_A} = \frac{(t_2 - t_3)}{(L_B/k_B)}$$

$$\therefore \quad 938.58 = \frac{1670 - t_1}{1/74}$$

or $$t_1 = 1670 - 938.58 \times \frac{1}{74} = 1657.3°C$$



$$938.58 = \frac{(1657.3 - t_2)}{0.2/1.25} \quad \text{or} \quad t_2 = 1657.3 - 938.58 \times \frac{0.2}{1.25} = \mathbf{1507.1°C} \quad \textbf{(Ans)}$$

$$938.58 = \frac{(1507.1 - t_3)}{(0.1/0.074)} \quad \text{or} \quad t_3 = 1507.1 - 938.58 \times \frac{0.1}{0.074} = \mathbf{238.7°C} \quad \textbf{(Ans.)}$$

Example No. 3

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Example. *The interior of a refrigerator having inside dimensions of 0.5 m × 0.5 m base area and 1m height, is to be maintained at 6°C. The walls of the refrigerator are constructed of two mild steel sheets 3mm thick (k = 46.5 W/m°C) with 50 mm of glass wool insulation (k = 0.046 W/m°C) between them. If the average heat transfer coefficients at the outer and inner surfaces are 11.6 W/m²°C and 14.5 W/m²°C respectively, calculate :*

(i) The rate at which heat must be removed from the interior to maintain the specified temperature in the kitchen at 25°C, and

(ii) The temperature on the outer surface of the metal sheet.

Solution.

$L_A = L_C = 3\text{mm} = 0.003\text{ m}$

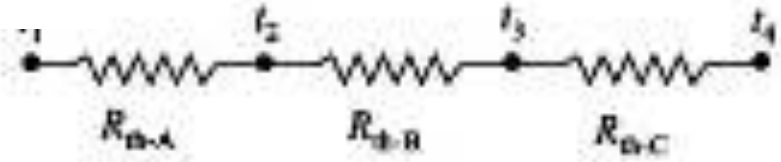
$L_B = 50\text{ mm} = 0.05\text{ m}$

$k_A = k_C = 46.5\text{ W/m°C};$

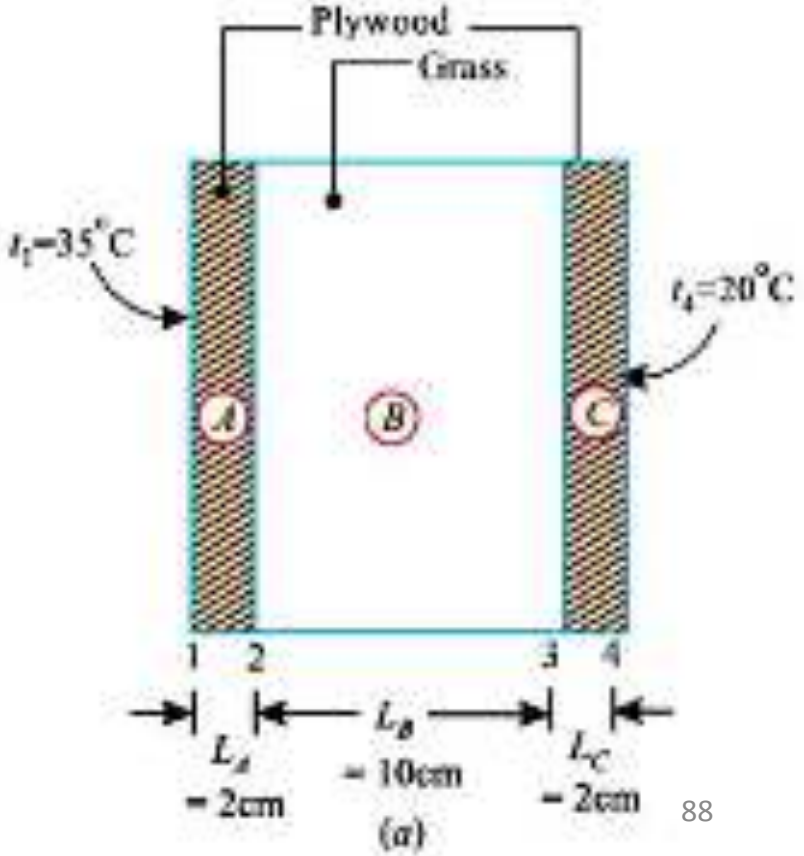
$k_B = 0.046\text{ W/m°C}$

$h_o = 11.6\text{ W/m}^2\text{°C};\ h_i = 14.5\text{ W/m}^2\text{°C}$

$5°C;\ t_1 = 6°C.$



(b)



(a)

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The total area through which heat is coming into the refrigerator,

$$A = 0.5 \times 0.5 \times 2 + 0.5 \times 1 \times 4 = 2.5\ \text{m}^2$$

(i) The rate of removal of heat, Q :

$$Q = \frac{A(t_0 - t_i)}{\frac{1}{h_0} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_i}}$$

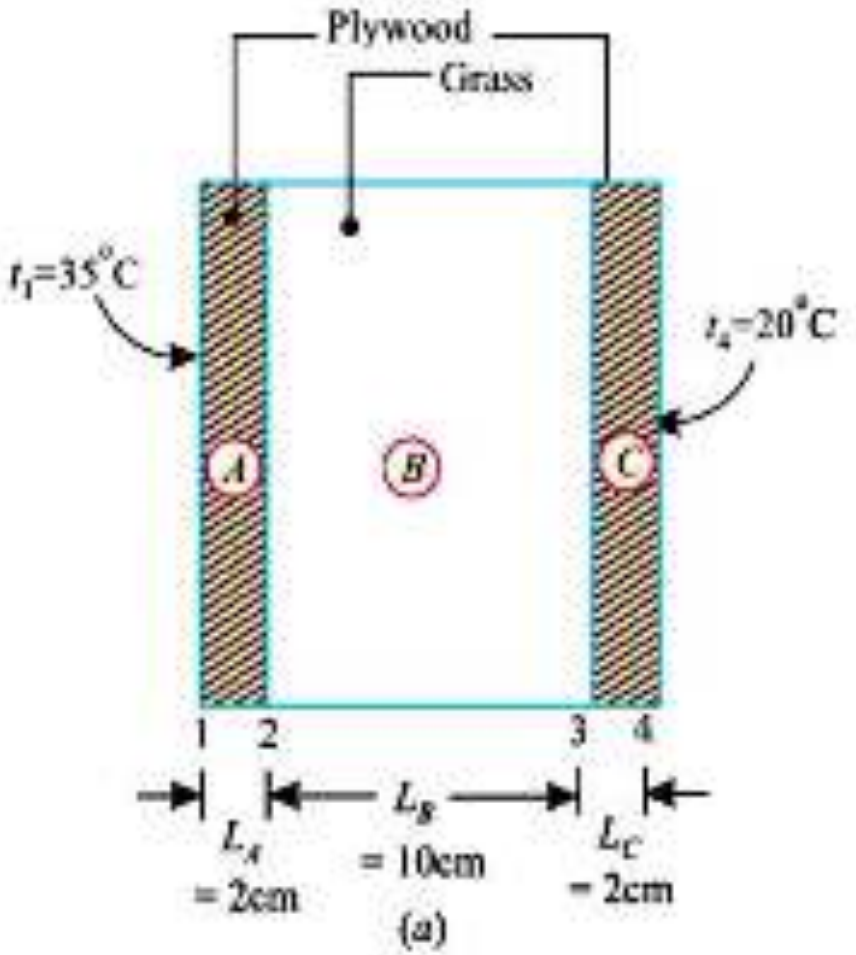
$$= \frac{2.5(25 - 6)}{\frac{1}{11.6} + \frac{0.003}{46.5} + \frac{0.05}{0.046} + \frac{0.003}{46.5} + \frac{1}{14.5}} = \mathbf{38.2\ W} \quad \textbf{(Ans.)}$$

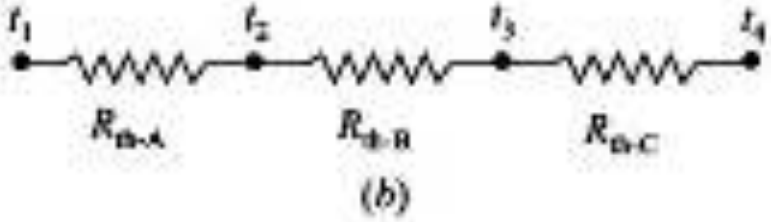
(ii) The temperature at the outer surface of the metal sheet, t :

$$Q = h_0 A (25 - t_1)$$

$$38.2 = 11.6 \times 2.5 (25 - t_1)$$

$$t_1 = 25 - \frac{38.2}{11.6 \times 2.5} = \mathbf{23.68^\circ C} \quad \textbf{(Ans.)}$$





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Complex Problems of Composite Walls

In order to solve more complex problems involving both series and parallel thermal resistances, the electrical analogy may be used. A typical problem and its analogous electric circuit are shown in Fig.

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$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \qquad R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv}$$

$$R_1 = \frac{L_1}{k_1 A_1} \qquad R_2 = \frac{L_2}{k_2 A_2}$$

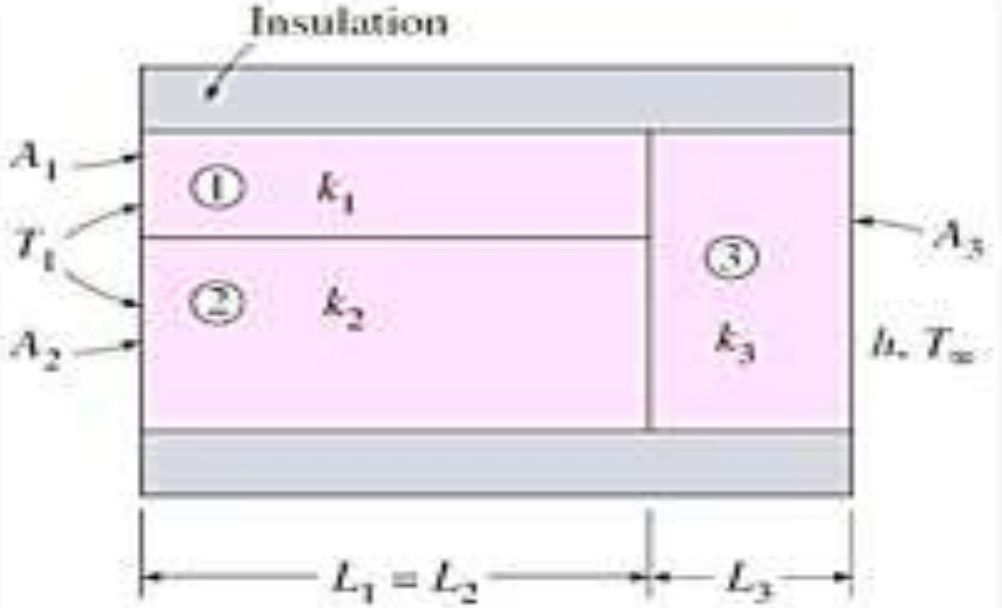
$$R_3 = \frac{L_3}{k_3 A_3} \qquad R_{conv} = \frac{1}{hA_3}$$

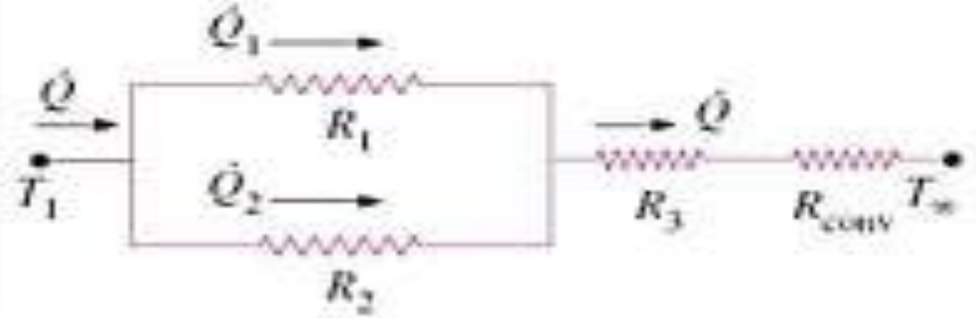
Two assumptions in solving complex multidimensional heat transfer problems by treating them as one-dimensional using the thermal resistance network are

(1) any plane wall normal to the *x*-axis is *isothermal* (i.e., to assume the temperature to vary in the *x*-direction only)

ny plane parallel to the *x*-axis is
diabatic (i.e., to assume heat transfer
o occur in the *x*-direction only)

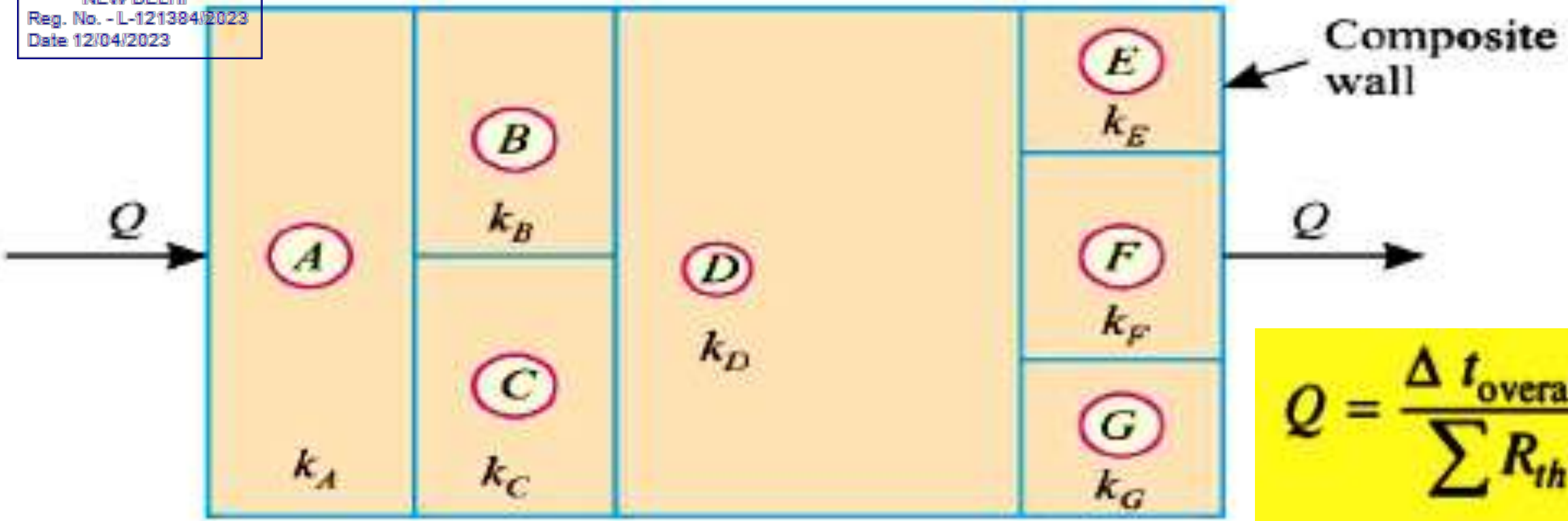
hey give the same result?



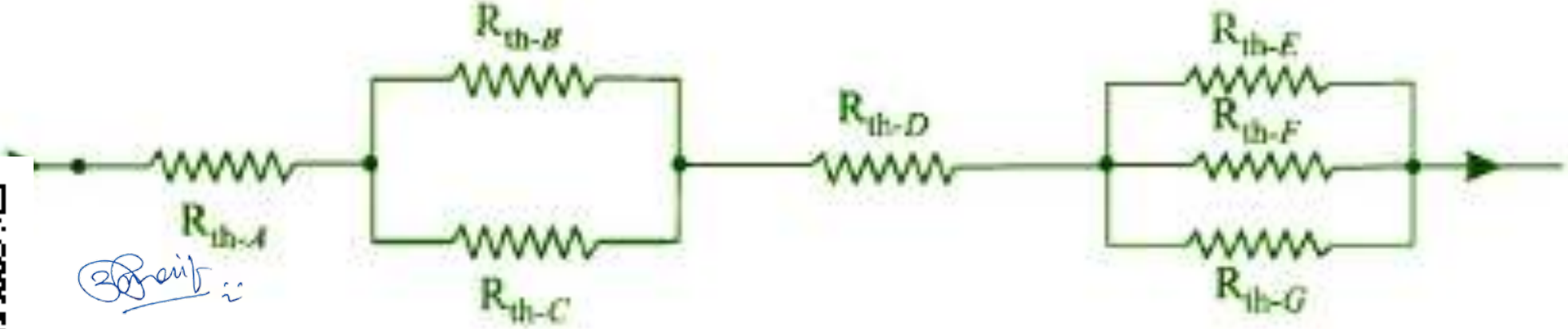


Thermal resistance network for combined series-parallel arrangement.

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$$Q = \frac{\Delta t_{overall}}{\sum R_{th}}$$



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Numerical On

Complex Problems of Composite Walls (Parallel OR Series)

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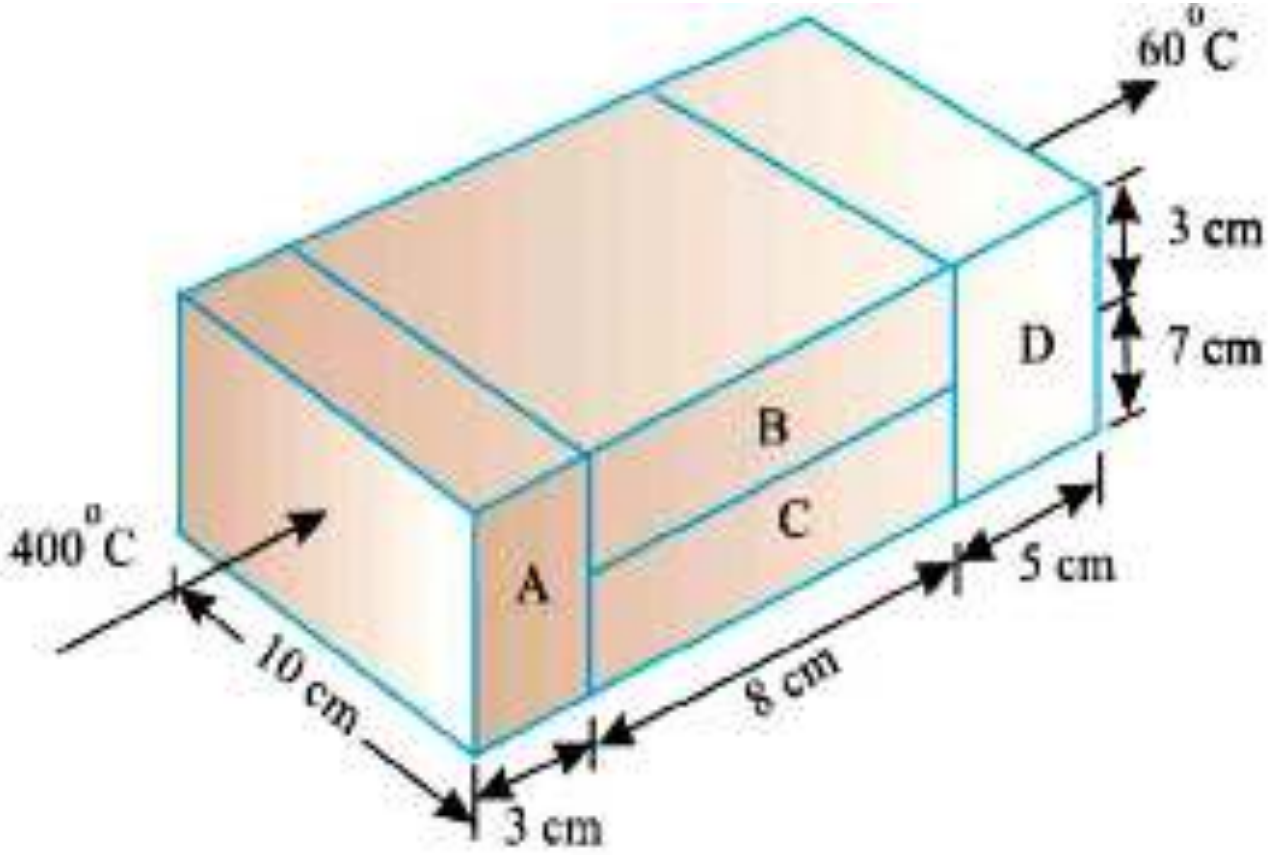
Example *Find the heat flow rate through the composite wall as shown in Fig. Assume one dimensional flow.*

$k_A = 150\ W/m°C,$

$k_B = 30\ W/m°C,$

$k_C = 65\ W/m°C,\ and$

$k_D = 50\ W/m°C$



Solution.

Given :

Thickness :

$L_A = 3\ cm = 0.03\ m;\ L_B = L_C = 8\ cm = 0.08\ m;\ L_D = 5\ cm = 0.05\ m$

$0.1 \times 0.1 = 0.01 m^2$; $A_B = 0.1 \times 0.03 = 0.003 m^2$

$0.1 \times 0.07 = 0.007 m^2$; $A_D = 0.1 \times 0.1 = 0.01 m^2$

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Heat flow rate, Q :

The thermal resistances are given by,

$$R_{th-A} = \frac{L_A}{k_A A_A} = \frac{0.03}{150 \times 0.01} = 0.02 \qquad R_{th-B} = \frac{L_B}{k_B A_B} = \frac{0.08}{30 \times 0.003} = 0.89$$

$$R_{th-C} = \frac{L_C}{k_C A_C} = \frac{0.08}{65 \times 0.007} = 0.176 \qquad R_{th-D} = \frac{L_D}{k_D A_D} = \frac{0.05}{50 \times 0.01} = 0.1$$

The equivalent thermal resistance for the parallel thermal resistance R_{th-B} and R_{th-C} is given by

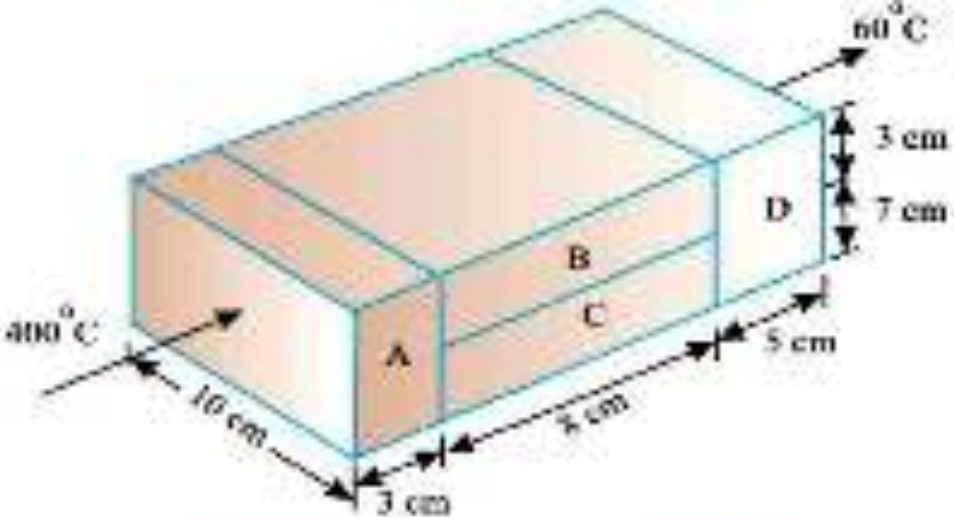
$$\frac{1}{(R_{th})_{eq}} = \frac{1}{R_{th-B}} + \frac{1}{R_{th-C}} = \frac{1}{0.89} + \frac{1}{0.176} = 6.805$$

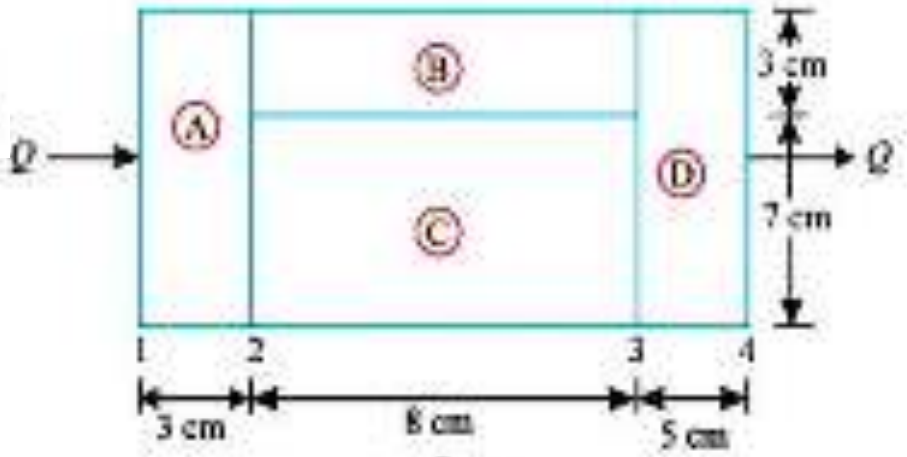
$$(R_{th})_{eq.} = \frac{1}{6.805} = 0.147$$

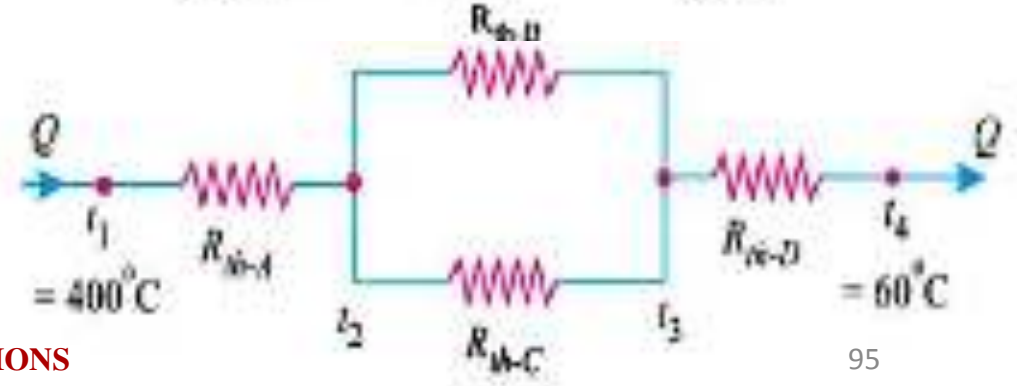
Now, the total thermal resistance is given by

$$R_{th-A} + (R_{th})_{eq} + R_{th-D} = 0.02 + 0.147 + 0.1 = 0.267$$

$$\frac{\Delta t)_{overall}}{R_{th})_{total}} = \frac{(400 - 60)}{0.267} = 1273.4\ \text{W (Ans.)}$$







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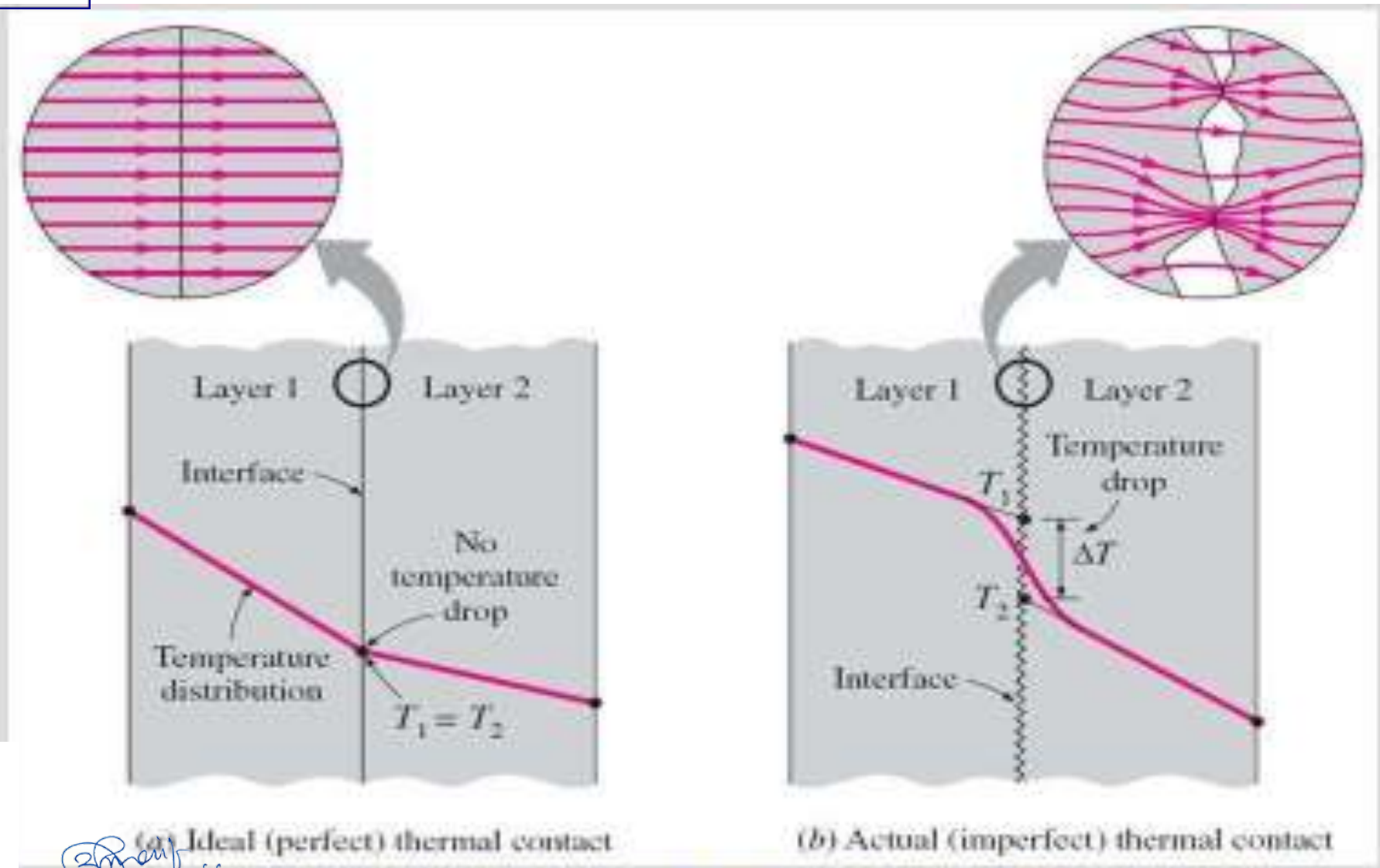
THERMAL CONTACT RESISTANCE

OR

Temperature Drop

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THERMAL CONTACT RESISTANCE



Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

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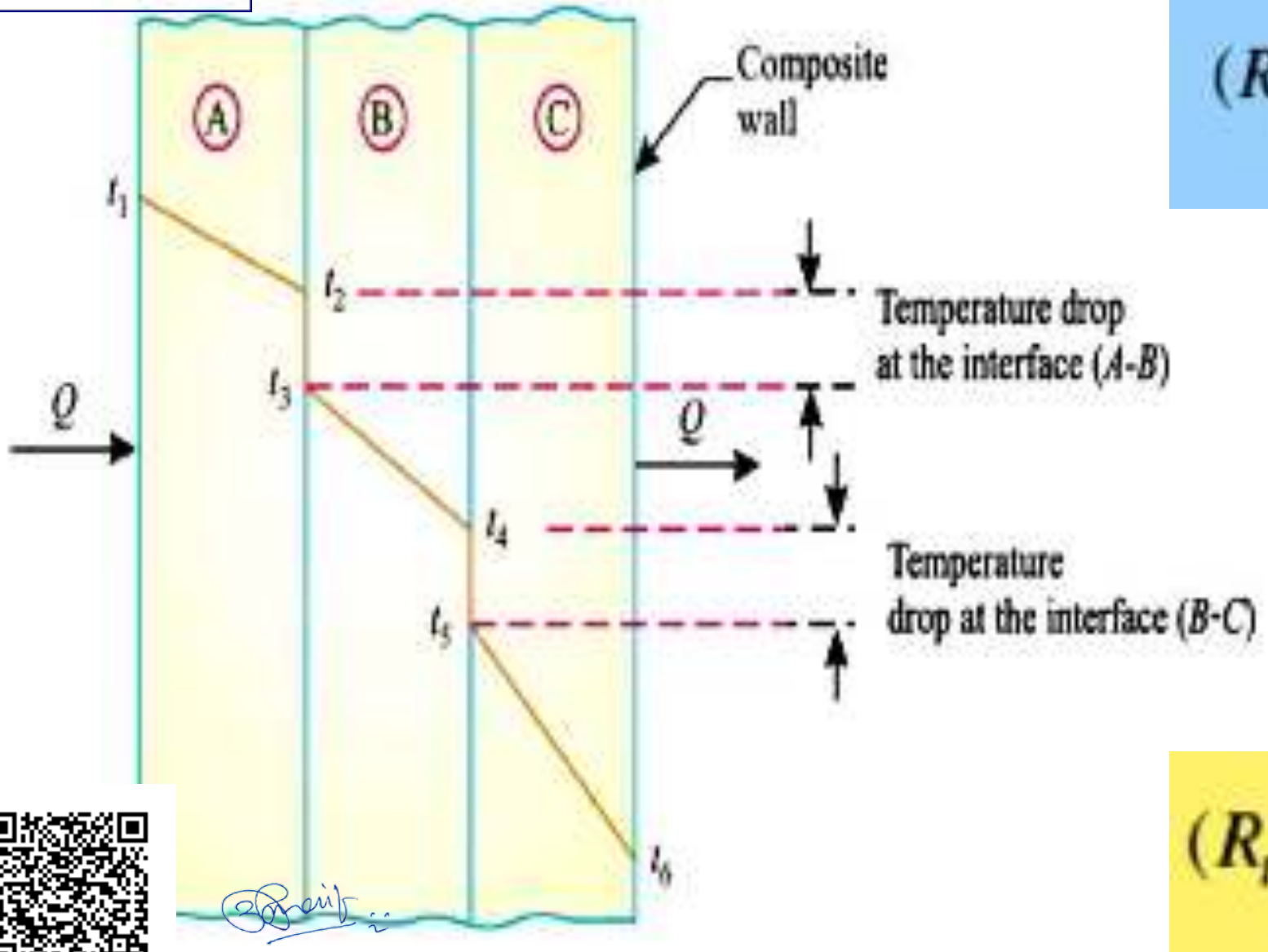






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$$(R_{th-AB})_{cont.} = \frac{(t_2 - t_3)}{Q/A}$$

$$(R_{th-BC})_{cont.} = \frac{(t_4 - t_5)}{Q/A}$$

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Numerical on

Thermal Contact Resistance or Temperature Drop



Example No.1

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Example *A wall of a furnace is made up of inside layer of silica brick 120 mm thick covered with a layer of magnesite brick 240 mm thick. The temperatures at the inside surface of silica brick wall and outside surface of magnesite brick wall are 725°C and 110°C respectively. The contact thermal resistance between the two walls at the interface is 0.0035°C/W per unit wall area. If thermal conductivities of silica and magnesite bricks are 1.7 W/m°C and 5.8 W/m°C, calculate.*

(i) The rate of heat loss per unit area of walls, and

(ii) The temperature drop at the interface.

Solution.

Given : $L_A = 120 \text{ mm} = 0.12 \text{ m};$

$L_B = 240 \text{ mm} = 0.24 \text{ m};$

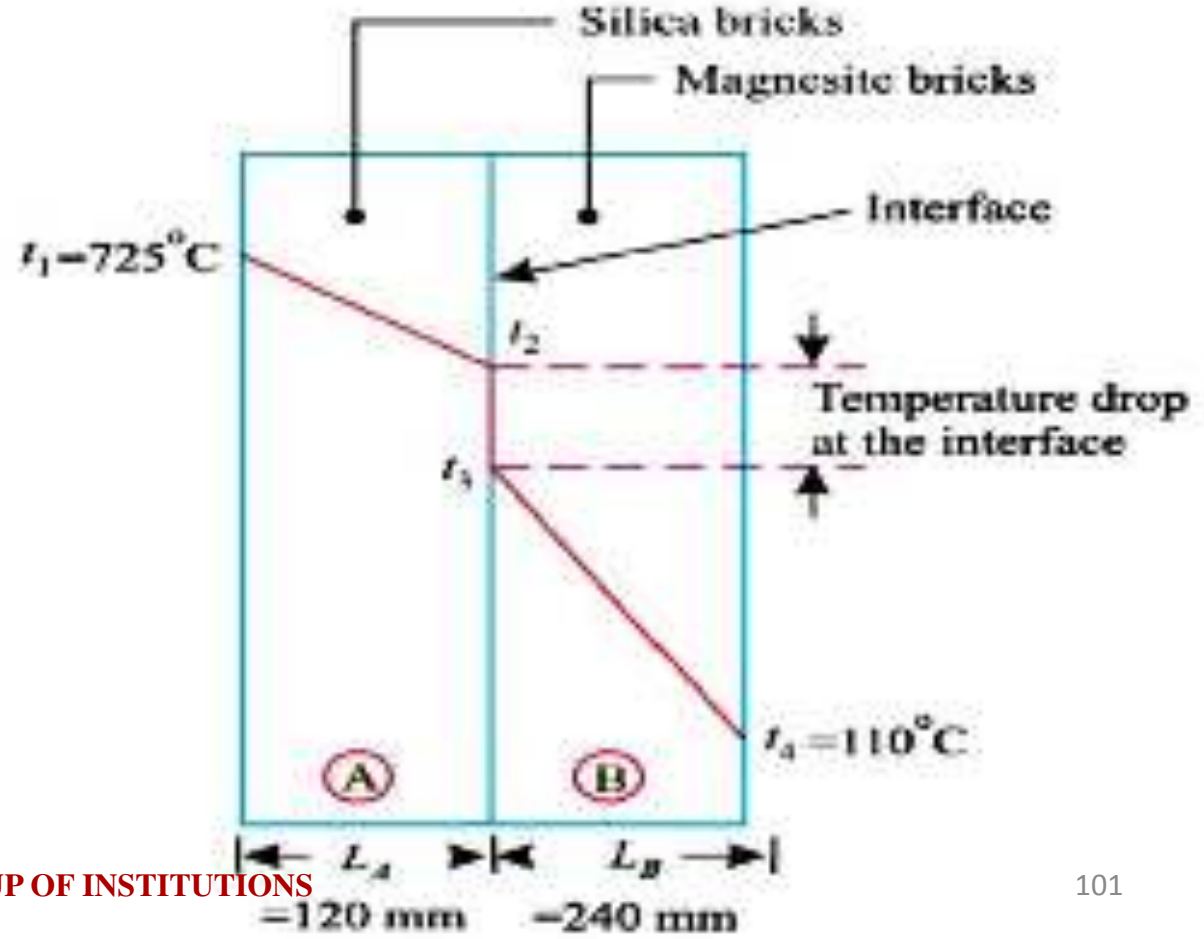
$k_A = 1.7 \text{ W/m°C}; k_B = 5.8 \text{ W/m°C}$

The contact thermal resistance $(R_{th})_{cont.}$

$= 0.0035°C/W$

s …erature at the inside surface of …all, $t_1 = 725°C$

…erature at the outside surface of t… brick wall, $t_4 = 110°C$



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(i) The rate of heat loss per unit area of wall, q :

$$q = \frac{\Delta t}{\Sigma R_{th}} = \frac{\Delta t}{R_{th-A} + (R_{th})_{cont.} + R_{th-B}}$$

$$= \frac{(t_1 - t_4)}{L_A / k_A + 0.0035 + L_B / k_B}$$

$$= \frac{(725 - 110)}{0.12/1.7 + 0.0035 + 0.24/5.8}$$

$$= \frac{615}{0.0706 + 0.0035 + 0.0414}$$

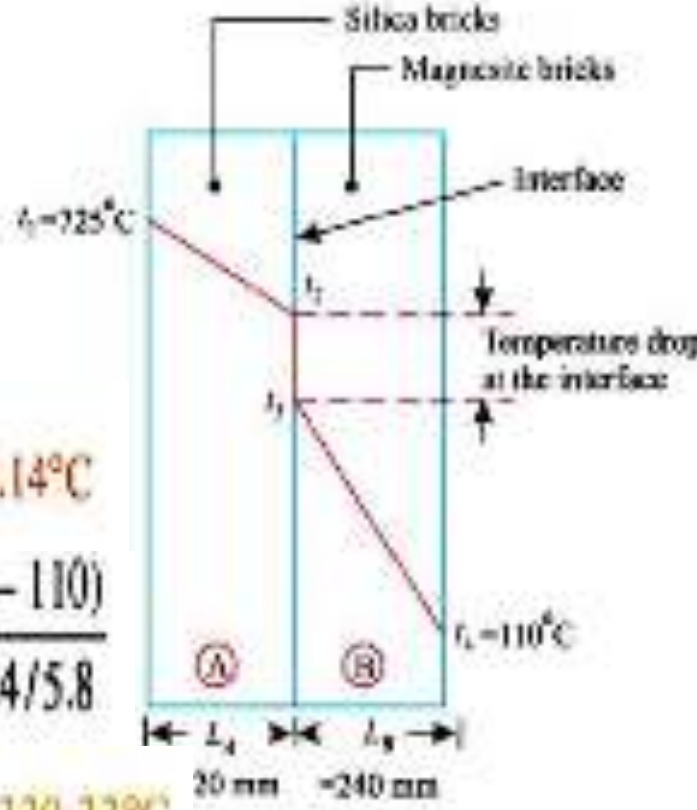
$$= 5324.67 \text{ W/m}^2$$

$\therefore$ Heat loss per unit area of wall, $q = 5324.67$ W/m^2

(ii) The temperature drop at the interface, $(t_2 - t_3)$:

As the same heat flows through each layer of composite wall, therefore,

$$q = \frac{t_1 - t_2}{L_A / k_A} = \frac{t_3 - t_4}{L_B / k_B}$$



or,

$$5324.67 = \frac{(725 - t_2)}{0.12/1.7}$$

or, $t_2 = 725 - 5324.67 \times \frac{0.12}{1.7} = 349.14°C$

Similarly, $$5324.67 = \frac{(t_3 - 110)}{0.24/5.8}$$

or, $t_3 = 110 + 5324.67 \times \frac{0.24}{5.8} = 330.33°C$

Hence, the temperature drop at the interface $= t_2 - t_3$

$= 349.14 - 330.33 = 18.81°C$ (Ans.)

Example No.2

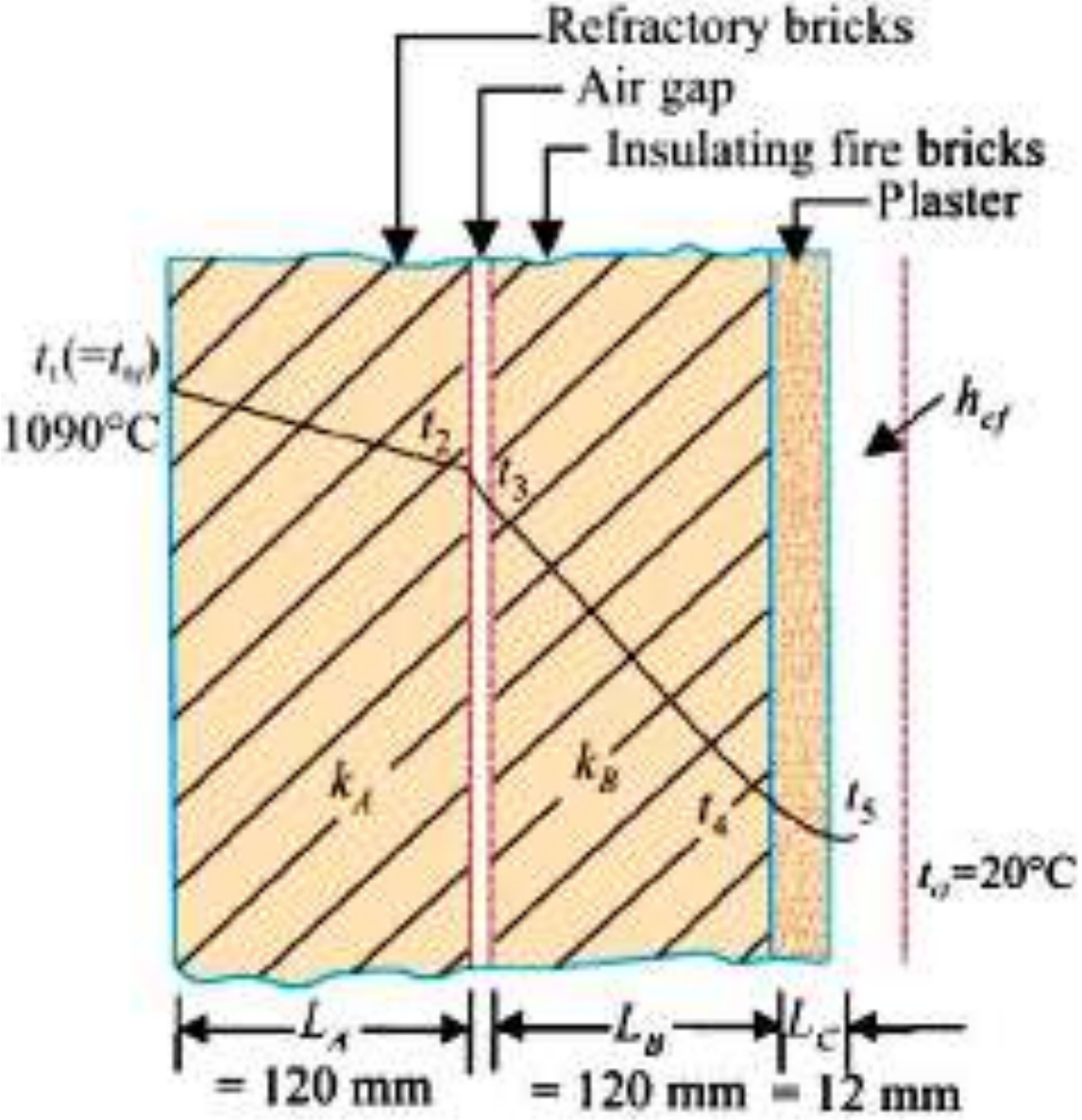
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Exampel *The furnace wall consists of 120 mm wide refractory brick and 120 mm wide insulating fire brick separated by an air gap. The outside wall is covered with a 12 mm thickness of plaster. The inner surface of the wall is at 1090°C and the room temperature is 20°C. The heat transfer coefficient from the outside wall surface to the air in the room is 18 W/m²°C, and the resistance to heat flow of the air gap is 0.16 K/W. If the thermal conductivities of the refractory brick, insulating fire brick, and plaster are 1.6, 0.3 and 0.14 W/mK, respectively calculate :*

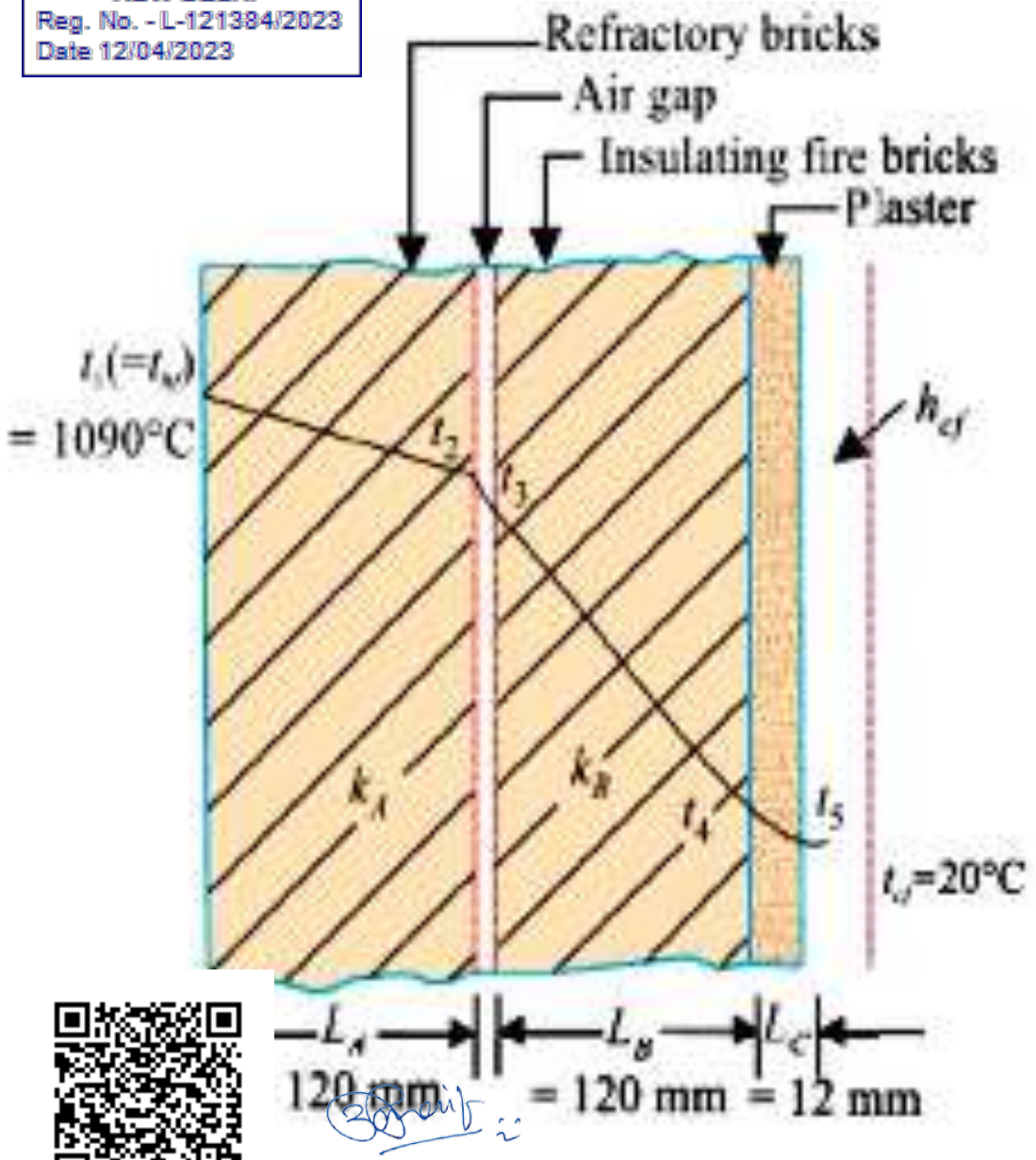
(i) Rate at which heat is lost per m² of the wall surface;

Each interface temperature; and

Temperature of the outside surface of the wall.



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Example No.2

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Solution:

Thickness of refractory brick, $L_A = 120\,mm = 0.12\,m$

Thickness of insulating fire brick, $L_B = 120\,mm = 0.12\,m$

Thickness of plaster, $L_C = 12\,mm = 0.012\,m$

Heat transfer coefficient from the outside wall surface to the air in the room,

$$h_{cf} = 18\,W/m^2{}^\circ C$$

Resistance of air gap to heat flow $= 0.16\,K/W$

Thermal conductivities :

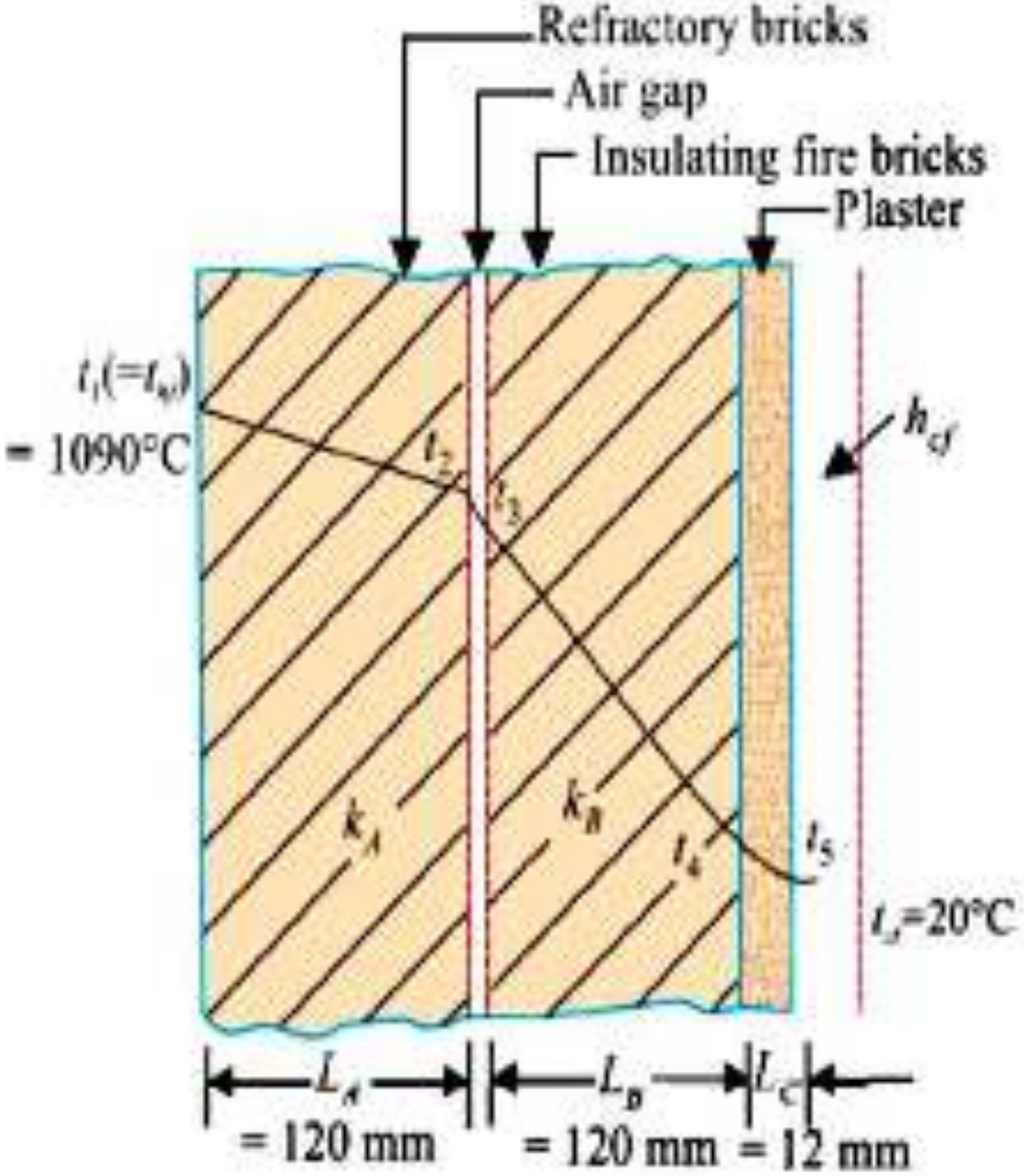
Refractory brick, $k_A = 1.6\,W/m^\circ C$

Insulating fire brick, $k_B = 0.3\,W/m^\circ C$

Plaster, $k_C = 0.14\,W/m^\circ C$.

Temperatures : $t_{if} = 1090^\circ C$; $t_{of} = 20^\circ C$

^{2} of surface area.



Example No.2

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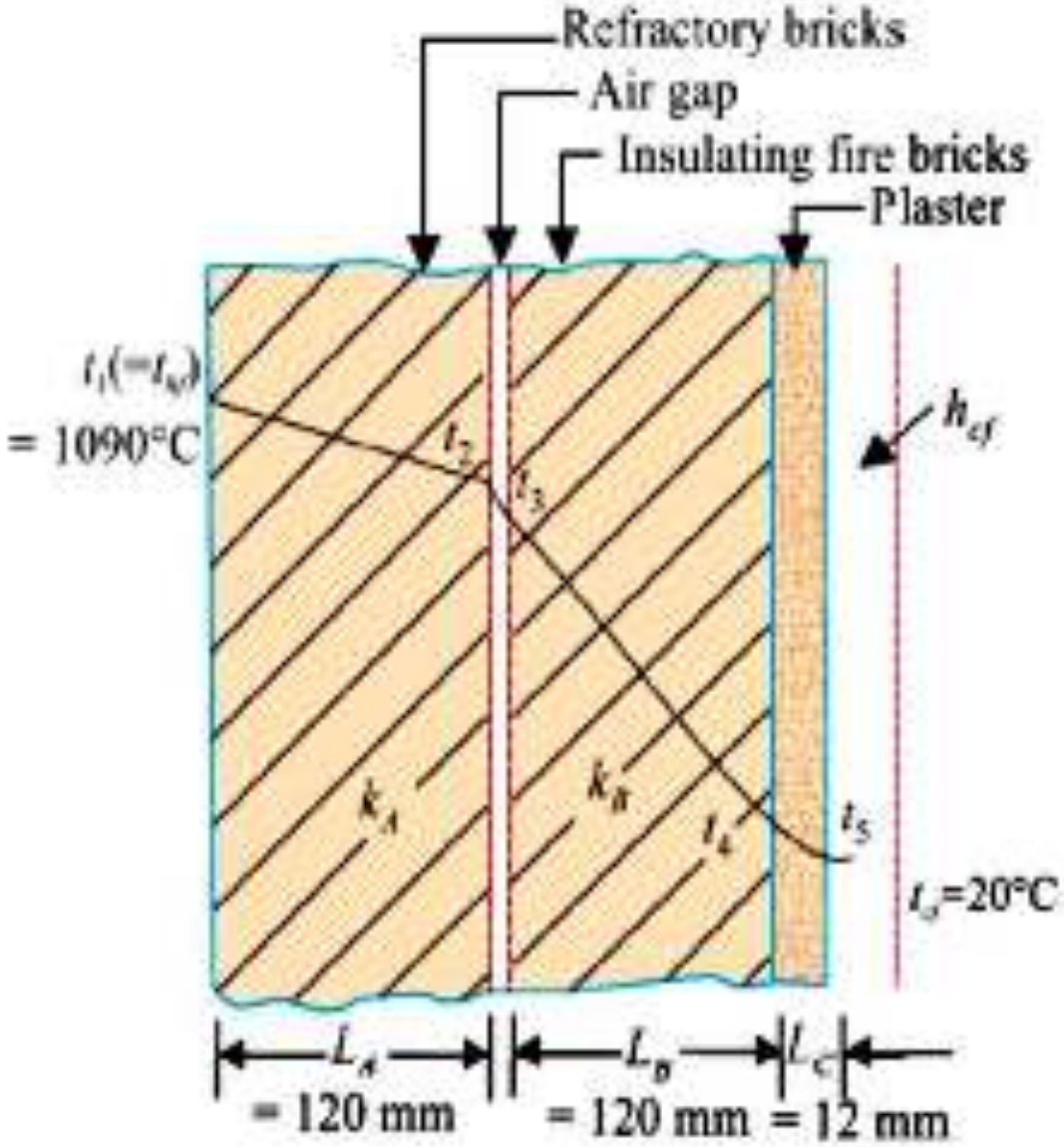
Solution:

(i) Rate of heat loss per m^2 of surface area, q :

$$q = \frac{(t_{hf} - t_{cf})}{\frac{L_A}{k_A} + \text{air gap resistance} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_{cf}}}$$

$$= \frac{(1090 - 20)}{\frac{0.12}{1.6} + 0.16 + \frac{0.12}{0.3} + \frac{0.012}{0.14} + \frac{1}{18}}$$

$$= \frac{1070}{0.075 + 0.16 + 0.4 + 0.0857 + 0.0555} = 1378.5 \text{ W or } 1.3785 \text{ kW}$$



of heat loss per m^2 of surface area = **1.3785 kW** **(Ans.)**

Example No.2

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Solution:

(ii) Temperatures at interfaces, t_2, t_3, t_4:

$$Q = 1378.5 = \frac{1090 - t_2}{L_A / k_A} = \frac{1090 - t_2}{0.12/1.6} = \frac{1090 - t_2}{0.075}$$

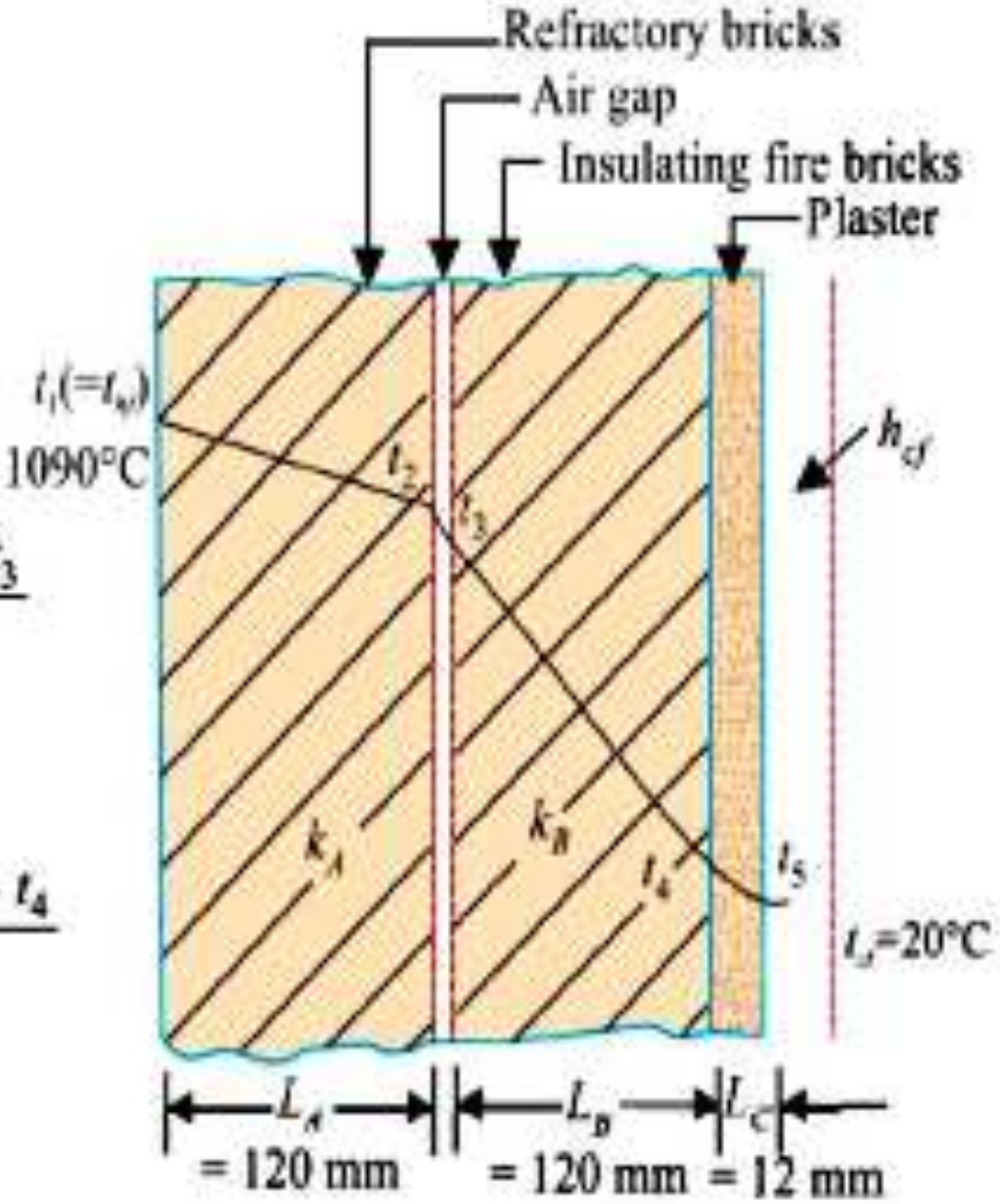
$$t_2 = 1090 - 1378.5 \times 0.075 = 986.6°C \quad \textbf{(Ans.)}$$

Also,
$$Q = 1378.5 = \frac{t_2 - t_3}{\text{air gap resistance}} = \frac{986.6 - t_3}{0.16}$$

$$t_3 = 986.6 - 1378.5 \times 0.16 = 766.04°C \quad \textbf{(Ans.)}$$

Again,
$$Q = 1378.5 = \frac{t_3 - t_4}{L_B / k_B} = \frac{766.04 - t_4}{0.12/0.3} = \frac{766.04 - t_4}{0.4}$$

$$6.04 - 1378.5 \times 0.4 = 214.64°C \quad \textbf{(Ans.)}$$



Example No.2

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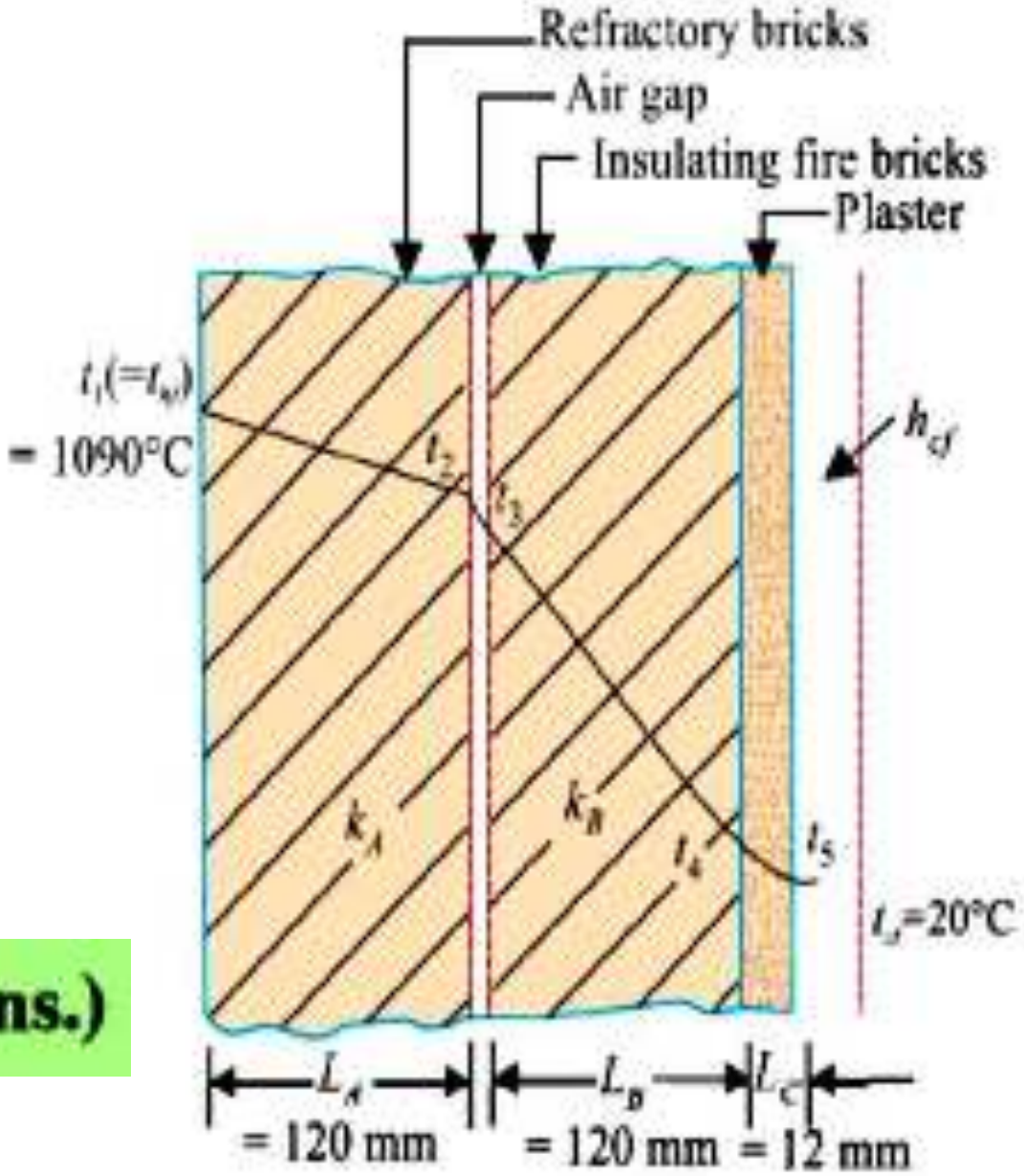
Solution:

(iii) Temperature of the outside surface of the wall, t_5 :

$$Q = 1378.5 = \frac{t_4 - t_5}{L_C / k_C}$$

$$= \frac{214.64 - t_5}{0.012/0.14} = \frac{214.64 - t_5}{0.0857}$$

$$64 - 1378.5 \times 0.0857 = 96.5°C \quad \textbf{(Ans.)}$$



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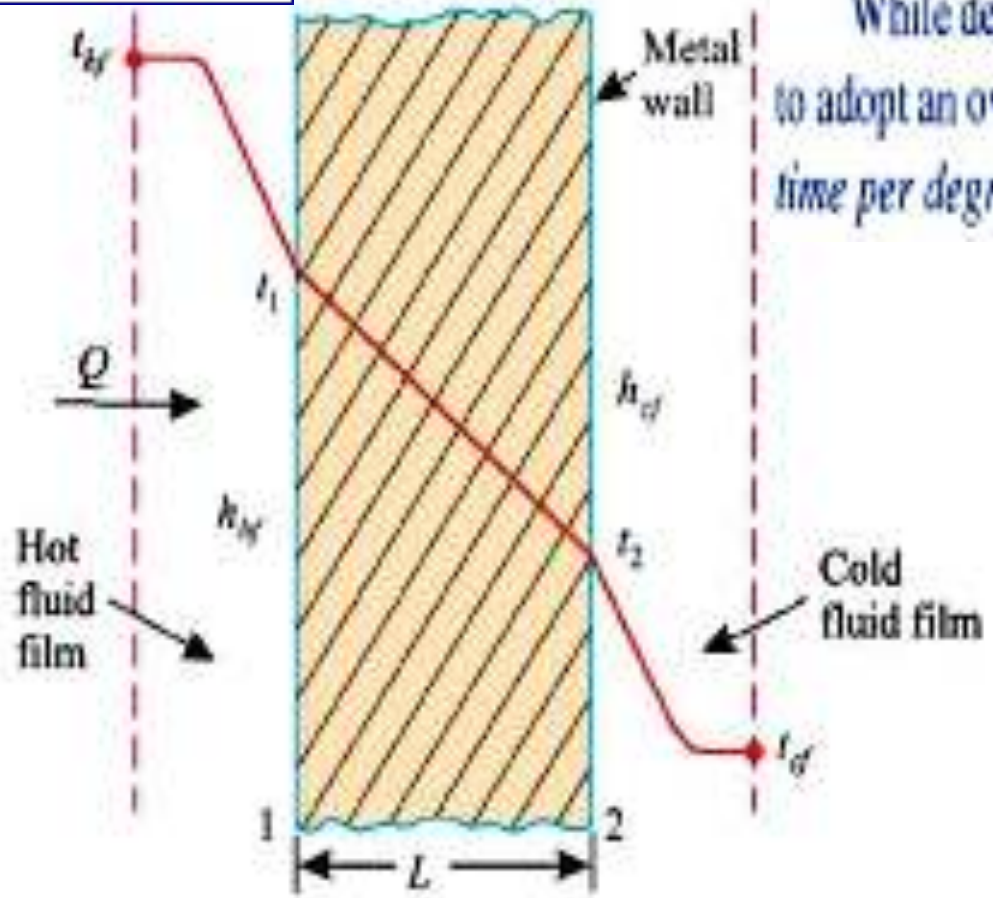
Overall Heat Transfer Coefficient





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Overall Heat Transfer Coefficient



While dealing with the problems of fluid to fluid heat transfer across a metal boundary, it is usual to adopt an overall heat transfer coefficient U which *gives the heat transmitted per unit area per unit time per degree temperature difference between the bulk fluids on each side of the metal.*

Let

L = Thickness of the metal wall,

k = Thermal conductivity of the wall material,

t_1 = Temperature of the surface–1,

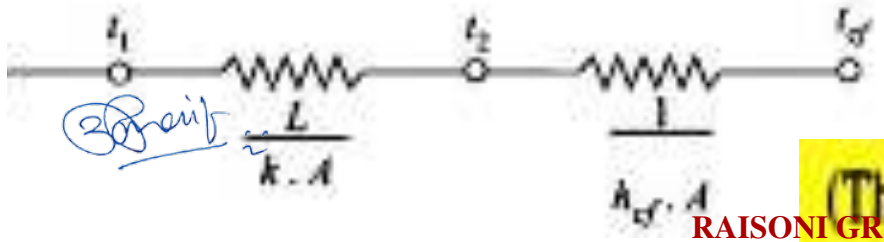
t_2 = Temperature of the surface–2,

t_{hf} = Temperature of the hot fluid,

t_{cf} = Temperature of the cold fluid,

h_{hf} = Heat transfer coefficient from hot fluid to metal surface, and

h_{cf} = Heat transfer coefficient from metal surface to cold fluid.



(The suffices *hf* and *cf* stand for hot fluid and cold fluid respectively.)

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The equations of heat flow through the fluid and the metal surface are given by

$$Q = h_{hf} A (t_{hf} - t_1) \quad \ldots(i)$$

$$Q = \frac{k.A\,(t_1 - t_2)}{L} \quad \ldots(ii)$$

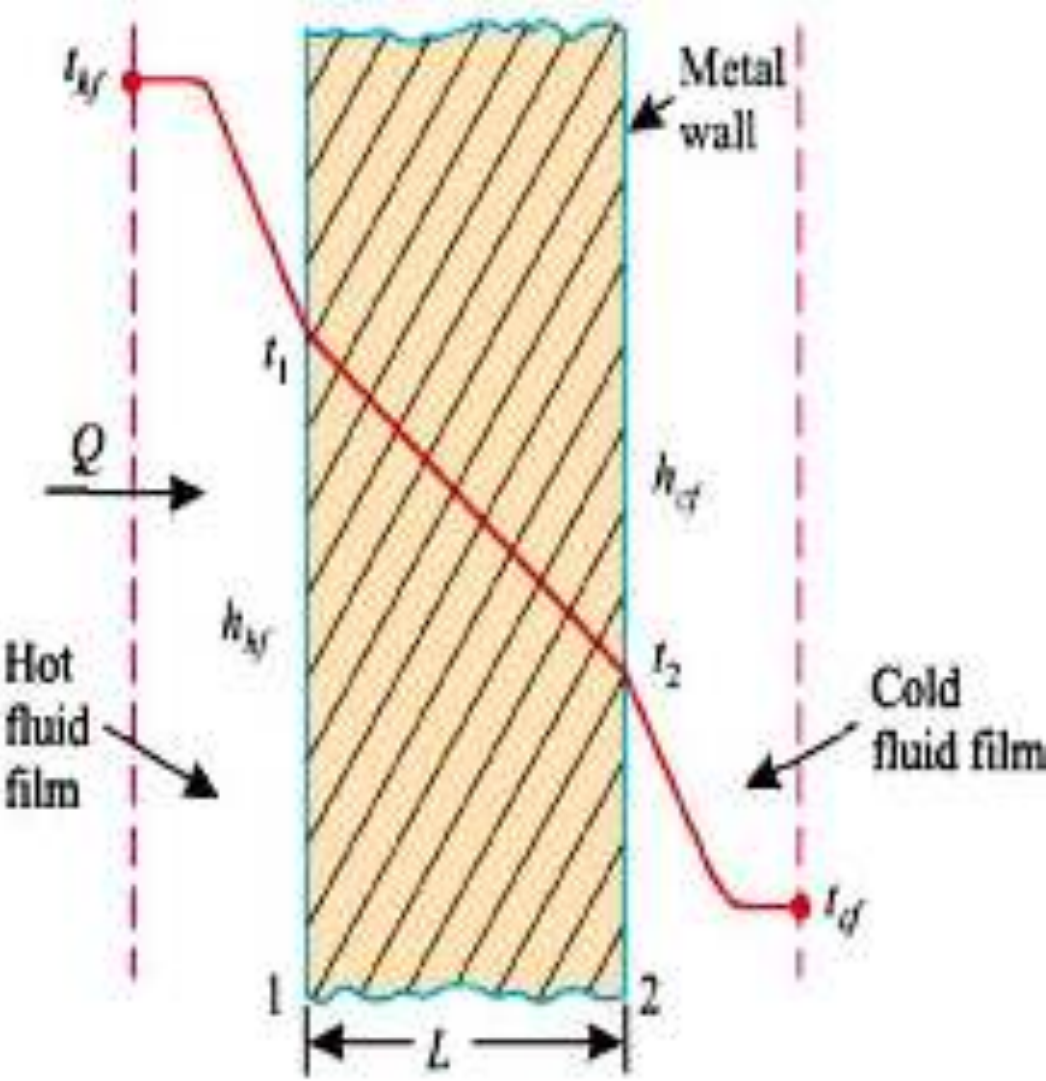
$$Q = h_{cf} A (t_2 - t_{cf}) \quad \ldots(iii)$$

By rearranging (i), (ii) and (iii), we get

$$t_{hf} - t_1 = \frac{Q}{h_{hf} . A} \quad \ldots(iv)$$

$$t_1 - t_2 = \frac{QL}{k . A} \quad \ldots(v)$$

$$t_2 - t_{cf} = \frac{Q}{h_{cf} . A} \quad \ldots(vi)$$



Adding (iv), (v) and (vi) we get

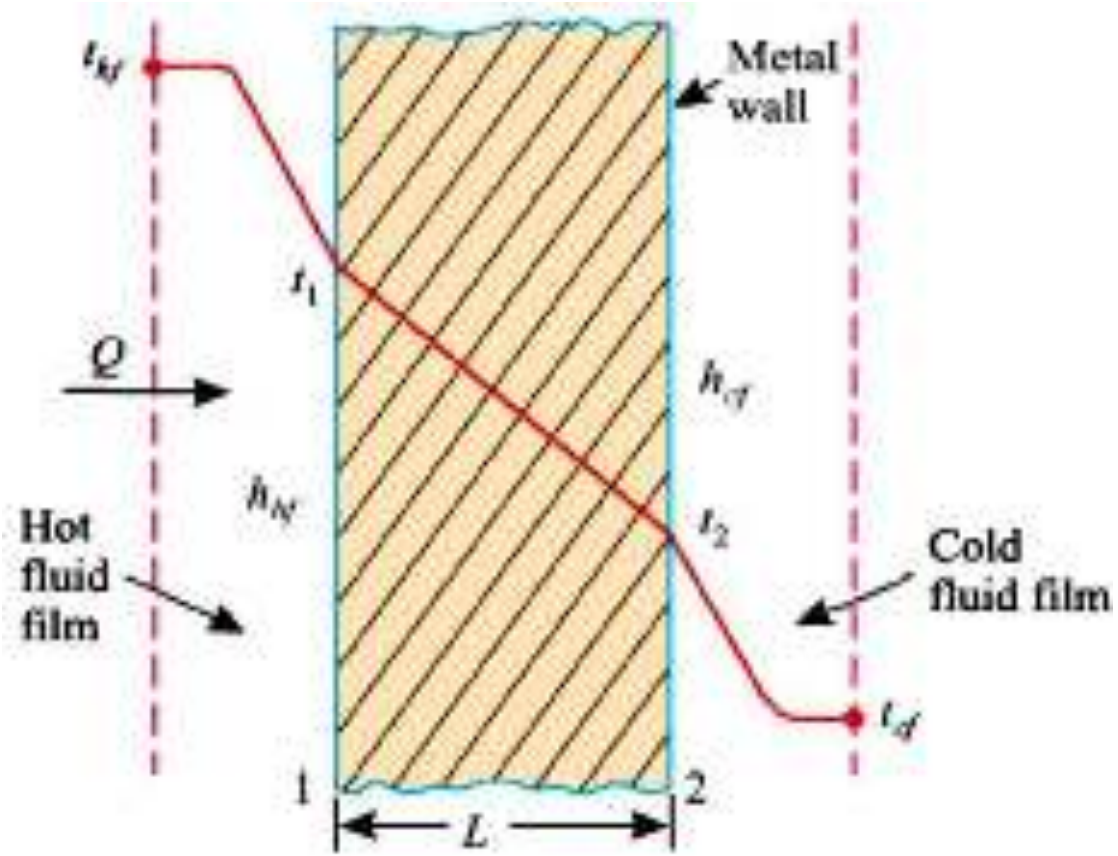
$$t_{hf} - t_{cf} = Q\left[\frac{1}{h_{hf} A} + \frac{L}{k . A} + \frac{1}{h_{cf} . A}\right]$$

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$$t_{hf} - t_{cf} = Q\left[\frac{1}{h_{hf} \cdot A} + \frac{L}{k \cdot A} + \frac{1}{h_{cf} \cdot A}\right]$$

or,

$$Q = \frac{A\,(t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$



If U is the overall coefficient of heat transfer, then

$$Q = U.A\,(t_{hf} - t_{cf}) = \frac{A\,(t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

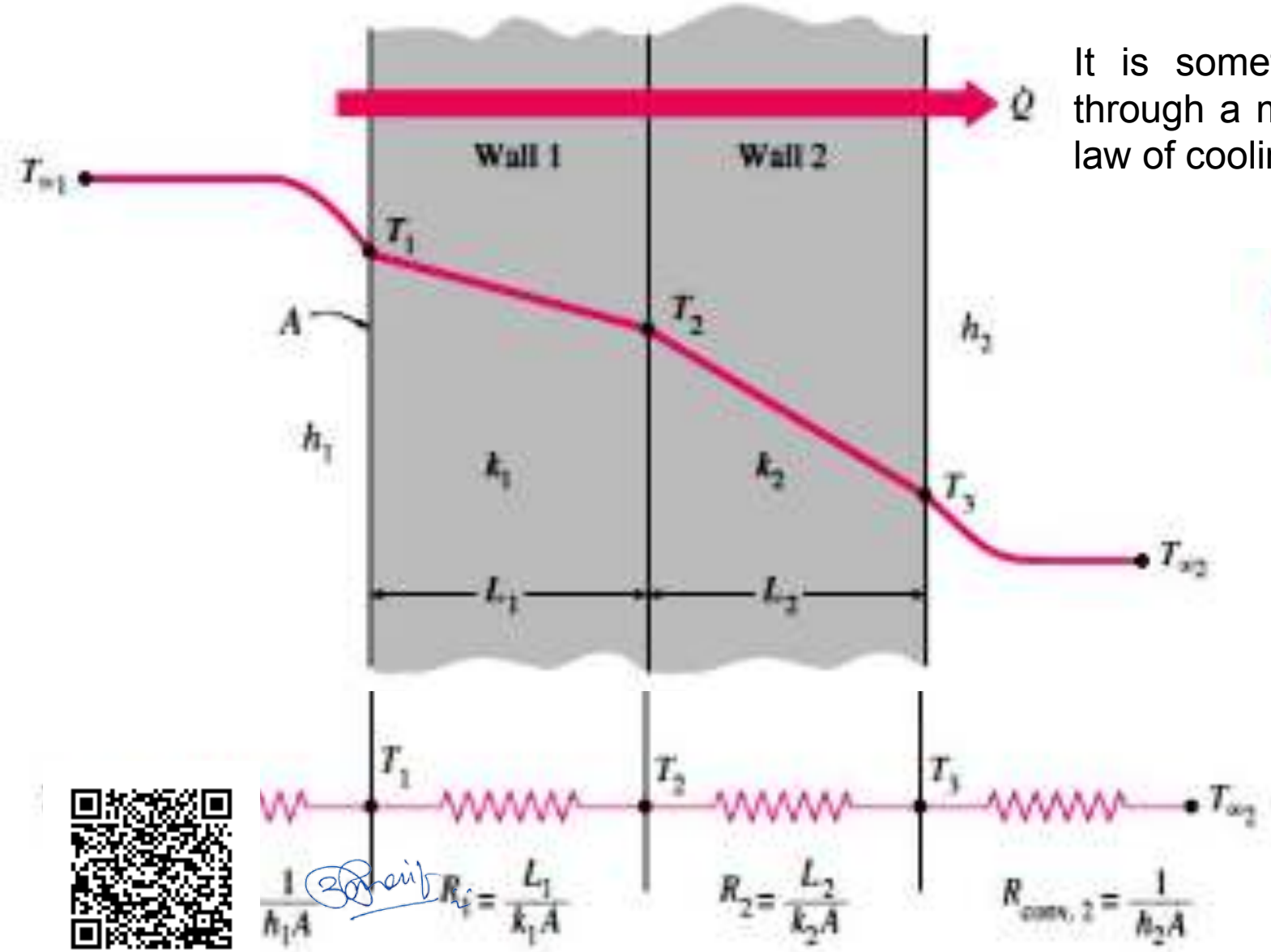
or,

$$U = \frac{1}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

be noticed from the above equation that if the individual coefficients differ greatly in magnitude only a change in the *least* will have any significant effect on the rate of heat transfer.

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Overall Heat Transfer Coefficient



It is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA\,\Delta T \qquad (W)$$

where U is the **overall heat transfer coefficient.**

$$UA = \frac{1}{R_{total}}$$

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Numerical on

Overall Heat Transfer Coefficient



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Example *A furnace wall is made up of three layers of thicknesses 250 mm, 100 mm and 150 mm of thermal conductivities of 1.65, k and 9.2 W/m°C respectively. The inside is exposed to gases at 1250°C with a convection coefficient of 25 W/m²°C and the inside surface is at 1100°C, the outside surface is exposed to air at 25°C with convection coefficient of 12 W/m²°C. Determine :*

(i) The unknown thermal conductivity 'k';

(ii) The overall heat transfer coefficient;

(iii) All surface temperatures.

Solution.

L_A = 250 mm = 0.25 m;

L_B = 100 mm = 0.1 m;

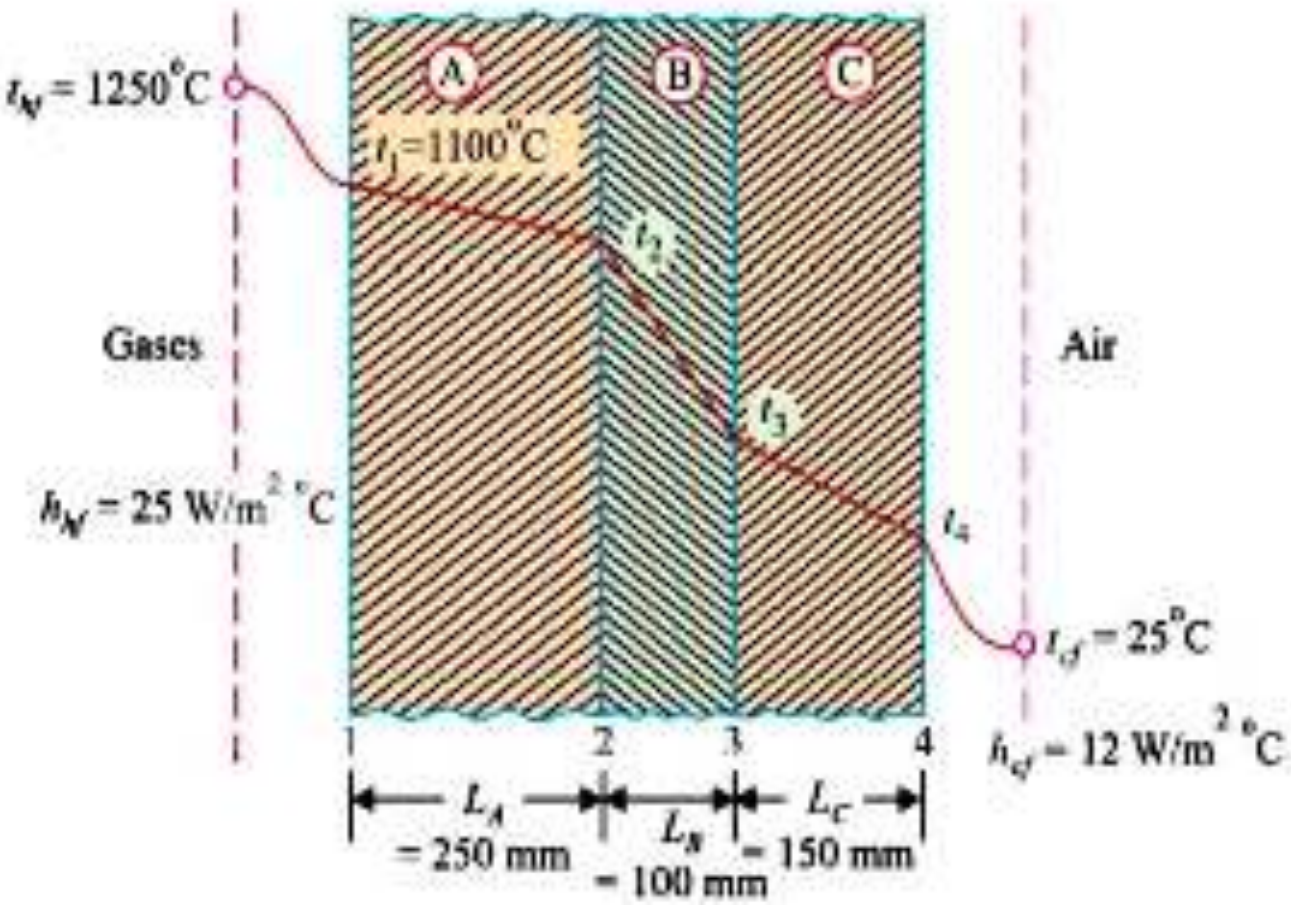
L_C = 150 mm = 0.15 m;

k_A = 1.65W/m°C;

W/m°C;

)°C, t_1 = 1100°C;

/m²°C; h_{of} = 12 W/m²°C.



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Solution

(i) Thermal conductivity, k ($= k_B$) :

The rate of heat transfer per unit area of the furnace wall,

$$q = h_{hf}(t_{hf} - t_1)$$

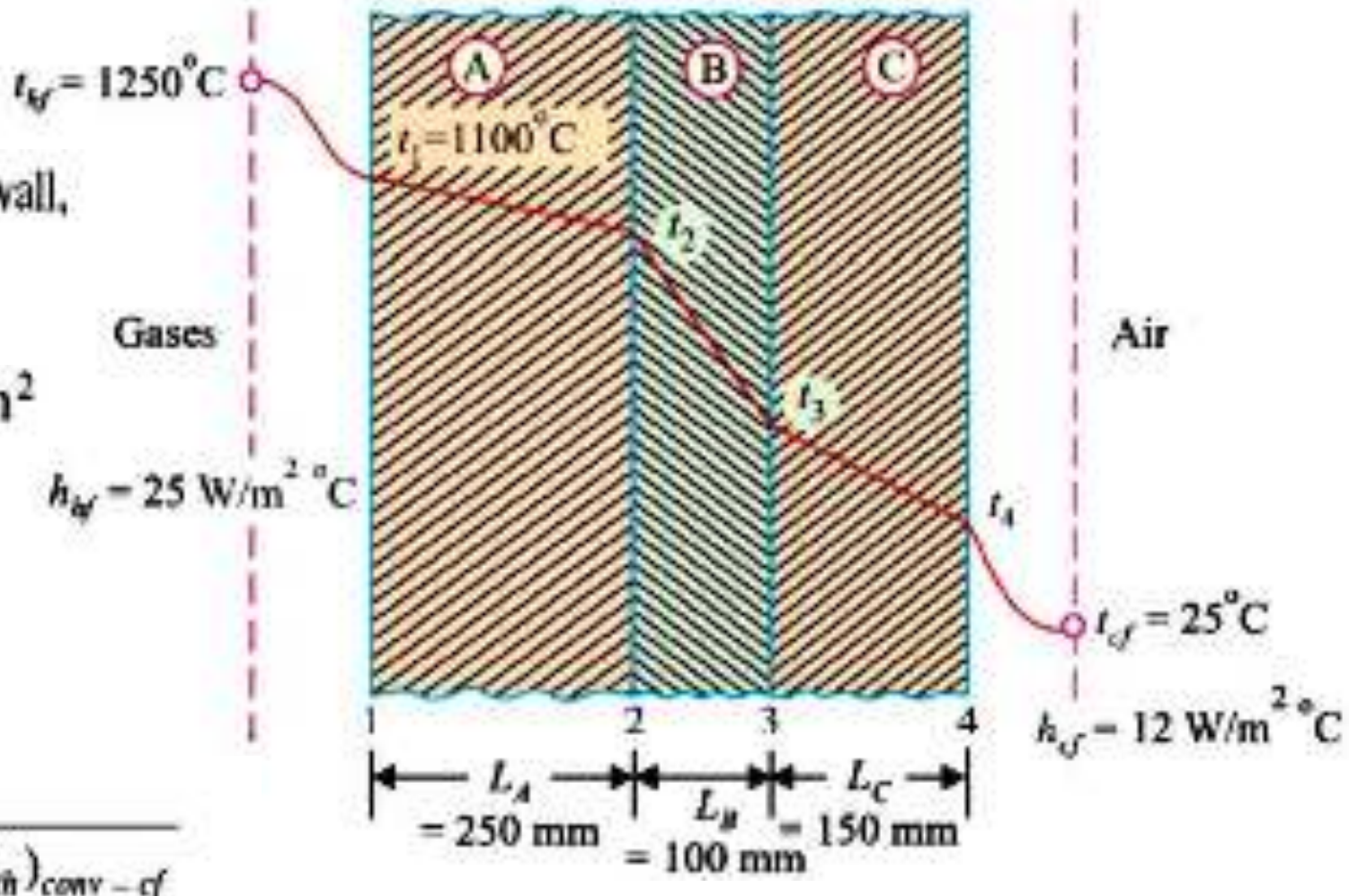
$$= 25(1250 - 1100) = 3750 \text{ W/m}^2$$

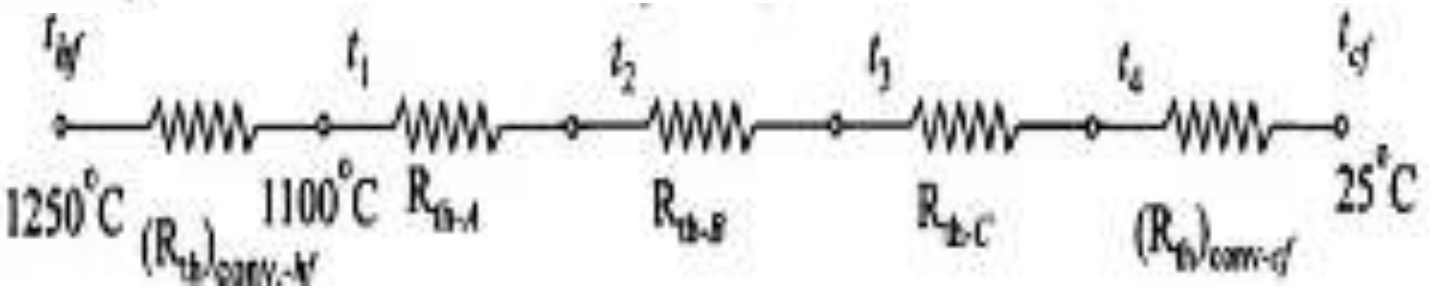
Also,

$$q = \frac{(\Delta t)_{overall}}{(R_{th})_{total}}$$

or,

$$q = \frac{(t_{hf} - t_{cf})}{(R_{th})_{conv-hf} - R_{th-A} + R_{th-B} + R_{th-C} + (R_{th})_{conv-cf}}$$





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Solution

(i) Thermal conductivity, k ($= k_B$) :

or,

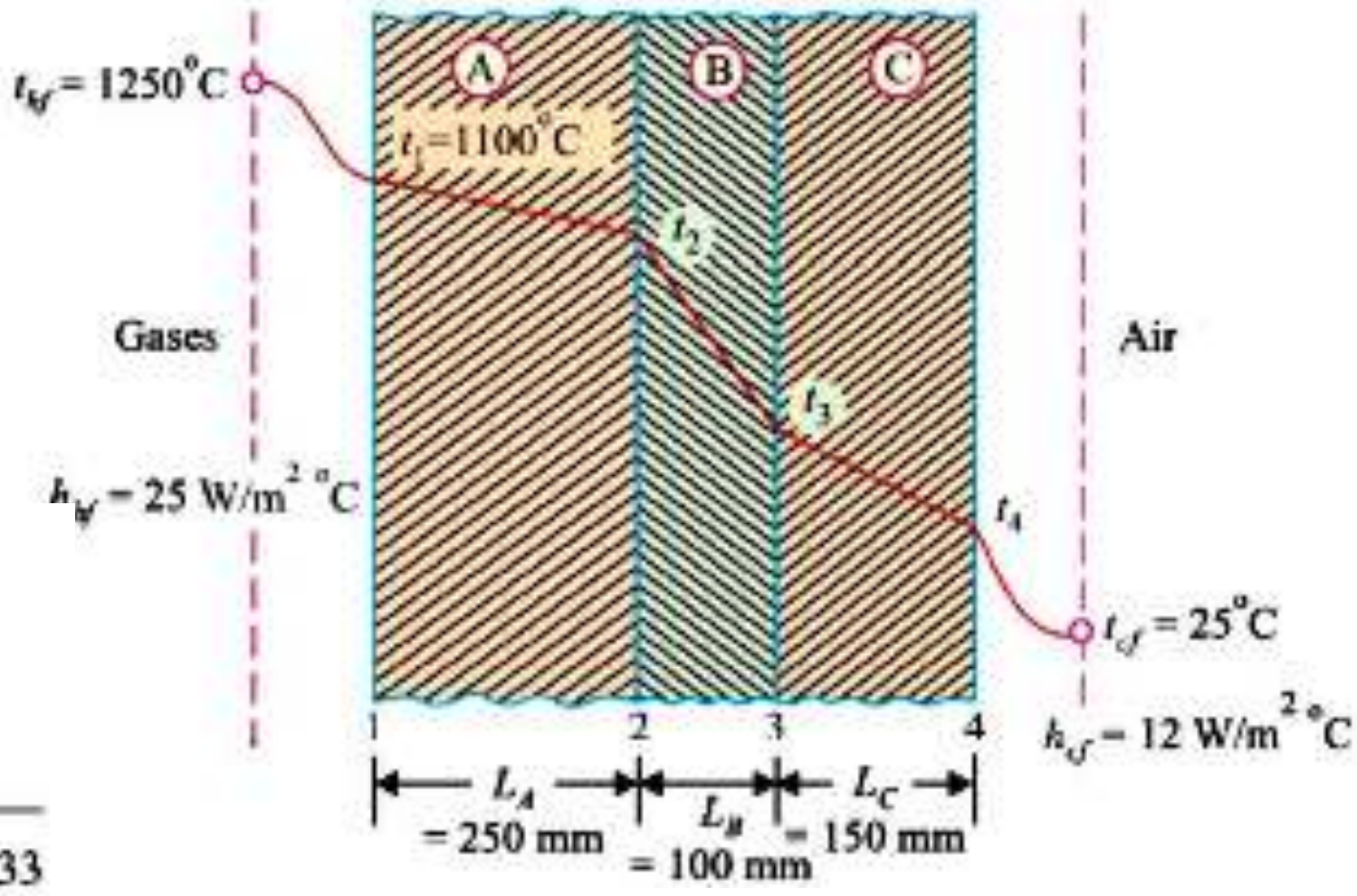
$$3750 = \frac{(1250 - 25)}{\frac{1}{h_{hf}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_{cf}}}$$

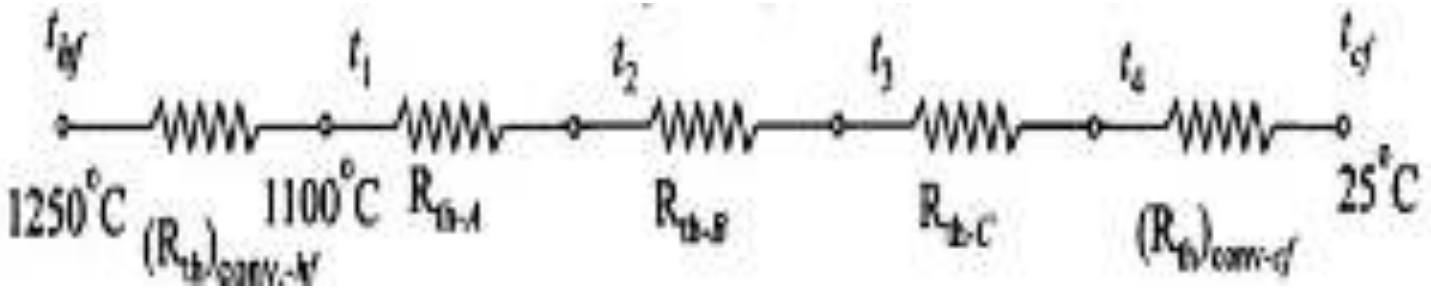
$$3750 = \frac{1225}{\frac{1}{25} + \frac{0.25}{1.65} + \frac{0.1}{k_B} + \frac{0.15}{9.2} + \frac{1}{12}}$$

$$= \frac{1225}{0.04 + 0.1515 + \frac{0.1}{k_B} + 0.0163 + 0.0833}$$

or,

$$\frac{1225}{11 + \frac{0.1}{k_B}}$$





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Solution

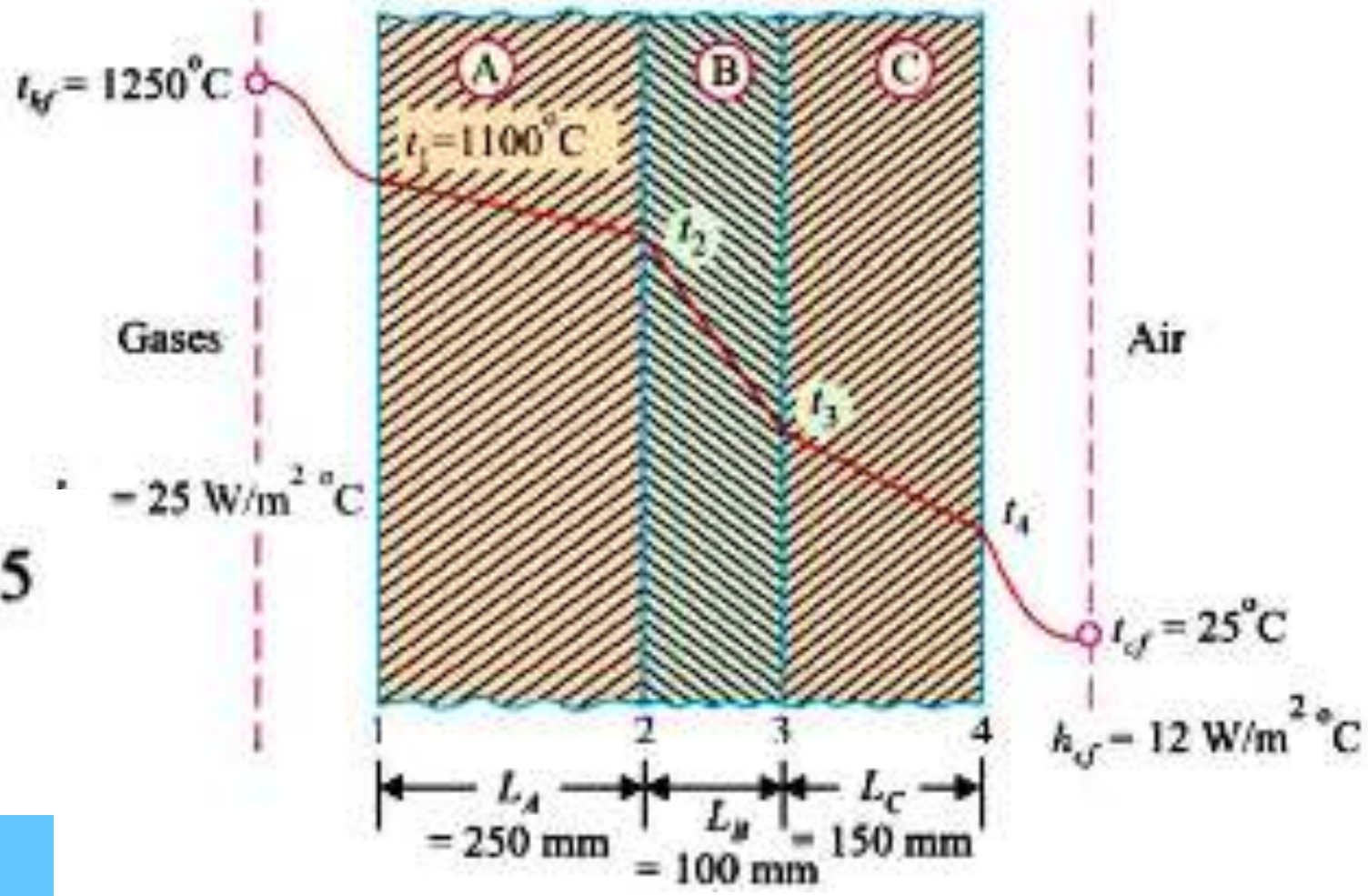
(i) **Thermal conductivity, k ($= k_B$) :**

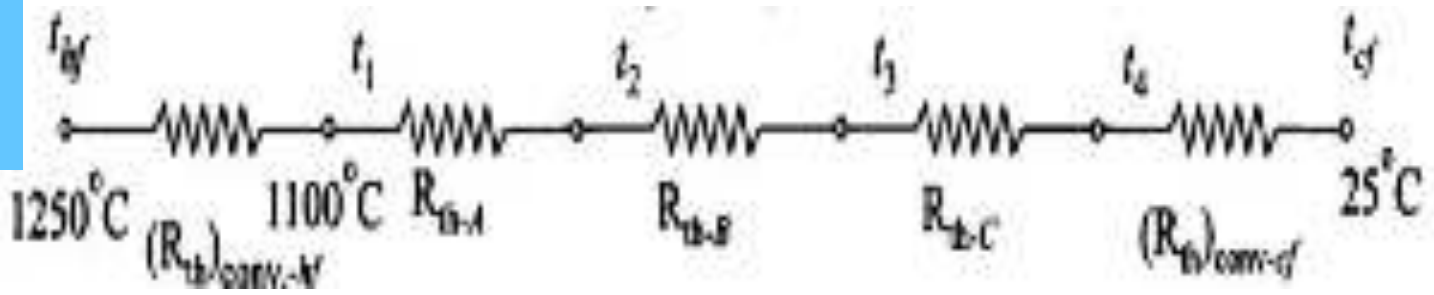
or,
$$3750\left(0.289 + \frac{0.1}{k_B}\right) = 1225$$

or,
$$\frac{0.1}{k_B} = \frac{1225}{3750} - 0.2911 = 0.0355$$

or,
$$k_B = k = \frac{0.1}{0.0355}$$

$$= \text{W/m°C} \qquad \textbf{(Ans.)}$$





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Solution

(i): The overall heat transfer coefficient, U :

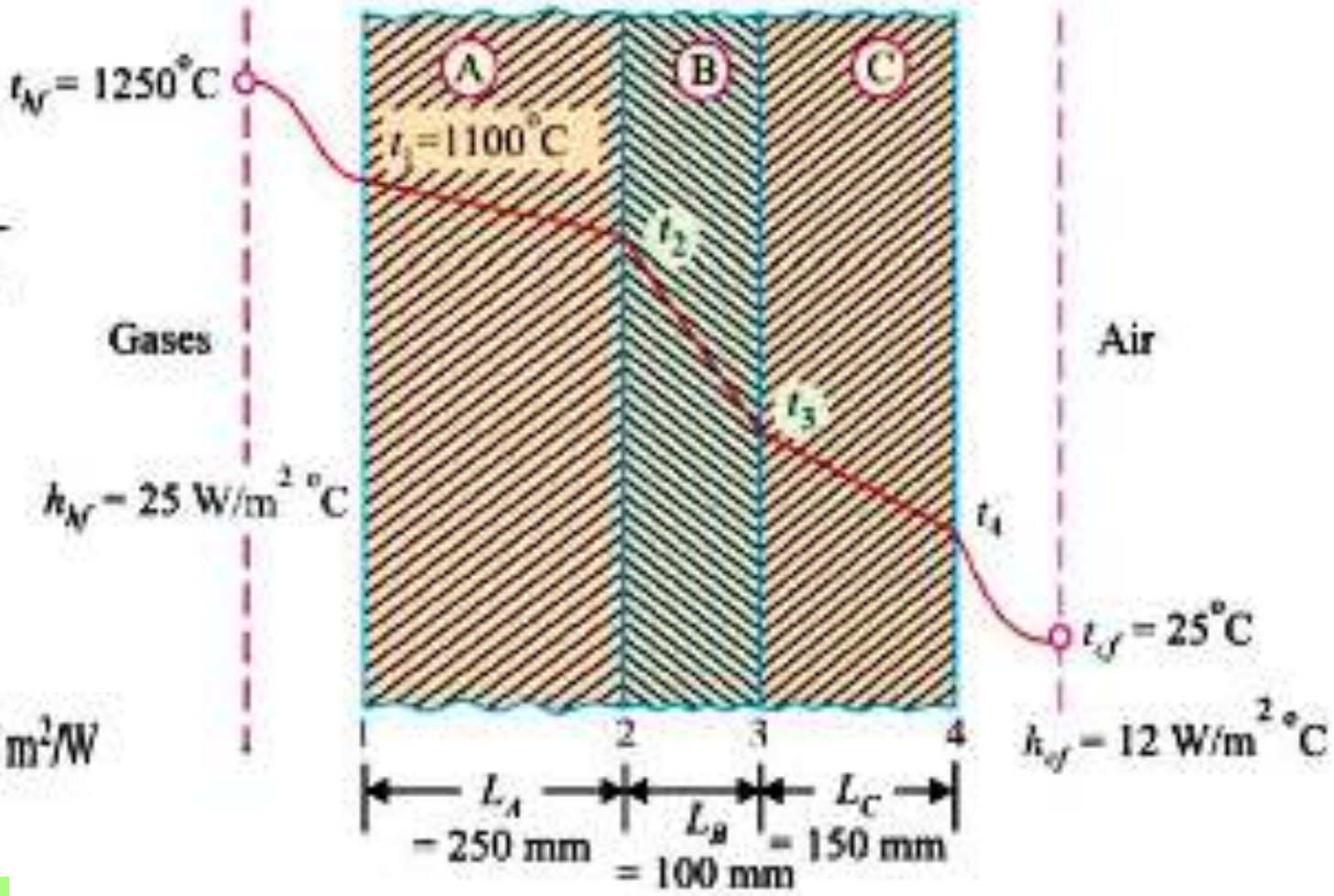
The overall heat transfer coefficient, $U = \dfrac{1}{(R_{th})_{total}}$

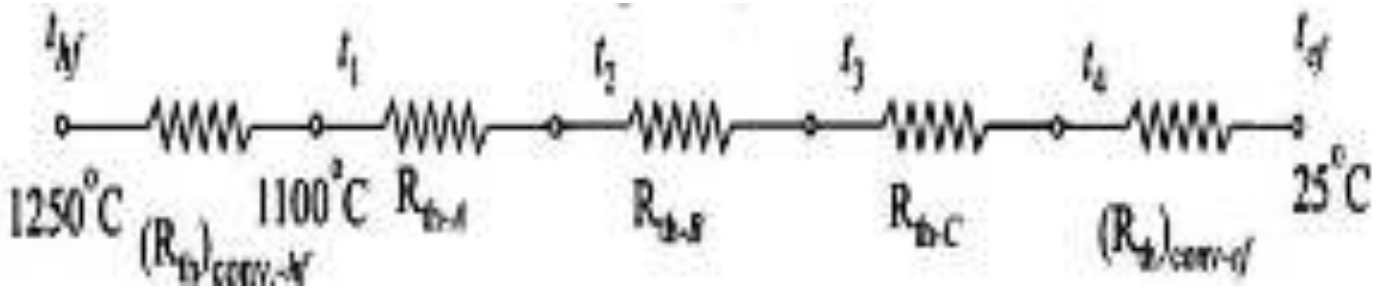
Now,

$$(R_{th})_{total} = \frac{1}{25} + \frac{0.25}{1.65} + \frac{0.1}{2.817} + \frac{0.15}{9.2} + \frac{1}{12}$$

$$= 0.04 + 0.1515 + 0.0355 + 0.0163 + 0.0833 = 0.3266\ °C\,m^2/W$$

$$U = \frac{1}{\text{total}} = \frac{1}{0.3266} = 3.06\ W/m^2\,°C\ \text{(Ans.)}$$





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Solution

(iii) Interface temperatures; t_1, t_2, t_3, t_4:

As we Know

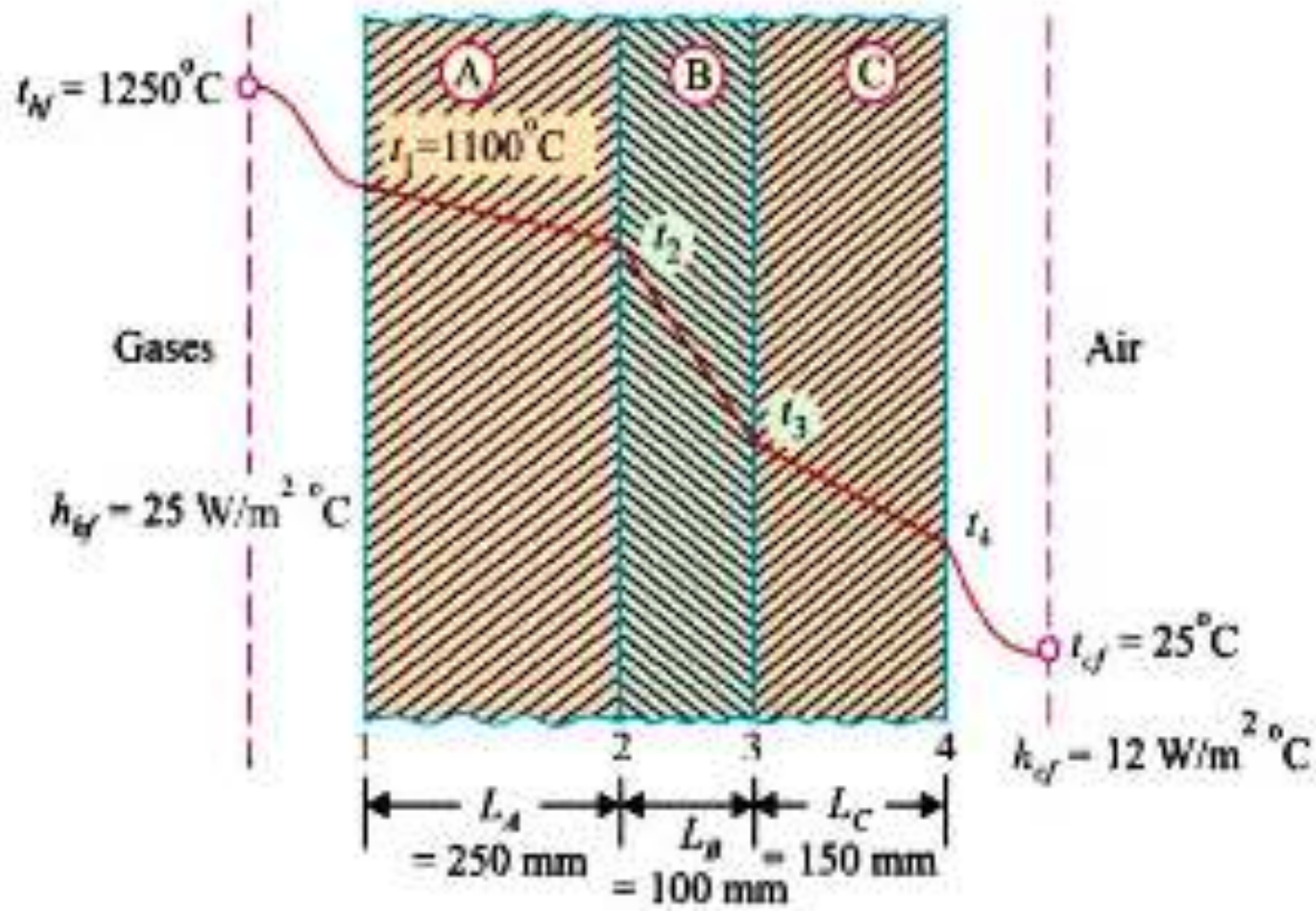
$$q = q_A = q_B = q_C$$

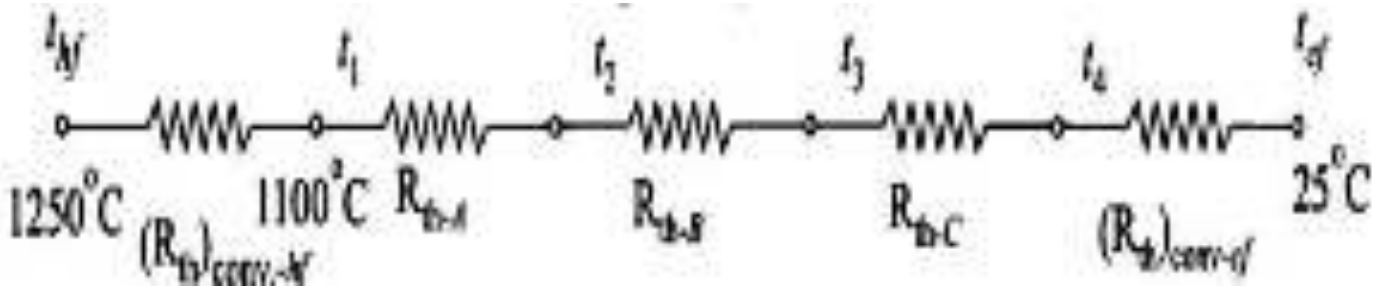
$$3750 = \frac{(t_1 - t_2)}{L_A / k_A} = \frac{(t_2 - t_3)}{L_B / k_B} = \frac{(t_3 - t_4)}{L_C / k_C}$$

For T_2

$$3750 = \frac{(1100 - t_2)}{0.25/1.65}$$

$$t_2 = 1100 - 3750 \times \frac{0.25}{1.65} = 531.8^\circ C \text{ (Ans.)}$$





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Solution

(iii) To find surface temperatures; t_1, t_2, t_3, t_4:

Similarly,

For T_3

$$3750 = \frac{(531.8 - t_3)}{0.1/2.817}$$

$$t_3 = 531.8 - 3750 \times \frac{0.1}{2.817} = 398.6^\circ C \text{ (Ans.)}$$

Similarly,

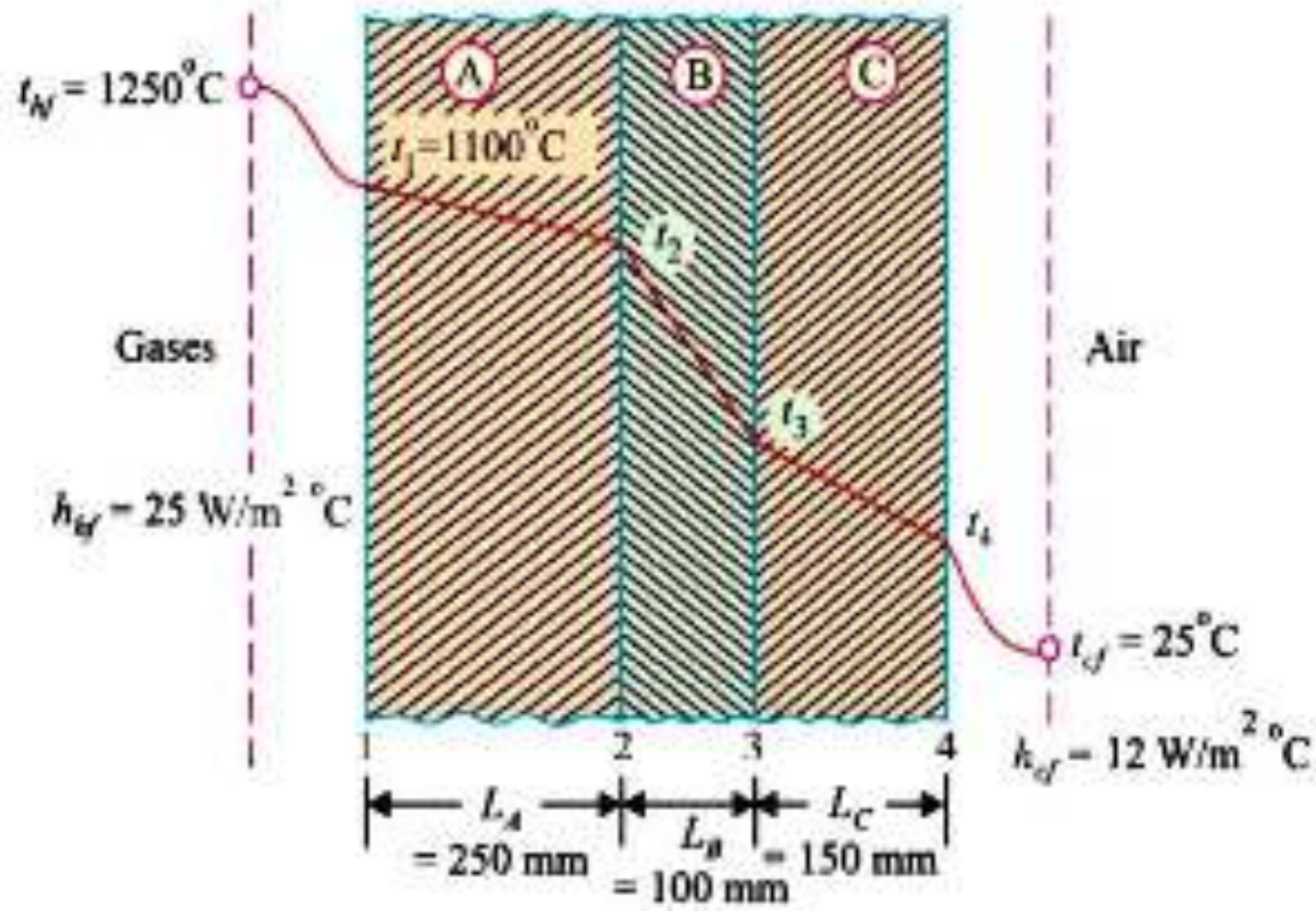
For T_4

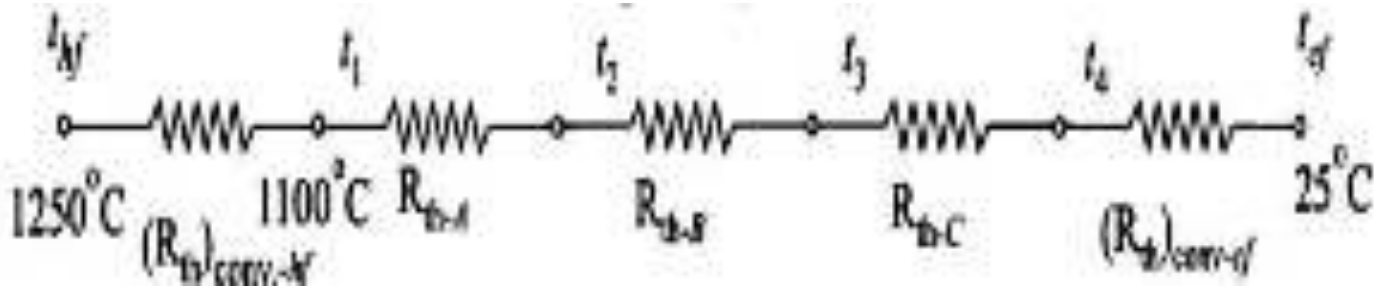
$$3750 = \frac{(398.6 - t_4)}{(0.15/9.2)}$$

$$t_4 = 398.6 - 3750 \times \frac{0.5}{9.2} = 337.5^\circ C \text{ (Ans.)}$$

C **outside convection,**

$$\frac{t_4 - 25}{1/12} = \frac{(337.5 - 25)}{1/12} = 3750 \text{ W/m}^2$$

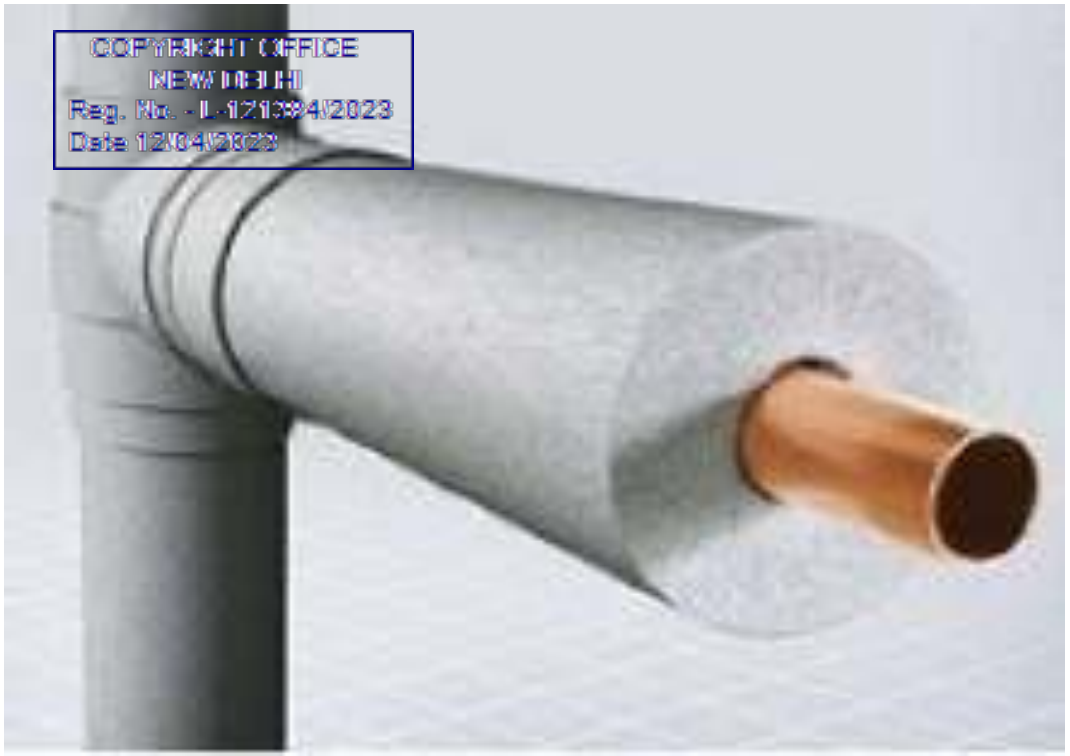




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One dimensional steady state heat conduction equation for the Cylinder and Sphere

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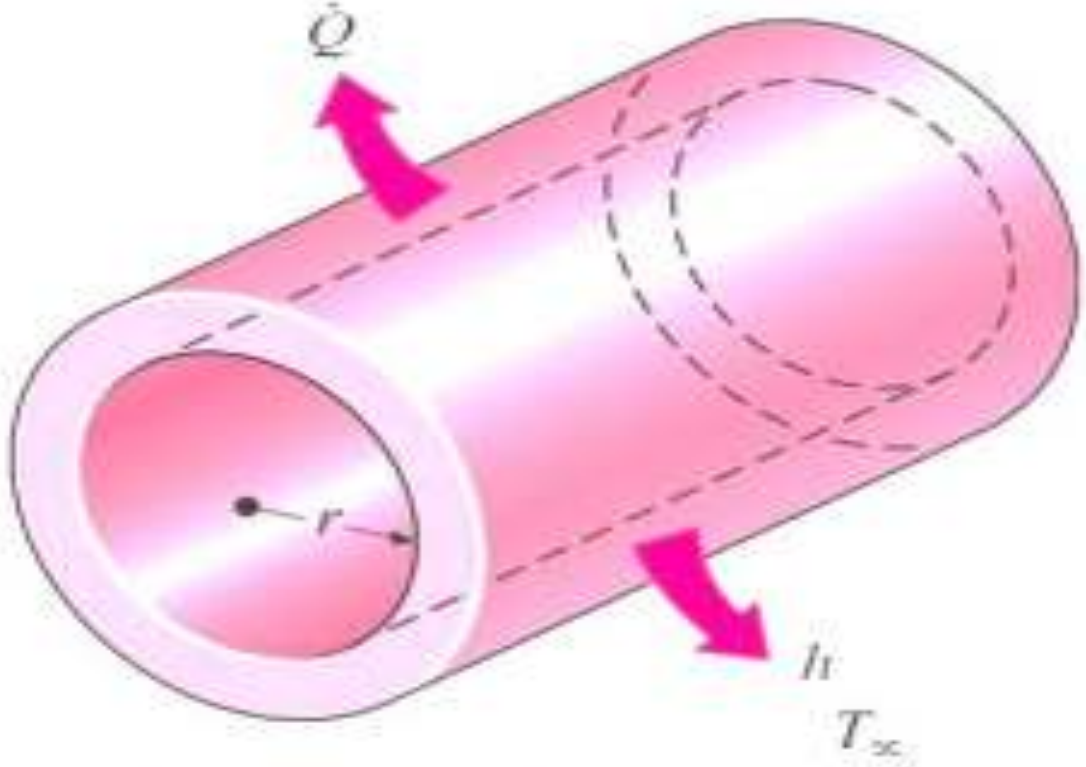






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HEAT CONDUCTION IN CYLINDERS AND SPHERES



s lost from a hot-water pipe to
outside in the radial direction,
us heat transfer from a long
one-dimensional.

Heat transfer through the pipe can be modeled as *steady* and *one-dimensional*.

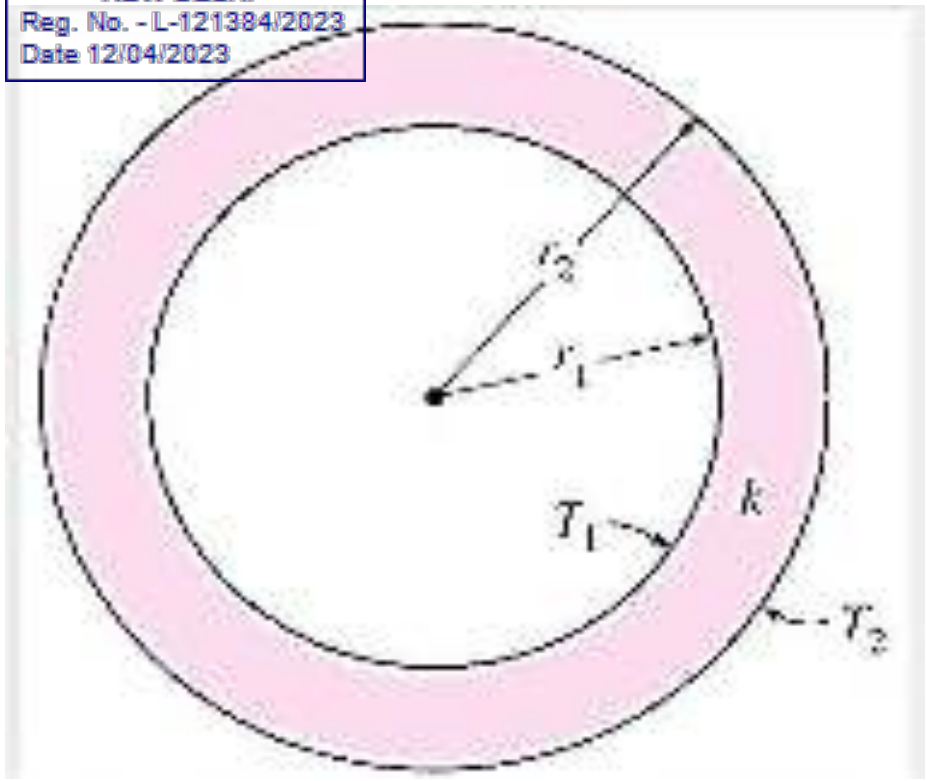
The temperature of the pipe depends on one direction only (the radial r-direction) and can be expressed as $T = T(r)$.

The temperature is independent of the azimuthal angle or the axial distance.

This situation is approximated in practice in long cylindrical pipes and spherical containers.

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For Single Layer Cylinder



A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \quad \text{(W)}$$

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\,dr = -\int_{T=T_1}^{T_2} k\,dT$$

But, $A = 2\pi rL$

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \quad \text{(W)}$$

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \quad \text{(W)}$$

$$= \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times \text{Length} \times \text{Thermal conductivity}}$$

conduction resistance of the cylinder layer

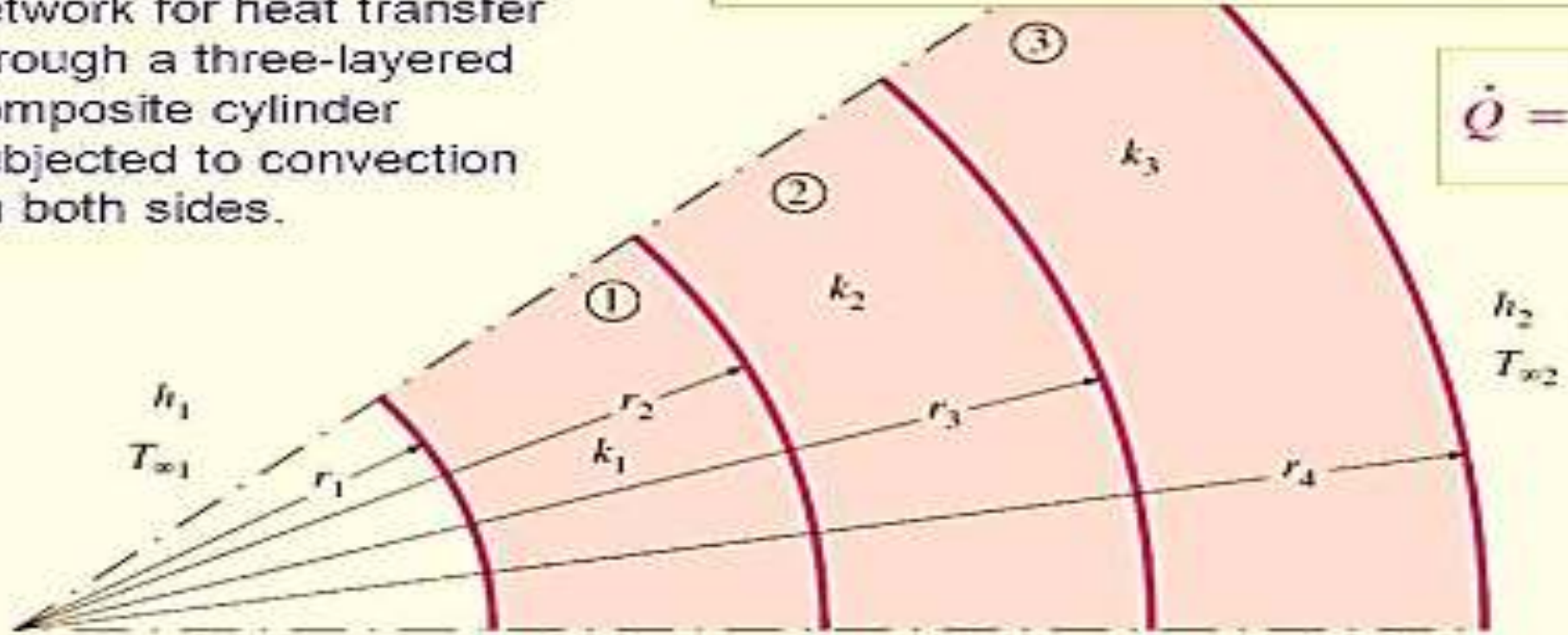
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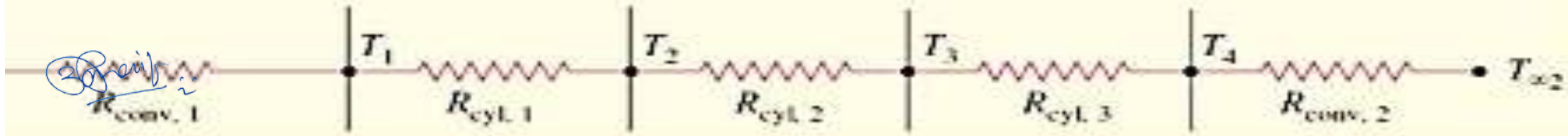
Multilayered Cylinders and Spheres

$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2}$$
$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.

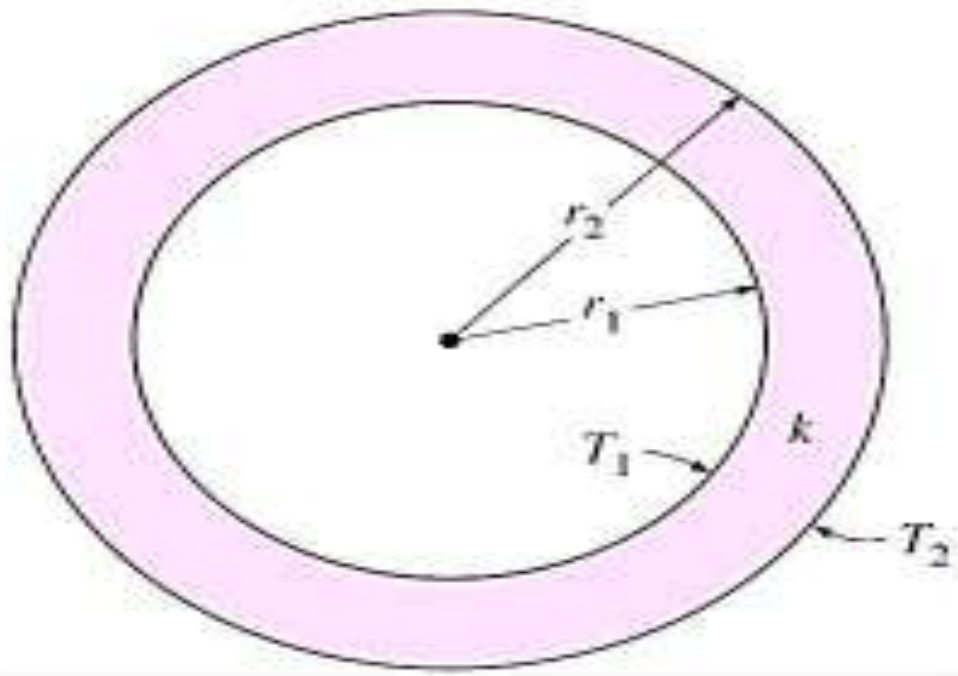
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$





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Single Spherical Layer



A spherical shell with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{\text{cond, sph}} = \frac{T_1 - T_2}{R_{\text{sph}}}$$

$$= \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$$

duction resistance of the spherical layer

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Numerical on

One dimensional steady state heat conduction equation for the Cylinder



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Example:

Steam at $T_{\infty 1}$ 320°C flows in a cast iron pipe ($k = 80$ W/m · °C) whose inner and outer diameters are $D_1 = 5$ cm and $D_2 = 5.5$ cm, respectively. The pipe is covered with 3 cm thick glass wool insulation with $k = 0.05$ W/m · °C. Heat is lost to the surroundings at $T_{\infty 2} = 5$°C by natural convection and radiation, with a combined heat transfer coefficient of $h_2 = 18$ W/m^2 · °C. Taking the heat transfer coefficient inside the pipe to be $h_1 = 60$ W/m^2 · °C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

SOLUTION A steam pipe covered with glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions

1. Heat transfer is steady since there is no indication of any change with time.
2. Heat transfer is one-dimensional since there is thermal symmetry about the centreline and no change in the axial direction.
3. Thermal conductivities are constant.
4. Thermal contact resistance at the interface is negligible.

SOLUTION

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Properties The thermal conductivities are given to be $k = 80$ W/m · °C for cast iron and $k = 0.05$ W/m · °C for glass wool insulation.

Analysis The thermal resistance network for this problem involves four resistances in series and is given in Fig. .Taking $L = 1$ m, the areas of the surfaces exposed to convection are determined to be

$$A_1 = 2\pi r_1 L = 2\pi(0.025\ \text{m})(1\ \text{m}) = 0.157\ \text{m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575\ \text{m})(1\ \text{m}) = 0.361\ \text{m}^2$$

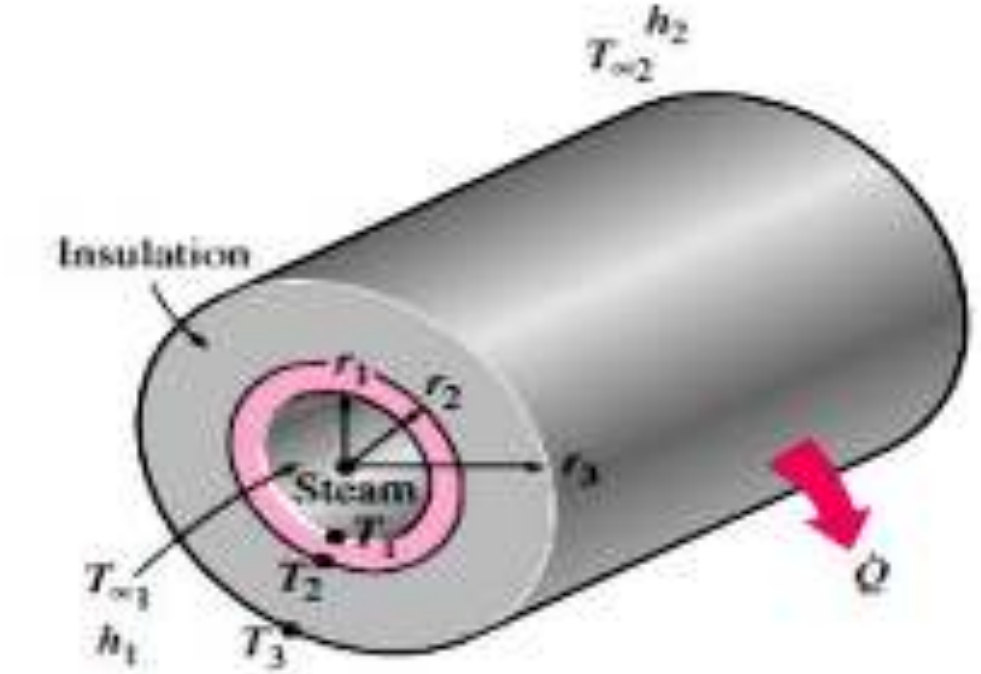
Then the individual thermal resistances become

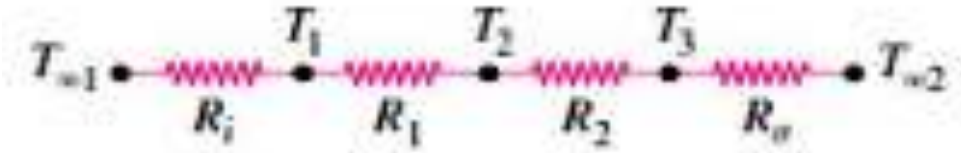
$$R_i = R_{\text{conv},1} = \frac{1}{h_1 A} = \frac{1}{(60\ \text{W/m}^2 \cdot °\text{C})(0.157\ \text{m}^2)} = 0.106°\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80\ \text{W/m} \cdot °\text{C})(1\ \text{m})} = 0.0002°\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05\ \text{W/m} \cdot °\text{C})(1\ \text{m})} = 2.35°\text{C/W}$$

$$R_o = \frac{1}{h_2 A_3} = \frac{1}{(18\ \text{W/m}^2 \cdot °\text{C})(0.361\ \text{m}^2)} = 0.154°\text{C/W}$$





SOLUTION

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Noting that all resistances are in series, the total resistance is determined to be

$$R_{total} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61°C/W$$

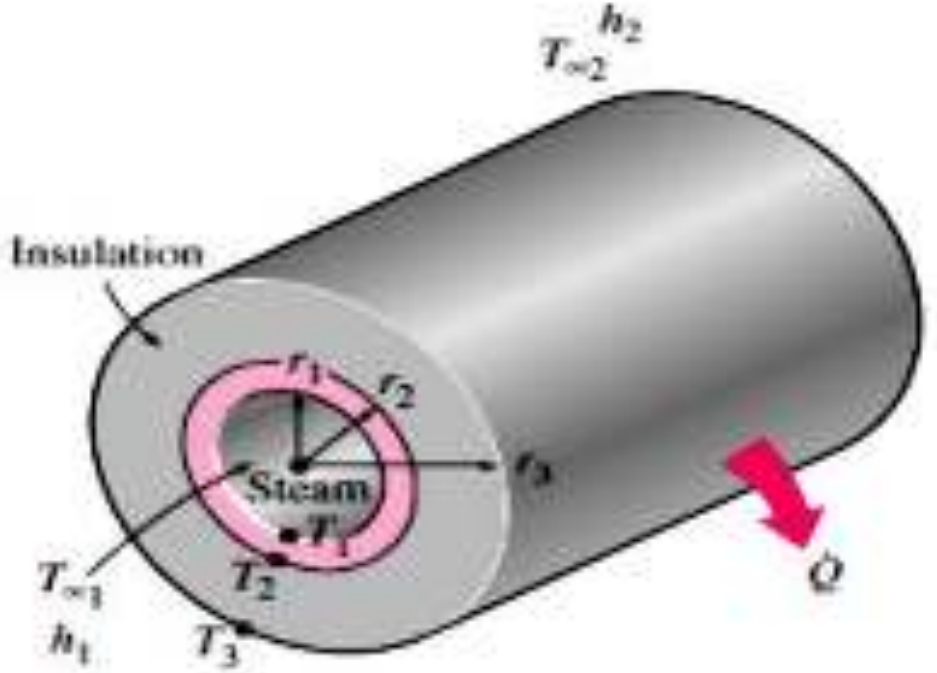
Then the steady rate of heat loss from the steam becomes

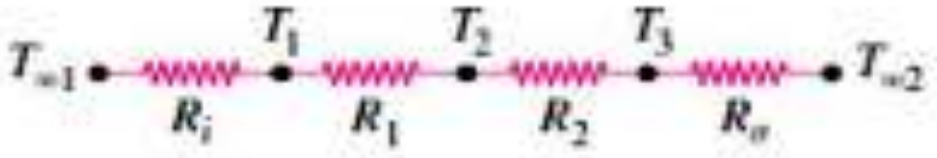
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(320 - 5)°C}{2.61°C/W} = 121\ W \qquad \text{(per m pipe length)}$$

The heat loss for a given pipe length can be determined by multiplying the above quantity by the pipe length *L*.The temperature drops across the pipe and the insulation are determined from Eq.

$$\Delta T_{pipe} = \dot{Q} R_{pipe} = (121\ W)(0.0002°C/W) = 0.02°C$$

$$\Delta T_{insulation} = \dot{Q} R_{insulation} = (121\ W)(2.35°C/W) = 284°C$$





The temperatures between the inner and the outer surfaces of the pipe differ by 0.02°C, whereas the temperatures between the inner and outer surfaces of the insulation differ by 284°C.

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CRITICAL RADIUS OF INSULATION



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CRITICAL RADIUS OF INSULATION

Definition. A *material which retards the flow of heat with reasonable effectiveness* is known as '***Insulation***'. Insulation serves the following *two purposes* :

(*i*) It prevents the heat flow from the system to the surroundings;

(*ii*) It prevents the heat flow from the surroundings to the system.

Applications :

The *fields of application* of insulations are :

(*i*) Boilers and steam pipes;

(*ii*) Airconditioning systems;

(*iii*) Food preserving stores and refrigerators;

nsulating bricks (employed in various types of furnaces);

reservation of liquid gases etc.

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CRITICAL RADIUS OF INSULATION

Factors affecting thermal conductivity :

Some of the important factors which affect thermal conductivity (k) of the insulators (*the value of k should be always low to reduce the rate of heat flow*) are as follows :

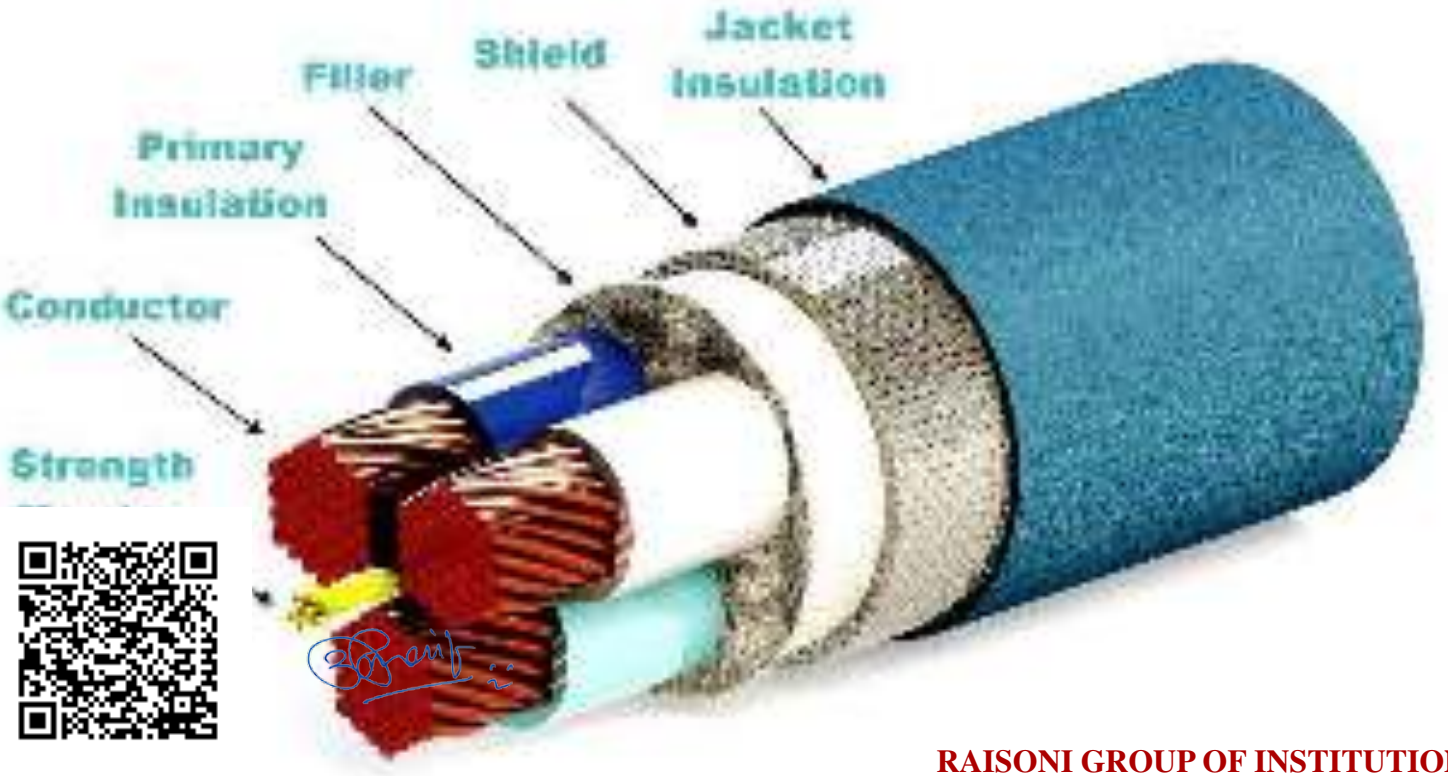
1. *Temperature.* For most of the insulating materials, the value of k *increases with increase in temperature.*
2. *Density.* There is no mathematical relationship between k and ρ (density). The common understanding that high density insulating materials will have higher values of k is not always true.
3. *Direction of heat flow.* For most of the insulating materials (except few like wood) the effect of direction of heat flow on the values of k is negligible.
4. *Moisture.* It is always considered necessary to prevent ingress of moisture in the insulating materials, during service. it is, however, difficult to find the effect of moisture on the values of k of different insulating materials.

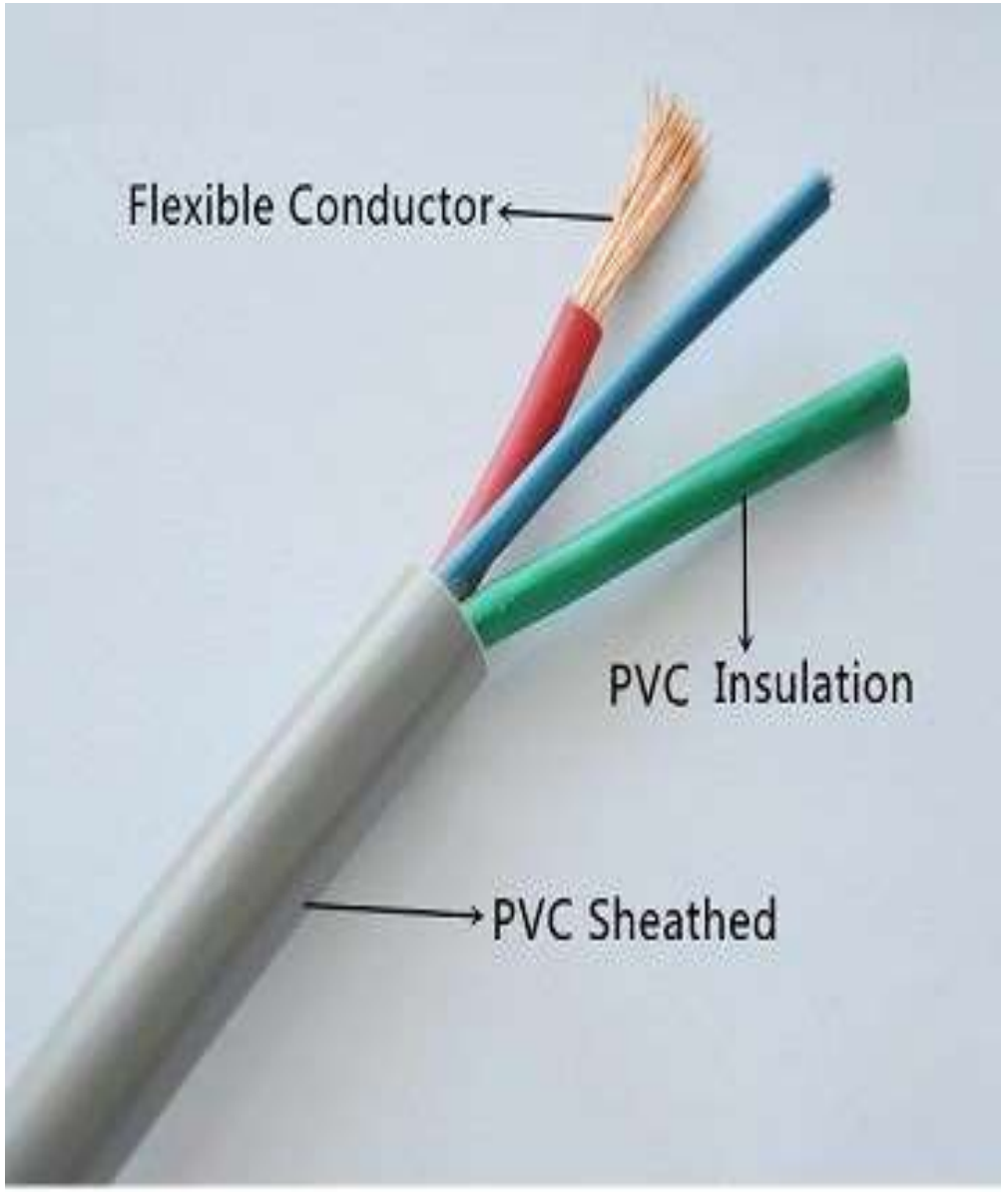
ure. It has been found that the value of k *decreases with decrease in pressure.*

n in insulators. The value of k increases due to the phenomenon of
n in insulators.

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PVC Insulated Cable

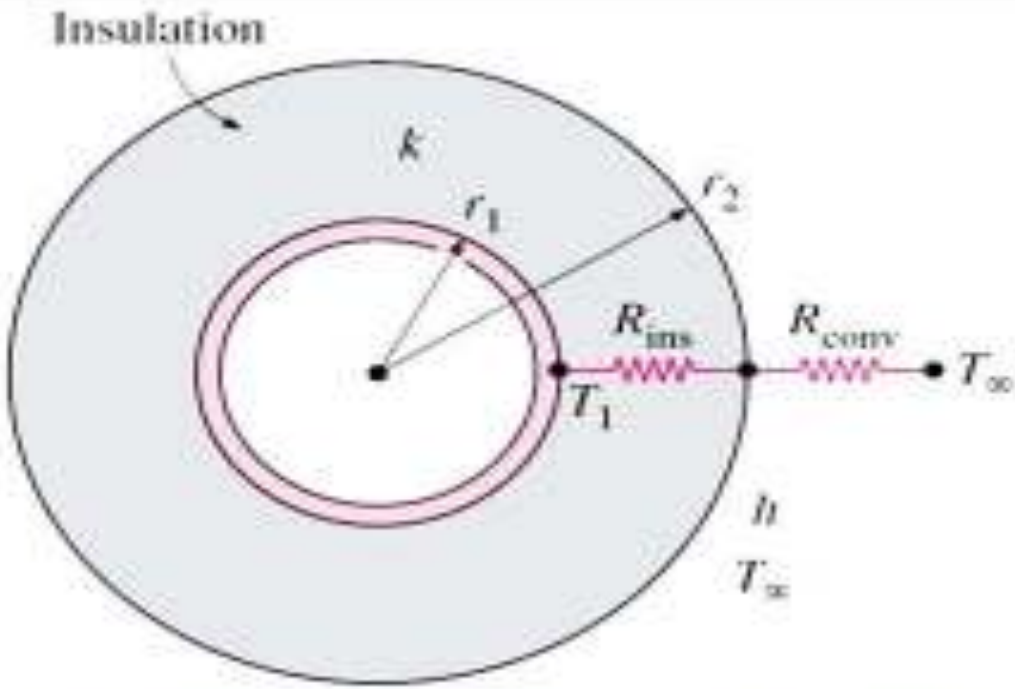
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CRITICAL RADIUS OF INSULATION

Adding more insulation to a wall or to the attic always decreases heat transfer since the heat transfer area is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

In a a cylindrical pipe or a spherical shell, the additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection.

...e heat transfer from the pipe
...ay increase or decrease,
...pending on which effect
...minates.



An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{h(2\pi r_2 L)}}$$

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The critical radius of insulation for a cylindrical body:

$$r_{cr,\,cylinder} = \frac{k}{h} \qquad \text{(m)}$$

The critical radius of insulation for a spherical shell:

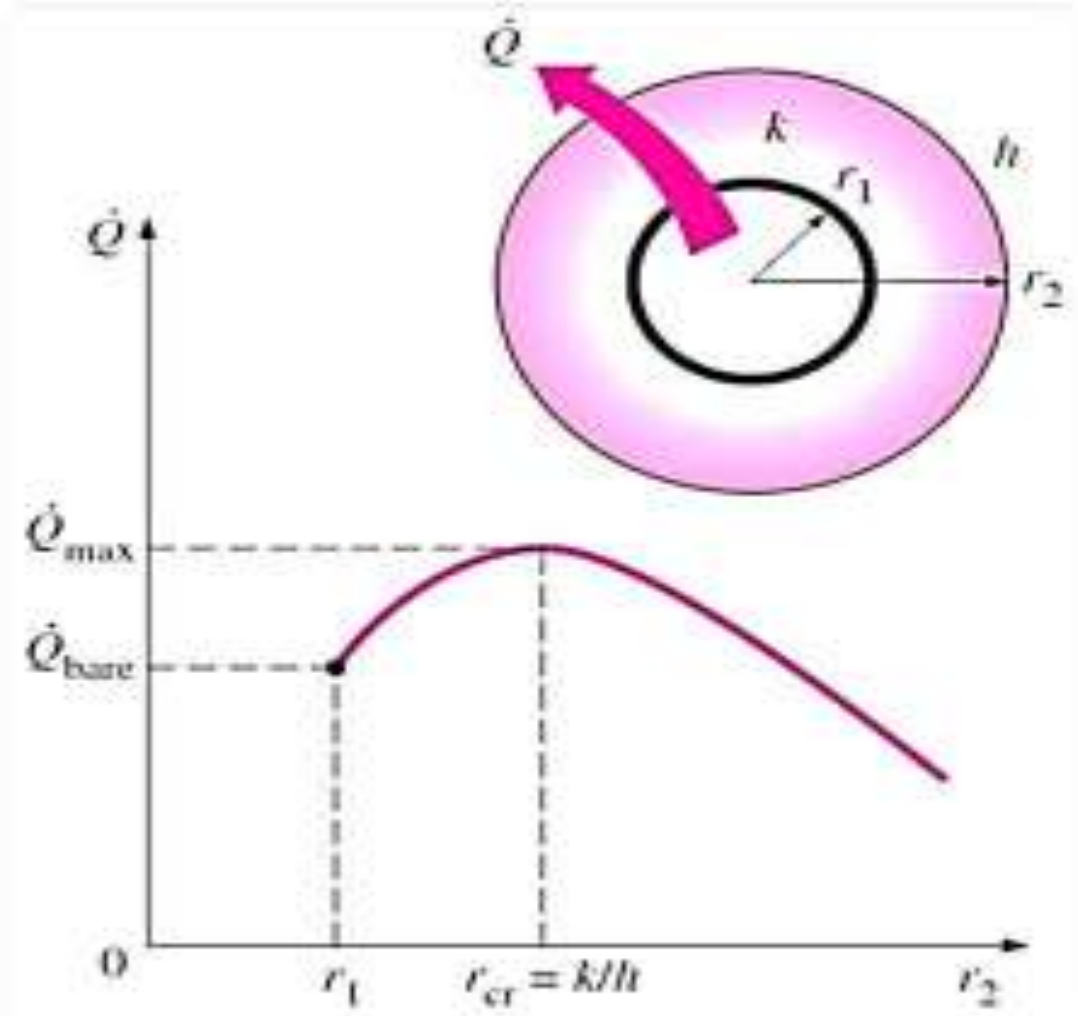
$$r_{cr,\,sphere} = \frac{2k}{h}$$

The largest value of the critical radius we are likely to encounter is

$$r_{cr,\,max} = \frac{k_{max.\,insulation}}{h_{min}} = \frac{0.05\ \text{W/m} \cdot {}^\circ\text{C}}{5\ \text{W/m}^2 \cdot {}^\circ\text{C}}$$

$$= 0.01\ \text{m} = 1\ \text{cm}$$

We can insulate hot-water or ...am pipes freely without ...rrying about the possibility of ...reasing the heat transfer by ...ulating the pipes.



The variation of heat transfer rate with the outer radius of the insulation r_2 when $r_1 < r_{cr}$

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Numerical on

Critical Radius of Insulation



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Example *Calculate the critical radius of insulation for asbestos [k = 0.172 W/m K] surrounding a pipe and exposed to room air at 300 K with h = 2.8 W/m K. Calculate the heat loss from a 475 K, 60 mm diameter pipe when covered with the critical radius of insulation and without insulation.*



Solution.

Given :

$$k = 0.172 \text{ W/m K}; T_1 = 475 \text{ K}; T_2 = 300 \text{ K};$$

$$h_0 = 2.8 \text{ W/m}^2 \text{ K}; \; r_i = \frac{60}{2} = 30 \text{ mm} = 0.03 \text{ m}.$$

The critical radius of insulation,

$$r_c = \frac{0.172}{2.8} = 0.06143 \text{m} \quad \text{or} \quad \mathbf{61.43 \text{ mm}} \quad \textbf{(Ans.)}$$



Solution.

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$$Q \text{ (with insulation)} = \frac{2\pi(T_1 - T_2)}{\frac{\ln(r_c / r_1)}{k} + \frac{1}{h_0 r_c}}$$

$$= \frac{2\pi(475 - 300)}{\frac{\ln(0.06143/0.03)}{0.172} + \frac{1}{2.8 \times 0.06143}} = \frac{1099.56}{4.167 + 5.814} = 110.16 \text{ W/m} \quad \text{(Ans.)}$$

$$Q \text{ (without insulation)} = h_0 \times 2\pi r_1 (T_1 - T_2)$$

$$= 2.8 \times 2\pi \times 0.03 (475 - 300) = 92.36 \text{ W/m} \quad \text{(Ans.)}$$





Example 2

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A 3-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is $k = 0.15$ W/m · °C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30$°C with a heat transfer coefficient of $h = 12$ W/m² · °C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

SOLUTION An electric wire is tightly wrapped with a plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal c ties are constant. **4** The thermal contact resistance at the interface is r **5** Heat transfer coefficient incorporates the radiation effects, if any.

P The thermal conductivity of plastic is given to be $k = 0.15$ W/m · °C.

Example 2

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Analysis Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

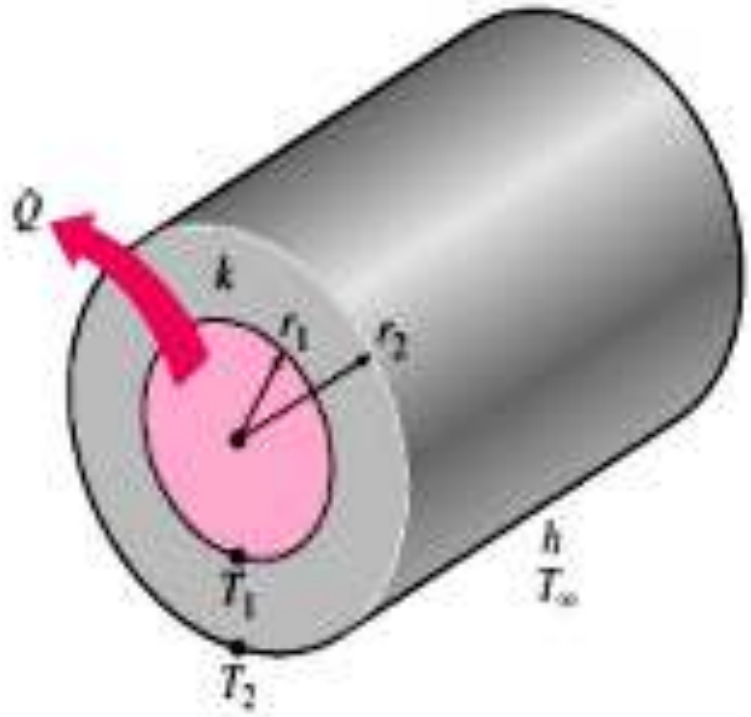
$$\dot{Q} = \dot{W}_e = VI = (8\text{ V})(10\text{ A}) = 80\text{ W}$$

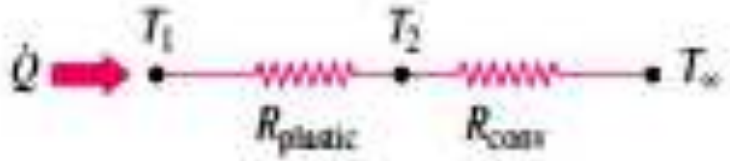
The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. 3–32. The values of these two resistances are determined to be

$$A_2 = (2\pi r_2)L = 2\pi(0.0035\text{ m})(5\text{ m}) = 0.110\text{ m}^2$$

$$R_{conv} = \frac{1}{hA_2} = \frac{1}{(12\text{ W/m}^2\cdot{}^\circ\text{C})(0.110\text{ m}^2)} = 0.76^\circ\text{C/W}$$

$$R_{plastic} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15\text{ W/m}\cdot{}^\circ\text{C})(5\text{ m})} = 0.18^\circ\text{C/W}$$





Example 2

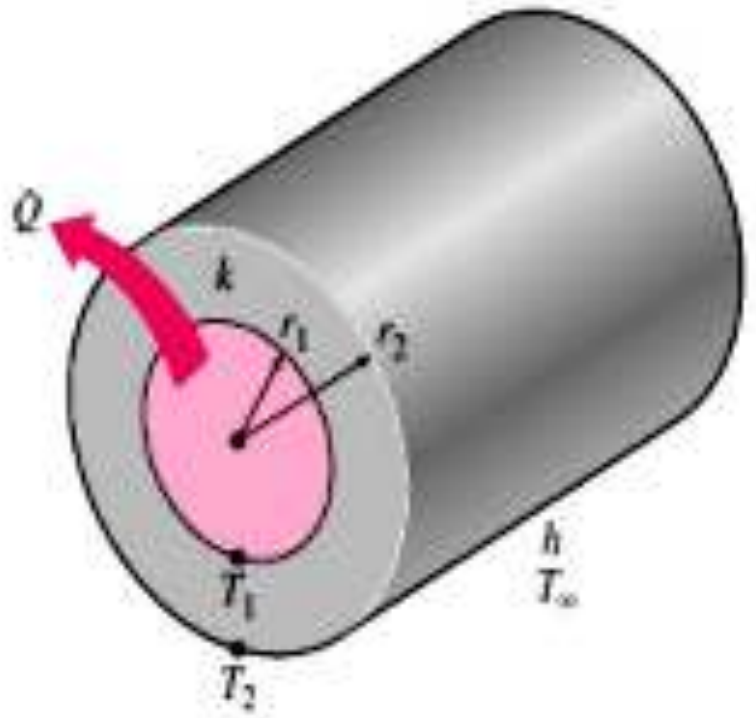
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and therefore

$$R_{total} = R_{plastic} + R_{conv} = 0.76 + 0.18 = 0.94°C/W$$

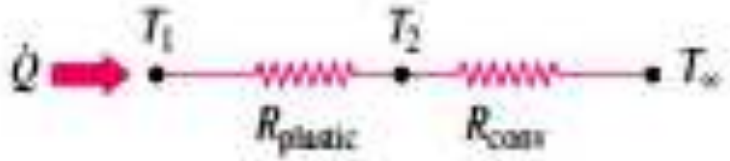
Then the interface temperature can be determined from

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{total} = 30°C + (80\ W)(0.94°C/W) = 105°C$$



Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 3–50 to be



$$= \frac{0.15\ W/m \cdot °C}{12\ W/m^2 \cdot °C} = 0.0125\ m = 12.5\ mm$$

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THEORETICAL QUESTIONS

1. Define the following terms :

   (*i*) Heat (*ii*) Heat transfer (*iii*) Thermodynamics.

2. What is the difference between thermodynamics and heat transfer ?
3. Enumerate the basic laws which govern the heat transfer.
4. Name and explain briefly the various modes of heat transfer.
5. What is conduction heat transfer ? How does it differ from convective heat transfer ?
6. What is the significance of heat transfer ?
7. Enumerate some important areas which are covered under the discipline of heat transfer.
8. What is the difference between the 'natural' and 'forced' convection ?
9. What is 'Fourier's law of conduction'? State also the assumptions on which this law is based.
10. State some essential features of Fourier's law.
11. How is thermal conductivity of a material defined ? What are its units ?
12. What is thermal resistance ?
13. What is 'Newton's law of cooling ?

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UNSOLVED EXAMPLES

1. The inner surface of a plane brick wall is at 40°C and the outer surface is at 20°C. Calculate the rate of heat transfer per m^2 of surface area of the wall, which is 250 mm thick. The thermal conductivity of the brick is 0.52 W/m°C. **(Ans. 41.6 W/m²)**

2. A plane wall (thermal conductivity = 10.2 W/m°C) of 100 mm thickness and area $3m^2$ has steady surface temperature of 170°C and 100°C. Determine :

   (*i*) The rate of heat flow across the plane wall;

   (*ii*) The temperature gradient in the flow direction. **[Ans. (*i*) 21.42 kW; (*ii*) – 700°C/m]**

3. Determine the heat transfer by convection over a surface of 0.75 m^2 if the surface is at 200°C and the fluid is at 80°C. The value of convective heat transfer is 25 W/m^2 °C. **[Ans. 2.25 kW]**

4. A surface of area $3m^2$ and at 200°C exchanges heat with another surface at 30°C by radiation. If the value of factor due to the geometric location and emissivity is 0.69, determine :

   (*i*) The rate of heat transfer,

   (*ii*) The value of thermal resistance, and

   (*iii*) The equivalent convection coefficient. **[Ans. (*i*) 4885.6W; (*ii*) 0.0348 °C/W; (*iii*) 9.58 W/m^2 °C]**

rface at 200°C exposed to the surroundings at 60°C convects and radiates heat to the surroundings. convection coefficient and radiation factor are 80W/m^2 °C and unity respectively. If the heat is lucted to the surface through a solid of conductivity 15W/m °C what is the temperature gradient e surface ? **(Ans. – 889.4 °C/m)**

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Unit No.2

Heat Transfer in Extended surface, FINS

&

Conduction with internal heat generation



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Extended Surface Heat Transfer





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EXTENDED SURFACES / FINS

Convection: Heat transfer between a solid surface and a moving fluid is governed by the Newton's cooling law: $q = hA(T_s-T_\infty)$. Therefore, to increase the convective heat transfer, one can

- ✓ Increase the temperature difference (T_s-T_∞) between the surface and the fluid.

- ✓ Increase the convection coefficient h. This can be accomplished by increasing the fluid flow over the surface since h is a function of the flow velocity and the higher the velocity, the higher the h. Example: a cooling fan.

- ncrease the contact surface area A. Example: a heat sink with ins.

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The following **assumptions** are made for the analysis of heat flow through the fin :

1. Steady state heat conduction.
2. No heat generation within the fin.
3. Uniform heat transfer coefficient (h) over the entire surface of the fin.
4. Homogeneous and isotropic fin material (*i.e.* thermal conductivity of material constant).
5. Negligible contact thermal resistance.
6. Heat conduction one-dimensional.
7. Negligible radiation.

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HEAT TRANSFER FROM FINNED SURFACES

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

Newton's law of cooling: The rate of heat transfer from a surface to the surrounding medium

When T_s and T_∞ are fixed, *two ways* to increase the rate of heat transfer are

- To increase the *convection heat transfer coefficient h*. This may require the installation of a pump or fan, or replacing the existing one with a larger one, but this approach may or may not be practical. Besides, it may not be adequate.
- To increase the *surface area* A_s by attaching to the surface *extended surfaces* called ***fins*** made of highly conductive materials such as aluminum.

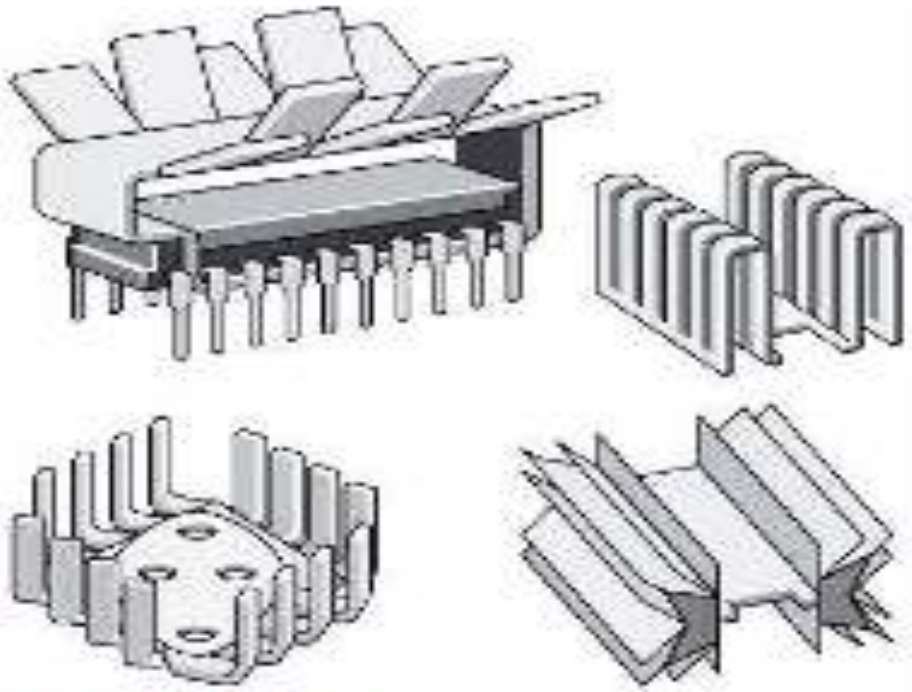


FIGURE 17–35
Some innovative fin designs.

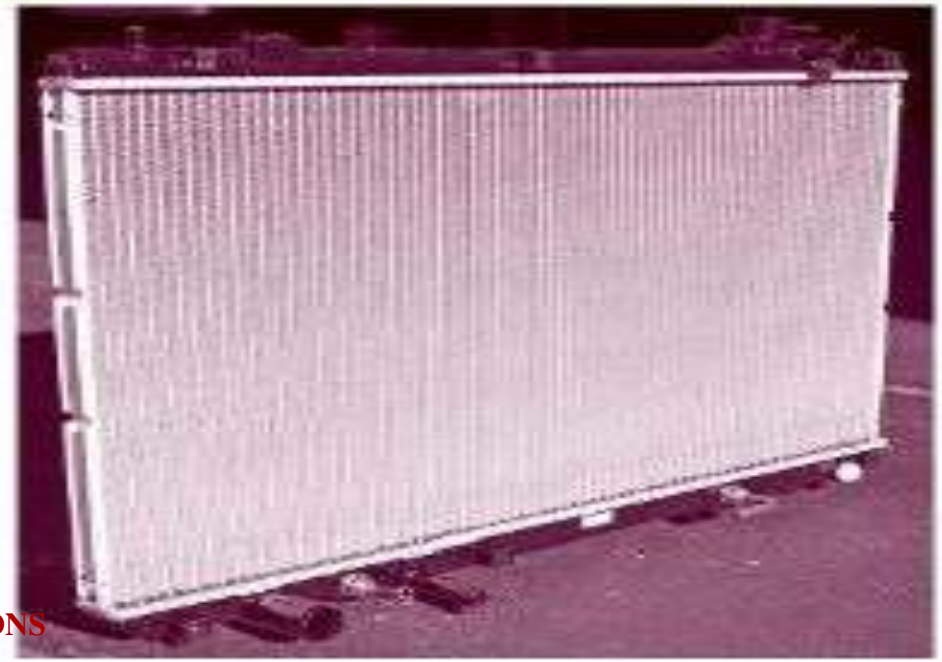
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Presumed cooling fins on dinosaur stegosaurus.

The thin plate fins of a car radiator greatly increase the rate of heat transfer to the air.



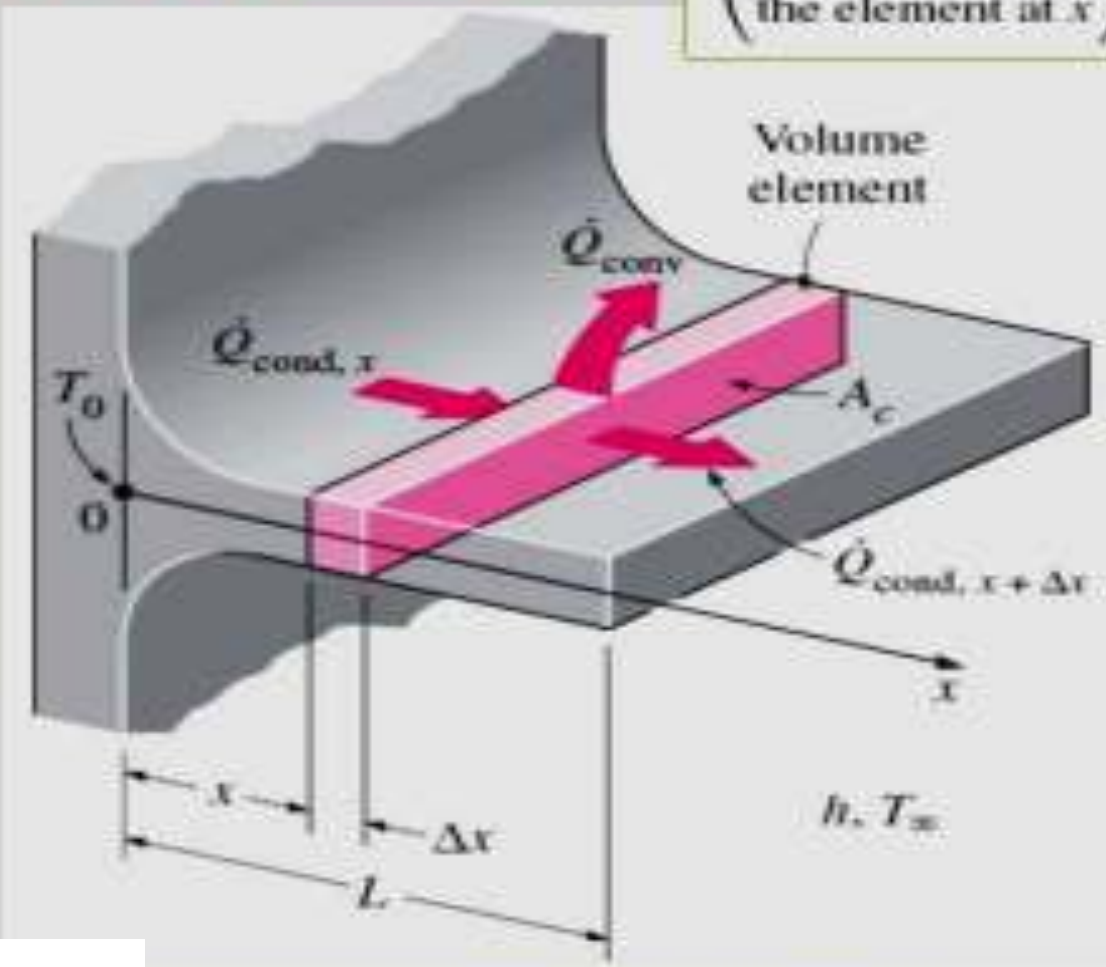


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Fin Equation

$$\left(\begin{array}{c}\text{Rate of } heat \\ conduction \text{ into} \\ \text{the element at } x\end{array}\right) = \left(\begin{array}{c}\text{Rate of } heat \\ conduction \text{ from the} \\ \text{element at } x + \Delta x\end{array}\right) + \left(\begin{array}{c}\text{Rate of } heat \\ convection \text{ from} \\ \text{the element}\end{array}\right)$$



$$\dot{Q}_{cond,\,x} = \dot{Q}_{cond,\,x+\Delta x} + \dot{Q}_{conv}$$

$$\dot{Q}_{conv} = h(p\,\Delta x)(T - T_\infty)$$

$$\frac{\dot{Q}_{cond,\,x+\Delta x} - \dot{Q}_{cond,\,x}}{\Delta x} + hp(T - T_\infty) = 0$$

$$\Delta x \to 0$$

$$\frac{d\dot{Q}_{cond}}{dx} + hp(T - T_\infty) = 0$$

$$\dot{Q}_{cond} = -kA_c \frac{dT}{dx}$$

$$\frac{d}{dx}\left(kA_c \frac{dT}{dx}\right) - hp(T - T_\infty) = 0$$

ne element of a fin at location x
g a length of Δx, cross-sectional
of A_c, and perimeter of p.

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

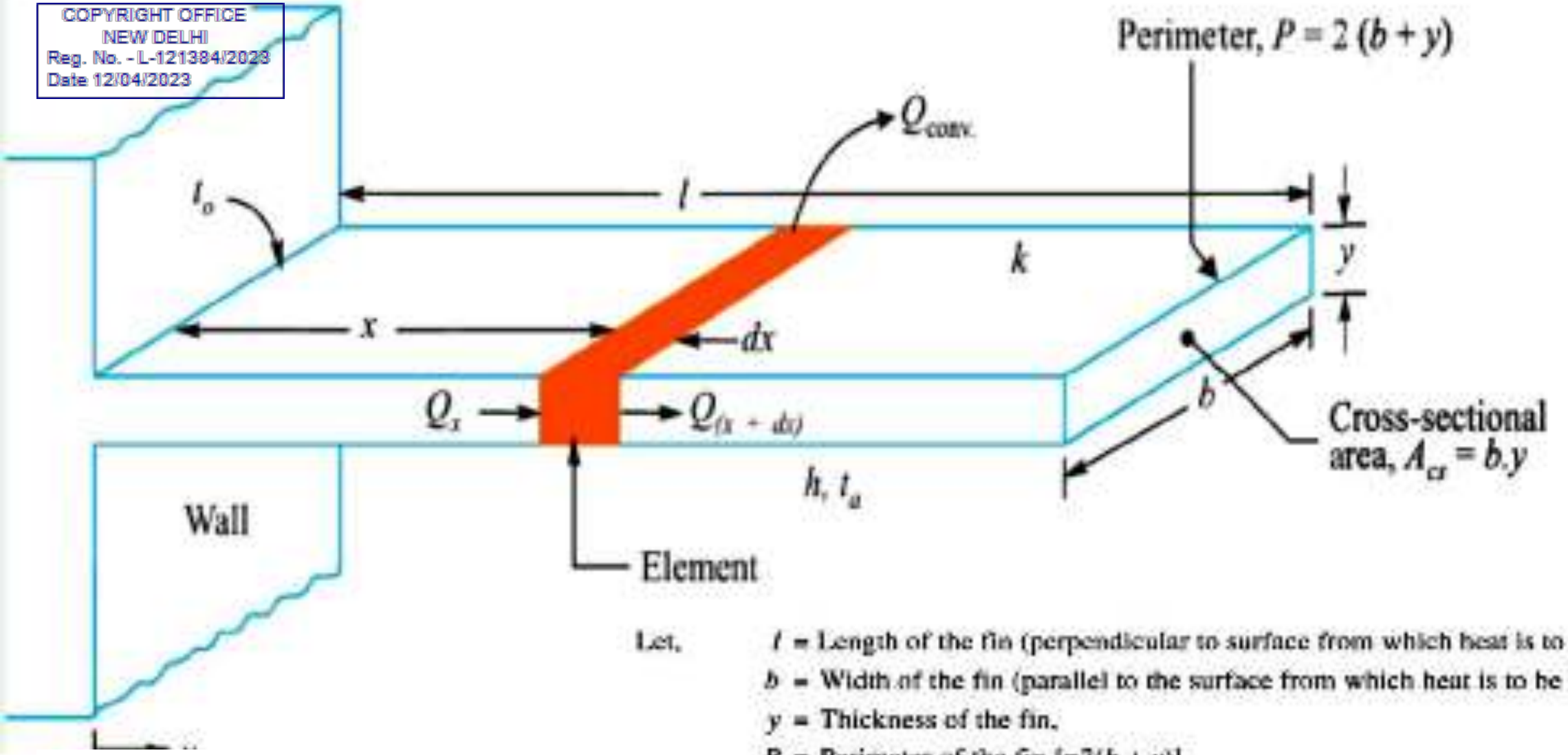
Differential equation

$$m^2 = \frac{hp}{kA_c}$$

$$\theta = T - T_\infty$$

Temperature excess

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Let, l = Length of the fin (perpendicular to surface from which heat is to be removed),
b = Width of the fin (parallel to the surface from which heat is to be removed),
y = Thickness of the fin,
P = Perimeter of the fin [$=2(b + y)$],
A_{cr} = Area of cross-section ($=by$),
t_b = Temperature at the base of the fin, and
t_a = Temperature of the ambient/surrounding fluid,
k = Thermal conductivity (constant), and
h = Heat transfer coefficient (convective).

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Case I. The *fin is infinitely long* and the temperature at the end of the fin is esentially that of the ambient/surrounding fluid.

Case II. The end of the *fin is insulated*.

Case III. The *fin is of finite length* and loses heat by convection.

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The general solution of the differential equation

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

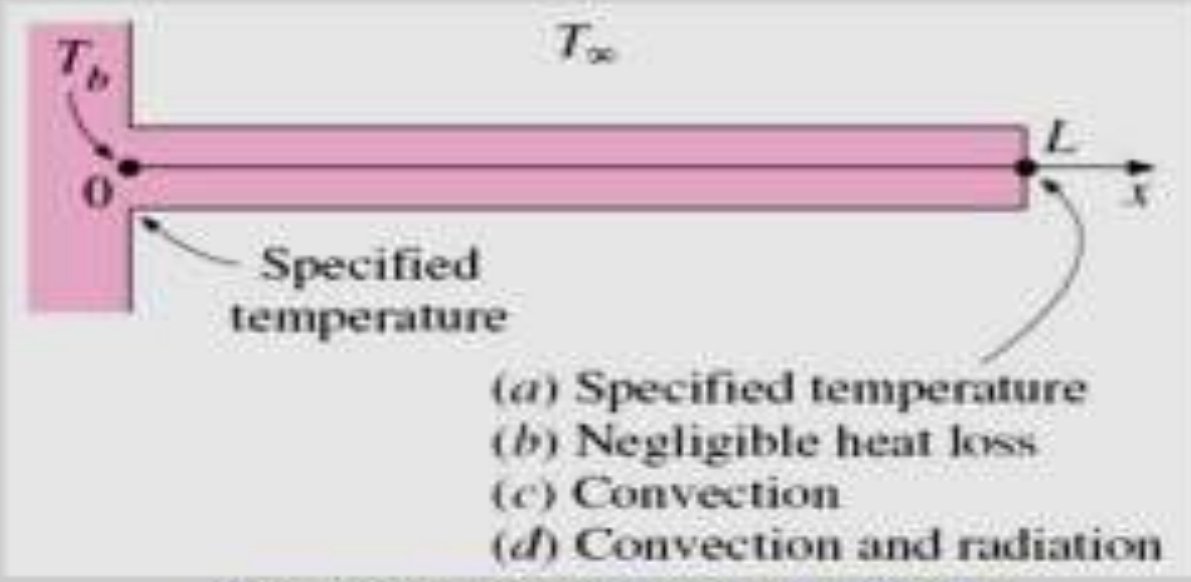
Boundary condition at fin base

$$\theta(0) = \theta_b = T_b - T_\infty$$

1 Infinitely Long Fin ($T_{\text{fin tip}} = T_\infty$)

Boundary condition at fin tip

$$\theta(L) = T(L) - T_\infty = 0 \quad L \rightarrow \infty$$



(*a*) Specified temperature
(*b*) Negligible heat loss
(*c*) Convection
(*d*) Convection and radiation

Boundary conditions at the fin base and the fin tip.

The variation of temperature along the fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}}$$

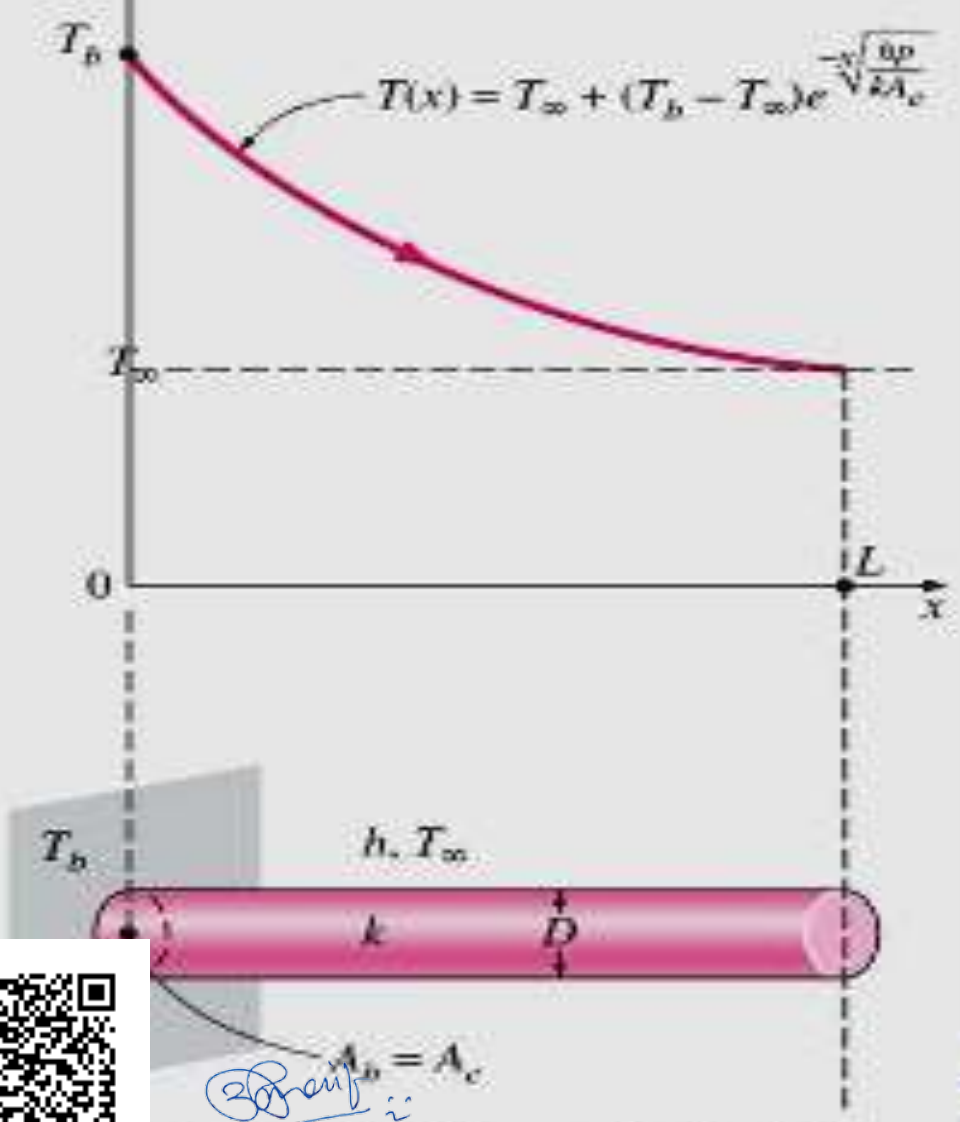
$$\theta = T - T_\infty$$

$$m = \sqrt{hp/kA_c}$$

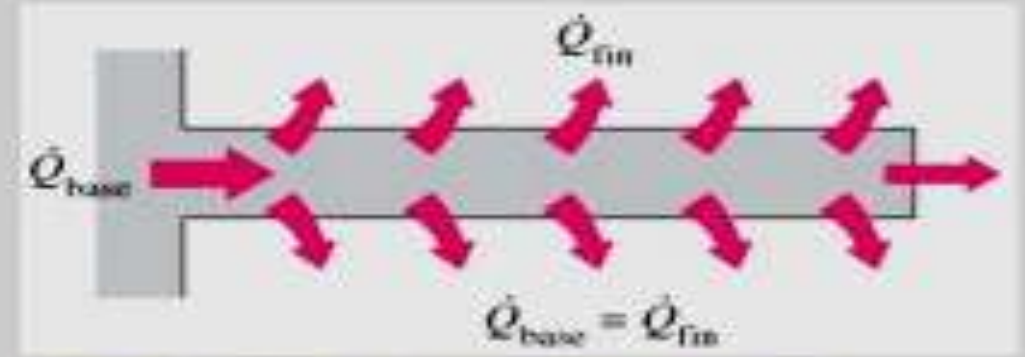
steady rate of *heat transfer* from the entire fin

$$q_{\text{fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$

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D, $A_c = \pi D^2/4$ for a cylindrical fin)



Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.

The rate of heat transfer from the fin could also be determined by considering heat transfer from a differential volume element of the fin and integrating it over the entire surface of the fin:

$$\dot{Q}_{fin} = \int_{A_{fin}} h[T(x) - T_\infty]\, dA_{fin} = \int_{A_{fin}} h\theta(x)\, dA_{fin}$$

A long circular fin of uniform cross section and the variation of temperature along it.

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2. Negligible Heat Loss from the Fin Tip (Adiabatic fin tip, $Q_{fin\ tip} = 0$)

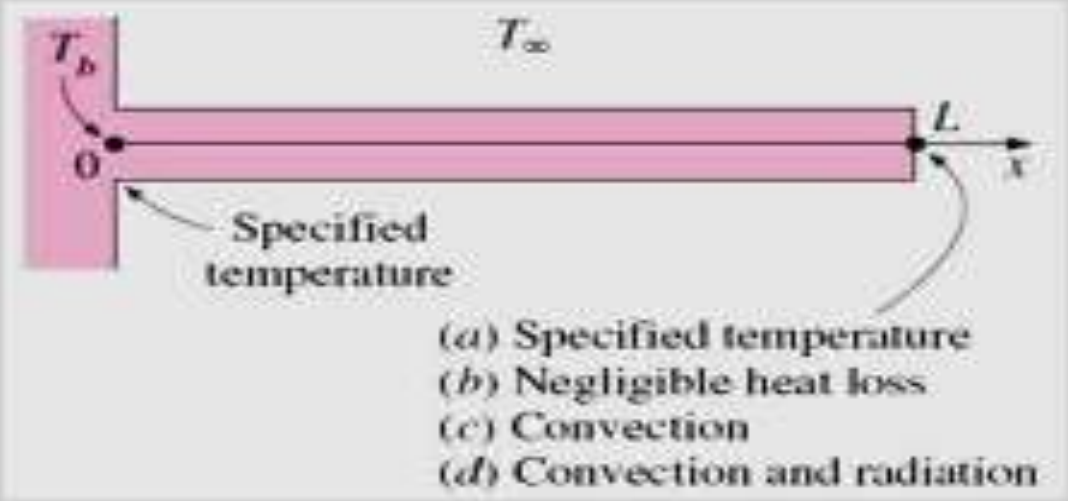
Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic assumption is for heat transfer from the fin tip to be negligible since the surface area of the fin tip is usually a negligible fraction of the total fin area.

Boundary condition at fin tip

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

The variation of temperature along the fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$$



(*a*) Specified temperature
(*b*) Negligible heat loss
(*c*) Convection
(*d*) Convection and radiation

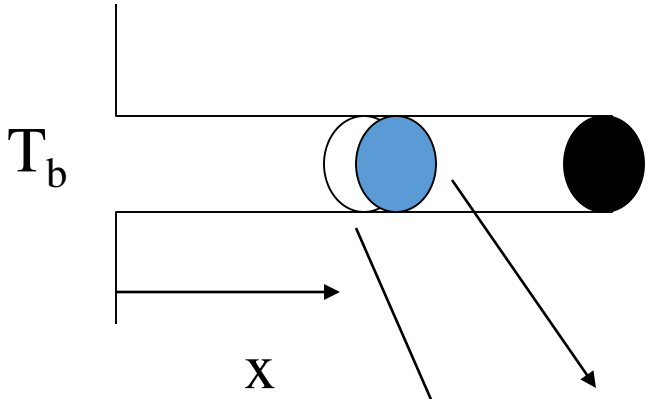
Heat transfer from the entire fin

$$\dot{Q}_{\text{batic tip}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$= \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL$$

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Extended Surface Analysis



P: the fin perimeter
A_c: the fin cross-sectional area

$$q_x = -kA_C \frac{dT}{dx}$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx} dx$$

A_C is the cross-sectional area

$dq_{conv} = h(dA_S)(T - T_\infty)$, where dA_S is the surface area of the element

Energy Balance: $q_x = q_{x+dx} + dq_{conv} = q_x + \frac{dq_x}{dx} dx + h dA_S (T - T_\infty)$

$$-kA_C \frac{d^2T}{dx^2} dx + hP(T - T_\infty)dx = 0, \text{ if k, } A_C \text{ are all constants.}$$

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Extended Surface Analysis (contd....)

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_C}(T - T_\infty) = 0,\ \text{A second-order, ordinary differential equation}$$

Define a new variable $\theta(x) = T(x) - T_\infty$, so that

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0,\ \text{where } m^2 = \frac{hP}{kA_C},\ (D^2 - m^2)\theta = 0$$

Characteristics equation with two real roots: +m & -m

The general solution is of the form

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

To evaluate the two constants C_1 and C_2, we need to specify two boundary conditions:

The first one is obvious: the base temperature is known as $T(0) = T_b$

The second condition will depend on the end condition of the tip

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Extended Surface Analysis (contd...)

For example: assume the tip is insulated and no heat transfer
dθ/dx(x=L)=0

The temperature distribution is given by

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\theta}{\theta_b}=\frac{\cosh m(L-x)}{\cosh mL}$$

The fin heat transfer rate is given by

$$q_f=-kA_c\frac{dT}{dx}(x=0)=\sqrt{hPkA_c}\tanh mL=M\tanh mL$$



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3. Specified Temperature ($T_{fin,tip} = T_L$)

In this case the temperature at the end of the fin (the fin tip) is fixed at a specified temperature T_L.

This case could be considered as a generalization of the case of *Infinitely Long Fin* where the fin tip temperature was fixed at T_∞.

Boundary condition at fin tip: $\theta(L) = \theta_L = T_L - T_\infty$

Specified fin tip temperature:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{[(T_L - T_\infty)/(T_b - T_\infty)]\sinh mx + \sinh m(L-x)}{\sinh mL}$$

Specified fin tip temperature:

pecified temp. $= -kA_c \left.\frac{dT}{dx}\right|_{x=0}$

$= \sqrt{hpkA_c}(T_b - T_\infty)\frac{\cosh mL - [(T_L - T_\infty)/(T_b - T_\infty)]}{\sinh mL}$

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Convection from Fin Tip

The fin tips, in practice, are exposed to the surroundings, and thus the proper boundary condition for the fin tip is convection that may also include the effects of radiation. Consider the case of convection only at the tip. The condition at the fin tip can be obtained from an energy balance at the fin tip.

$$(\dot{Q}_{cond} = \dot{Q}_{conv})$$

Boundary condition at fin tip:

$$-kA_c \left.\frac{dT}{dx}\right|_{x=L} = hA_c[T(L) - T_\infty]$$

Convection from fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x) + (h/mk)\sinh m(L - x)}{\cosh mL + (h/mk)\sinh mL}$$

Convection from fin tip:

$$\dot{Q}_{convection} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$= \sqrt{hpkA_c}(T_b - T_\infty)\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

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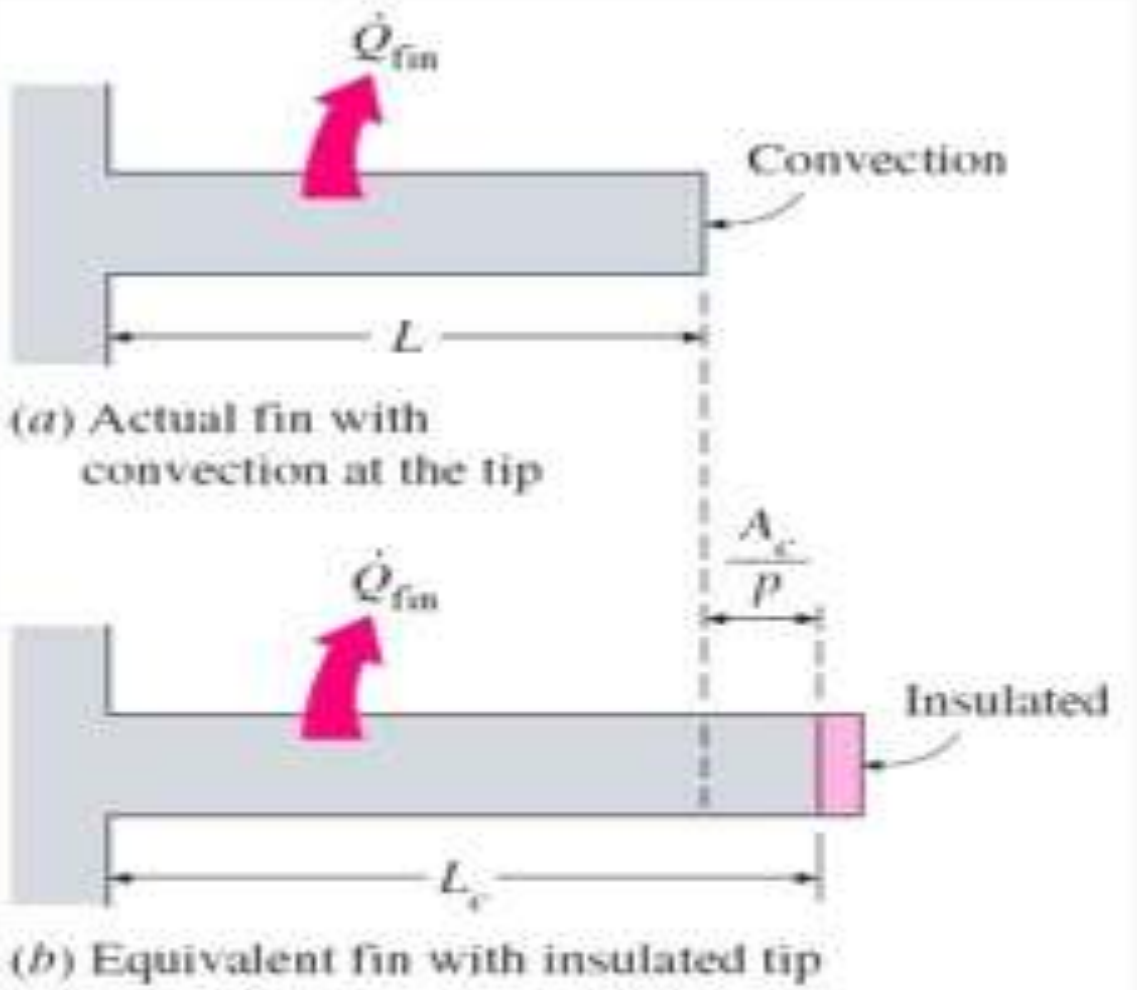
A practical way of accounting for the heat loss from the fin tip is to replace the *fin length* L in the relation for the *insulated tip* case by a **corrected length** defined as

$$L_c = L + \frac{A_c}{p}$$

$$L_{c,\text{ rectangular fin}} = L + \frac{t}{2}$$

$$L_{c,\text{ cylindrical fin}} = L + \frac{D}{4}$$

t the thickness of the rectangular fins
D the diameter of the cylindrical fins



Corrected fin length L_c is defined such that heat transfer from a fin of length L_c with insulated tip is equal to heat transfer from the actual fin of length L with convection at the fin tip.

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Temperature distribution for fins of different configurations

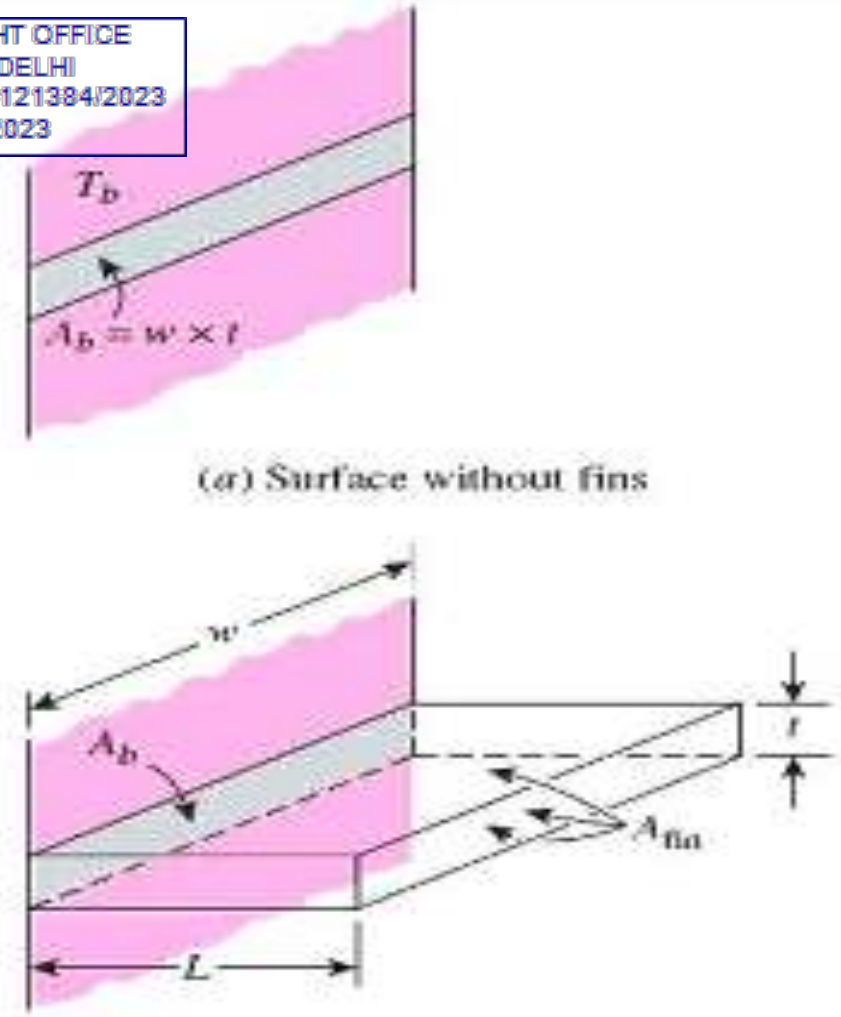
Case	Tip Condition	Temp. Distribution	Fin heat transfer
A	Convection heat transfer: $h\theta(L)=-k(d\theta/dx)_{x=L}$	$\frac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL}$	$M\theta_o\frac{\sinh mL+(h/mk)\cosh mL}{\cosh mL+(h/mk)\sinh mL}$
B	Adiabatic $(d\theta/dx)_{x=L}=0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M\theta_0\tanh mL$
C	Given temperature: $\theta(L)=\theta_L$	$\frac{(\theta_L/\theta_b)\sinh m(L-x)+\sinh m(L-x)}{\sinh mL}$	$M\theta_0\frac{(\cosh mL-\theta_L/\theta_b)}{\sinh mL}$
D	Infinitely long fin $\theta(L)=0$	e^{-mx}	$M\theta_0$

$$\theta\equiv T-T_\infty,\quad m^2\equiv\frac{hP}{kA_c}$$

$$\theta_b=\theta(0)=T_b-T_\infty,\quad M=\sqrt{hPkA_c}\theta_b$$

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Fin Efficiency



$$A_{fin} = 2 \times w \times L + w \times t \equiv 2 \times w \times L$$

FIGURE
Fins enhance heat transfer from face by enhancing surface area.

$$\dot{Q}_{fin,\,max} = hA_{fin}(T_b - T_\infty)$$

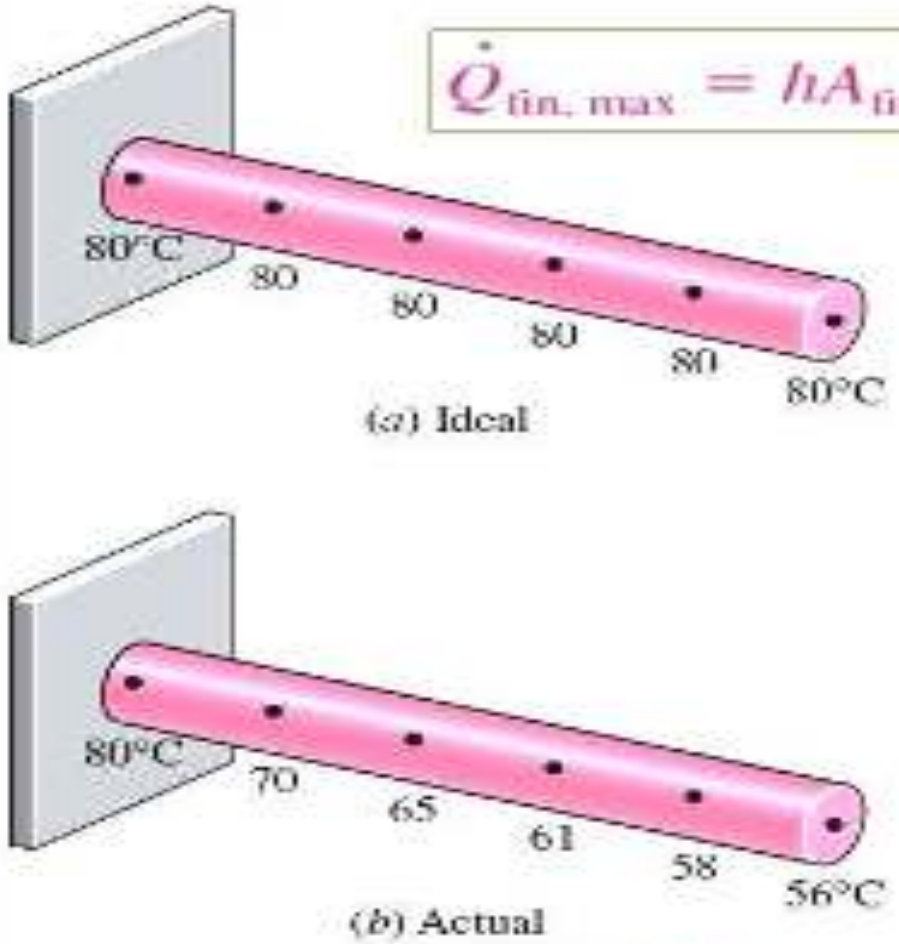


FIGURE
Ideal and actual temperature distribution along a fin.

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$$Q_{\text{fin, max}} = hA_{\text{fin}}(T_b - T_\infty)$$

Zero thermal resistance or infinite thermal conductivity ($T_{\text{fin}} = T_b$)

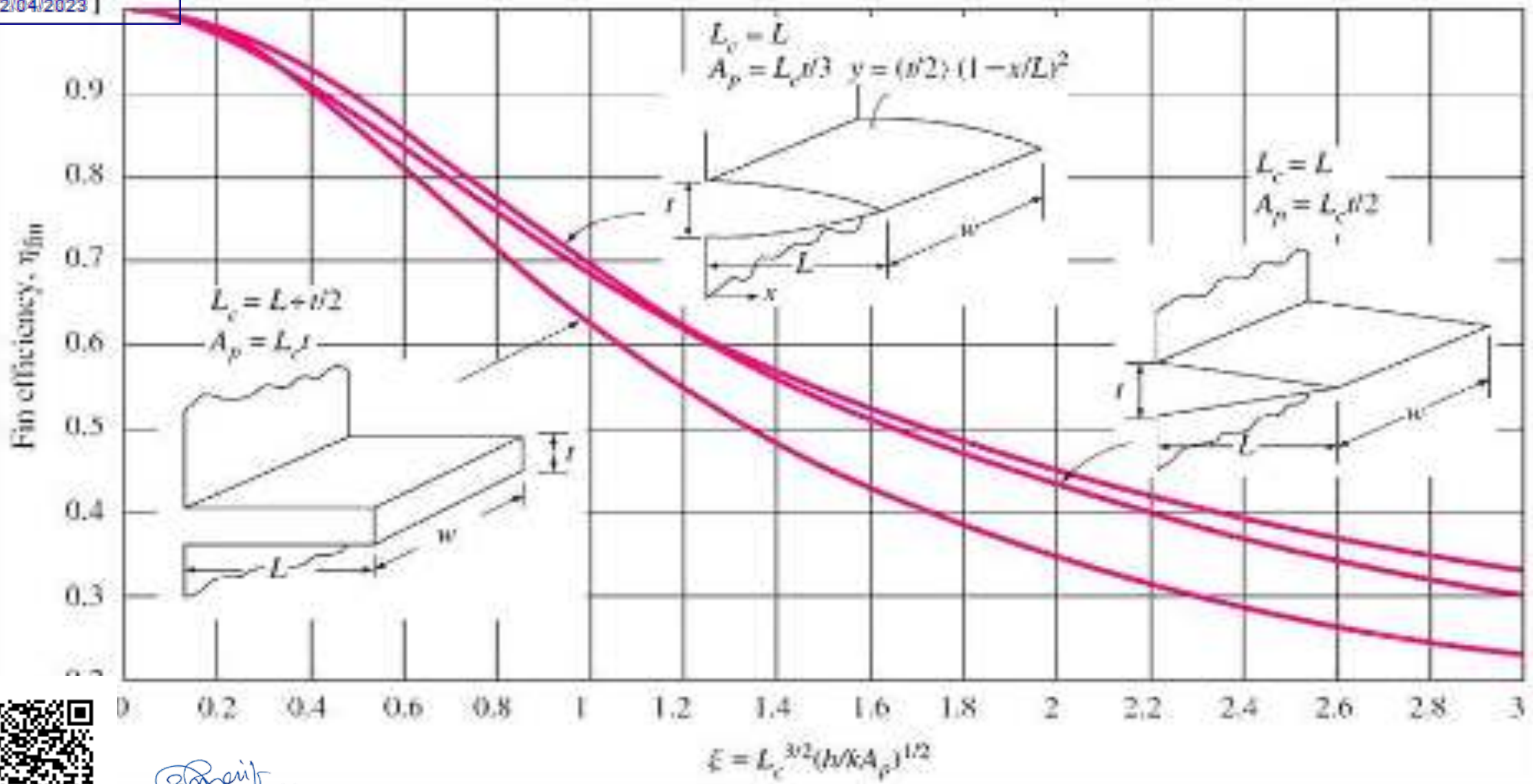
$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{Q_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} \dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}(T_b - T_\infty)}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{mL}$$

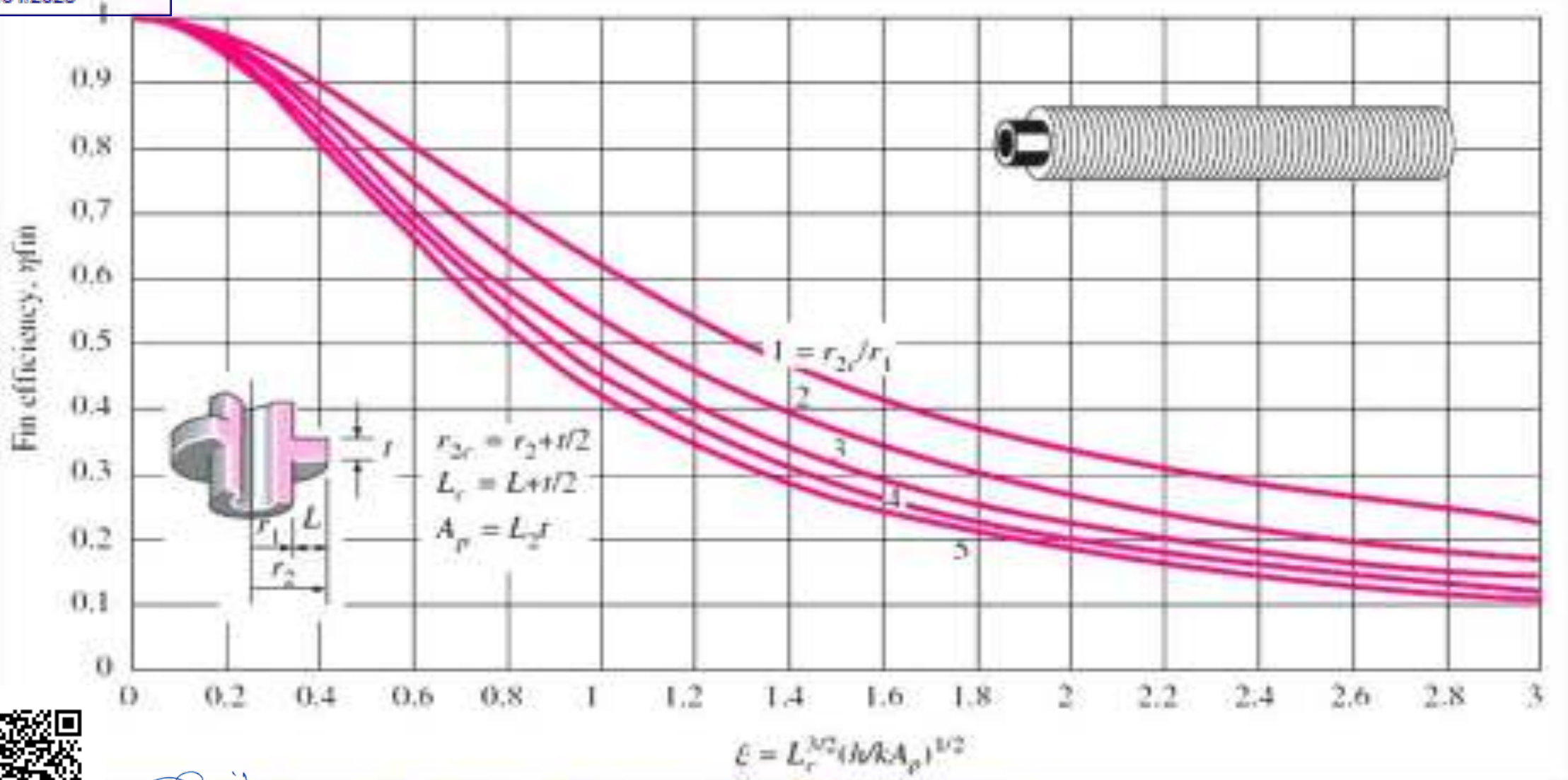
$$\text{adiabatic tip} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}(T_b - T_\infty)\tanh aL}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{\tanh mL}{mL}$$

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ency of straight fins of rectangular, triangular, and parabolic profiles.

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iency of annular fins of constant thickness t.

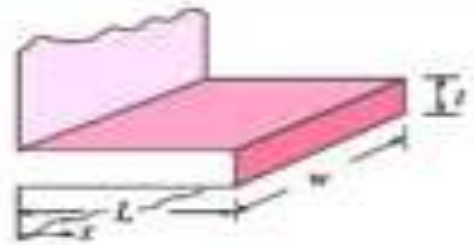
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...and surface areas of common fin configurations

...rectangular fins

$$m = \sqrt{2h/kt}$$
$$L_c = L + t/2$$
$$A_{fin} = 2wL_c$$

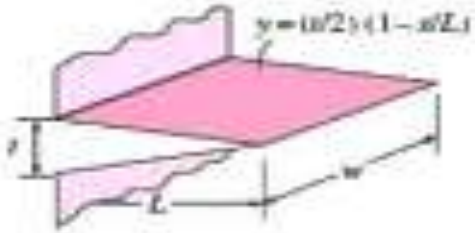
$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$



Straight triangular fins

$$m = \sqrt{2h/kt}$$
$$A_{fin} = 2w\sqrt{L^2 + (t/2)^2}$$

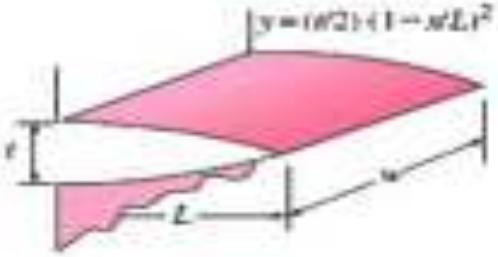
$$\eta_{fin} = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$



Straight parabolic fins

$$m = \sqrt{2h/kt}$$
$$A_{fin} = wL[C_1 + (L/t)\ln(t/L + C_1)]$$
$$C_1 = \sqrt{1 + (t/L)^2}$$

$$\eta_{fin} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}}$$



Circular fins of rectangular profile

$$m = \sqrt{2h/kt}$$
$$r_{2c} = r_2 + t/2$$
$$A_{fin} = 2\pi(r_{2c}^2 - r_1^2)$$

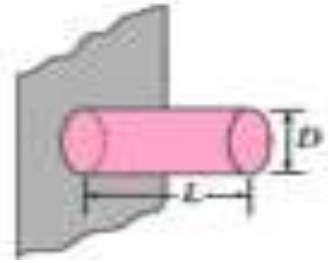
$$\eta_{fin} = C_2\frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$
$$C_2 = \frac{2r_1/m}{r_{2c}^2 - r_1^2}$$



...rectangular profile

$$/kD$$
$$D/4$$
$$L_c$$

$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$



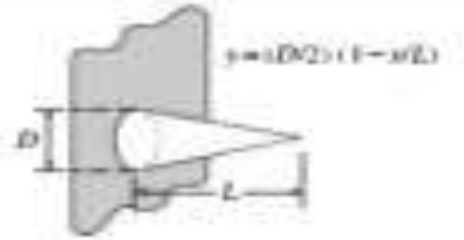
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Pin fins of triangular profile

$$m = \sqrt{4h/kD}$$

$$A_{fin} = \frac{\pi D}{2}\sqrt{L^2 + (D/2)^2}$$

$$\eta_{fin} = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$



Pin fins of parabolic profile

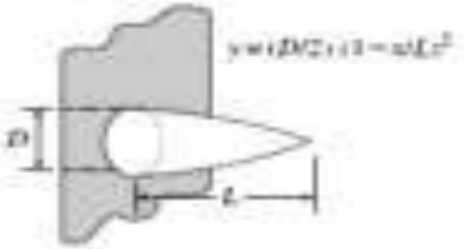
$$m = \sqrt{4h/kD}$$

$$A_{fin} = \frac{\pi L^2}{8D}\left[C_3 C_4 - \frac{L}{2D}\ln(2DC_4/L + C_3)\right]$$

$$C_3 = 1 + 2(D/L)^2$$

$$C_4 = \sqrt{1 + (D/L)^2}$$

$$\eta_{fin} = \frac{2}{1 + \sqrt{(2mL/3)^2 + 1}}$$

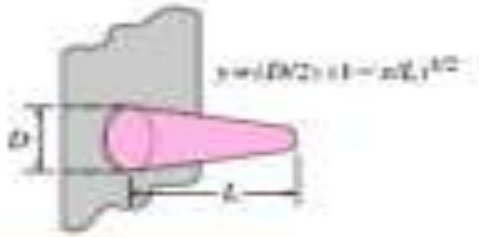


Pin fins of parabolic profile (blunt tip)

$$m = \sqrt{4h/kD}$$

$$A_{fin} = \frac{\pi D^2}{96L^2}\left\{[16(L/D)^2 + 1]^{3/2} - 1\right\}$$

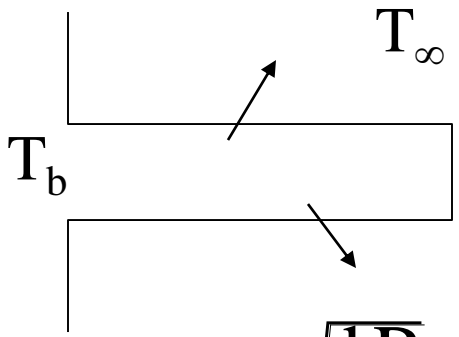
$$\eta_{fin} = \frac{3}{2mL}\frac{I_2(4mL/3)}{I_0(4mL/3)}$$



- Fins with triangular and parabolic profiles contain less material and are more efficient than the ones with rectangular profiles.
- The fin efficiency decreases with increasing fin length. Why?
- How to choose fin length? Increasing the length of the fin beyond a certain value cannot be justified unless the added benefits outweigh the added cost.
- Fin lengths that cause the fin efficiency to drop below 60 percent usually cannot be justified economically.
- The efficiency of most fins used in practice is above 90 percent.

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Fin Design



Total heat loss: q_f=Mtanh(mL) for an adiabatic fin, or q_f=Mtanh(mL_C) if there is convective heat transfer at the tip

where $m=\sqrt{\frac{hP}{kA_c}}$, and $M=\sqrt{hPkA_c}\,\theta_b=\sqrt{hPkA_c}(T_b-T_\infty)$

Use the thermal resistance concept:

$$q_f=\sqrt{hPkA_c}\tanh(mL)(T_b-T_\infty)=\frac{(T_b-T_\infty)}{R_{t,f}}$$

where $R_{t,f}$ is the thermal resistance of the fin.

For a fin with an adiabatic tip, the fin resistance can be expressed as

$$R_{t,f}=\frac{(T_b-T_\infty)}{q_f}=\frac{1}{\sqrt{hPkA_c}[\tanh(mL)]}$$

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Fin Effectiveness

How effective a fin can enhance heat transfer is characterized by the fin effectiveness ε_f: Ratio of fin heat transfer and the heat transfer without the fin. For an adiabatic fin:

$$\varepsilon_f = \frac{q_f}{q} = \frac{q_f}{hA_c(T_b - T_\infty)} = \frac{\sqrt{hPkA_c}\tanh(mL)}{hA_c} = \sqrt{\frac{kP}{hA_c}}\tanh(mL)$$

If the fin is long enough, $mL > 2$, $\tanh(mL) \to 1$,
it can be considered an infinite fin

$$\varepsilon_f \to \sqrt{\frac{kP}{hA_c}} = \sqrt{\frac{k}{h}\left(\frac{P}{A_c}\right)}$$

In order to enhance heat transfer, $\varepsilon_f > 1$.
However, $\varepsilon_f \geq 2$ will be considered justifiable
If $\varepsilon_f < 1$ then we have an insulator instead of a heat fin

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Fin Effectiveness

(contd...)

$$\varepsilon_f \rightarrow \sqrt{\frac{kP}{hA_c}} = \sqrt{\frac{k}{h}\left(\frac{P}{A_c}\right)}$$

- To increase ε_f, the fin's material should have higher thermal conductivity, k.
- It seems to be counterintuitive that the lower convection coefficient, h, the higher ε_f. But it is not because if h is very high, it is not necessary to enhance heat transfer by adding heat fins. Therefore, heat fins are more effective if h is low. Observation: If fins are to be used on surfaces separating gas and liquid. Fins are usually placed on the gas side. (Why?)

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Fin Effectiveness (contd...)

- P/AC should be as high as possible. Use a square fin with a dimension of W by W as an example: P=4W, AC=W2, P/AC=(4/W). The smaller W, the higher the P/AC, and the higher εf.
- Conclusion: It is preferred to use thin and closely spaced (to increase the total number) fins.

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Fin Effectiveness (contd...)

The effectiveness of a fin can also be characterized as

$$\varepsilon_f = \frac{q_f}{q} = \frac{q_f}{hA_c(T_b - T_\infty)} = \frac{(T_b - T_\infty)/R_{t,f}}{(T_b - T_\infty)/R_{t,h}} = \frac{R_{t,h}}{R_{t,f}}$$

It is a ratio of the thermal resistance due to convection to the thermal resistance of a fin. In order to enhance heat transfer, the fin's resistance should be lower than that of the resistance due only to convection.

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Fin Efficiency

Define Fin efficiency: $\eta_f = \frac{q_f}{q_{max}}$

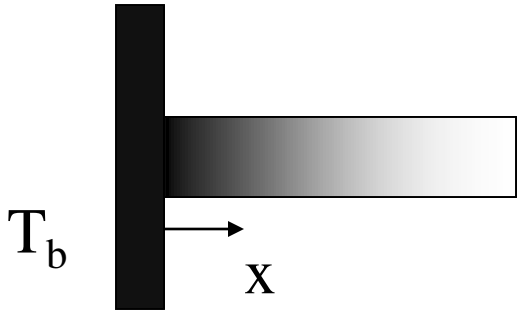
where q_{max} represents an idealized situation such that the fin is made up of material with infinite thermal conductivity. Therefore, the fin should be at the same temperature as the temperature of the base.

$q_{max} = hA_f(T_b - T_\infty)$

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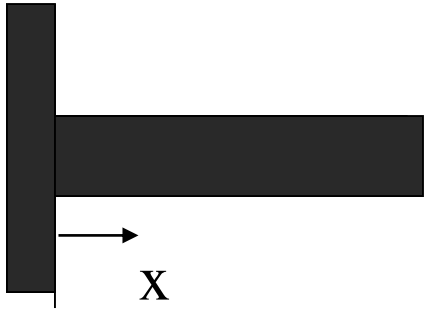
Fin Efficiency (contd…)

$T(x)<T_b$ for heat transfer to take place



Total fin heat transfer q_f

For infinite k
$T(x)=T_b$, the heat transfer is maximum



Ideal heat transfer q_{max}

Real situation

Ideal situation

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Fin Efficiency (cont.)

Use an adiabatic rectangular fin as an example:

$$\eta_f = \frac{q_f}{q_{\max}} = \frac{M \tanh mL}{hA_f(T_b - T_\infty)} = \frac{\sqrt{hPkA_c}(T_b - T_\infty)\tanh mL}{hPL(T_b - T_\infty)}$$

$$= \frac{\tanh mL}{\sqrt{\frac{hP}{kA_c}}L} = \frac{\tanh mL}{mL}$$

The fin heat transfer: $q_f = \eta_f q_{\max} = \eta_f hA_f(T_b - T_\infty)$

$$q_f = \frac{T_b - T_\infty}{1/(\eta_f hA_f)} = \frac{T_b - T_\infty}{R_{t,f}}, \text{ where } R_{t,f} = \frac{1}{\eta_f hA_f}$$

Thermal resistance for a single fin.

As compared to convective heat transfer: $R_{t,b} = \frac{1}{hA_b}$

In order to have a lower resistance as that is required to enhance heat transfer: $R_{t,b} > R_{t,f}$ or $A_b < \eta_f A_f$

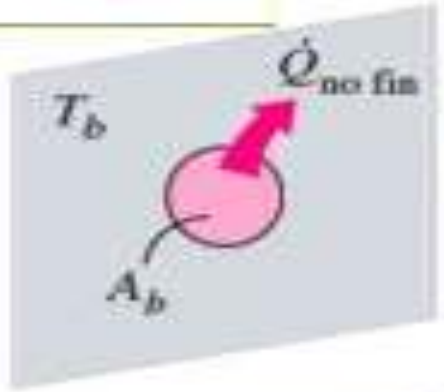
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Fin Effectiveness

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b}$$

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\eta_{\text{fin}}$$

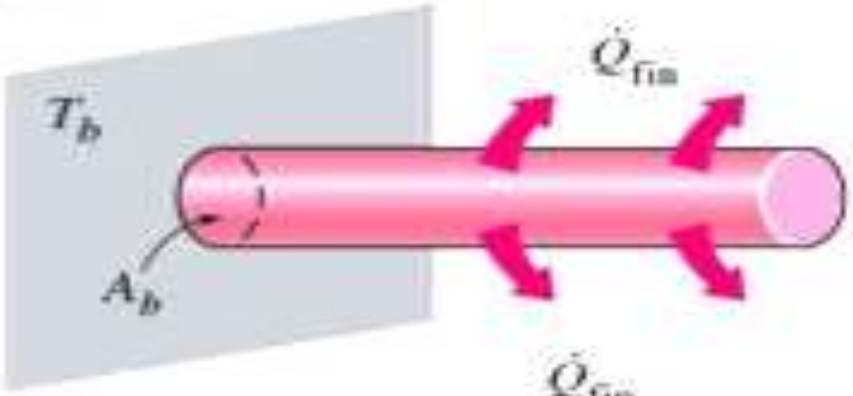
$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}$$



The effectiveness of a fin

- The *thermal conductivity* ***k*** of the fin should be as high as possible. Use aluminum, copper, iron.
- The ratio of the *perimeter* to the *cross-sectional area* of the fin p/A_c should be high as possible. Use slender pin fins.

w convection heat transfer coefficient
Place fins on gas (air) side.



$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}}$$

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The total rate of heat transfer from a finned surface

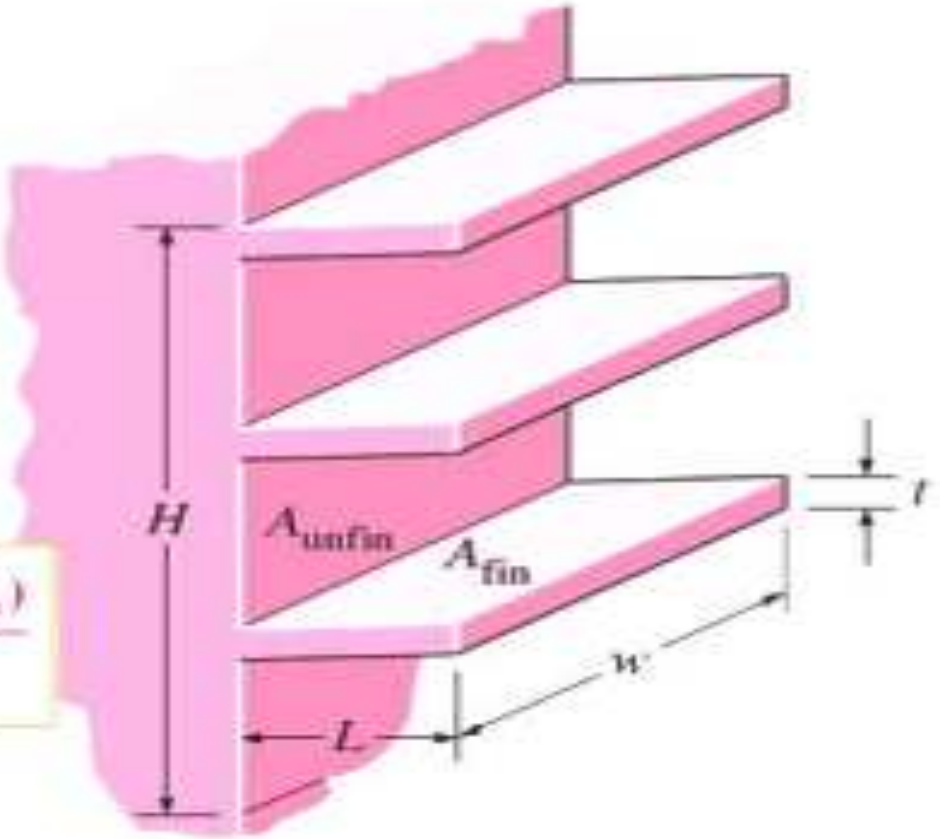
$$\dot{Q}_{\text{total, fin}} = \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}}$$
$$= hA_{\text{unfin}}(T_b - T_\infty) + \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$
$$= h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)$$

Overall effectiveness for a finned surface

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}(T_b - T_\infty)}$$

The overall fin effectiveness depends on the fin density (number of fins per unit length) as well as the effectiveness of the individual fins.

overall effectiveness is a better sure of the performance of a d surface than the effectiveness e individual fins.



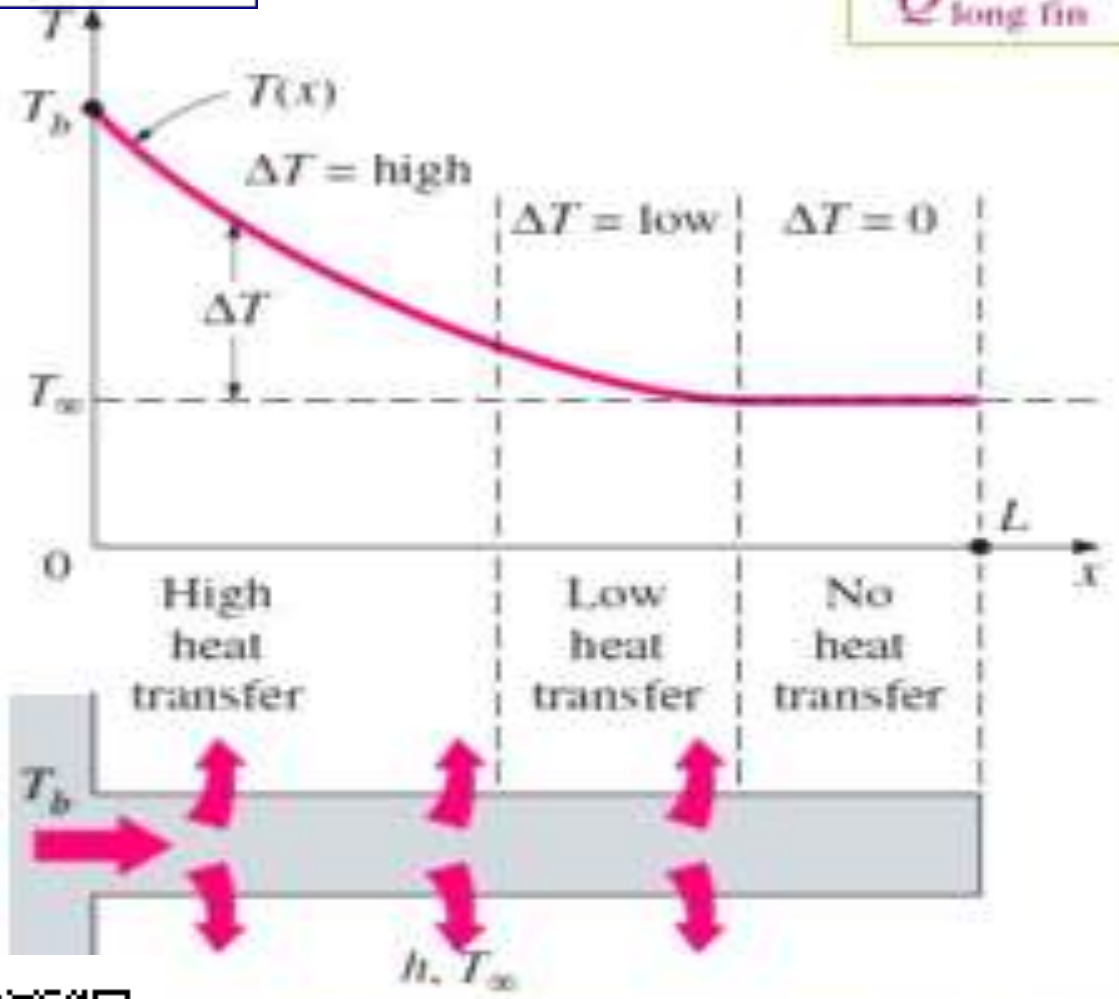
$$A_{\text{no fin}} = w \times H$$
$$A_{\text{unfin}} = w \times H - 3 \times (t \times w)$$
$$A_{\text{fin}} = 2 \times L \times w + t \times w$$
$$\cong 2 \times L \times w \text{ (one fin)}$$

Various surface areas associated with a rectangular surface with three fins.

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Fin Length of a Fin

$$\frac{\dot{Q}_{fin}}{\dot{Q}_{long\ fin}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL}{\sqrt{hpkA_c}\,(T_b - T_\infty)} = \tanh mL$$



The variation of heat transfer from a fin relative to that from an infinitely long fin

mL	$\frac{\dot{Q}_{fin}}{\dot{Q}_{long\ fin}} = \tanh mL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

se of the gradual temperature drop
the fin, the region near the fin tip makes
no contribution to heat transfer.

$mL = 5$ → an infinitely long fin
$mL = 1$ offer a good compromise between heat transfer performance and the fin size.

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A common approximation used in the analysis of fins is to assume the fin temperature to vary in one direction only (along the fin length) and the temperature variation along other directions is negligible.

Perhaps you are wondering if this one-dimensional approximation is a reasonable one.

This is certainly the case for fins made of thin metal sheets such as the fins on a car radiator, but we wouldn't be so sure for fins made of thick materials.

Studies have shown that the error involved in one-dimensional fin analysis is negligible (less than about 1 percent) when

$$\frac{h\delta}{k} < 0.2$$

where δ is the characteristic thickness of the fin, which is taken to be the plate thickness t for rectangular fins and the diameter D for cylindrical ones.

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Heat sinks: Specially designed finned surfaces which are commonly used in the cooling of electronic equipment, and involve one-of-a-kind complex geometries.

- The heat transfer performance of heat sinks is usually expressed in terms of their *thermal resistances R*.
- A small value of thermal resistance indicates a small temperature drop across the heat sink, and thus a high fin efficiency.

$$\dot{Q} = \frac{T_b - T_\infty}{R} = hA_{fin}\,\eta_{fin}\,(T_b - T_\infty)$$

Combined natural convection and radiation thermal resistance of various heat sinks used in the cooling of electronic devices between the heat sink and the surroundings. All fins are made of aluminum 6063T-5, are black anodized, and are 76 mm (3 in) long.

HS 5030



R = 0.9°C/W (vertical)
R = 1.2°C/W (horizontal)
Dimensions: 76 mm × 105 mm × 44 mm
Surface area: 677 cm²

HS 6065



R = 5°C/W
Dimensions: 76 mm × 38 mm × 24 mm
Surface area: 387 cm²

HS 6071



R = 1.4°C/W (vertical)
R = 1.8°C/W (horizontal)
Dimensions: 76 mm × 92 mm × 26 mm
Surface area: 968 cm²

HS 6105



R = 1.8°C/W (vertical)
R = 2.1°C/W (horizontal)
Dimensions: 76 mm × 127 mm × 91 mm
Surface area: 677 cm²

HS 6115



R = 1.1°C/W (vertical)
R = 1.3°C/W (horizontal)
Dimensions: 76 mm × 102 mm × 25 mm
Surface area: 929 cm²

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Example *Aluminium fins of rectangular profile are attached on a plane wall with 5 mm spacing. The fins have thickness y = 1 mm, length l = 10 mm, and the thermal conductivity, k = 200 W/m K. The wall is maintained at a temperature 200°C, and the fins dissipate heat by convection into the ambient air at 40°C, with heat transfer coefficient h = 50 W/m²K. Determine the heat loss.*

Solution. *Given* : $t = 1\text{ mm} = 0.001\text{ m}$; $l = 10\text{ mm} = 0.01\text{ m}$; $y = 1\text{ mm} = 0.001\text{ m}$;

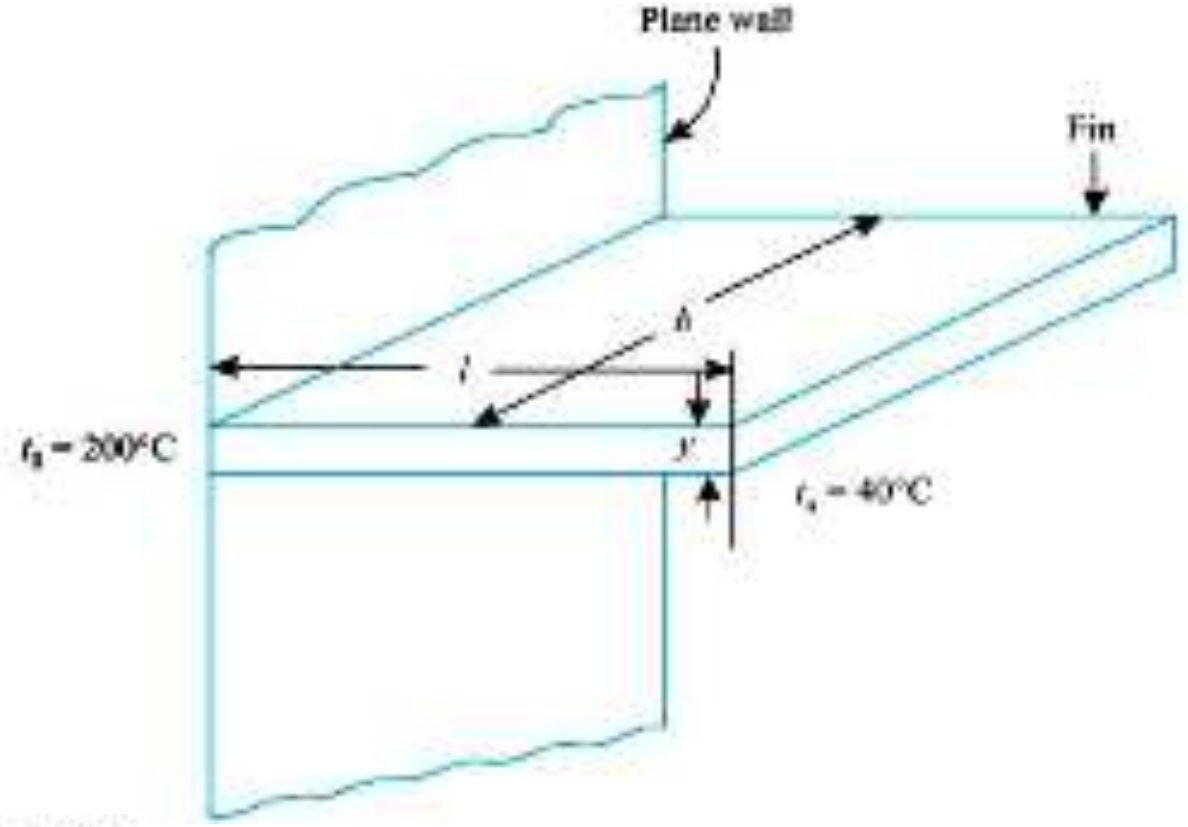
$k = 200\text{ W/mK}$; $t_o = 200°C$; $t_a = 40°C$; $h = 50\text{ W/m}^2\text{ K}$.

Heat loss, Q :

$$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h(b + y) \times 2}{k(b \times y)}}$$

$$= \sqrt{\frac{h \times 2b}{k \times by}}, \text{ assuming } b >> y$$

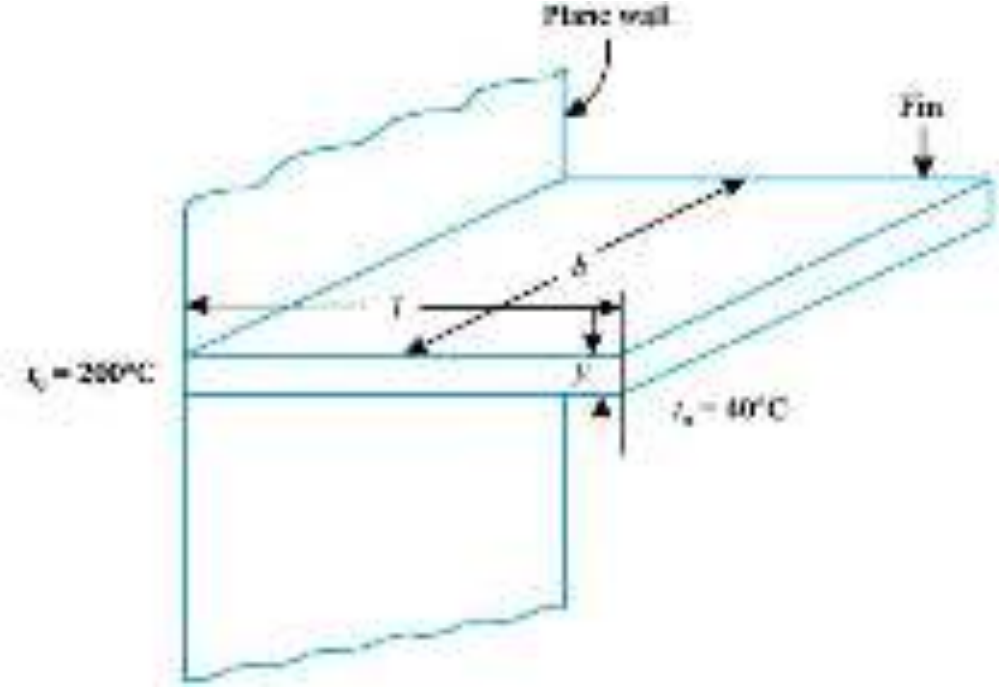
$$\sqrt{\frac{2 \times 50}{200 \times 0.001}} = 22.36$$



For unit width of the fin, (b = 1 m)

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$$Q = \sqrt{PhkA_{cs}}\,(t_0 - t_a)\,\tanh(ml)$$



$$= \sqrt{(2 \times 1) \times 50 \times 200 \times (1 \times 0.001)} \times (200 - 40) \times \tanh(22.36 \times 0.01)$$

$$= \tanh(0.2236) = \mathbf{157.38\ W/m} \qquad \textbf{(Ans.)}$$

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Example *It is required to heat oil to about 300°C for frying purpose. A laddle is used in the frying. The section of the handle is 5 mm × 18 mm. The surroundings are at 30°C. The conductivity of the material is 205 W/m°C. If the temperature at a distance of 380 mm from the oil should not reach 40°C, determine the convective heat transfer coefficient.*

Solution.

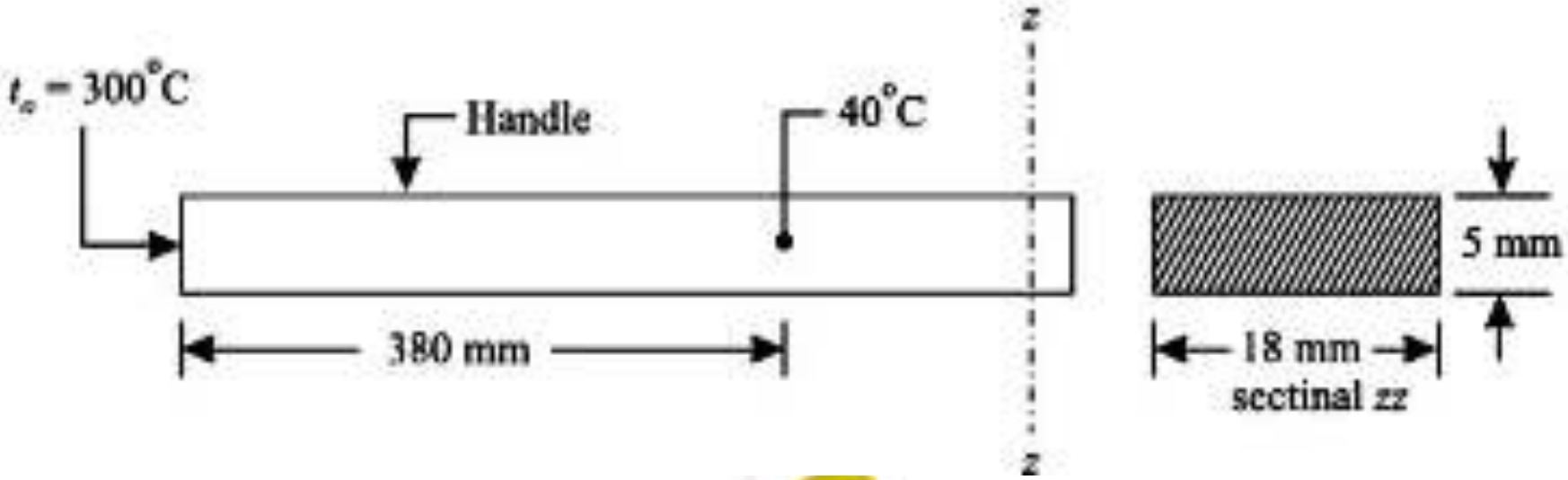
$t_o = 300°C$

$b = 18 \text{ mm} = 0.018 \text{ m}$

$y = 5 \text{ mm} = 0.005 \text{ m}$

$l = 380 \text{ mm} = 0.38 \text{ m}$

$k = 205 \text{ W/m°C}$





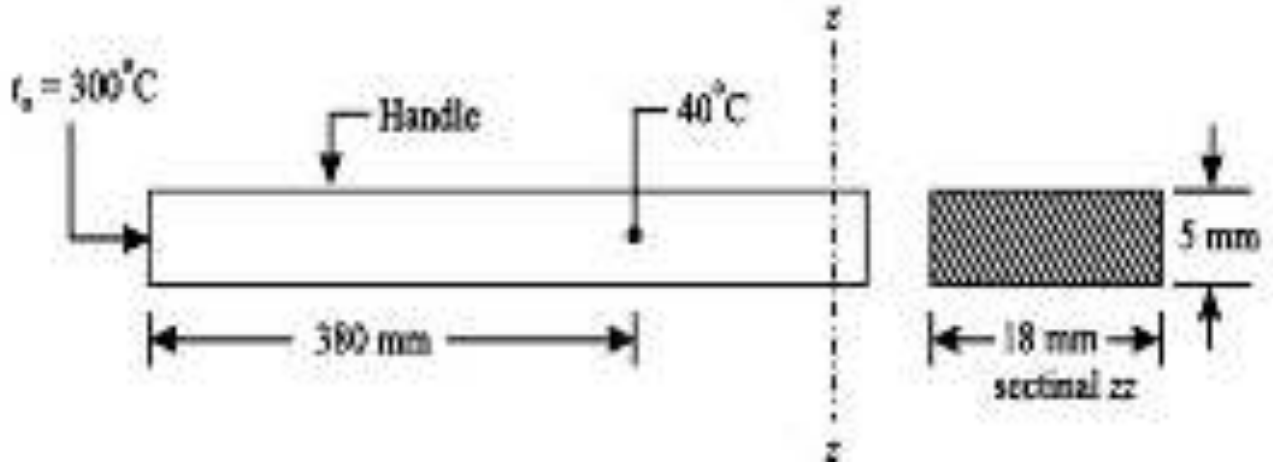
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Solution :

Convective heat transfer coefficient, h :

Assumming the *fin* to be *long one*, we have

$$\frac{t - t_a}{t_o - t_a} = e^{-mx} \quad \text{or,} \quad \frac{t_o - t_a}{t - t_a} = e^{mx}$$



or, $$\frac{300 - 30}{40 - 30} = e^{m \times 0.38} \quad (\because\ x = 380 \text{ mm} = 0.38 \text{ m})$$

or, $$e^{0.38m} = 27$$

or, $$m = 8.673$$

$$n \sqrt{\frac{hP}{k A_{cs}}} = 8.673$$

or, $$\frac{hP}{k A_{cs}} = 75.22$$

$$\frac{h \times [0.018 + 0.005) \times 2]}{205 \times (0.018 \times 0.005)} = 75.22$$

or,

$$h = \frac{75.22 \times 205 \times (0.018 \times 0.005)}{(0.018 + 0.005) \times 2} = 30.17 \text{ W/m}^2\text{°C} \quad \text{(Ans.)}$$

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Example *A very long 25 mm diameter copper rod ($k = 380$ W/m°C) extends horizontally from a plane heated wall at 120°C. The temperature of the surrounding air is 25°C and the convective heat transfer coefficient is 9.0W/m²°C.*

(i) Determine the heat loss;

(ii) How long the rod be in order to be considered infinite?

Solution. $d = 25$ mm $= 0.025$ m; $k = 380$ W/m°C; $t_0 = 120$°C; $t_a = 25$°C; $h = 9.0$ W/m²°C.

(i) Heat loss:

Heat dissipation from an *infinitely long fin* is given by

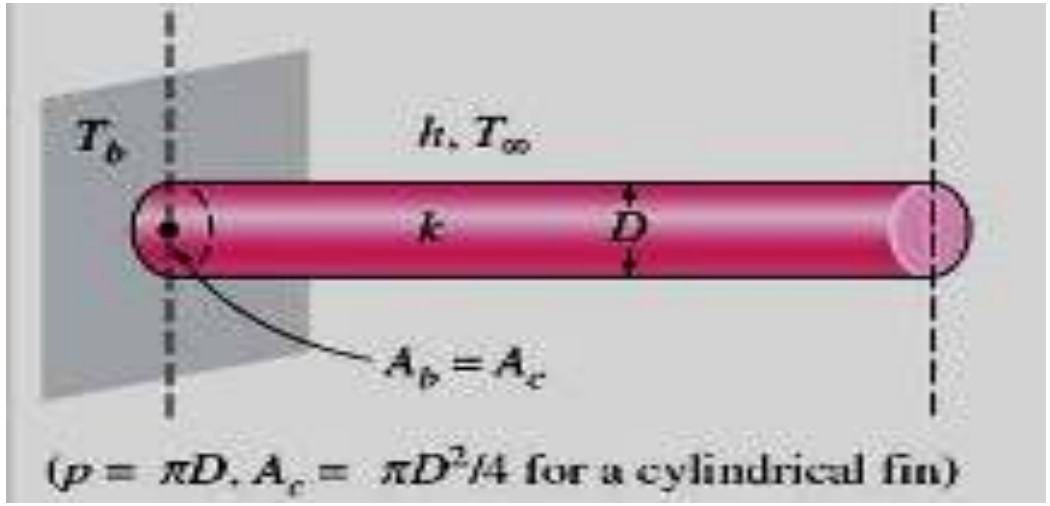
$$Q_{fin} = k A_{cs} m (t_o - t_a)$$

where,

$$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h \times \pi d}{k \times \frac{\pi}{4} d^2}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 9.0}{380 \times 0.025}} = 1.946$$

$$\frac{\pi}{4} d^2 \times \sqrt{\frac{4h}{kd}} (t_o - t_a)$$

$$\frac{\pi}{4} \times 0.025^2 \times 1.946 (120 - 25) = \mathbf{34.48\ W} \quad \textbf{(Ans.)}$$



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(ii) Length of the rod (to be considered infinite) :

Since there is no heat loss from the tip of an infinitely long rod and as such it behaves as if the tip *were insulated*; therefore, an estimate of the validity of this approximation can be made by comparing the following two expressions for the fin :

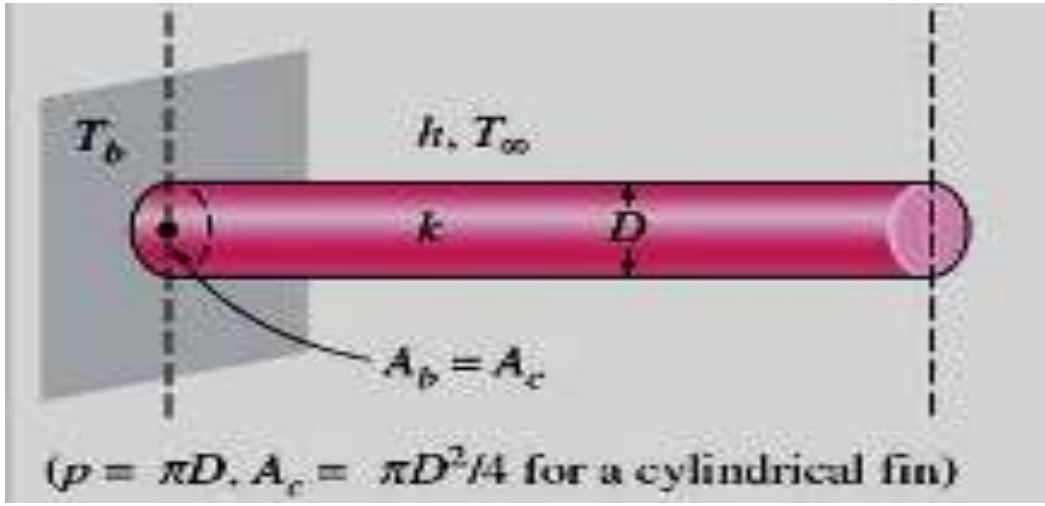
$$Q_{fin} = kA_{cs}\, m\, (t_o - t_a) \quad \text{... infinitely long fin;}$$

$$Q_{fin} = k_{Acs}\, m\, (t_o - t_a) \tanh (ml) \quad \text{... fin with tip insulated.}$$

Equivalent results are obtained if $\tanh (ml) \geq 0.99$ or $ml \geq 2.646$.

Hence the rod can be considered infinite if

$$l \geq \frac{2.646}{m} = \frac{2.646}{1.946} = \mathbf{1.36\,m} \qquad \textbf{(Ans.)}$$

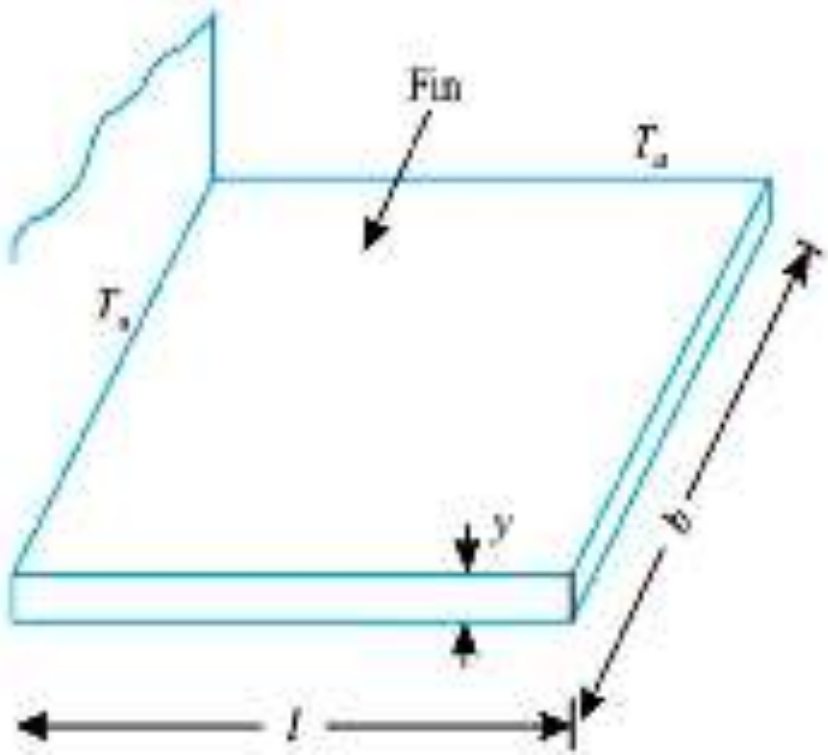


($p = \pi D$, $A_c = \pi D^2/4$ for a cylindrical fin)

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Example. *Find out the amount of heat transferred through an iron fin of length 50 mm, width 100 mm and thickness 5 mm. Assume k = 210 kJ/mh°C and h = 42 kJ/m²h°C for the material of the fin and the temperature at the base of the fin as 80°C. Also determine the temperature at tip of the fin, if the atmosphere temperature is 20°C.*

Solution. Given : l = 50 mm = 0.05 m; b = 100 mm = 0.1 m; y = 5 mm = 0.005 m; k = 210 kJ/mh°C; h = 42 kJ/m²h°C; t_0 = 80°C; t_a = 20°C.



Amount of heat transferred through the fin, Q :

Perimeter, $P = 2(b + y) = 2(0.1 + 0.005) = 0.21$ m

Area, $A_{cs} = b \times y = 0.1 \times 0.005 = 0.0005$ m²

$$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{42 \times 0.21}{210 \times 0.0005}} = 9.165$$

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Amount of heat transferred through the fin, Q :

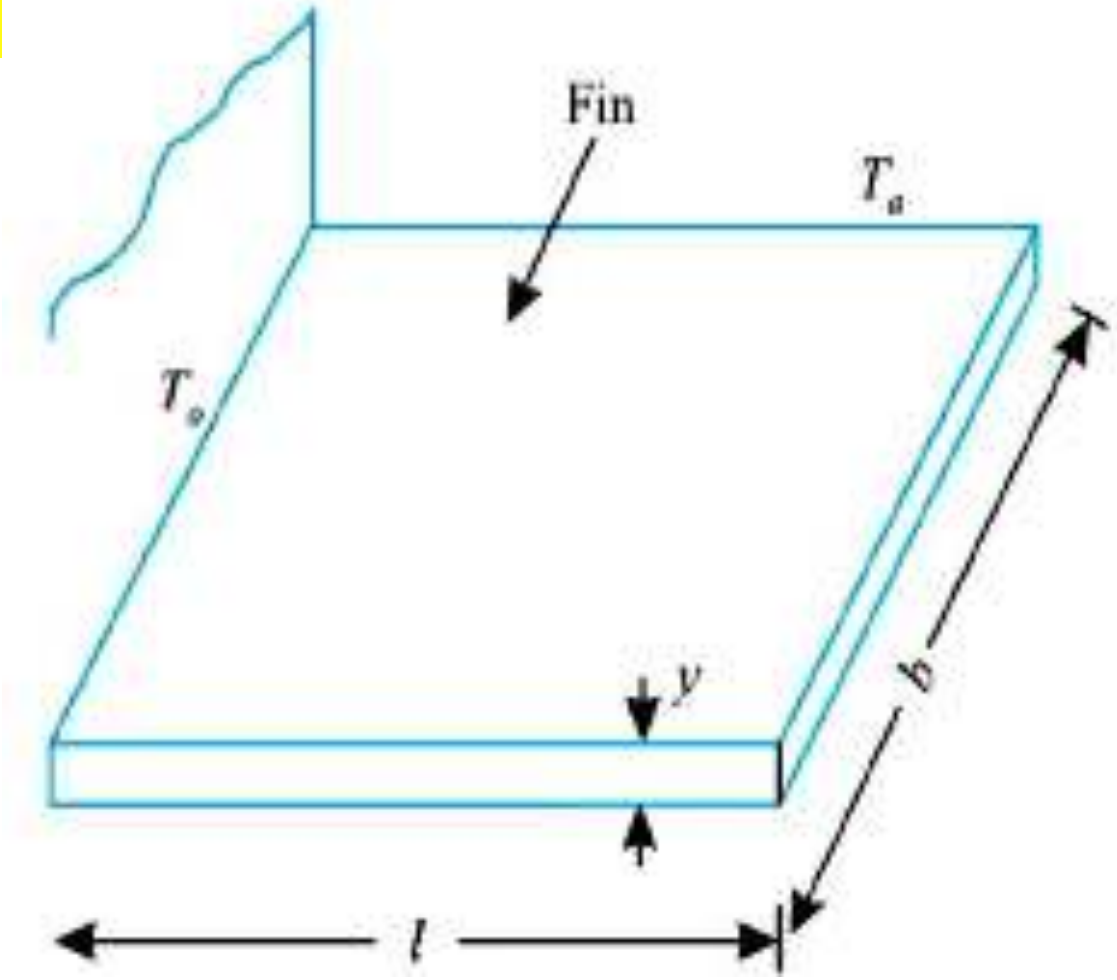
$$\therefore \quad Q_{fin} = \sqrt{PhkA_{cs}}\,(t_0 - t_a)\tanh(ml)$$

$$= \sqrt{0.21 \times 42 \times 210 \times 0.0005}$$

$$\times (80 - 20)\tanh(9.165 \times 0.05)$$

$$= 0.9263 \times 60 \times 0.4286$$

$$= \mathbf{24.75\ kJ/h\ (Ans.)}$$



Temperature at the tip of the fin, Q_L :

We know that,

$$\frac{\theta}{\theta_0} = \frac{t - t_a}{t_0 - t_a} = \frac{\cosh\{m(l - x)\}}{\cosh(ml)}$$

ave

$$\frac{t - 20}{80 - 20} = \frac{1}{\cosh\ ml}$$

or,

$$t = \frac{60}{\cosh(9.165 \times 0.05)} + 20 = \mathbf{74.21°C\ (Ans.)}$$

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Example *A carbon steel (k = 54 W/m°C) rod with a cross-section of an equilateral triangle (each side 5 mm) is 80 mm long. It is attached to a plane wall which is maintained at a temperature of 400°C. The surrounding environment is at 50°C and unit surface conductance is 90 W/m²°C. Compute the heat dissipated by the rod.*

Solution. $a = 5$ mm $= 0.005$ m; $l = 80$ mm $= 0.08$ m; $t_o = 400$°C; $t_a = 50$°C; $h = 90$ W/m²°C; $k = 54$ W/m°C.

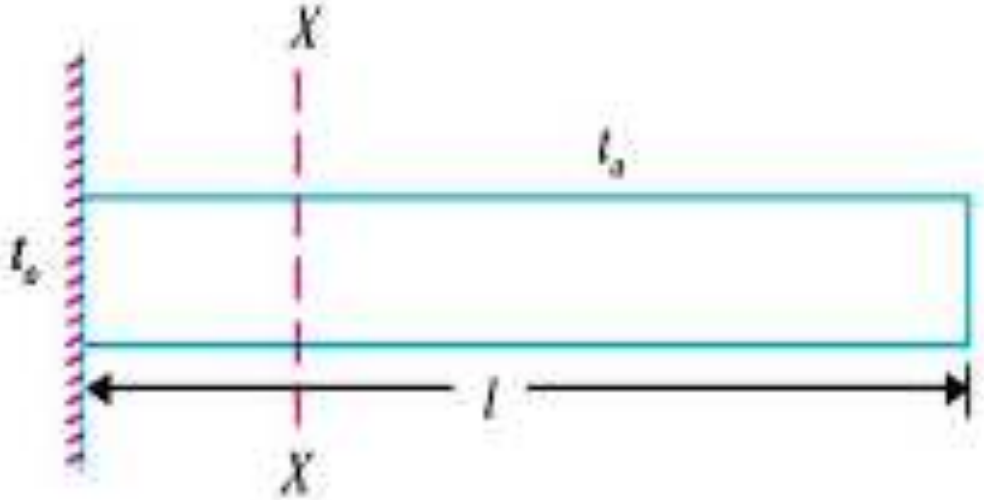
Heat dissipated by the rod, Q :

The heat flow from the rod (considering tip of the fin to be *insulated*) is given by

$$Q = kA_{cs}\, m\, (t_o - t_a) \tanh (ml)$$

where,

$$P = 3a, A_{cs} = \frac{1}{2} \times a \times \left(\frac{\sqrt{3}}{2} a\right) = \frac{\sqrt{3}}{4} a^2$$





Section at XX

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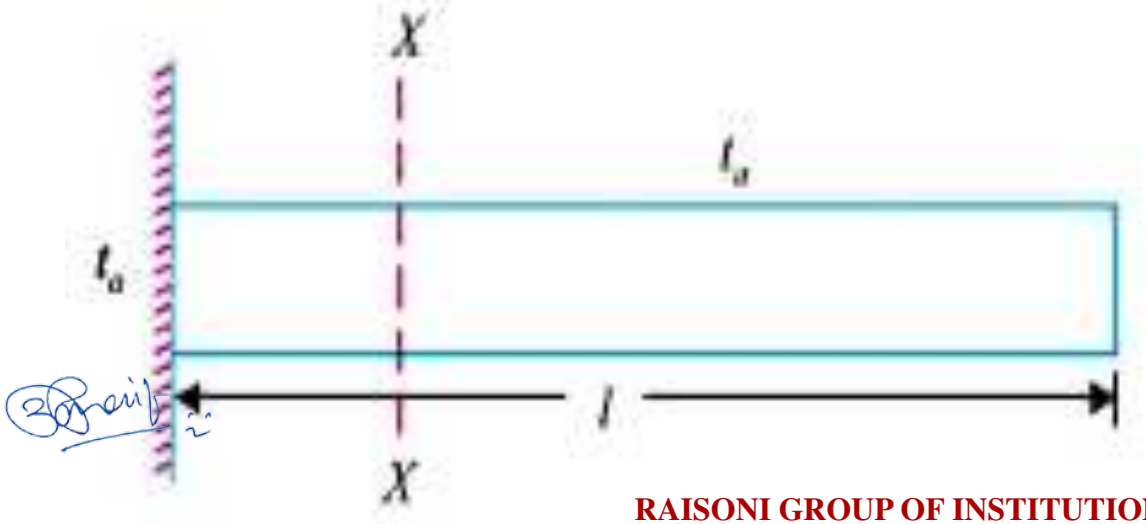
$$m = \sqrt{\frac{hP}{kA_{cr}}}, \quad P = 3a, \quad A_{cr} = \frac{1}{2} \times a \times \left(\frac{\sqrt{3}}{2}a\right) = \frac{\sqrt{3}}{4}a^2$$

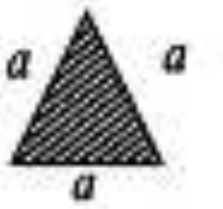
and,

$$\therefore \quad \frac{P}{A_{cr}} = \frac{3a}{\frac{\sqrt{3}}{4} \times a^2} = \frac{4\sqrt{3}}{a} = \frac{6.93}{0.005} = 1386$$

Substituting the values in the above equation, we have

$$Q = 54 \times \left[\frac{\sqrt{3}}{4} \times (0.005)^2\right] \times 48.06 \times (400 - 50) \tanh(48.06 \times 0.08) = \mathbf{9.82\ W} \quad \textbf{(Ans.)}$$





Section at *XX*

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Example *Pin fins are provided to increase the heat transfer rate from a hot surface. Which of the following arrangement will give higher heat transfer rate ?*

(i) 6 – fins of 10 cm length.

(ii) 12 – fins of 5 cm length.

Take k (fins material) = 200 W/m°C, h = 20 W/m²°C

Cross-sectional area of fin = 2 cm²

Perimeter of fin = 4 cm

Fin base temperature = 230°C

Surrounding air temperature = 30°C

Solution. *Given* : $A_{cs} = 2 \times 10^{-4}$ m²; $P = 0.04$ m, $t_o = 230°C$; $t_a = 30°C$; $k = 200$ W/m°C; $h = 20$ W/m²°C.

The heat flow through 'n' number of fins is given by

where,

$$[k A_{cs} m (t_0 - t_a) \tanh (ml)]$$

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(i) *Case I.* $n = 6$ and $l = 10\text{ cm} = 0.1\text{ m}$

$ml = 4.47 \times 0.1 = 0.447$

$\therefore \quad Q_1 = 6\,[200 \times 2 \times 10^{-4} \times 4.47 \times (230 - 30) \tanh (0.447)] = 89.99\text{ W}$

(ii) *Case II.* $n = 12$ and $l = 5\text{ cm} = 0.05\text{ m}$ $ml = 4.47 \times 0.05 = 0.2235$

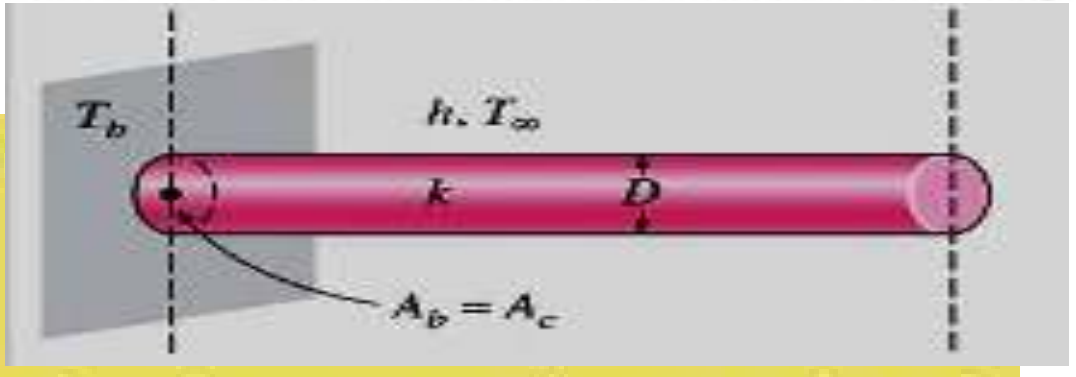
$\therefore \quad Q_2 = 12\,[200 \times 2 \times 10^{-4} \times 4.47\,(230 - 30) \tanh (0.2235)] = 94.34\text{ W}$

This shows that the rate of heat transfer is higher in second case, therefore, this arrangement

(Case II) is better.

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Example *One end of a long rod, 35 mm in diameter, is inserted into a furnace with the other end projecting in the outside air. After the steady state is reached, the temperature of the rod is measured at two points 180 mm apart and found to be 180°C and 145°C. The atmospheric air temperature is 25°C. If the heat transfer coefficient is 65 W/m² °C, calculate the thermal conductivity of the rod.*



Solution. Diameter of the rod, $d = 35$ mm $= 0.035$ m

The atmospheric air temperature, $t_a = 25°C$

Heat transfer coefficient; $h = 65$ W/m²°C

The starting point $x = 0$ is considered at the first point where the temperature is measured; $x = l$ is considered at the outer point. *Assume* that the *end of the fin is insulated.*

For *insulated end*, we have

$$\frac{t - t_a}{t_a} = \frac{\cosh\{m(l - x)\}}{\cosh ml}$$

At $x = l$, this equation, reduces to

$$\frac{\theta_l}{\theta_o} = \frac{1}{\cosh ml}$$

Here,

$\theta_l = 145 - 25 = 120°C$

$\theta_o = t_o - t_a = 180 - 25 = 155°C$

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$$\frac{\theta_l}{\theta_o} = \frac{1}{\cosh ml}$$

or,

$$\frac{120}{155} = \frac{1}{\cosh ml}$$

or,

$$\cosh ml = \frac{155}{120} = 1.292$$

or,

$$ml = 0.747$$

or,

$$m = \frac{0.747}{l}$$

$$m = \sqrt{\frac{hP}{kA_{cs}}}$$

$$\sqrt{\frac{hP}{kA_{cs}}} = \frac{0.747}{l}$$

$$\therefore \quad \frac{hP}{kA_{cs}} = \frac{0.747^2}{l^2}$$

or,

$$\frac{h}{k} \cdot \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{0.747^2}{l^2} \text{ or } \frac{h}{k} \times \frac{4}{d} = \frac{0.558}{l^2}$$

$$k = \frac{4hl^2}{0.558d} = \frac{4 \times 65 \times 0.18^2}{0.558 \times 0.035} = \mathbf{431.34\ W/m°C} \qquad \textbf{(Ans.)}$$

[where, l = 180 mm = 0.18 m (*given*)]

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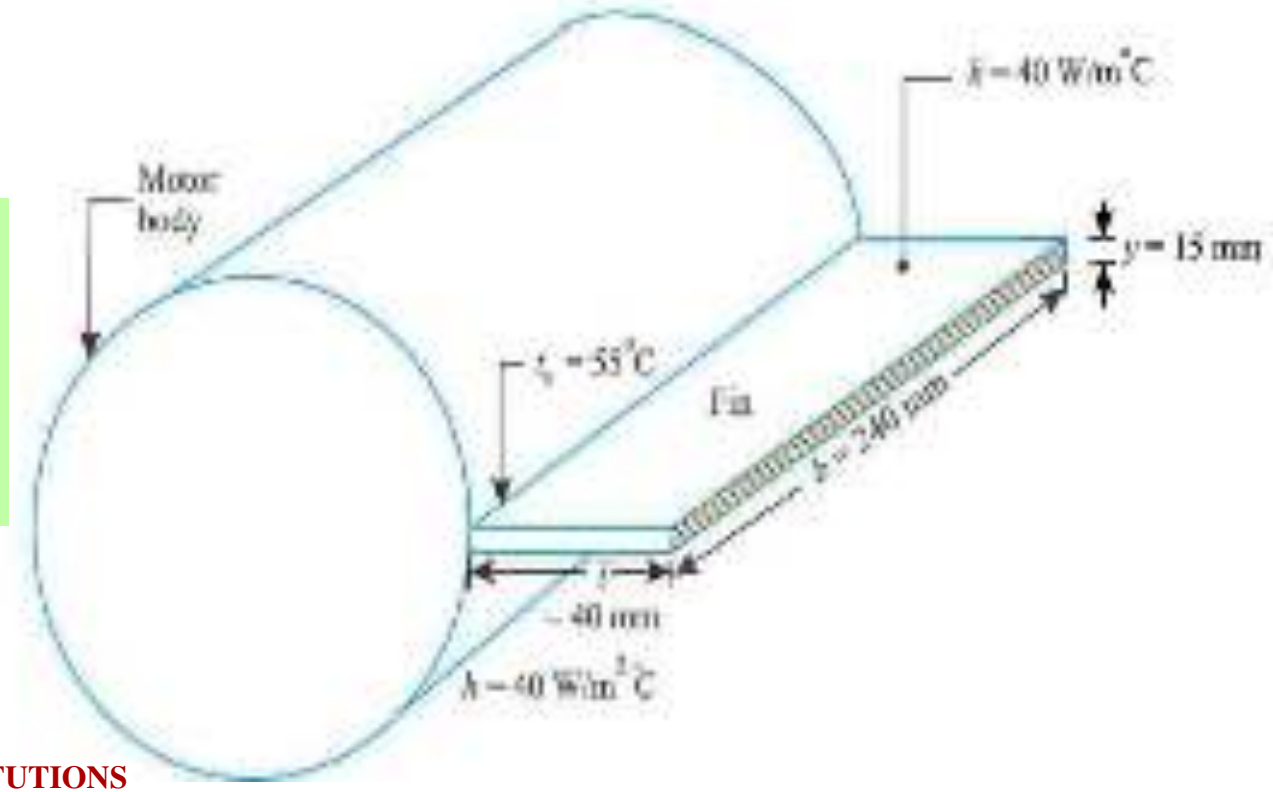
Example *A motor body is 360 mm in diameter (outside) and 240 mm long. Its surface temperature should not exceed 55°C when dissipating 340 W. Longitudinal fins of 15 mm thickness and 40 mm height are proposed. The convection coefficient is 40 W/m²°C. Determine the number of fins required. Atmospheric temperature is 30°C. Thermal conductivity = 40 W/m°C.*

Solution. Refer to Fig. 2.133, $l = 40$ mm $= 0.04$ m; $b = 240$ mm $= 0.24$ m; $y = 15$ mm $= 0.015$ m; $k = 40$ W/m°C; $h = 40$ W/m²°C, $t_0 = 55$°C; $t_a = 30$°C; $Q_{total} = 340$ W.

The given problem is a case of heat dissipation from a fin losing heat at the tip (*short fin situation*).

The rate of heat transfer in such a case is given by

$$Q_{fin} = \sqrt{P h k A_{cs}}\,(t_o - t_a)\left[\frac{\tanh(ml) + \dfrac{h}{km}}{1 + \dfrac{h}{km} \times \tanh(ml)}\right]$$

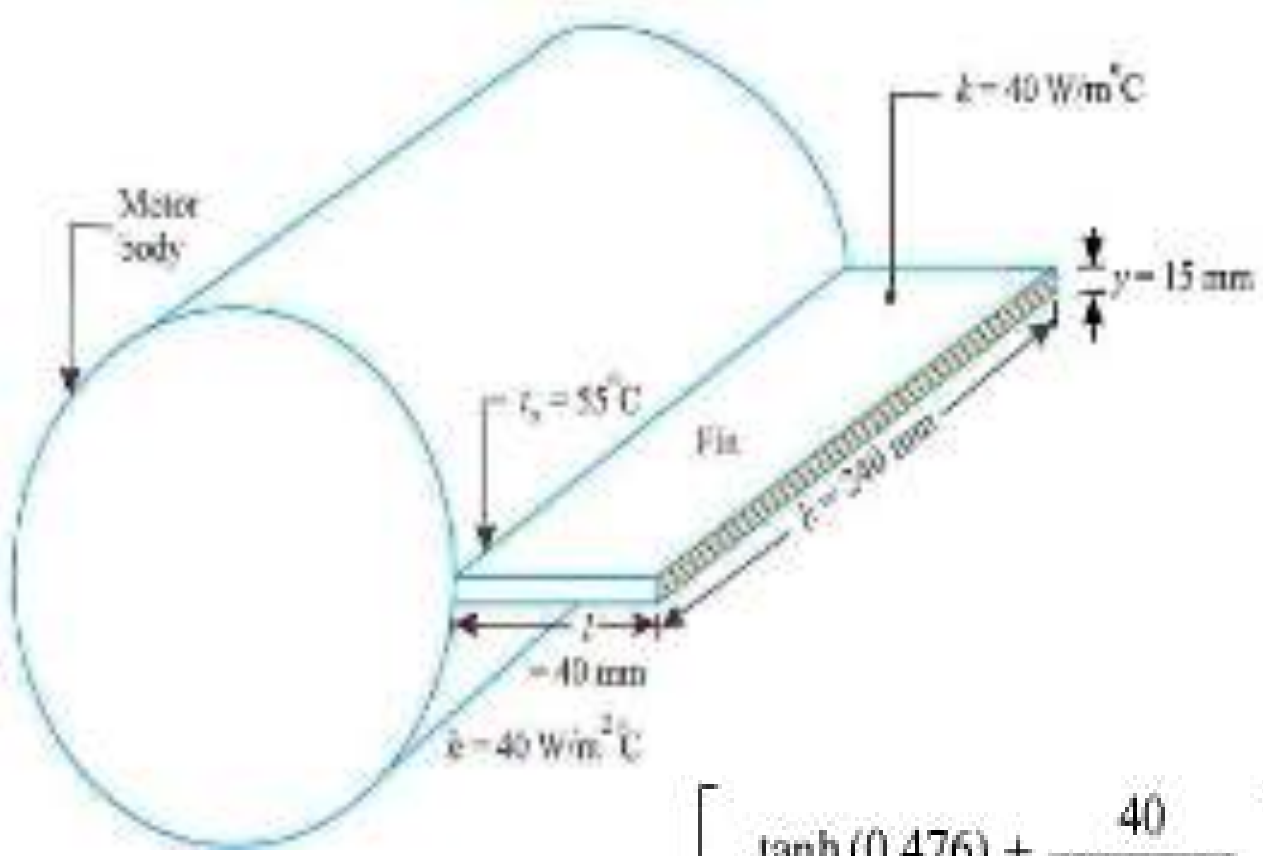


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where,

$$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h(b+y)\times 2}{k(b\times y)}} = \sqrt{\frac{40\times(0.24+0.015)\times 2}{40\times(0.24\times 0.015)}} = 11.9$$

and, $ml = 11.9 \times 0.04 = 0.476$



$$Q_{fin} = \sqrt{P\,h\,k\,A_{cs}}\,(t_o - t_a)\left[\frac{\tanh(ml)+\frac{h}{km}}{1+\frac{h}{km}\times\tanh(ml)}\right]$$

$$Q = [(0.24 + 0.015) \times 2] \times 40 \times 40 \times (0.24 \times 0.015) \times (55 - 30) \times \left[\frac{\tanh(0.476)+\frac{40}{40\times 11.9}}{1+\frac{40}{40\times 11.9}\times\tanh(0.476)}\right]$$

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$$= 1.714 \times 25 \times \left[\frac{0.443 + 0.084}{1 + 0.084 \times 0.443} \right] = 21.77 \text{ W}$$

$$\therefore \quad \text{Number of fins required} = \frac{Q_{total}}{Q_{fin}} = \frac{340}{21.77} = \textbf{16 fins} \quad \textbf{(Ans.)}$$

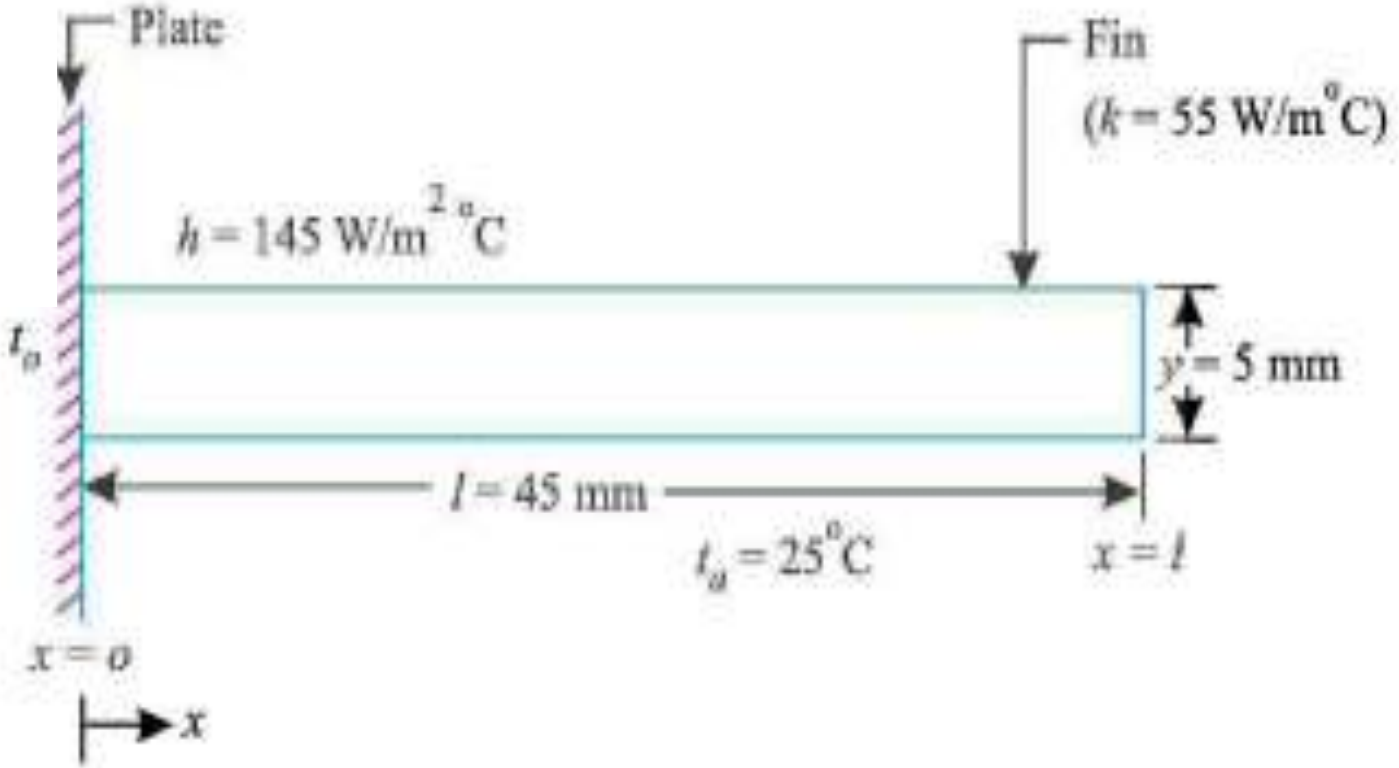
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Example *A fin 5 mm thick and 45 mm long has its base on a plane plate which is maintained at 125°C. The ambient temperature is 25°C. The conductivity of fin material is 55 W/m°C and the heat transfer coefficient is 145 W/m²°C. Determine :*

(i) Temperature at the end of the fin,

(ii) Temperature at the middle of the fin, and

(iii) Heat dissipated by the fin (per metre width).



Solution.

mm = 0.045 m; b = 1m; y = 5mm = 0.005m; k = 55 W/m°C; h = 145 W/m²°C; t_o = 125°C;

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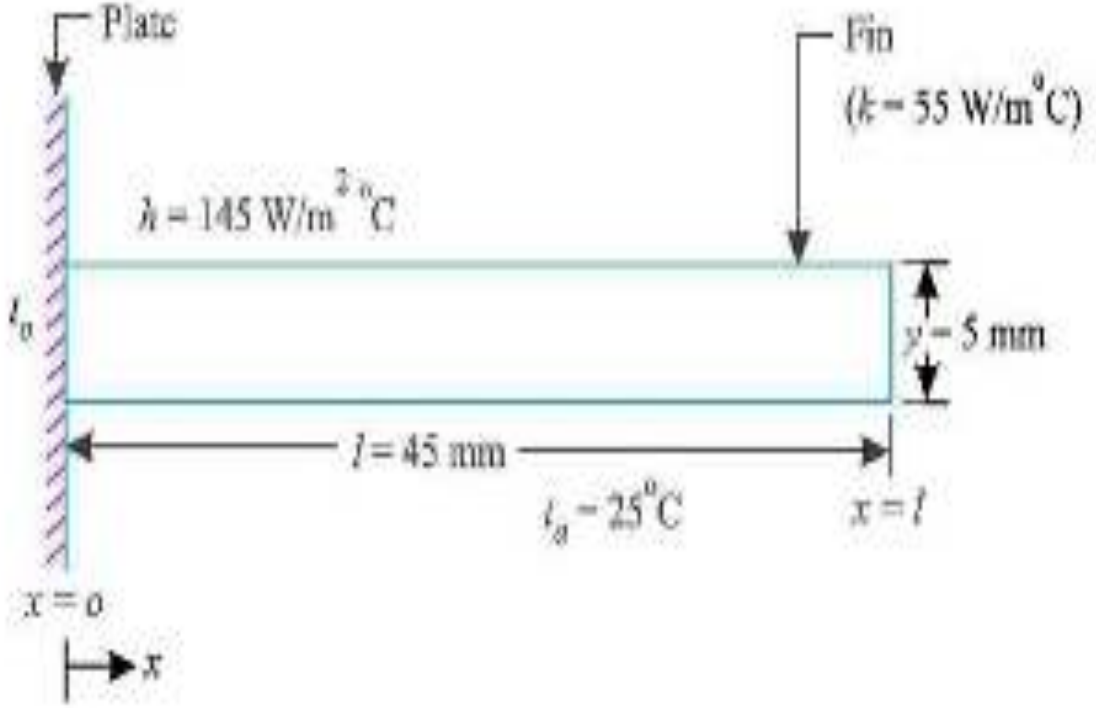
(i) Temperature at the end of the fin, t_l :

Assuming heat loss by convection from the end of the fin; under this condition, temperature at the end of the fin is given by

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \left[\frac{\cosh\{m(l-x)\} + \frac{h}{km}[\sinh\{m(l-x)\}}{\cosh(ml) + \frac{h}{km}[\sinh(ml)]} \right]_{x=l}$$

or,

$$\frac{t_l - t_a}{t_o - t_a} = \frac{1}{\cosh(ml) + \frac{h}{km}[\sinh(ml)]}$$



where,

$$\sqrt{\frac{hP}{A_{cs}}} = \sqrt{\frac{h \times (2b + 2y)}{k \times (b \times y)}} = \sqrt{\frac{h}{k} \times \frac{2}{y}} = \sqrt{\frac{145}{55} \times \frac{2}{0.005}} = 32.47$$

$(\because 2y << 2b)$

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or,

$$ml = 32.47 \times 0.045 = 1.461$$

$$\therefore \quad \frac{t_l - 25}{125 - 25} = \frac{1}{\cosh(1.461) + \frac{145}{55 \times 32.47}[\sinh(1.461)]} = 0.41$$

or,

$$t_l = 25 + 0.41\ (125 - 25) = \mathbf{66^\circ C} \quad \textbf{(Ans.)}$$

(ii) Temperature at the middle of the fin, $t_{l/2}$:

$$\frac{\theta}{\theta_0} = \frac{t - t_a}{t_0 - t_a} = \left[\frac{\cosh\{m(l - x)\} + \frac{h}{km}[\sinh\{m(l - x)\}]}{\cosh(ml) + \frac{h}{km}\{\sinh(ml)\}}\right]_{x = \frac{l}{2}}$$

or,

$$\frac{t_{l/2} - t_a}{t_0 - t_a} = \left[\frac{\cosh\left(\frac{ml}{2}\right) + \frac{h}{km}\left\{\sinh\left(\frac{ml}{2}\right)\right\}}{\cosh(ml) + \frac{h}{km}\{\sinh(ml)\}}\right]$$

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$$\text{or} \quad \frac{t_{l/2} - 25}{125 - 25} = \left[\frac{\cosh(0.7305) + \frac{145}{55 \times 32.47}\{\sinh(0.7305)\}}{\cosh(1.461) + \frac{145}{55 \times 32.47}\{\sinh(1.461)\}} \right]$$

$$= \left[\frac{1.2789 + 0.0647}{2.2711 + 0.1655} \right] = 0.5514$$

$$t_{l/2} = 25 + 0.5514\,(125 - 25) = \mathbf{80.14^{\circ}C\ (Ans.)}$$

(iii) Total heat dissipated by the fin, Q_{fin} (per metre width) :

$$Q_{fin} = \sqrt{PhkA_{cs}}\,(t_o - t_a)\left[\frac{\tanh(ml) + \frac{h}{km}}{1 + \frac{h}{km} \cdot \tanh(ml)} \right]$$

$$= \sqrt{[(b + y) \times 2] \times hk \times (b \times y)}\,(t_o - t_a)\left[\frac{\tanh(ml) + \frac{h}{km}}{1 + \frac{h}{km} \cdot \tanh(ml)} \right]$$

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$$= \sqrt{(1 + 0.005) \times 2 \times 145 \times 55 \times (1 \times 0.005)}\ (125 - 25) \left[\frac{\tanh(1.461) + \frac{145}{55 \times 32.47}}{1 + \frac{145}{55 \times 32.47} \times \tanh(1.461)} \right]$$

or,

$$Q_{fin} = 8.9526 \times 100 \times \left[\frac{0.8978 + 0.0812}{1 + 0.0729} \right] = \mathbf{816.9\ W} \quad \textbf{(Ans.)}$$

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Numerical on

Efficiency and effectiveness of fin

Efficiency of fin (η_{fin}) :

The efficiency of a fin is defined as the ratio of the actual heat transferred by the fin to the maximum heat transferable by fin, if entire fin area were at base temperature.

Effectiveness of fin (ε_{fin}) :

Effectiveness of fin is the ratio of the fin heat transfer rate to the heat transfer rate that would

ut a fin.

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Example *A longitudinal copper fin ($k = 380$ W/m°C) 600 mm long and 5 mm diameter is exposed to air stream at 20°C. The convective heat transfer coefficient is 20W/m²°C. If the fin base temperature is 150°C, determine :*

(i) The heat transferred, and

(ii) The efficiency of the fin.

Solution. Length of the fin, $l = 600$ mm $= 0.6$ m

Diameter of the fin, $d = 5$ mm $= 0.005$ m

The fin base temperature, $t_0 = 150°C$

Air stream temperature, $t_a = 20°C$

Thermal conductivity of fin material, $k = 380$ W/

:ctive heat transfer coefficient $h = 20$ W/m²°C

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(i) The heat transferred, Q :

Neglecting the heat loss from the end surface, the heat transfer from the fin is given by

$$Q = kA_{cs}\, m\, (t_o - t_a) \tanh (ml)$$

where

$$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h}{2} \times \frac{\pi d}{\frac{\pi}{4} d^2}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 20}{380 \times 0.005}} = 6.49$$

$\therefore$

$$Q = 380 \times \left(\frac{\pi}{4} \times 0.005^2 \right) \times 6.49 \times (150 - 20) \tanh (6.49 \times 0.6) = 6.29 \text{ W}$$

$3600 = 22644$ J/h or **22.644 kJ/h** (Ans.)

(ii) The efficiency of the fin, η_{fin} :

For a fin which is insulated at the tip is given by

$$\eta_{fin} = \frac{\tanh (ml)}{ml}$$

$$= \frac{\tanh (6.49 \times 0.6)}{(6.49 \times 0.6)} = 0.2566 \text{ or } \mathbf{25.66\%} \quad \text{(Ans.)}$$

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Example *A steel rod (k = 32 W/m°C), 12 mm in diameter and 60 mm long, with an insulated end, is to be used as a spine. It is exposed to surroundings with a temperature of 60°C and a heat transfer coefficient of 55 W/m²°C. The temperature at the base of fin is 95°C. Determine :*

(i) The fin efficiency;

(ii) The temperature at the edge of the spine;

(iii) The heat dissipation.

Solution. *Given* : d = 12 mm = 0.012 m; l = 60 mm = 0.06 m; t_a = 60°C; (k = 32 W/m°C); h = 55 W/m²°C; t_o = 95°C.

(i) The fin efficiency, η_{fin} :

$$\eta_{fin} = \frac{\tanh (ml)}{(ml)}$$

where,

$$m = \sqrt{\frac{hP}{kA_{cr}}} = \sqrt{\frac{h \times \pi d}{k \times \frac{\pi}{4} d^2}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 55}{32 \times 0.012}} = 23.93$$

$$\therefore \quad \eta_{fin} = \frac{\tanh (23.93 \times 0.06)}{(23.93 \times 0.06)} = 0.6218 \text{ or } \mathbf{62.18\%} \quad \textbf{(Ans.)}$$

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(ii) The temperature at the edge of the spine, t_l :

$$\frac{\theta}{\theta_0} = \frac{t - t_a}{t_0 - t_a} = \frac{\cosh m(l - x)}{\cosh ml}$$

At $x = l$, the above equation reduces to

$$\frac{\theta_l}{\theta_0} = \frac{t_l - t_a}{t_0 - t_a} = \frac{1}{\cosh ml}$$

or,

$$\frac{t_l - 60}{95 - 60} = \frac{1}{\cosh (23.93 \times 0.06)} = 0.45$$

$\therefore$

$$t_l = 60 + (95 - 60) \times 0.45 = 75.75°C \quad \text{(Ans.)}$$

(iii) The heat dissipation, Q_{fin} :

$$Q_{fin} = k_{Acs}\, m\, (t_o - t_a) \tanh (ml)$$

$$= 32 \times \frac{\pi}{4} \times (0.012)^2 \times 23.93(95 - 60) \tanh (23.93 \times 0.06) = 2.7\ W$$

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Estimation of Error in Temperature Measurement in a Thermometer Well

For estimating error in the value of temperature measured by a thermometer dipped in a thermometer well, the theory of extended surfaces is very helpful. A *thermometer well* is defined as a *small tube welded radially into a pipeline through which a fluid whose temperature is to be measured is flowing.*

Let, l = Length of the well/tube,

d = Internal diameter of the well/tube,

δ = Thickness of well/tube,

t_f = Temperature of the fluid flowing through the pipe (which is to be measured), and

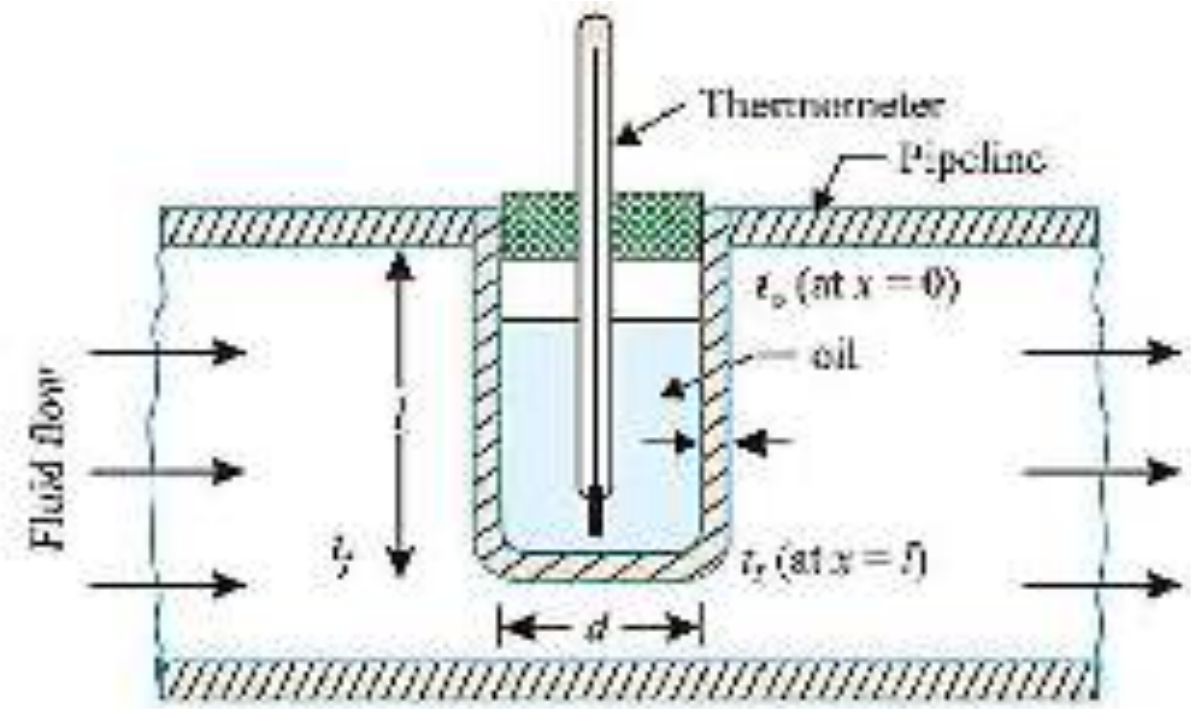
t_o = Temperature of the pipe-wall.

When the temperature of the fluid flowing through the pipeline is higher than the ambient temperature, the heat flows from the fluid towards the tube walls along the well. Consequently the temperature at the bottom of well becomes colder than the fluid flowing around, obviously the

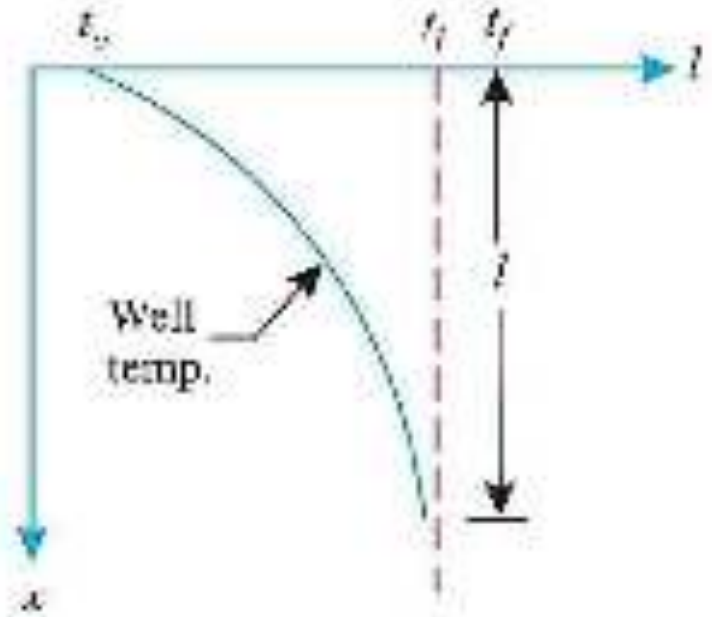
ure shown by the thermometer will not be the true temperature of the fluid. This error may be
d by assuming the well to be a spine protruding from the wall of a pipe in which fluid is
It may be assumed, for simplicity, that there is no flow of heat from the tip of the well (*i.e.*,
f the well is *insulated*). The temperature distribution at any distance x measured from pipe
g the temperature well is given by

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Estimation of Error in Temperature Measurement in a Thermometer Well



Thermometer well.



Temperature variation in well.

$$\frac{t_x - t_f}{t_o - t_f} = \frac{\cosh[m(l-x)]}{\cosh(ml)}$$

At $x = l$, we have

$$\frac{t_l - t_f}{t_o - t_f} = \frac{\cosh[m(l-l)]}{\cosh(ml)} = \frac{1}{\cosh(ml)}$$

[Thermometeric error]

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Estimation of Error in Temperature Measurement in a Thermometer Well

(where, t_i = Temperature recorded by the thermometer at the bottom of the well.)

Now, perimeter of the well, $P = \pi (d + 2\delta) = \pi d$,

and cross-sectional area, $A_{cs} = \pi d \delta$

$$\therefore \quad \frac{P}{A_{cs}} = \frac{\pi d}{\pi d \delta} = \frac{1}{\delta}$$

Then, $$m = \sqrt{\frac{hP}{k A_{cs}}} = \sqrt{\frac{h}{k\delta}}$$

Thus, the temperature measured by the thermometer is *not* affected by the diameter of the well.

From the Eqn. it is obvious that in order to reduce the temperature measurement error, ml should be large necessitating the following :

(*i*) Large value of h (heat transfer coefficient).

(*ii*) Small value of of k (thermal conductivity).

(*iii*) Long and thin well, the pocket (protruding small tube) may be placed obliquely/inclined, if ssary, to provide a longer insertion of thermometer.

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Example *A mercury thermometer placed in oil well is required to measure temperature of compressed air flowing in a pipe. The well is 140 mm long and is made of steel ($k = 50$ W/m°C) of 1 mm thickness. The temperature recorded by the well is 100°C while pipe wall temperature is 50°C. Heat transfer coefficient between the air and well wall is 30 W/m²°C. Estimate true temperature of air.*

Solution. *Given :* $l = 140$ mm $= 0.14$ m, $\delta = 1$ mm $= 0.001$ m; $t_i = 100$°C; $t_o = 50$°C; $h = 30$ W/m²°C; $k = 50$ W/m°C.

True temperature of air (fluid) t_f :

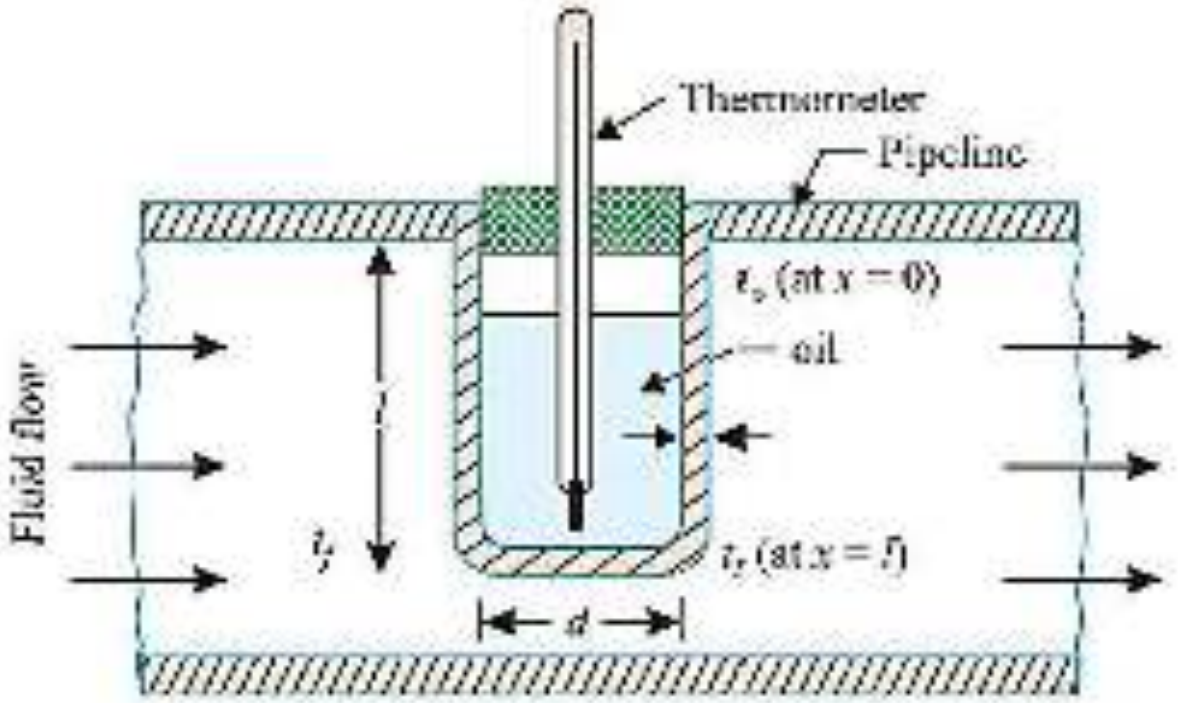
The value of t_f is found from the relation,

$$\frac{t_i - t_f}{t_o - t_f} = \frac{1}{\cosh(ml)}$$

where,

$$\sqrt{\frac{h}{k\delta}} = \sqrt{\frac{30}{50 \times 0.001}} = 24.5$$

$$24.5 \times 0.14 = 3.43$$



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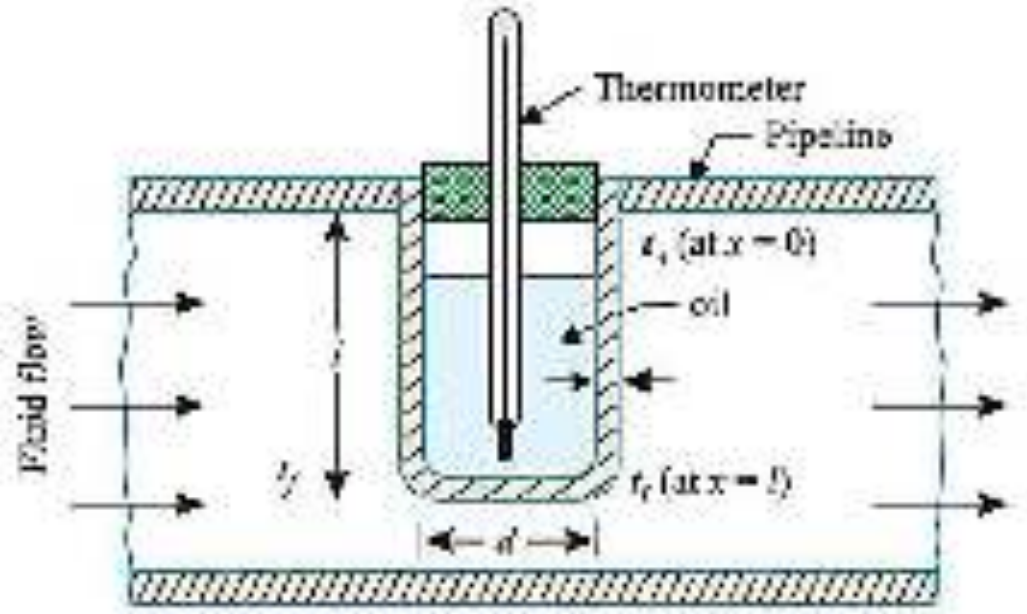
Substituting the values, we have

$$\frac{100 - t_f}{50 - t_f} = \frac{1}{\cosh(3.43)} = \frac{1}{15.4545} = 0.0647$$

or, $$(100 - t_f) = 0.0647\,(50 - t_f)$$

or, $$100 - t_f = 3.235 - 0.0647\,t_f$$

or, $$t_f = \mathbf{103.46°C\ (Ans.)}$$



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- **Unsteady state heat transfer,**
- **lumped heat capacity analysis,**
- **Heisler's charts.**
- **Biot Number, Fourier's Number & its significance.**



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Unsteady state heat transfer

What is Unsteady state heat transfer ?

Heat transfer is the **transfer** of **thermal** energy from a body, at a high temperature, to another at a lower temperature. ... Conversely, under **unsteady state** conditions the temperature within the system does vary with time. **Unsteady state** conditions are a precursor to steady **state** conditions.

Heat transfer is the transfer of thermal energy from a body, at a high temperature, to another at a lower temperature. This transfer of thermal energy may occur under steady or unsteady state conditions. Under Steady state conditions the temperature within the system does not change with time. Conversely, under unsteady state conditions the temperature within the system does vary with time.

Unsteady state conditions are a precursor to steady state conditions. No system exists initially under steady state conditions. Some time must pass, after heat transfer is initiated, before the system reaches steady state. During that period of transition the system is under unsteady state conditions.

Clearly, no system can remain under unsteady state conditions perpetually. The temperature of stem will eventually reach the temperature of the heat source, and once this happens, the n will be at steady state. Even if the amount of heat being transferred into the system is sed, at some point the system reaches its critical temperature and the energy transferred the starts causing phase changes within the system rather than temperatures increases.

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Practical Examples of Unsteady State Heat Transfer

1. Heat treatment of metals
2. Starting and shutting down of any Heat Transfer equipment like Lab Eqpt
3. Starting & shutting down of engines/motors
4. Starting & shutting down of Electric Furnace
5. Starting & shutting down of Electric heater

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Transient Heat Conduction (Lumped System Analysis)



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Lumped heat capacity analysis

Lumped system approximation provides a great convenience in **heat transfer analysis**. ... **Lumped system analysis** assumes a uniform **temperature** distribution throughout the body, which implies that the **conduction heat** resistance is zero. Thus, the **lumped system analysis** is exact when Bi = 0.

What is lumped system?

A **lumped**-capacitance model, also called **lumped system** analysis, reduces a thermal **system** to a number of discrete “lumps” and assumes that the temperature difference inside each lump is negligible. This approximation is useful to simplify otherwise complex differential heat equations.

lumped system in heat transfer practical examples..

Temperature of such bodies can be taken to be a function of time only. Heat transfer analysis that utilizes this idealization is known as **LUMPED SYSTEM ANALYSIS**. If a piece of meat when roasted you will see that the outer parts of roast are well done while the centre part is barely warm.

OR

इस तरह के निकायों का तापमान केवल समय के कार्य के लिए लिया जा सकता है। हीट ट्रांसफर विश्लेषण जो इस उपयोग करता है उसे LUMPED SYSTEM ANALYSIS के रूप में जाना जाता है। यदि भुना हुआ मांस टुकड़ा है, तो आप देखेंगे कि भूनने के बाहरी हिस्सों को अच्छी तरह से किया जाता है, जबकि केंद्र का शेकल से गर्म होता है।

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What is lumped system analysis when is it applicable?

The temperature in this process is considered to be uniform at a given time. Such an **analysis** is called **Lumped** parameter **analysis** because the whole solid, whose energy at any time is a function of its temperature and total heat capacity is treated as one lump. We can apply the **lumped** parameter **analysis** if $\beta i<1$.

Transient Heat Conduction

In general, temperature of a body varies with time as well as position.





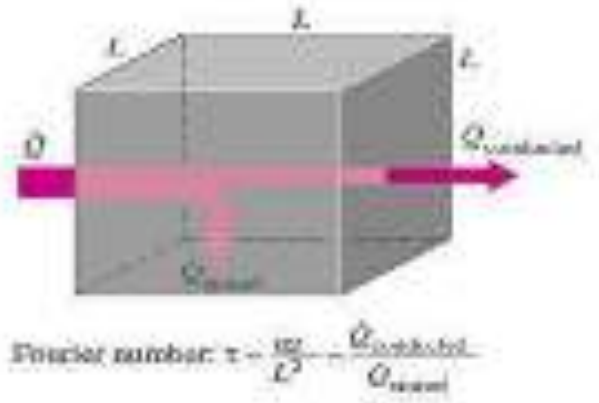
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Biot Number, Fourier's Number & its significance

Fourier Number indicates **the** degree of penetration of hotness or coldness into body. **Biot Number** is a ratio of internal conduction resistance to external convective resistance. If biots **number** is small it indicates that **the** whole body is at uniform temperature and lumped analysis can be applied.

What is Fourier number in heat transfer?

In physics and engineering, the **Fourier number** (Fo) or **Fourier** modulus, named after Joseph **Fourier**, is a dimensionless **number** that characterizes transient **heat conduction**.

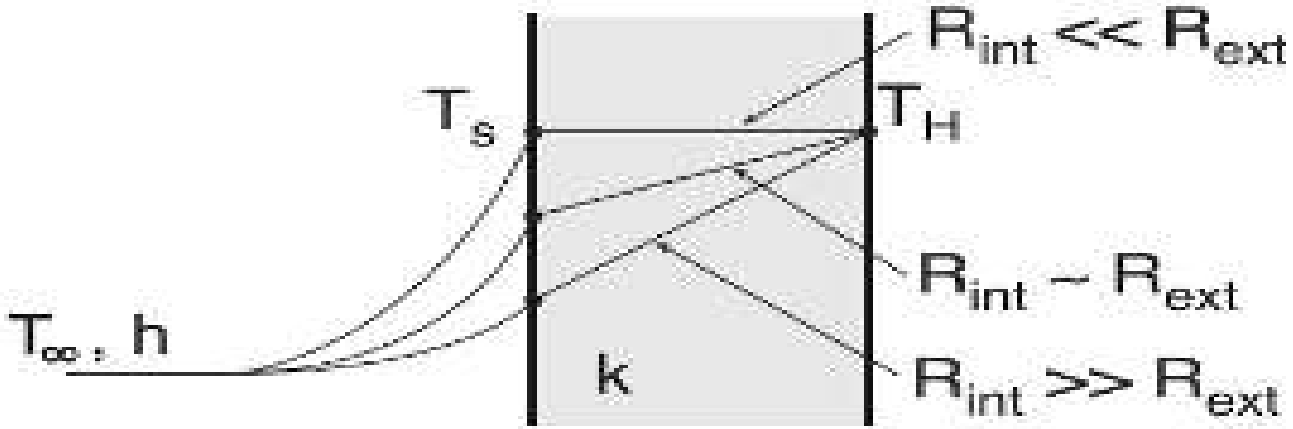


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Biot number

$$\frac{T_H - T_s}{T_s - T_\infty} = \frac{L/(k \cdot A)}{1/(h \cdot A)} = \frac{\text{internal resistance to H.T}}{\text{external resistance to H.T}} = \frac{hL}{k} = Bi \equiv \text{Biot number}$$

constant cross se



$R_{int} << R_{ext}$: the Biot number is small and we can conclude

$$T_H - T_s << T_s - T_\infty \qquad \text{and in the limit} \quad T_H \approx T_s$$

$R_{ext} << R_{int}$: the Biot number is large and we can conclude

$$T_s - T_\infty << T_H - T_s \qquad \text{and in the limit} \quad T_s \approx T_\infty$$

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Fourier number

$$Bi = \frac{hL}{k}$$

- In physics and engineering, the Fourier number (*Fo*) or Fourier modulus, named after Joseph Fourier, is a dimensionless number that characterizes heat conduction. Conceptually, it is the ratio of diffusive/conductive transport rate by the quantity storage rate and arises from non-dimensionalization of the heat equation. The general Fourier number is defined as:

Fo = Diffusive transport rate (α/L^2)/storage rate (1/t)

- The thermal Fourier number is defined by the conduction rate to the rate of thermal energy storage:

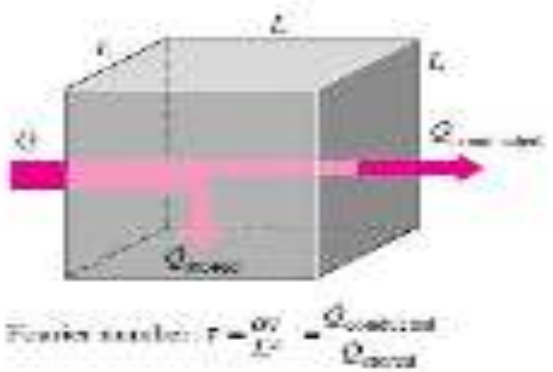
- Compare with non-dimensionless time parame

$$Fo = \alpha t / L^2$$

$$\tau = \frac{\alpha t}{L^2}$$

So Fo=τ

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau} \longrightarrow \frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial Fo}$$



To understand the physical significance of the *Fourier number* t, we may express it as

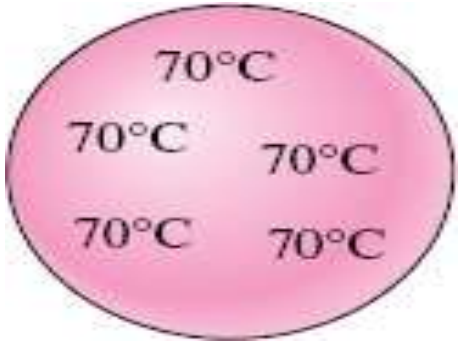
$$\tau = \frac{\alpha t}{L^2} = \frac{kL^2\,(1/L)}{\rho c_p L^3/t}\frac{\Delta T}{\Delta T} = \frac{\text{The rate at which heat is } \textit{conducted} \text{ across } L \text{ of a body of volume } L^3}{\text{The rate at which heat is } \textit{stored} \text{ in a body of volume } L^3}$$

ıin the Fourier number is a measure of *heat conducted* through a body relative to *heat stored.* Thus, a large value of the Fourier tes faster propagation of heat through a body.

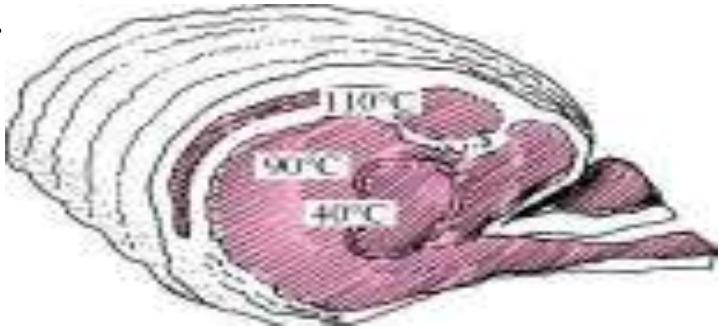
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Lumped System Analysis

- In heat transfer analysis, some bodies are observed to behave like a “lump” whose interior temperature remains essentially uniform at all times during a heat transfer process.
- The temperature of such bodies can be taken to be a function of time only, T(t).
- Heat transfer analysis that utilizes this idealization is known as lumped system analysis,
- Consider a small hot copper ball coming out of an oven. Measurements indicate that the temperature of the copper ball changes with time, but it does not change much with position at any given time.
- Now consider a large roast in an oven. You notice that the temperature distribution within the roast is not even close to being uniform. You will see that the outer of the roast are well done while the center part is barely warm. Thus, lumped system analysis is not applicable in this case.



Copper ball



Roast beef

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Lumped System Analysis

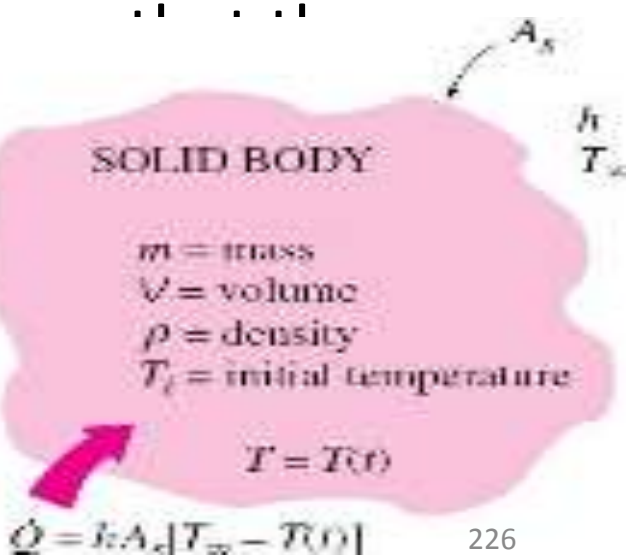
- Consider a body of arbitrary shape of mass *m*, volume *V*, surface
- area *As*, density ρ, and specific heat C_p initially at a uniform temperature T_i at time $t = 0$.
- The body is placed into a medium at temperature T_∞, and heat
- transfer takes place between the body and its environment, with a heat transfer coefficient *h*. Assume that $T_\infty > T_i$.
- We assume lumped system analysis to be applicabl
- temperature remains uniform within the body and only, $T = T(t)$.

$$\left(\begin{array}{c}\text{transfer into the body}\\ \text{during } dt\end{array}\right) = \left(\begin{array}{c}\text{The increase in the}\\ \text{energy of the body}\\ \text{during } dt\end{array}\right)$$

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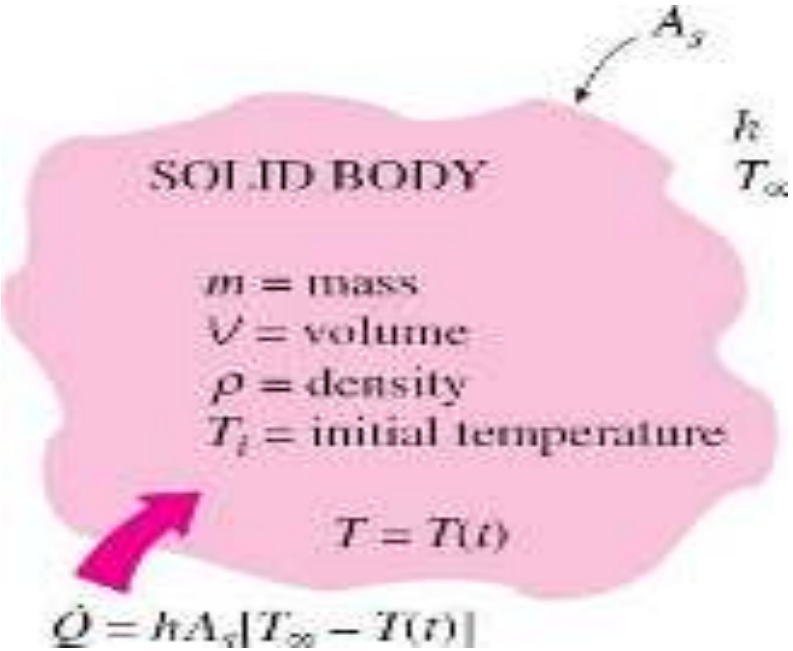


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$$hA_s(T_\infty - T)\,dt = mc_p\,dT$$

$$m = \rho V \text{ and } dT = d(T - T_\infty) \text{ since } T_\infty = \text{const}$$

$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V c_p}\,dt$$



Integrating from t = 0, at which $T = T_i$, to any time t, at which $T = T(t)$, gives

$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p}\,t$$

Taking the exponential of both sides and rearranging, we obtain

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

Where $b = \dfrac{hA_s}{\rho V c_p}$ (1/s)



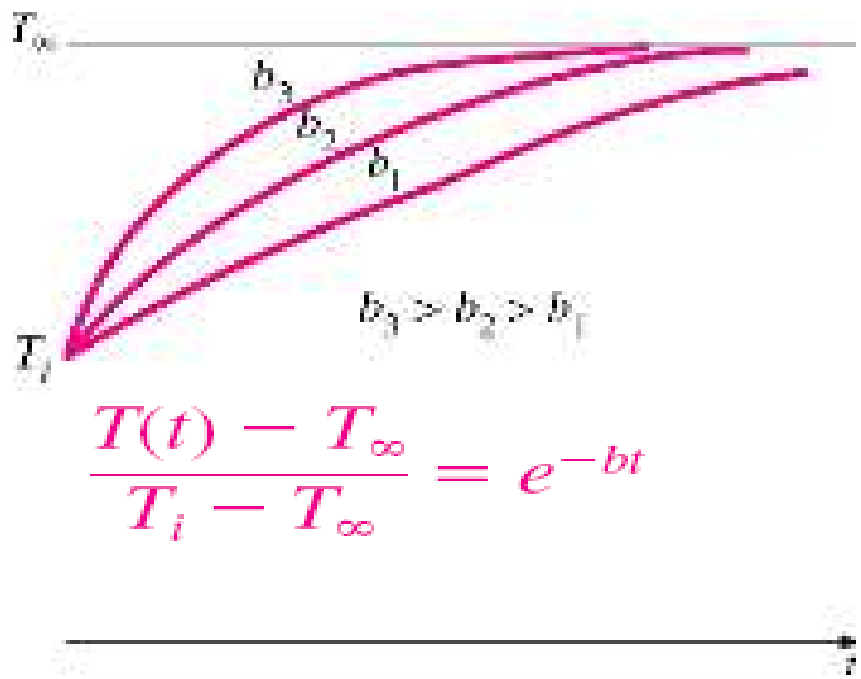
...rocal of b has time unit (usually s), and is called the **time constant**

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There are two observations:

1. Equation enables us to determine the temperature $T(t)$ of a body at time t, or alternatively, the time t required for the temperature to reach a specified value $T(t)$.
2. The temperature of a body approaches the ambient temperature T_∞ exponentially. The temperature of the body changes rapidly at the beginning, but rather slowly later on.

$$b = \frac{hA_s}{\rho V c_p}$$



$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

- The larger the value of the exponent b, the higher the rate of decay in temperature.
- A large value of b indicates that the body approaches the environment temperature in a short time.
- Note that b is proportional to the surface area and heat transfer coefficient, but inversely proportional to the d the specific heat of the body.

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- Once the temperature T(t) at time t is available, the *rate* of convection heat transfer between the body and its environment at that time can be determined from Newton's law of cooling as

$$\dot{Q}(t) = h A_s [T(t) - T_\infty] \quad (W)$$

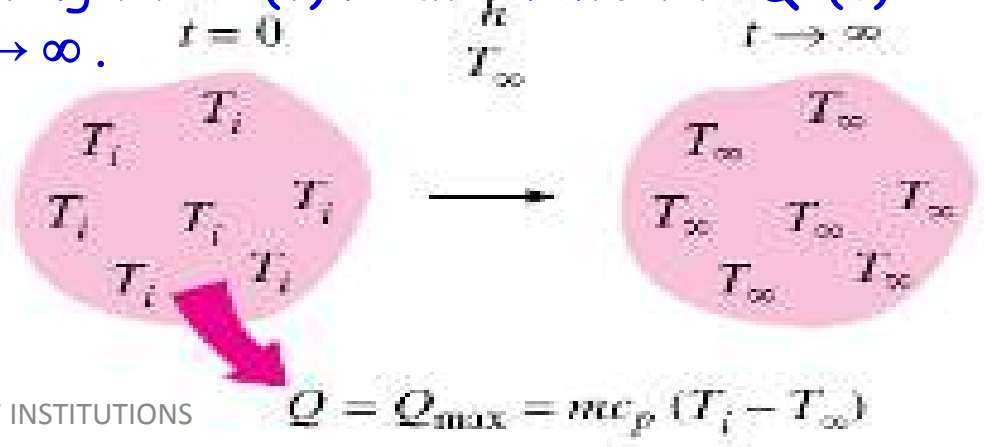
- The *total amount* of heat transfer between the body and the surrounding medium over the time interval $t = 0$ to t is simply the change in the energy content of the body:

$$Q = m C_p [T(t) - T_i] \quad (kJ)$$

- The amount of heat transfer reaches its upper limit when the body reaches the surrounding temperature T_∞. Therefore, the *maximum* heat transfer between the body and its surroundings is:

$$Q_{max} = m C_p (T_\infty - T_i) \quad (kJ)$$

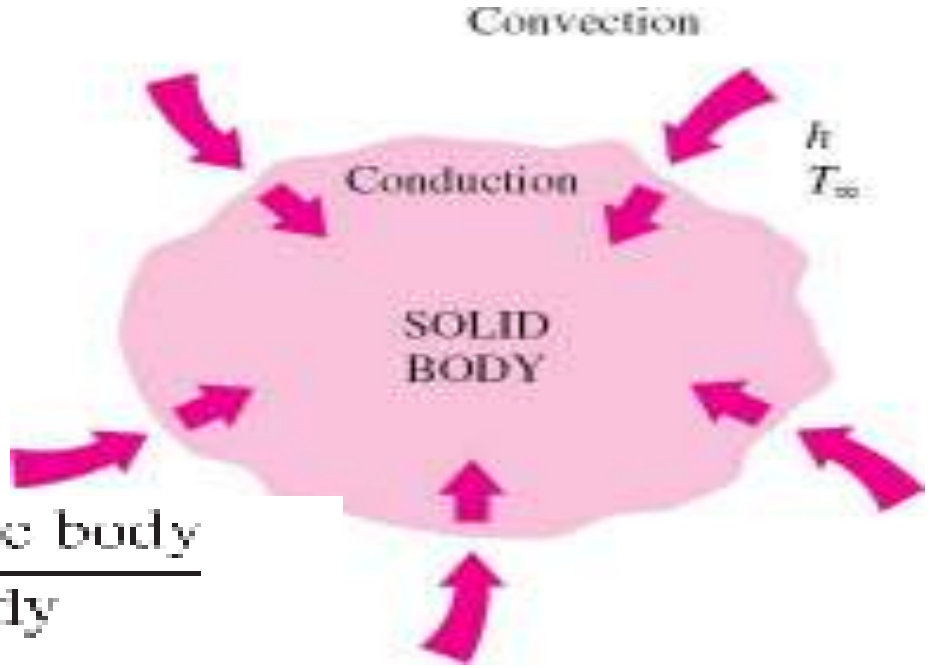
We could also obtain this equation by substituting the T(t) relation into the $\dot{Q}(t)$ relation and integrating it from $t = 0$ to $t \to \infty$.



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Criteria for Lumped System Analysis

- The first step in establishing a criterion for the applicability of the lumped system analysis is to define a **characteristic length** as: $L_c = V/A_s$
- and a **Biot number** B_i as: $B_i = hL_c/k$
- It can also be expressed as



$$\mathrm{Bi} = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

$$\mathrm{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

Biot number represents small resistance to heat conduction, and thus small ture gradients within the body. Thus, lumped system analysis is exact when $B_i = 0$ 'oximate when $B_i > 0$

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- It is generally accepted that lumped system analysis is *applicable* if $B_i \leq 0.1$
  - When this criterion is satisfied, the temperatures within the body relative to the surroundings (i.e., $T - T_\infty$) remain within 5% of each other even for well-rounded geometries such as a spherical ball.
  - Small bodies with high thermal conductivities and low convection coefficients are most likely to satisfy the criterion for lumped system analysis.

The first step in the application of lumped system analysis is the calculation of the *Biot number*, and the assessment of the applicability of this approach.

One may still wish to use lumped system analysis even when the criterion $B_i \leq 0.1$ is not satisfied, if high accuracy is not a major concern.



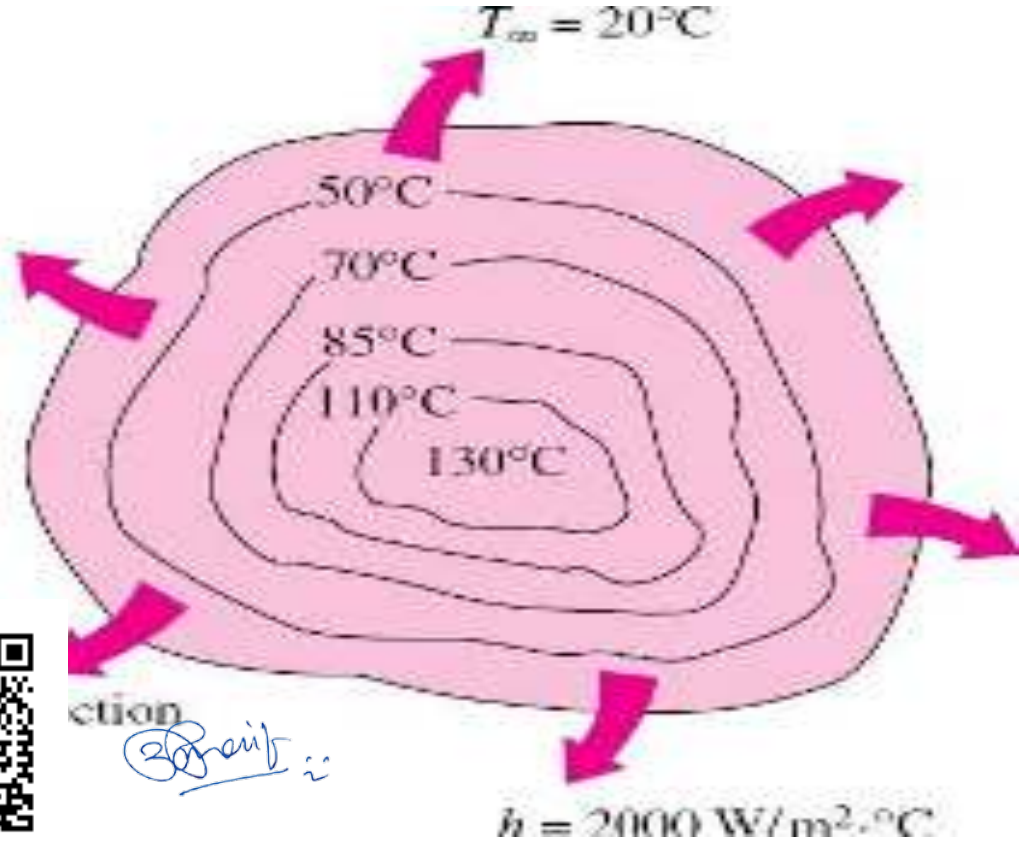
$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = 0.02 \text{ m}$$

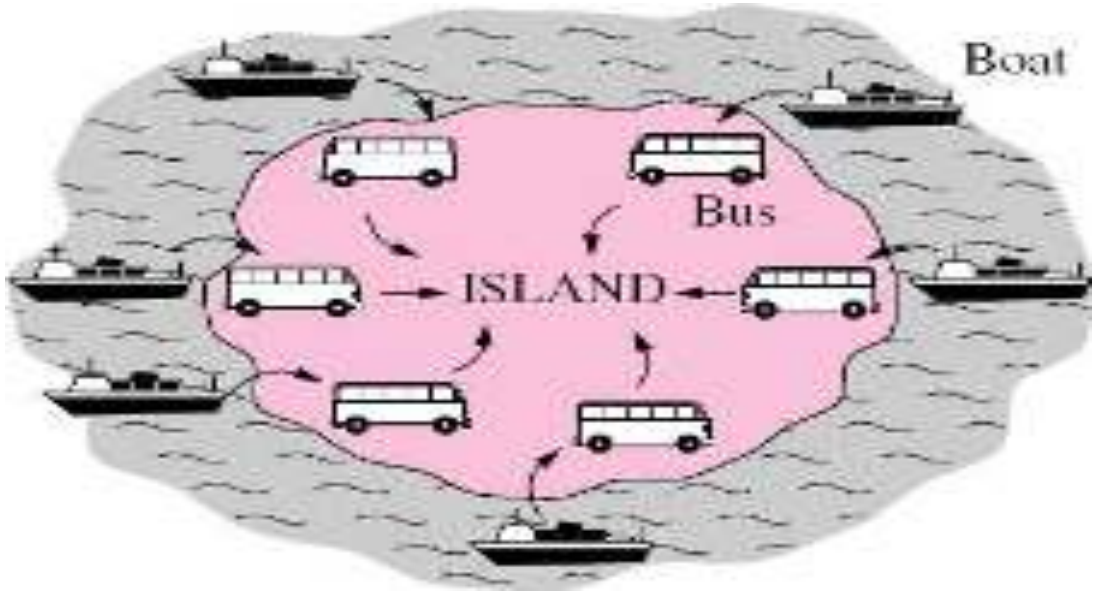
$$\text{Bi} = \frac{hL_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 < 0.1$$

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Some Remarks on Heat Transfer in Lumped Systems

- Analogy between heat transfer to a solid and passenger traffic to an island.





When the convection coefficient h is high and k is low, large temperature differences occur between the inner and outer regions of a large solid.

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Graphical Solutions : Heisler Charts

The solutions obtained for 1D non homogeneous problems with Neumann boundary conditions in Cartesian coordinate systems using the method of separation of variables have been collected and assembled in the form of transient temperature nomographs by Heisler. The given charts are a very useful baseline against, which to validate one's own analytical or numerical computations.

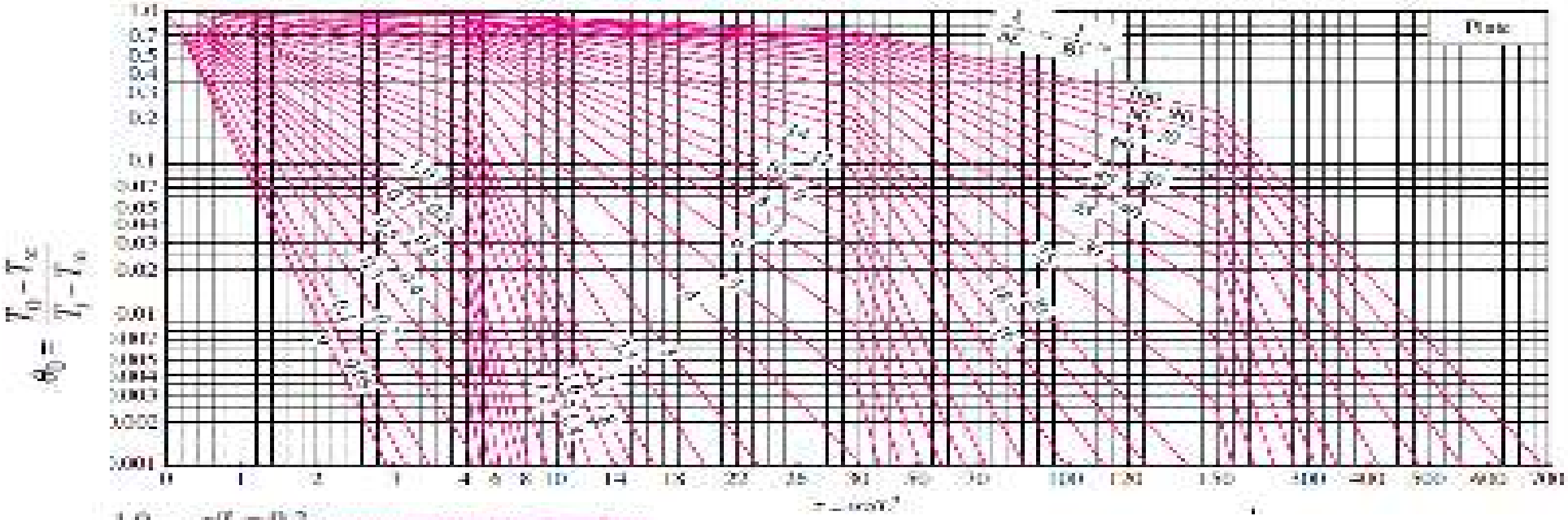
- Indeed, the determination of the constants A_1 and l_1 usually requires interpolation. For those who prefer reading charts to interpolating, these relations are plotted and the one-term approximation solutions are presented in graphical form, known as the *transient temperature charts.*
- The transient temperature charts shown in next slides for a large plane wall, long cylinder, and sphere were presented by M. P. Heisler in 1947 and are called **Heisler charts**.
- There are *three* charts associated with each geometry: the first chart is to determine the temperature T_0 at the *center* of the geometry at a given time *t.* The second chart is to determine the temperature at *other locations* at the same time in terms of *T*0. The third chart is to determine

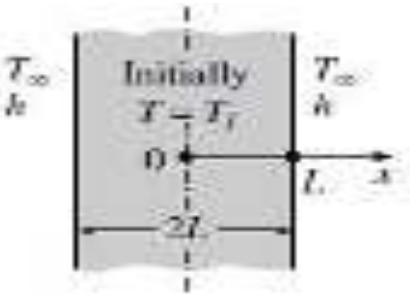
  the total amount of *heat transfer* up to the time *t.* **These plots are valid for t > 0.2.**

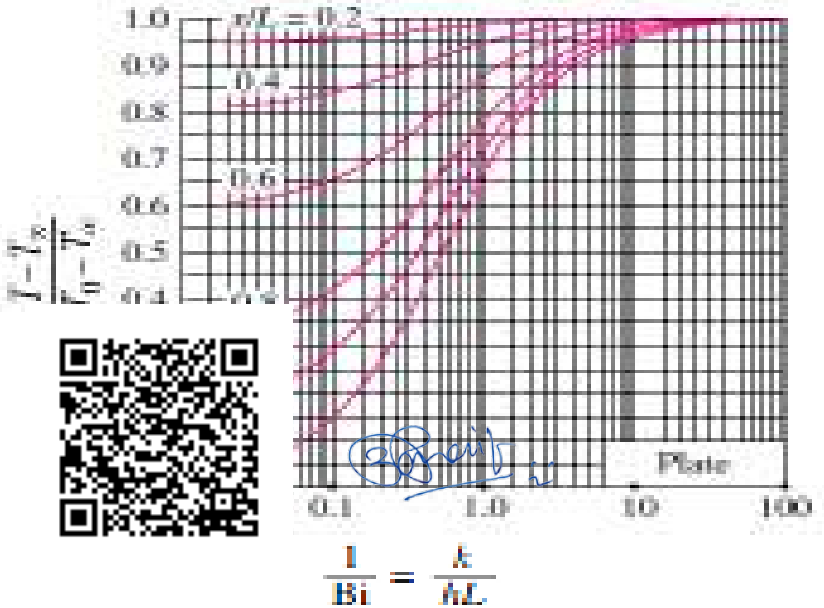


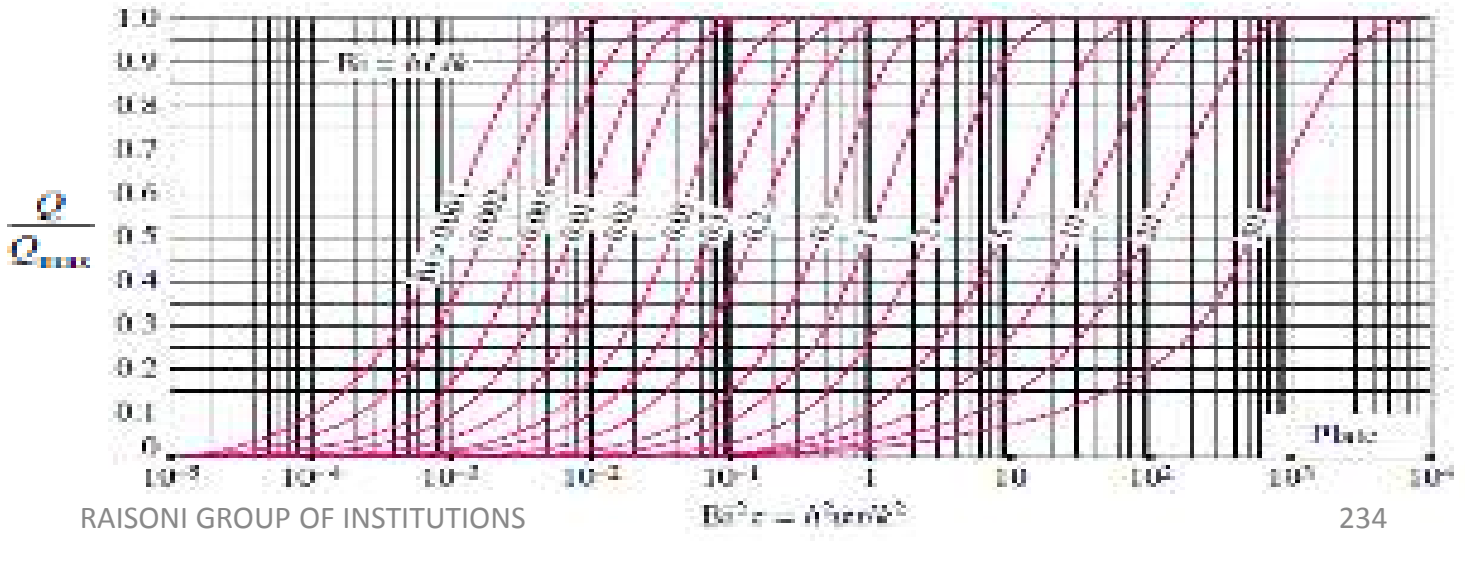
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Transient temperature and heat transfer charts for a **plane wall of thickness 2*L*** initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of *h.*



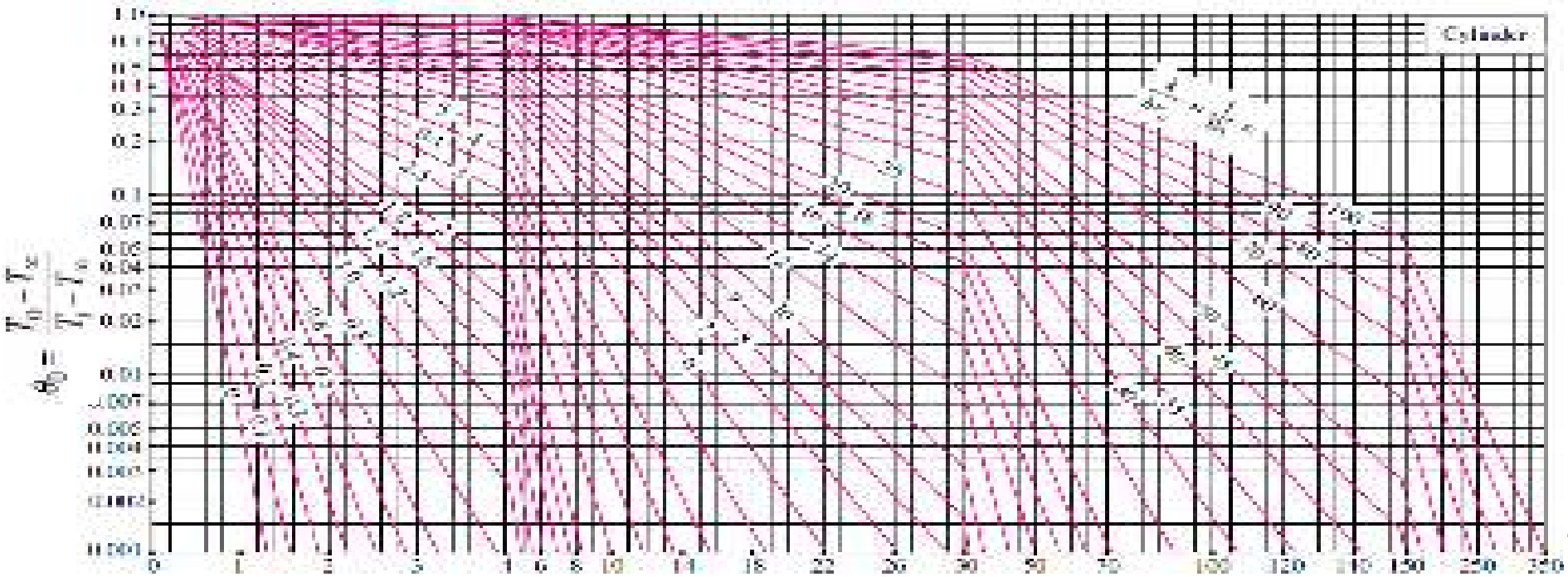




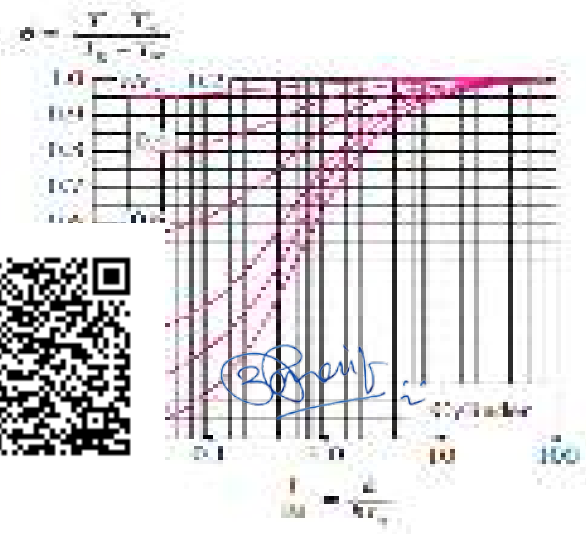


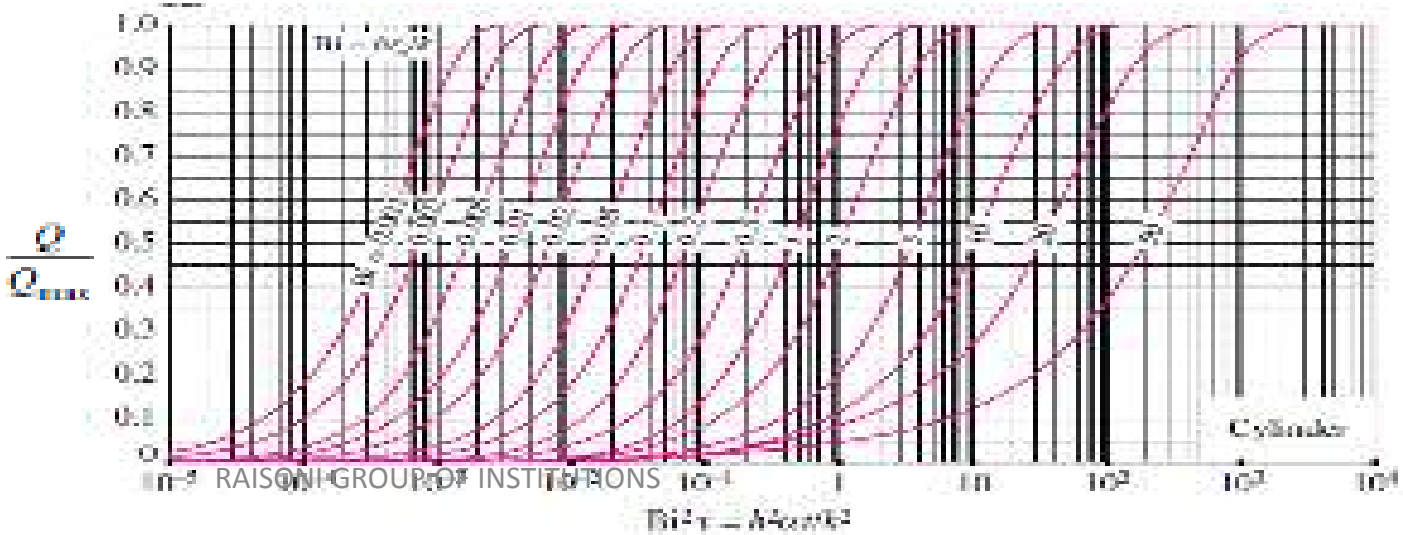
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Transient temperature and heat transfer charts for a ***long cylinder*** of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.



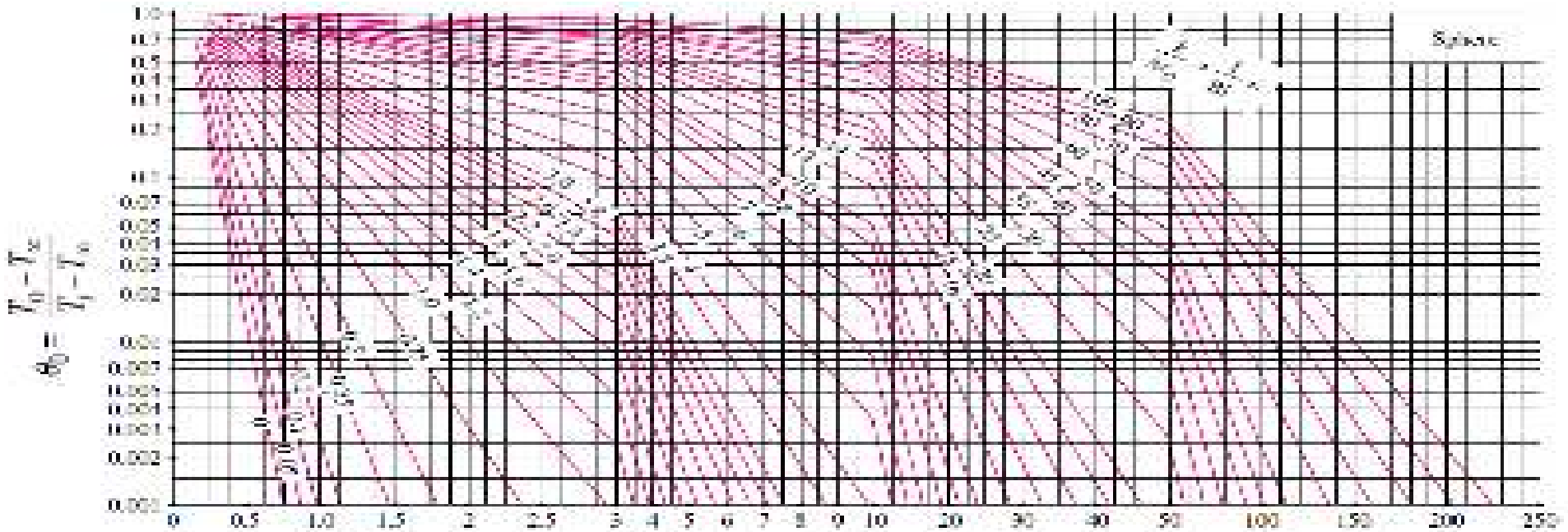


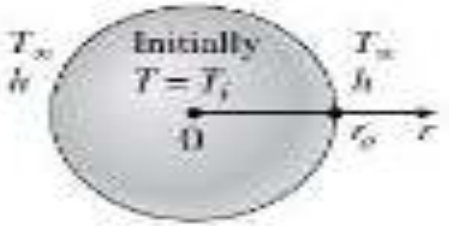




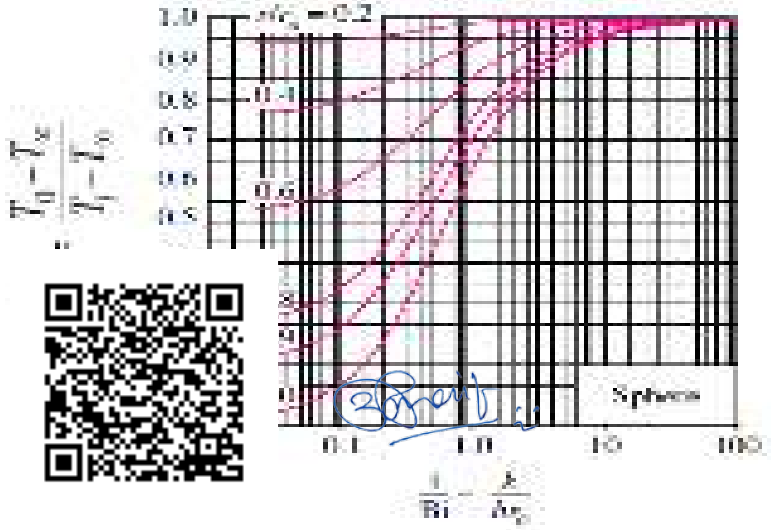
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Transient temperature and heat transfer charts for a ***sphere of radius*** r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

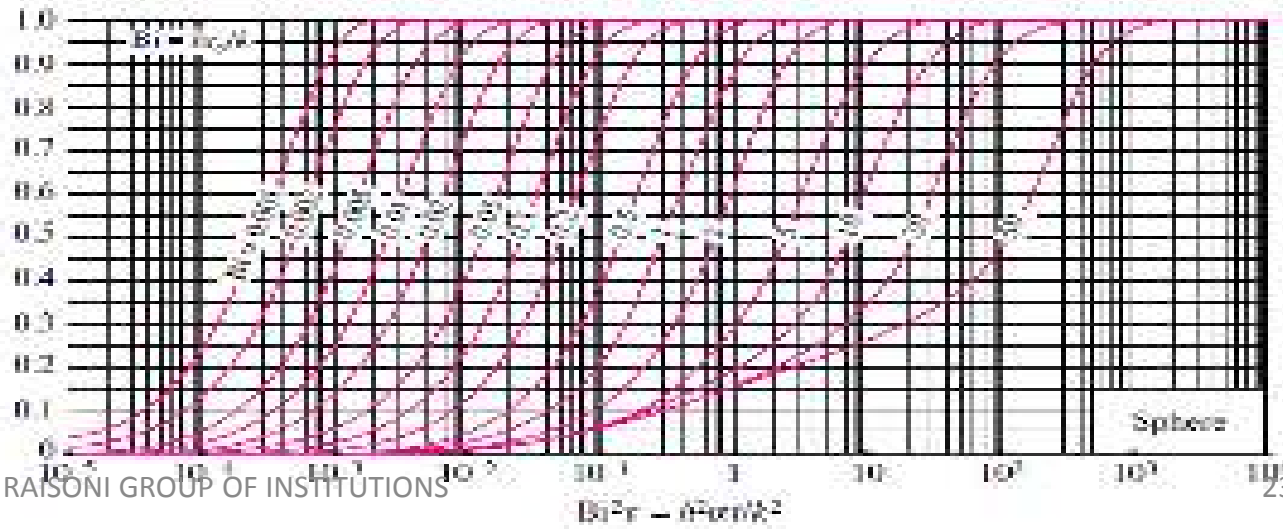




$\tau = \alpha t / r_o^2$



$\frac{Q}{Q_{max}}$



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Useful Relationship

- Again the temperature of the body changes from the initial temperature T_i to the temperature of the surroundings T_∞ at the end of the transient heat conduction process and the *maximum* amount of heat that a body can gain (or lose) is simply the *change* in the *energy content* of the body:

$$Q = \int \rho c_p [T(x,t) - T_i] dV$$

- The amount of heat transfer Q at a finite time t is

$$Q_{max} = mc_p(T_\infty - T_i) = \rho V c_p (T_\infty - T_i) \quad \text{(kJ)}$$

- Assuming constant properties, the ratio of *Q/Qmax* becomes

$$\frac{Q}{Q_{max}} = \frac{\int_V \rho c_p [T(x,t) - T_i] dV}{\rho c_p (T_\infty - T_i) V} = \frac{1}{V} \int_V (1 - \theta) dV$$

- Using the appropriate non-dimensional temperature relations based on the one term

approximation for the plane wall, cylinder, and sphere, and performing the indicated integrations, we obtain the following relations for the fraction of heat transfer in those geometries:

Plane wall:
$$\left(\frac{Q}{Q_{max}}\right)_{wall} = 1 - \theta_{0,wall} \frac{\sin \lambda_1}{\lambda_1}$$

Cylinder:
$$\left(\frac{Q}{Q_{max}}\right)_{cyl} = 1 - 2\theta_{0,cyl} \frac{J_1(\lambda_1)}{\lambda_1}$$

Sphere:
$$\left(\frac{Q}{Q_{max}}\right)_{sph} = 1 - 3\theta_{0,sph} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

/*Q*max ratio relations based on the one-term approximation are also plotted in **Heisler charts**, against the variables Bi and or the large plane wall, long cylinder, and sphere, respectively. Note that once the *fraction* of heat transfer *Q/Q*max has been ned from these charts or equations for the given *t*, the actual amount of heat transfer by that time can be evaluated by ing this fraction by *Q*max.

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Example 2.117. *An electric motor drives a centrifugal pump which circulates a hot liquid metal at 480°C. The motor is coupled to the pump impeller by a horizontal steel shaft (k = 32 W/m°C) 25 mm in diameter. If the ambient air temperature is 20°C, the temperature of the motor is limited to a maximum value of 55°C and the heat transfer coefficient between the steel shaft and the ambient air is 14.8 W/m²°C, what length of shaft should be specified between the motor and the pump?*

Example 2.119. *A heating unit is made in the form of a vertical tube fitted with rectangular section steel fins. The tube height is 1.2 m and its outer diameter is 60 mm. The fins are 50 mm in height and their thickness is 3 mm. The total number of the fins used is 20. The temperature at the base of a fin is 80°C and surrounding air temperature is 18°C. The heat transfer coefficient on the fin l tube surface to the surrounding air is 9.3 W/m²°C. k (fin material) = 55.7 W/m°C*

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Unit No.3

Forced Convection

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HEAT TRANSFER BY CONVECTION

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CONDUCTION

Mechanism of heat transfer through a solid or fluid in the absence any fluid motion.

CONVECTION

Mechanism of heat transfer through a fluid in the presence of bulk fluid motion

- **Natural (free) Convection**
- **Forced Convection**

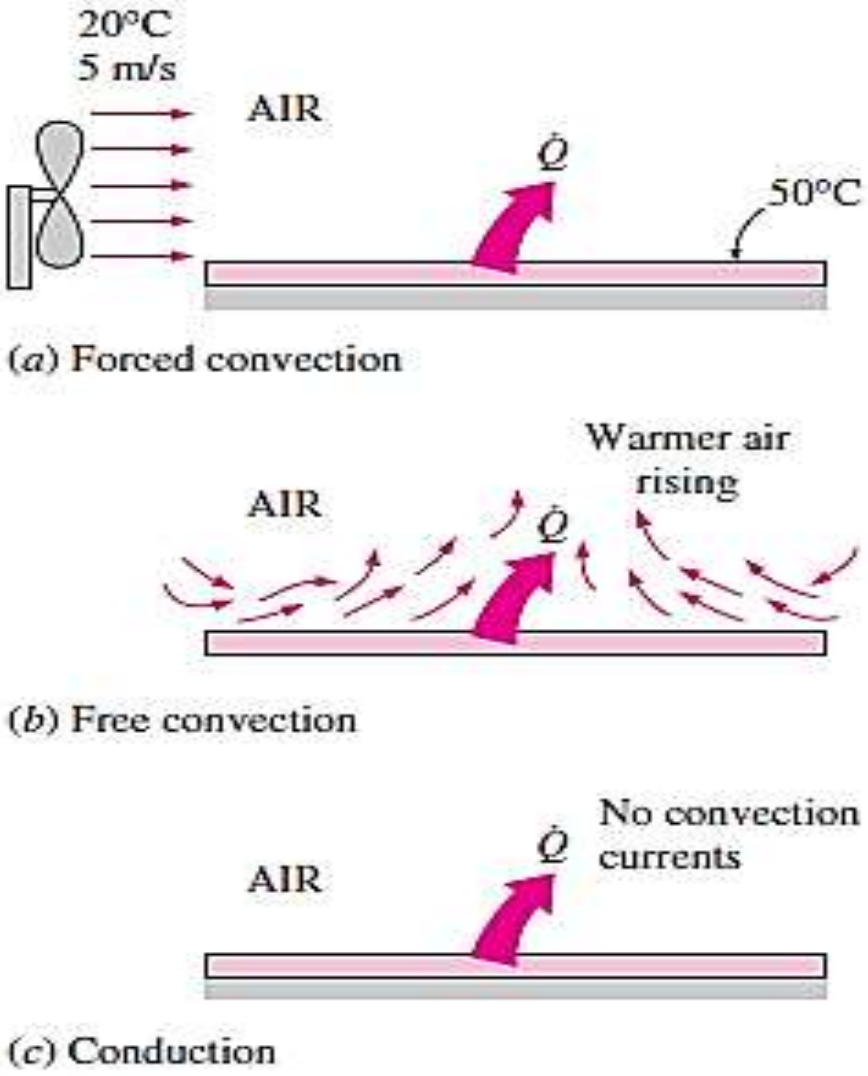
(depending on how the fluid motion is initiated)

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HEAT TRANSFER BY CONVECTION

We mentioned earlier that there are three basic mechanisms of heat transfer: conduction, convection, and radiation. Conduction and convection are similar in that both mechanisms require the presence of a material medium. But they are different in that convection requires the presence of a material medium. But they are different in that convection requires the presence of fluid motion. Heat transfer through a solid is always by conduction, since the molecules of a solid remain at relatively fixed positions. Heat transfer through a liquid or gas, however, can be by conduction or convection, depending on the presence of any bulk fluid motion. Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it. Therefore, conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid

Convection is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion. Convection is classified as *natural (or free*) and *forced* convection depending on how the fluid motion is initiated. In natural convection, any fluid motion is caused by natural h as the buoyancy effect, i.e. the rise of warmer fluid and er fluid. Whereas in forced convection, the fluid is forced to surface or in a tube by external means such as a pump or

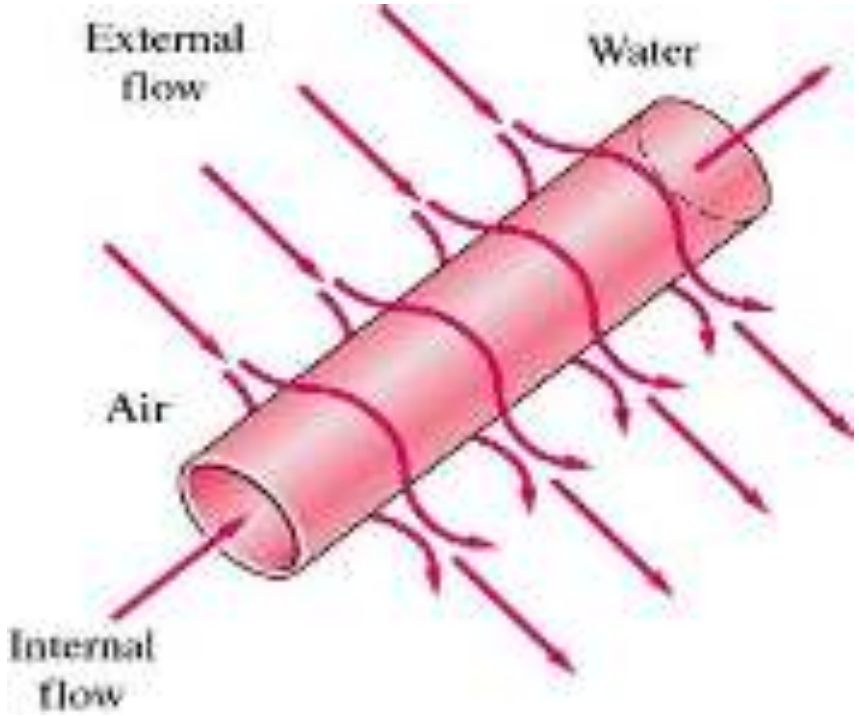


Heat transfer from a hot surface to the surrounding fluid by convection and conduction.

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CLASSIFICATION OF FLUID FLOWS

- **Viscous-inviscid**
- **Internal flow- External flow**
- **Open-closed channel**
- **Compressible-Incompressible**
- **Laminar- Turbulent**
- **Natural- Forced**
- **Steady- Unsteady**
- **One-,two-,three-dimensional**



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VISCOSITY

When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer.

internal resistance to flow

- cohesive forces between the molecules in liquid
- molecular collisions in gases.

Viscous flows: viscous effects are significant

Inviscid flow regions: viscous forces are negligibly small compared to inertial or pressure forces.

measure of stickness or resistance to deformation

1. **Kinematic viscosity**
2. **Dynamic viscosity**



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VISCOSITY DEPENDS ON

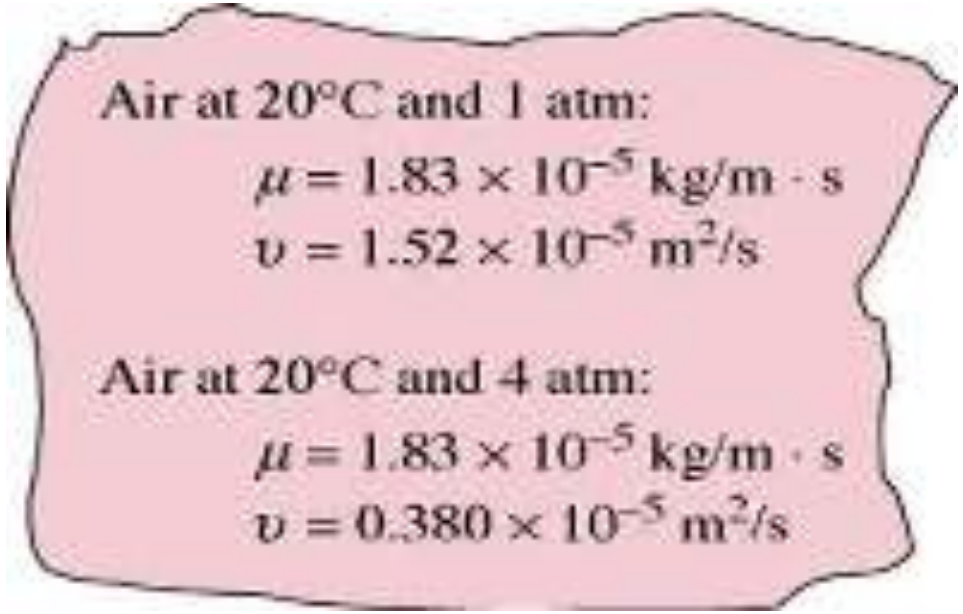
- **TEMPERATURE**
- **PRESSURE**

For liquids dependence of pressure is negligible

For gases kinematic viscosity depends on pressure since its relation to density

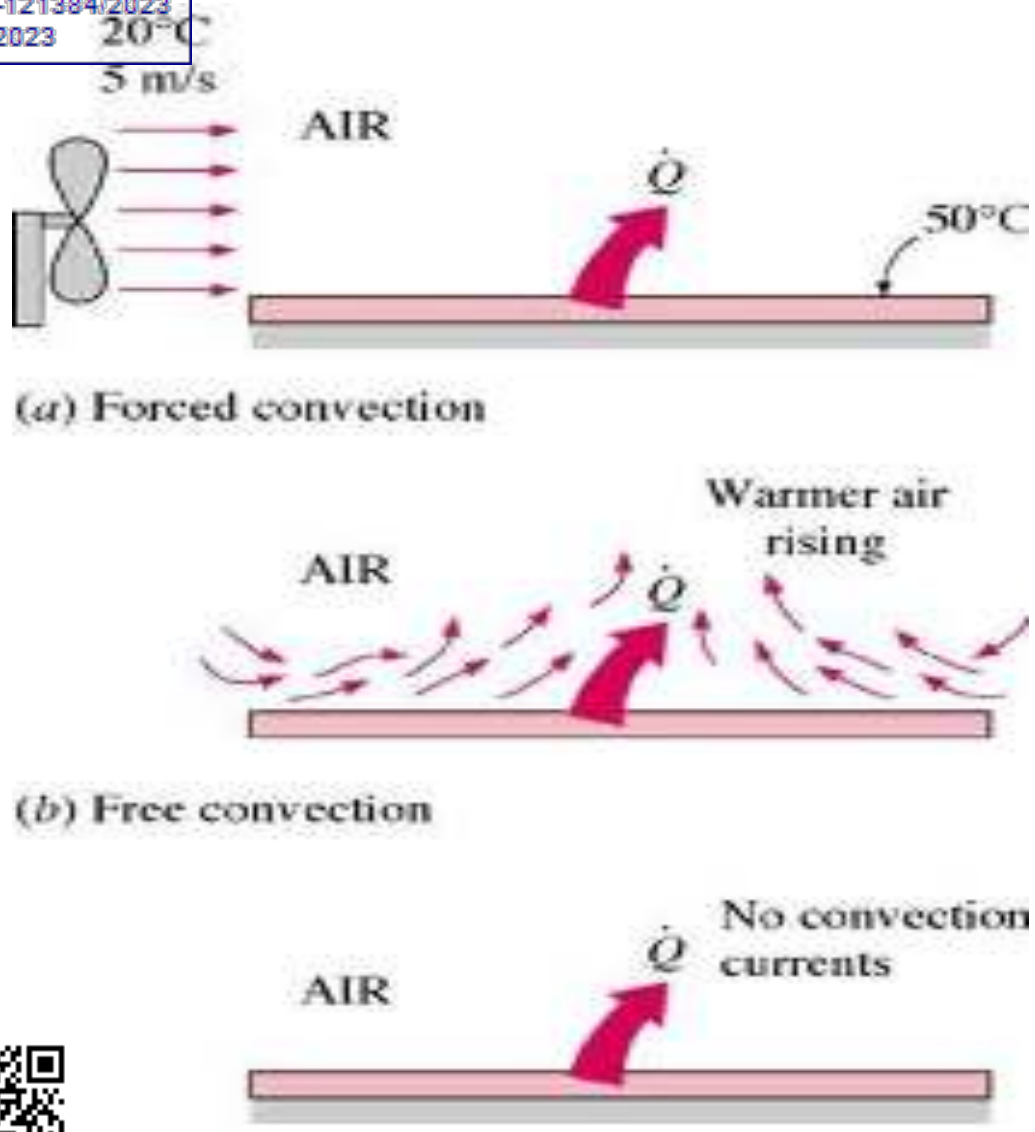
μ Dynamic viscosity (kg/m.s or poise)

$$\nu = \frac{\mu}{\rho}$$



ν Kinematic viscosity, m^2/s or stroke

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Convection heat transfer

- Dynamic viscosity
- Thermal conductivity
- Density
- Specific heat
- Fluid velocity
- Geometry
- Roughness
- Type of fluid flow

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NEWTON'S LAW OF COOLING

$$\dot{Q}_{conv} = hA_S(T_S - T_\infty) \qquad \text{(W)}$$

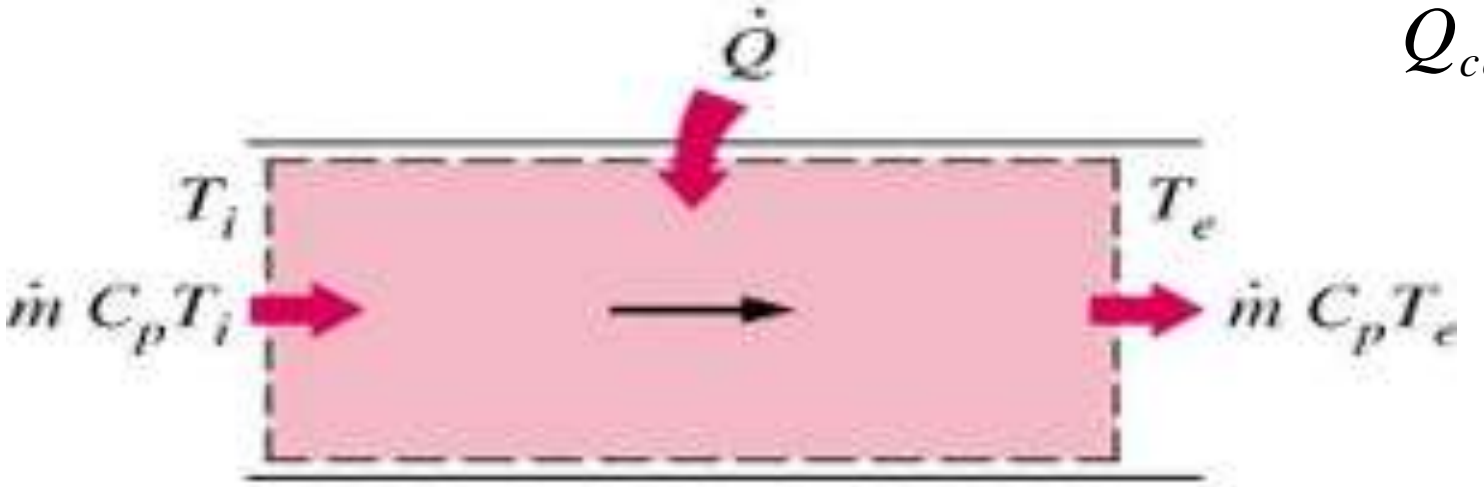
h Convection heat transfer coefficient ($W/m^2.^0C$)

The rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference

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GENERAL THERMAL ANALYSIS

$$\dot{Q}_{conv} = hA_S(T_S - T_\infty)$$



$$\dot{Q} = \dot{m}\,C_p(T_e - T_i)$$

Energy balance:

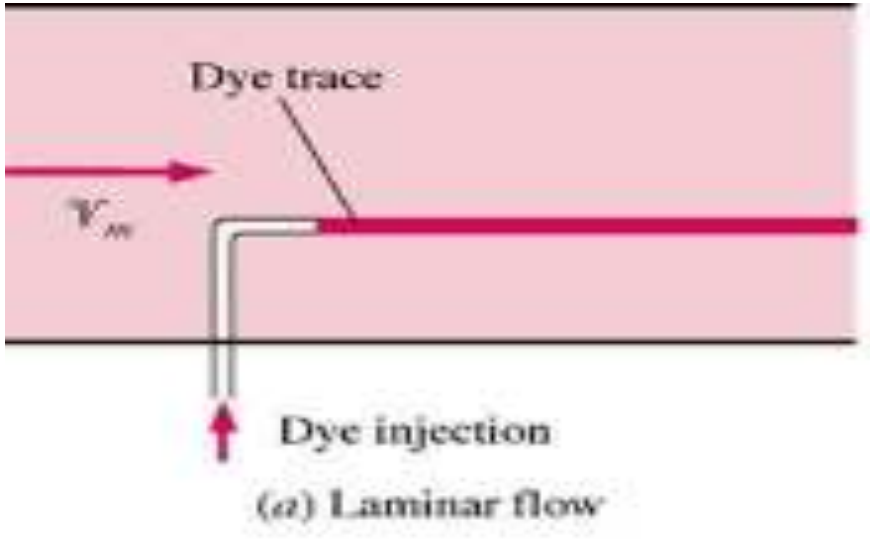
$$\dot{Q} = \dot{m}\,C_p(T_e - T_i)$$

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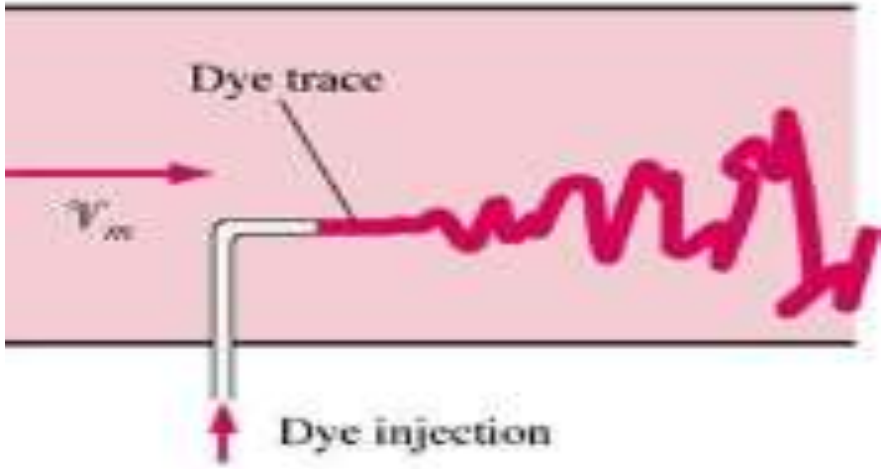
•**LAMINAR FLOW** Smooth tramlines Highly-ordered motion
(highly viscous fluids in small pipes)

•**TURBULENT FLOW** Velocity fluctuations Highly-disordered motion

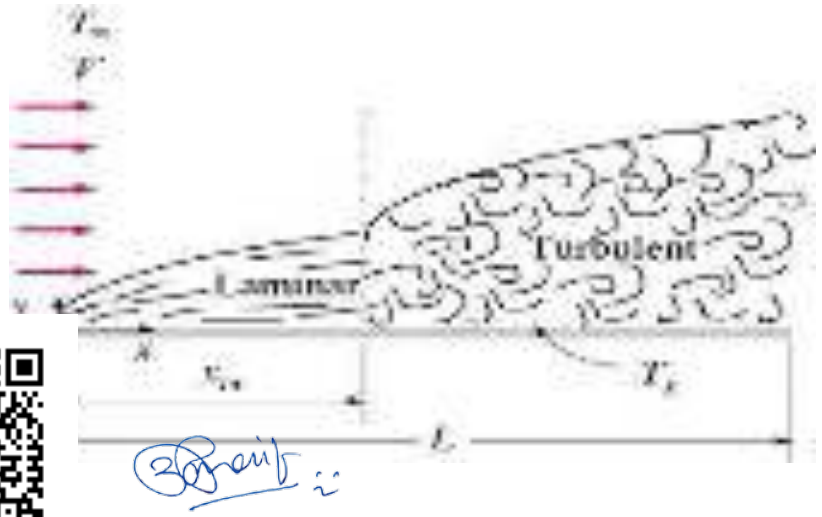
•**TRANSITIONAL FLOW**



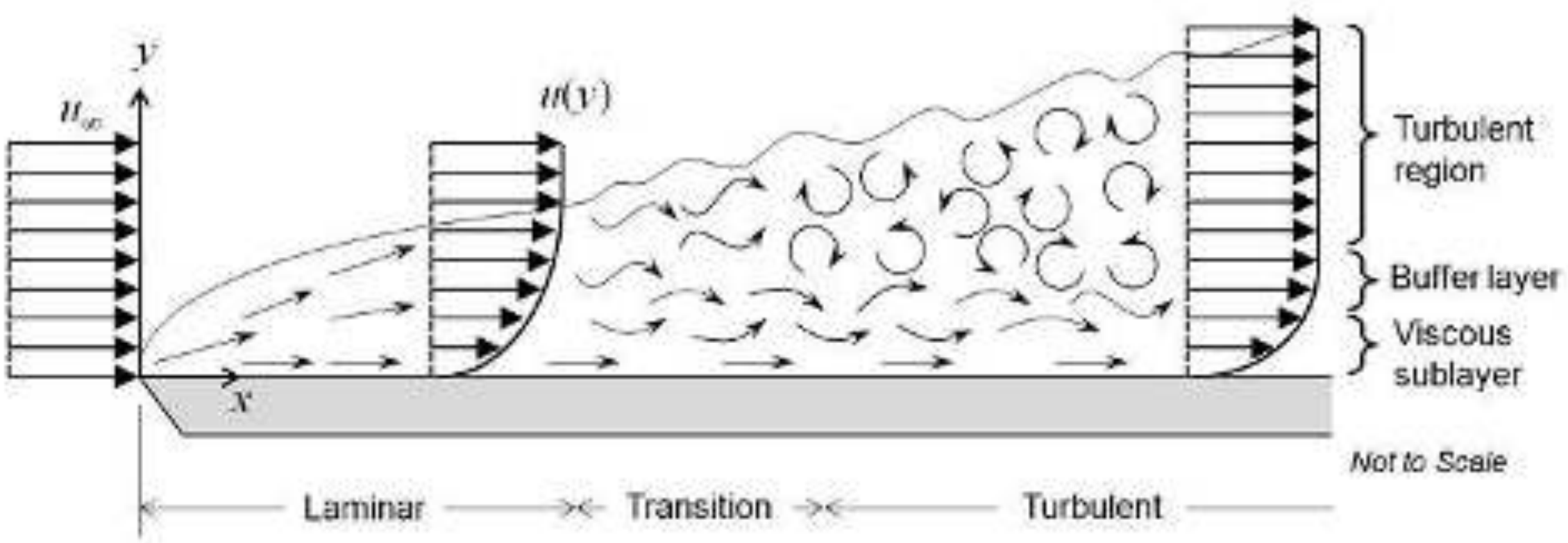
(a) Laminar flow



(b) Turbulent flow



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REYNOLDS NUMBER

Flow Regime: Geometry

Surface roughness

Flow velocity

Surface temperature type of fluid

Ratio of the inertial forces to viscous forces in the fluid

$$\mathrm{Re} = \frac{\upsilon_m D}{\nu} = \frac{\rho \upsilon_m D}{\mu}$$

υ_m Mean flow velocity

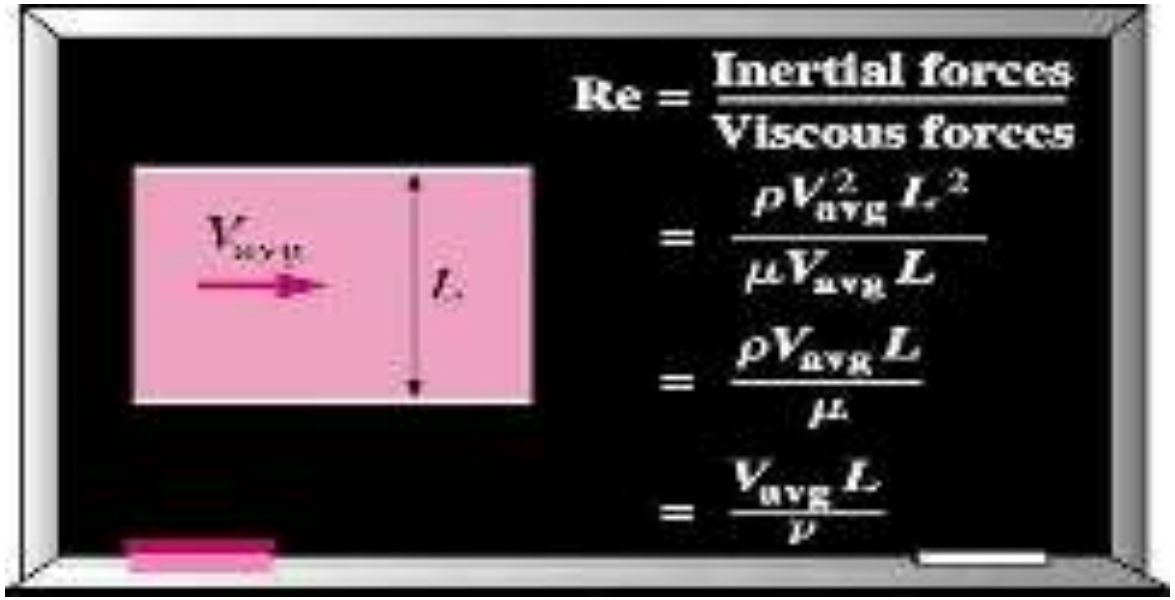
D Characteristic length of the geometry

$\nu = \mu / \rho$ Kinematic viscosity



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Definition of Reynolds number



- Critical Reynolds number (Re_{cr}) for flow in a round pipe

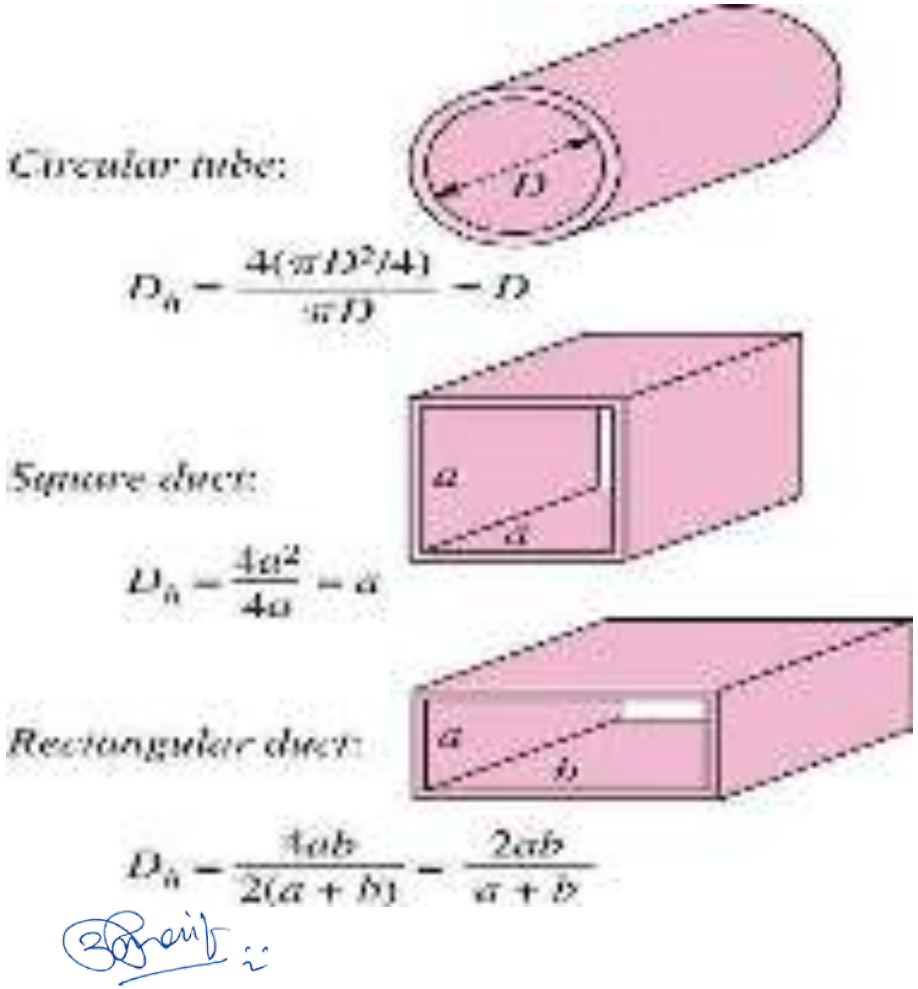
Re < 2300 ⇒ laminar

2300 ≤ Re ≤ 4000 ⇒ transitional Re > 4000 ⇒ turbulent

- Note that these values are approximate.
- For a given application, Re_{cr} depends upon
  - Pipe roughness
  - Vibrations
  - Upstream fluctuations, disturbances (valves, elbows, etc. that may disturb the flow)

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HYDRAULIC DIAMETER

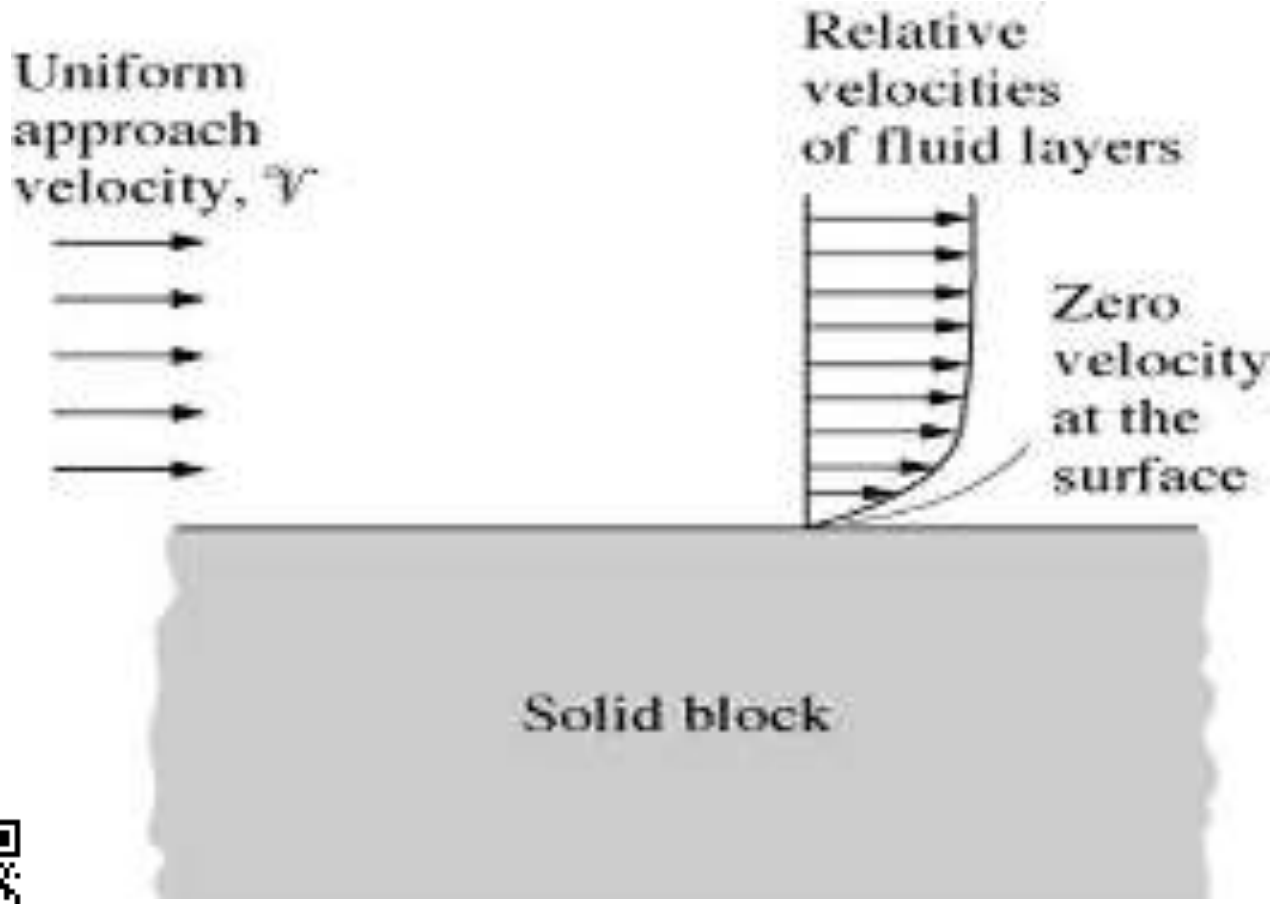


- For <u>non-round</u> pipes,
- the hydraulic diameter
  $D_h = 4A_c/P$
  A_c = cross-section area
  P = wetted perimeter

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Velocity

No-slip condition

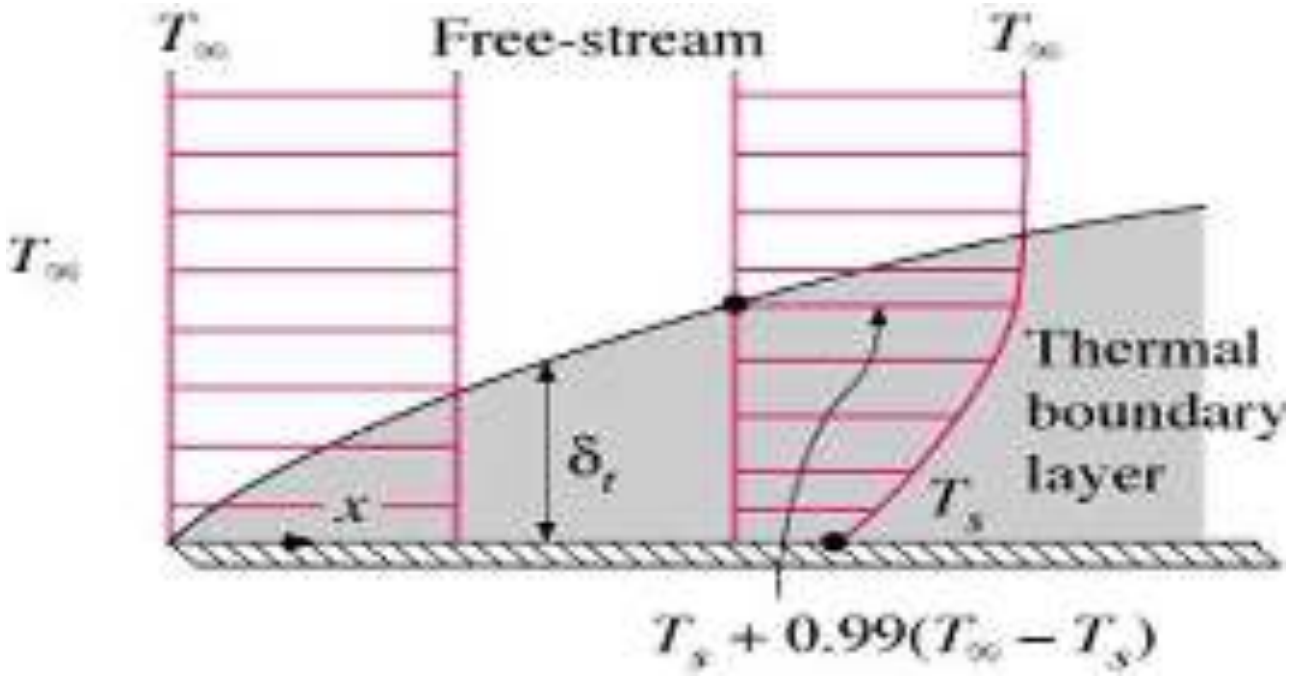


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THERMAL BOUNDARY LAYER

Flow region over the surface in which the temperature variation in the direction normal to the surface

Velocity profile influences temperature profile

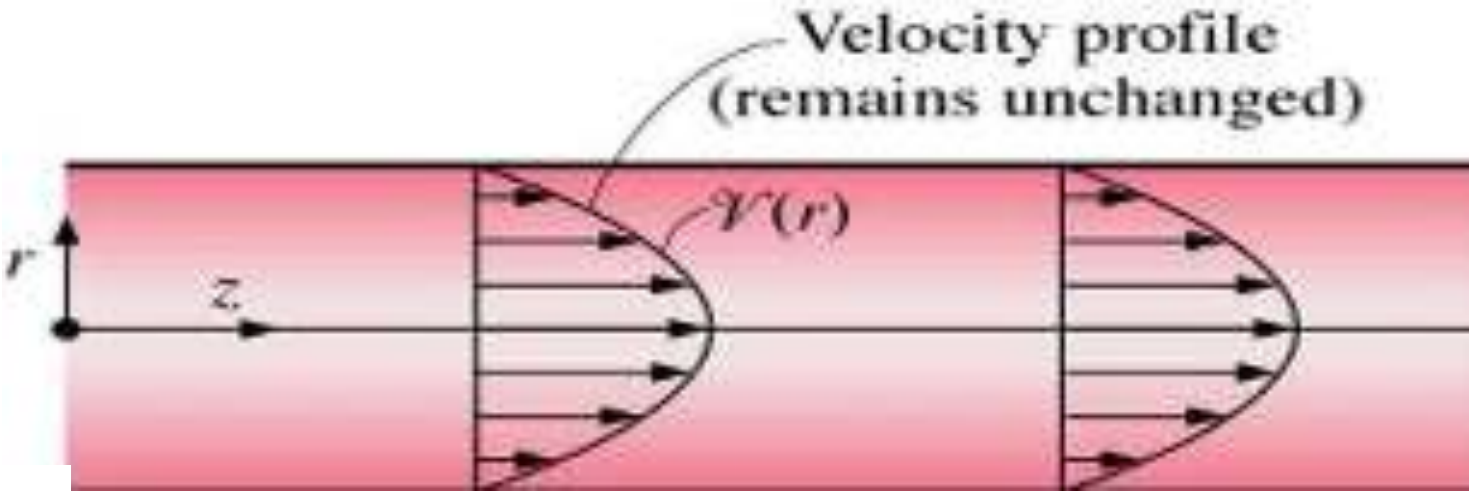


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VELOCITY

A flow field is best characterized by the velocity distribution, and velocity may vary in three dimension

$\vec{\upsilon}(x, y, z)$ in rectangular $\vec{\upsilon}(r, \theta, z)$ in cylinderical coordinates



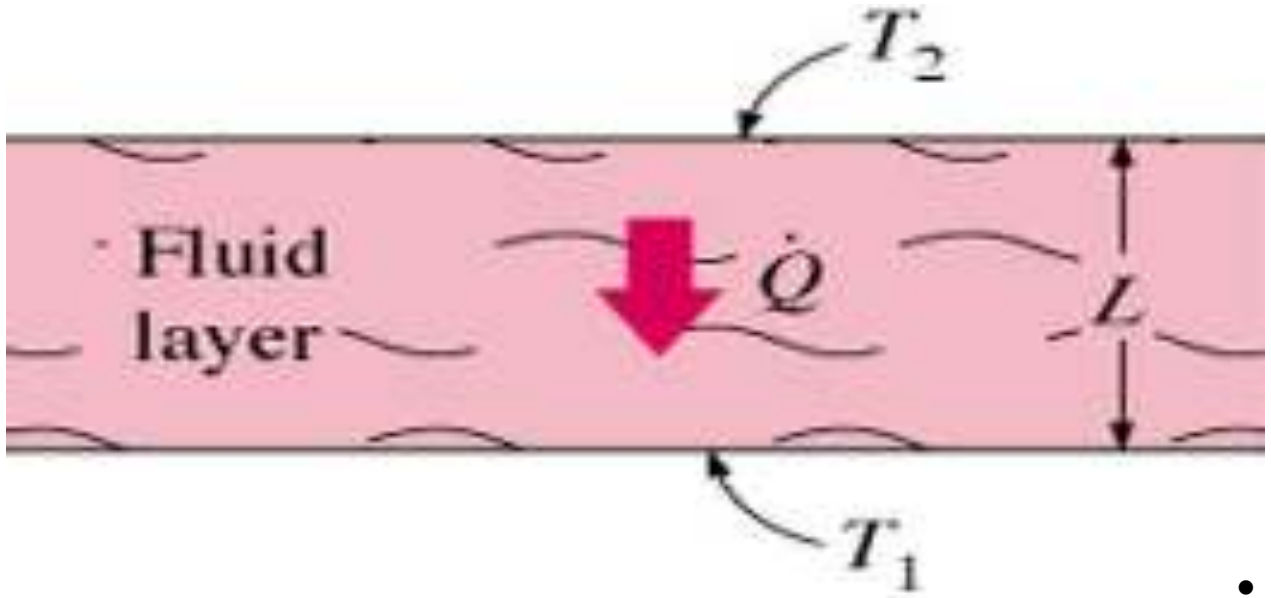
One dimensional flow in a circular pipe

In which direction does the velocity change in this figure???

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NUSSELT NUMBER

(Dimensionless number)



$$\Delta T = T_2 - T_1$$

$$Nu = \frac{hL_c}{k}$$

$$\dot{q}_{cond} = k \frac{\Delta T}{L}$$

$$\dot{q}_{conv} = h\Delta T$$

$$\frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h\Delta T}{k\Delta T / L} = \frac{hL}{k} = Nu$$

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PRANDTL NUMBER

- Boundary layer theory

$$\Pr = \frac{\mu C_p}{k}$$

$$\Pr = \frac{molecular \quad diffusivity \quad of \quad momentum}{molecular \quad diffusivity \quad of \quad heat} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

Pr<<1 heat diffuses very quickly in liquid metals, *tbl* thicker Pr>>1 heat diffuses very slowly in oils relative to momentum, *tbl* than *vbl*



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PARALLEL FLOW OVER FLAT PLATES

$$\mathrm{Re}_{cr} = \frac{\rho \upsilon\, x_{cr}}{\mu} = 5\times10^5$$

$$Nu = \frac{hL}{k} = 0.664\ \mathrm{Re}_L^{0.5}\mathrm{Pr}^{1/3} \qquad \mathrm{Re}_L < 5\times10^5 \qquad \text{laminar}$$

$$Nu = \frac{hL}{k} = 0.037\ \mathrm{Re}_L^{0.8}\mathrm{Pr}^{1/3} \qquad 0.6 \le \mathrm{Pr} \le 60 \qquad \text{turbulent}$$

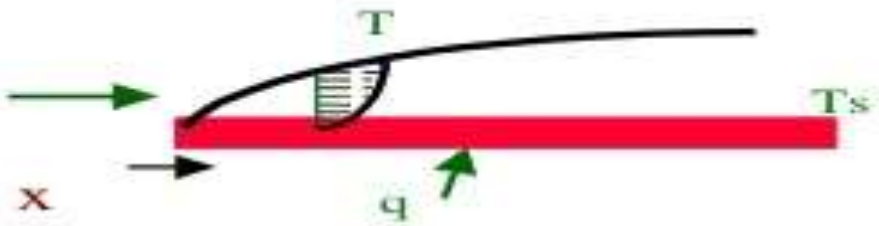
$$5\times10^5 \le \mathrm{Re}_L \le 10^7$$

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FORCED CONVECTION:

external flow (over flat plate)

An internal flow is surrounded by solid boundaries that can restrict the development of its boundary layer, for example, a pipe flow. An external flow, on the other hand, are flows over bodies immersed in an unbounded fluid so that the flow boundary layer can grow freely in one direction. Examples include the flows over airfoils, ship hulls, turbine blades, etc.



- Fluid particle adjacent to the solid surface is at rest
- These particles act to retard the motion of adjoining layers
- ⇒ **boundary layer effect**

Momentum balance: inertia forces, pressure gradient, viscous forces, body forces

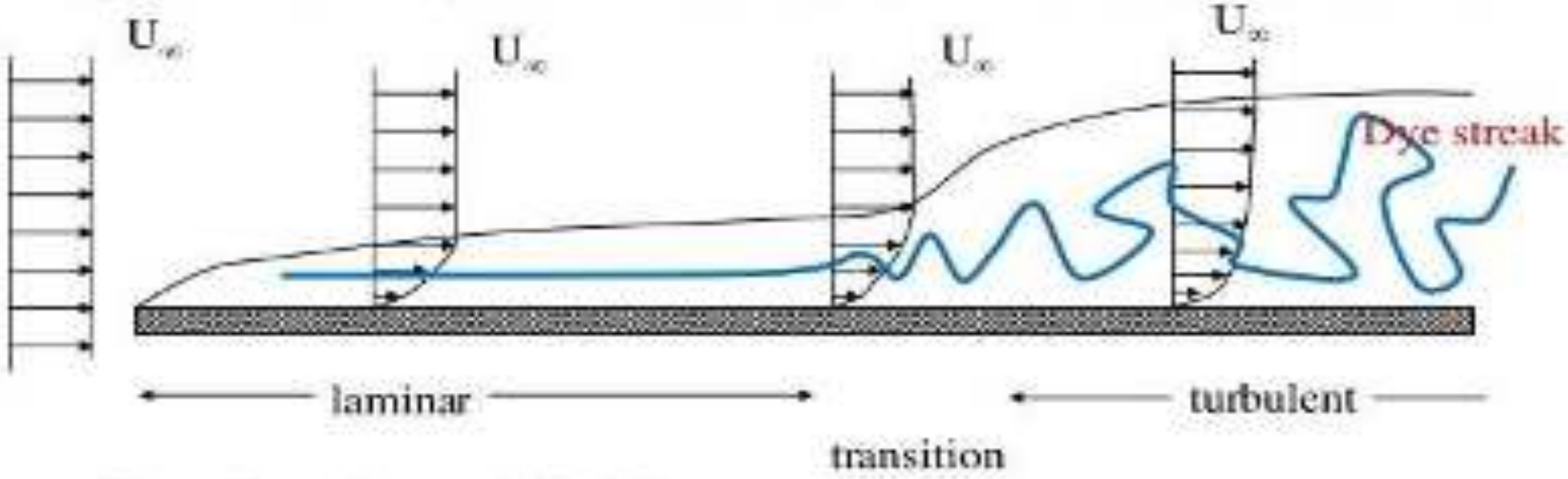
Energy balance: convective flux, diffusive flux, heat generation, energy storage

h=f(Fluid, Vel ,Distance,Temp)

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Hydrodynamic boundary layer

One of the most important concepts in understanding the external flows is the boundary layer development. For simplicity, we are going to analyze a boundary layer flow over a flat plate with no curvature and no external pressure variation.



Boundary layer definition

- Boundary layer thickness (δ): defined as the distance away from the surface where the local velocity reaches to 99% of the free-stream velocity, that is $u(y=\delta)=0.99U_\infty$. Somewhat an easy to understand but arbitrary definition.
- Boundary layer is usually very thin: δ/x usually $<< 1$.

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Hydrodynamic and Thermal boundary layers

- As we have seen earlier,the hydrodynamic boundary layer is a region of a fluid flow, near a solid surface, where the flow patterns are directly influenced by viscous drag from the surface wall.
- $0<u<U, \quad 0<y<\delta$
- The Thermal Boundary Layer is a region of a fluid flow, near a solid surface, where the fluid temperatures are directly influenced by heating or cooling from the surface wall.
- $0<t<T, \quad 0<y<\delta_t$
- *The two boundary layers may be expected to have similar characteristics but do not normally coincide.* Liquid metals tend to conduct heat from the wall easily and temperature changes are observed well outside the dynamic boundary layer. Other materials tend to show velocity changes well outside the thermal layer.

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Effects of Prandtl number, Pr



Pr >>1	Pr = 1	Pr <<1
$\nu >> \alpha$	$\nu = \alpha$	$\nu << \alpha$
e.g., oils	e.g., air and gases have Pr ~ 1 (0.7 - 0.9)	e.g., liquid metals

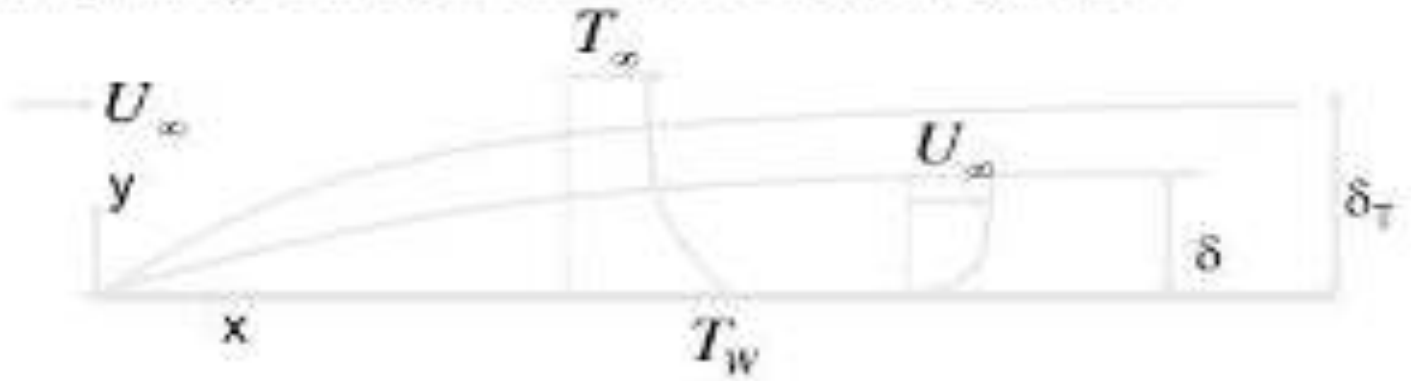
$$\frac{u}{U_\infty} \text{ similar to } \frac{T - T_w}{T_\infty - T_w}$$

(Reynold's analogy)

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Boundary layer equations (laminar flow)

- Simpler than general equations because boundary layer is thin



- Equations for 2D, laminar, steady boundary layer flow

$$\text{Conservation of mass:}\quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\text{Conservation of x-momentum:}\quad u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right)$$

$$\text{Conservation of energy:}\quad u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}\right)$$

- Note: for a flat plate, U_∞ is constant hence $\frac{dU_\infty}{dx}=0$

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Exact solutions: Blasius

Boundary layer thickness $\frac{\delta}{x} = \frac{4.99}{\sqrt{\mathrm{Re}_x}}$

Skin friction coefficient $C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$

$$\left(\mathrm{Re}_x = \frac{U_\infty x}{\nu}, \quad \tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} \right)$$

Average drag coefficient $C_D = \frac{1}{L}\int_0^L C_f dx = \frac{1.328}{\sqrt{\mathrm{Re}_L}} \quad \left(\mathrm{Re}_L = \frac{U_\infty L}{\nu} \right)$

Local Nusselt number $Nu_x = 0.339\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$

Average Nusselt number $\overline{Nu} = 0.678\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$

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Heat transfer coefficient

- Local heat transfer coefficient:

$$h_x = \frac{Nu_x k}{x} = \frac{0.339\, k\, \mathrm{Re}_x^{1/2}\, \mathrm{Pr}^{1/3}}{x}$$

- Average heat transfer coefficient:

$$\bar{h} = \frac{\overline{Nu}\, k}{L} = \frac{0.678 k\, \mathrm{Re}_L^{1/2}\, \mathrm{Pr}^{1/3}}{L}$$

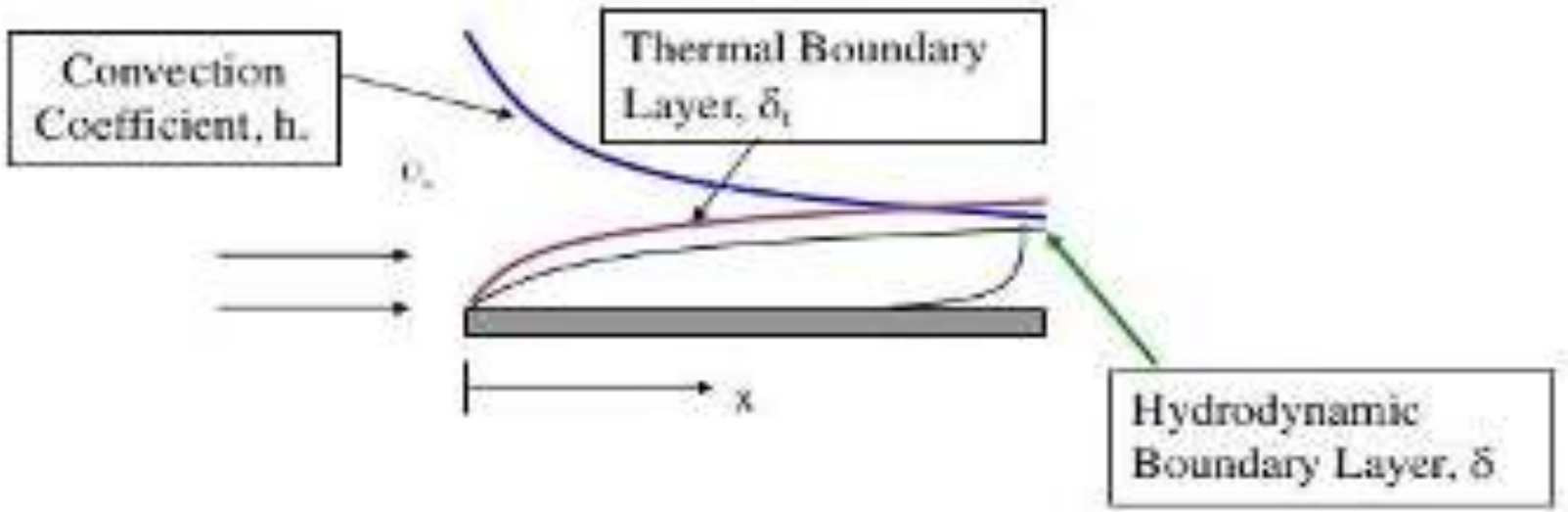
- Recall: $q_w = \bar{h} A (T_w - T_\infty)$, heat flow rate from wall

- Film temperature, T_{film}

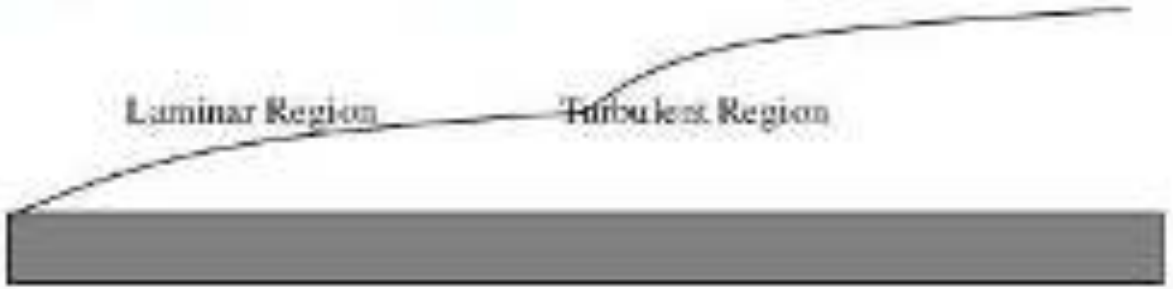
For heated or cooled surfaces, the thermophysical properties within the boundary layer should be selected based on the average temperature of the wall and the free stream; $T_{film} = \frac{1}{2}(T_w + T_\infty)$

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Heat transfer coefficient



Laminar and turbulent b.l.



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Turbulent boundary layer

* Re_x increases with x. Beyond a critical value of Reynolds number ($Re_x = Re_{xc}$), the flow becomes transitional and eventually turbulent

$$Re_{xc} = \frac{U_\infty x_c}{\nu} \quad \text{(For flow over flat plate, } x_c \approx 5\times10^5\text{)}$$

* Turbulent b.L equations are similar to laminar ones, but infinitely more difficult to solve.

* We will mainly use correlations based on experimental data:

$$C_f = 0.059\,Re_x^{-0.2} \qquad (Re_x > 5\times10^5)$$

$$C_D = 0.072\,Re_L - \frac{1}{Re_L}\left(0.072\,Re_{xc}^{0.8} - 1.328\,Re_{xc}^{0.5}\right)$$

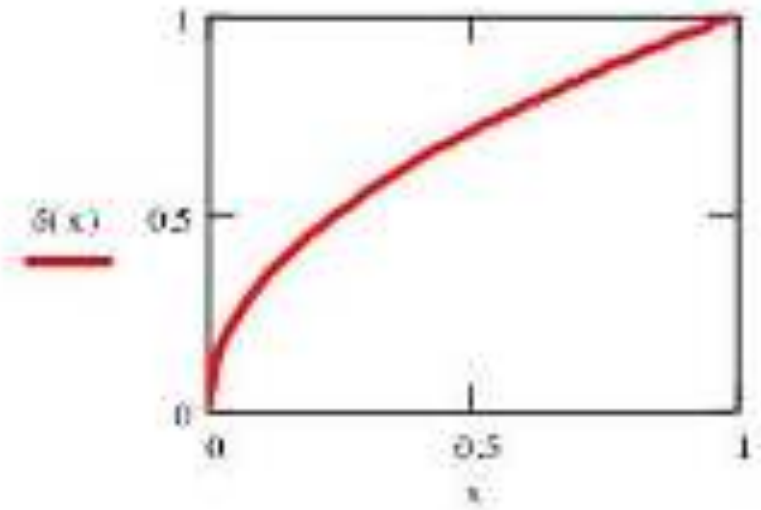
$$Nu_x = 0.029\,Re_x^{0.8}\,Pr^{1/3}$$

$$\overline{Nu} = 0.036\,Re_L^{0.8}\,Pr^{1/3} - Pr^{1/3}\left(0.036\,Re_{xc}^{0.8} - 0.664\,Re_{xc}^{0.5}\right)$$

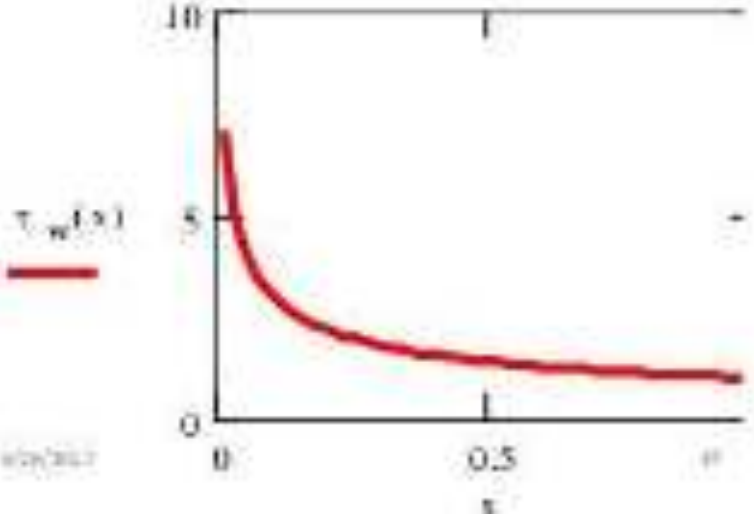
* Calculate heat transfer coefficient in usual way: $h = \frac{Nu\,k}{x}$ etc.

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Laminar Boundary Layer Development



- Boundary layer growth: $\delta \propto \sqrt{x}$
- Initial growth is fast
- Growth rate $d\delta/dx \propto 1/\sqrt{x}$, decreasing downstream.



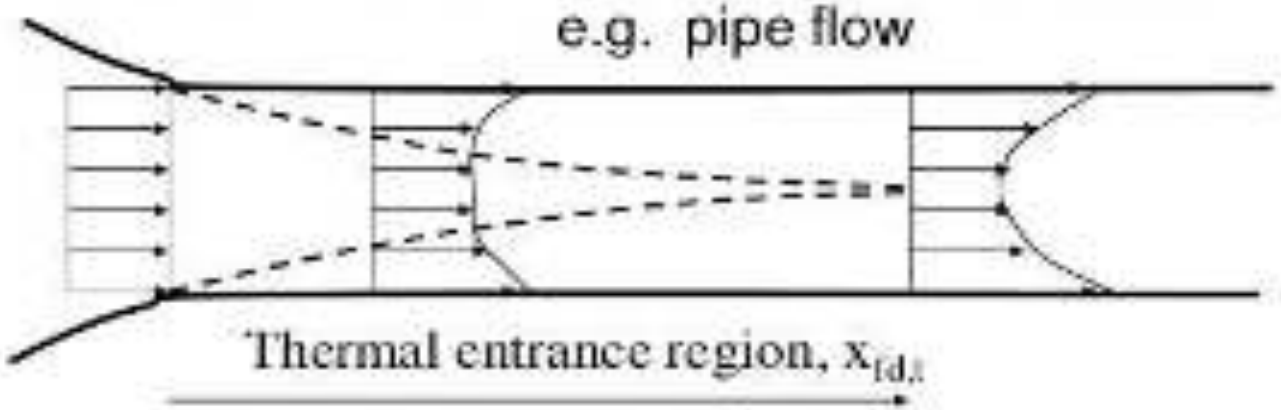
- Wall shear stress: $\tau_w \propto 1/\sqrt{x}$
- As the boundary layer grows, the wall shear stress decreases as the velocity gradient at the wall becomes less steep.

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FORCED CONVECTION: Internal flow

- Thermal conditions
  - ⇨ Laminar or turbulent
  - ⇨ entrance flow and fully developed thermal condition

e.g. pipe flow



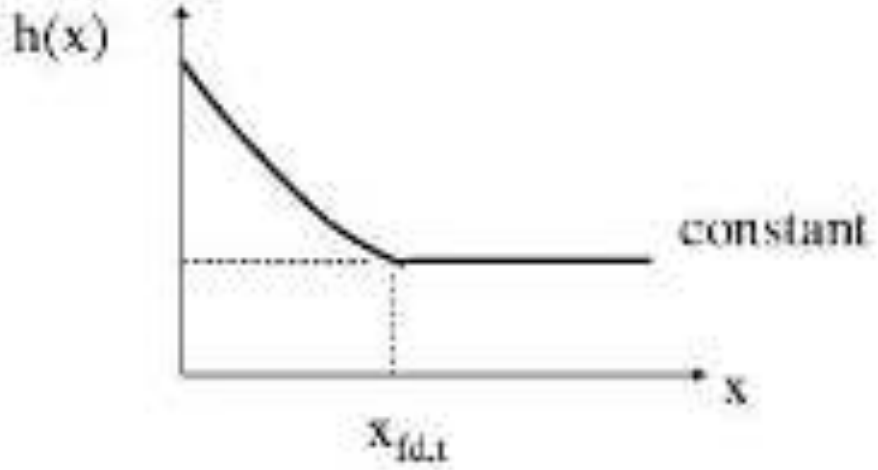
For laminar flows the thermal entrance length is a function of the Reynolds number and the Prandtl number: $x_{fd,t}/D \approx 0.05 Re_D Pr$, where the Prandtl number is defined as $Pr = \nu/\alpha$ and α is the thermal diffusitivity.

For turbulent flow, $x_{fd,t} \approx 10D$.

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Thermal Conditions

- For a fully developed pipe flow, the convection coefficient does not vary along the pipe length. (provided all thermal and flow properties are constant)



- Newton's law of cooling: $q''_s = hA(T_S - T_m)$

Question: since the temperature inside a pipe flow does not remain constant, we should use a mean temperature T_m, which is defines as follows:

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Energy Transfer

Consider the total thermal energy carried by the fluid as

$$\int_A \rho V C_v T dA = (\text{mass flux})\ (\text{internal energy})$$

Now imagine this same amount of energy is carried by a body of fluid with the same mass flow rate but at a uniform mean temperature T_m. Therefore T_m can be defined as

$$T_m = \frac{\int_A \rho V C_v T dA}{\dot{m} C_v}$$

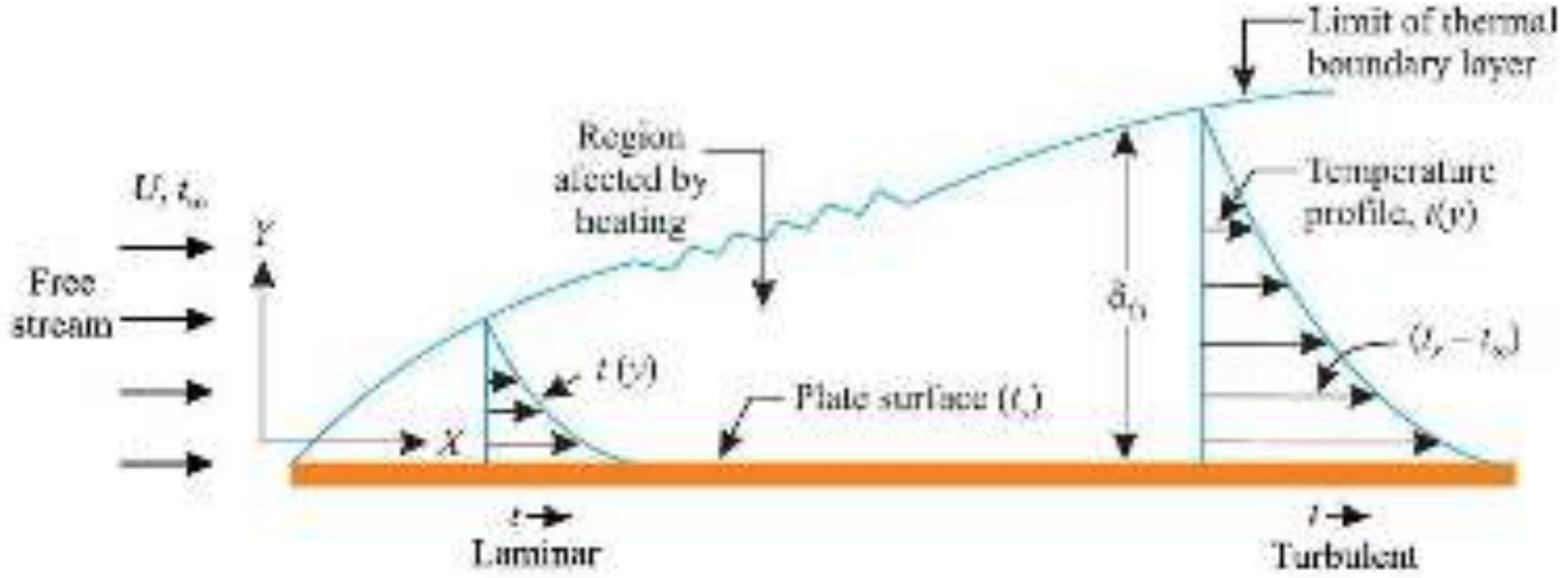
Consider T_m as the reference temperature of the fluid so that the total heat transfer between the pipe and the fluid is governed by the Newton's cooling law as: $q_s'' = h(T_s - T_m)$, where h is the local convection coefficient, and T_s is the local surface temperature. Note: usually T_m is not a constant and it varies along the pipe depending on the condition of the heat transfer.



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THERMAL BOUNDARY LAYER

Whenever a flow of fluid takes place past a heated or cold surface, a temperature field is set up in the field next to the surface. If the surface of the plate is hotter than fluid, the temperature distribution will be as shown in the Fig. *The zone or this layer wherein the temperature field exists is called the* **thermal boundary layer**. Due to the exchange of heat between the plate and the fluid, temperature gradient occurs/results.

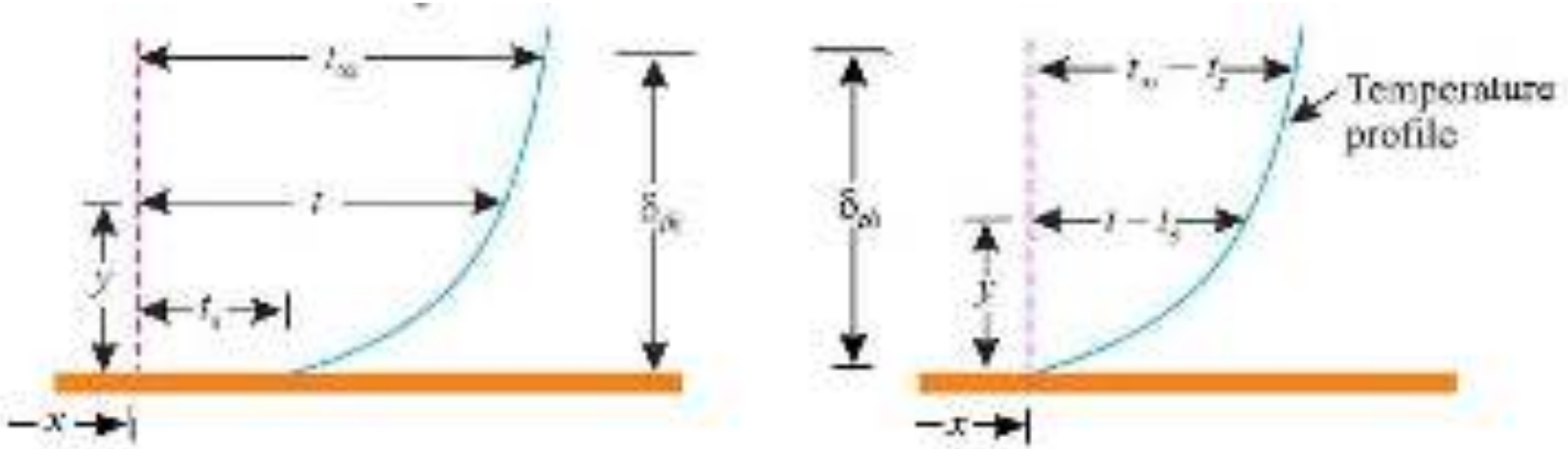


Thermal boundary layer formed during flow of cool fluid over a warm plate.

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The thermal boundary layer thickness (δ_{th}) is arbitrarily defined as the distance y from the plate surface at which

$$\frac{t_s - t}{t_s - t_\infty} = 0.99$$



Thermal boundary layer formed flow of warm fluid over a cool plate.

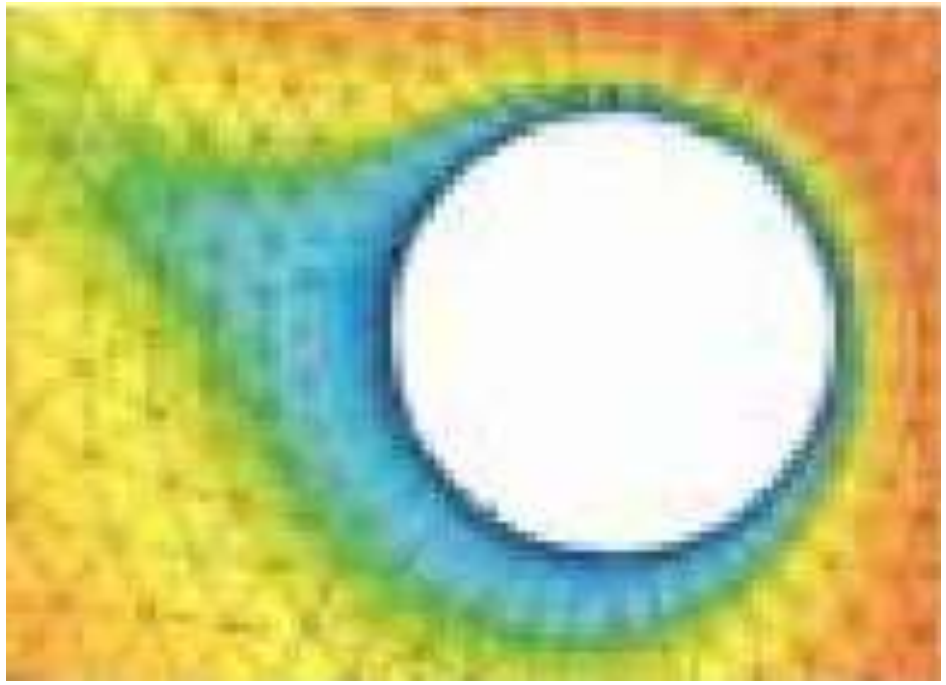


dary layer at the coil surface.

Figure shows the shape of the thermal boundary layer when the free stream temperature t_∞ is above the plate surface temperature t_s.

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The thermal boundary layer concept is analogous to hydrodynamic boundary layer; the parameters which affect their growth are however different. Whereas the velocity profile of the hydrodynamic boundary layer depends mainly on the fluid viscosity, the temperature profile of the thermal boundary layer depends upon the viscosity, velocity of flow, specific heat and thermal conducitivity of the fluid. The relative magnitude of δ and δ_{th} are affected by the thermo-physical properties of the



fluid; the governing parameter, however, is the non-dimensional Prandtel number, $Pr = \frac{\mu c_p}{k}$.

(i) $\delta_{th} = \delta$ ..... when $Pr = 1$; (ii) $\delta_{th} < \delta$ .....when $Pr > 1$

$\delta_{th} > \delta$ ..... when $Pr < 1$.

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List of Formulas used for Force Convection Heat Transfer

Non-dimensional Groups

1. Nusselt number: non-dimensional heat transfer coefficient

$$Nu = \frac{h\delta}{k} = \frac{q^{\bullet}_{conv}}{q^{\bullet}_{cond}}$$

where δ is the characteristic length, i.e. D for the tube and L for the flat plate. Nusselt number represents the enhancement of heat transfer through a fluid as a result of convection relative to conduction across the same fluid layer.

2. Reynolds number

$$\mathrm{Re} = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho V \delta}{\mu} = \frac{V\delta}{\nu}$$

At large *Re* numbers, the inertia forces, which are proportional to the density and the velocity of the fluid, are large relative to the viscous forces; thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (turbulent regime).

The Reynolds number at which the flow becomes turbulent is called the critical *Reynolds number*. For flat plate the critical Re is experimentally determined to be approximately *Re* critical = 5×10^5.

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List of Formulas used for Force Convection Heat Transfer

3. Prandtl number: is a measure of relative thickness of the velocity and thermal boundary layer

$$Pr = \frac{\text{molecular diffusivity of momentum}}{\text{molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

where fluid properties are:

mass density : ρ, (kg/m^3)

specific heat capacity : C_p, $(J/kg \cdot K)$

dynamic viscosity : μ, $(N \cdot s/m^2)$

kinematic viscosity : ν, μ / ρ (m^2/s)

thermal conductivity : k, $(W/m \cdot K)$

thermal diffusivity : α, $k/(\rho \cdot C_p)$ (m^2/s)

4. Thermal Boundary Layer

Similar to velocity boundary layer, a thermal boundary layer develops when a fluid at specific temperature flows over a surface which is at different temperature

The thickness of the thermal boundary layer δt is defined as the distance at which:

$$\frac{T - T_s}{T_\infty - T_s} = 0.99$$

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List of Formulas used for Force Convection Heat Transfer

- ✓ The relative thickness of the velocity and the thermal boundary layers is described by the Prandtl number.

- ✓ For low Prandtl number fluids, i.e. liquid metals, heat diffuses much faster than momentum flow (remember $Pr = \nu/\alpha << 1$) and the velocity boundary layer is fully contained within the thermal boundary layer. On the other hand, for high Prandtl number fluids, i.e. oils, heat diffuses much slower than the momentum and the thermal boundary layer is contained within the velocity boundary layer.

Flow Over Flat Plate

- ✓ The friction and heat transfer coefficient for a flat plate can be determined by solving the conservation of mass, momentum, and energy equations (either approximately or numerically). They can also be measured experimentally. It is found that the Nusselt number can be expressed as:

$$Nu = \frac{hL}{k} = C\,Re_L^m\,Pr^n$$

where *C, m*, and *n* are constants and *L* is the length of the flat plate. The properties of the fluid are usually evaluated at the *film temperature* defined as:

$$T_f = \frac{T_s + T_\infty}{2}$$

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Flow Over Flat Plate

List of Formulas used for Force Convection Heat Transfer

Laminar Flow

The local friction coefficient and the Nusselt number at the location *x* for laminar flow over a flat plate are

$$Nu_x = \frac{hx}{k} = 0.332 Re_x^{1/2} Pr^{1/3} \quad Pr \geq 0.6$$

$$C_{f,x} = \frac{0.664}{Re_x^{1/2}}$$

where *x* is the distant from the leading edge of the plate and $Rex = \rho V_\infty x / \mu$.

The *averaged* friction coefficient and the Nusselt number over the entire isothermal plate for laminar regime are:

$$Nu = \frac{hL}{k} = 0.664 Re_L^{1/2} Pr^{1/3} \quad Pr \geq 0.6$$

$$C_f = \frac{1.328}{Re_L^{1/2}}$$

Taking the critical Reynolds number to be 5 x10$_5$, the length of the plate X_{cr} over which the flow is laminar can be determined from

$$Re_{cr} = 5 \times 10^5 = \frac{V_\infty x_{cr}}{\nu}$$

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List of Formulas used for Force Convection Heat Transfer

Turbulent Flow

The local friction coefficient and the Nusselt number at location *x* for turbulent flow over a flat isothermal plate are:

$$Nu_x = \frac{hx}{k} = 0.0296\,Re_x^{4/5}\,Pr^{1/3} \qquad 0.6 \le Pr \le 60 \qquad 5\times10^5 \le Re_x \le 10^7$$

$$C_{f,x} = \frac{0.0592}{Re_x^{1/5}} \qquad 5\times10^5 \le Re_x \le 10^7$$

The averaged friction coefficient and Nusselt number over the isothermal plate in turbulent region are:

$$Nu = \frac{hL}{k} = 0.037\,Re_x^{4/5}\,Pr^{1/3} \qquad 0.6 \le Pr \le 60 \qquad 5\times10^5 \le Re_L \le 10^7$$

$$C_f = \frac{0.074}{Re_L^{1/5}} \qquad 5\times10^5 \le Re_L \le 10^7$$

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Boundary layer parameters for different velocity profiles

S. No.	Velocity profile	Boundary conditions		δ	$\overline{C_f}$
		At $y = 0$	At $y = \delta$		
1.	$\frac{u}{U} = \frac{y}{\delta}$	$u = 0$	$u = U$	$\frac{3.46x}{\sqrt{Re_x}}$	$\frac{1.155}{\sqrt{Re_L}}$
2.	$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$	$u = 0$	$u = U$ $\frac{\partial u}{\partial y} = 0$	$\frac{5.48x}{\sqrt{Re_x}}$	$\frac{1.46}{\sqrt{Re_L}}$
3.	$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$	$u = 0$ $\frac{d^2u}{dy^2} = 0$	$u = U$ $\frac{du}{dy} = 0$	$\frac{4.64x}{\sqrt{Re_x}}$	$\frac{1.292}{\sqrt{Re_L}}$
4.	$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$	$u = 0$	$u = U$	$\frac{4.795x}{\sqrt{Re_x}}$	$\frac{1.31}{\sqrt{Re_L}}$
5.	Blasius exact solution			$\frac{5x}{\sqrt{Re_x}}$	$\frac{1.328}{\sqrt{Re_L}}$

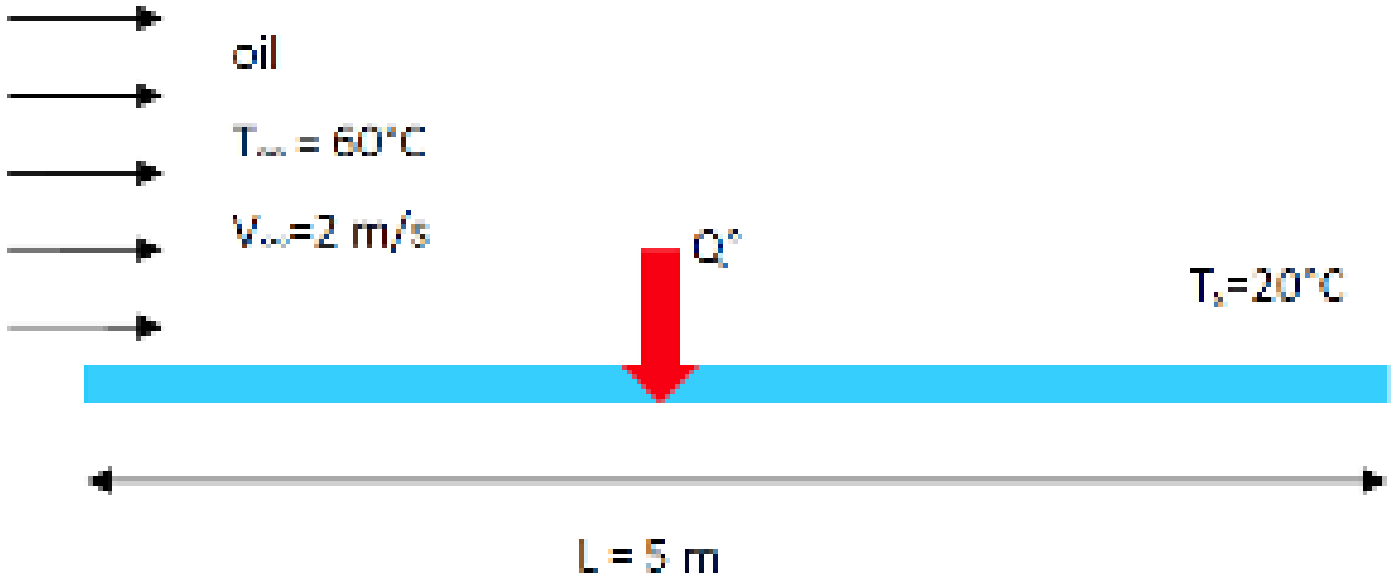
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Example 1

Engine oil at 60°C flows over a 5 m long flat plate whose temperature is 20°C with a velocity of 2 *m/s*. Determine the total drag force and the rate of heat transfer per unit width of the entire plate.

Solution:

We assume the critical Reynolds number is 5×10^5. The properties of the oil at the film temperature are:



$$T_f = \frac{T_s + T_\infty}{2} = 40^\circ C$$

$$\rho = 876\ kg/m^3$$

$$= 0.144\ W/(m.K)$$

$$= 2870$$

$$= 242 \times 10^{-6}\ m^2/s$$

The Re number for the plate is:

$$Re_L = V_\infty L/\nu = 4.13 \times 10^6$$

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which is less than the critical Re. Thus we have laminar flow. The friction coefficient and the drag force can be found from:

$$C_f = 1.328\,\mathrm{Re}_L^{-0.5} = 0.00653$$

$$F_D = C_f A \frac{\rho V_\infty^2}{2} = 0.00653 \times \left(5 \times 1m^2\right) \frac{\left(876kg/m^3\right)\left(2m/s\right)^2}{2} = 57.2N$$

The Nusselt number is determined from:

$$Nu = \frac{hL}{k} = 0664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} = 1918$$

Then,

$$h = 55.2 \frac{W}{m^2 K}$$

$$\dot{Q} = hA(T_\infty - T_s) = 11040W$$



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Example: *The velocity profile for laminar boundary is in the form given below :*

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Find the thickness of boundary layer at the end of the plate 1.5 m long and 1 m wide when placed in water flowing with a velocity of 0.12 m/s. Calculate the value of coefficient of drag also.

Take μ for water = 0.001 N-s/m^2

Solution:

. Velocity distribution $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

The length of the plate, $L = 1.5$ m

The width of the plate, $B = 1$ m

Free stream velocity, $U = 0.12$ m/s

μ for water = 0.001 N-s/m^2

Thickness of the boundary layer, δ :

Reynolds number at the end of the plate (*i.e.*, at a distance of 1.5 m from the leading edge) is given

$$Re_L = \frac{\rho U L}{\mu} = \frac{1000 \times 0.12 \times 1.5}{0.001} = 180000$$

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Since $Re_L < 5 \times 10^5$, therefore, this is case of laminar boundary layer. Thickness of boundary layer at a distance of 1.5 m is given by

$$\delta = \frac{5.48\,x}{\sqrt{Re_x}}$$

$$= \frac{5.48 \times 1.5}{\sqrt{180000}} = 0.01937 \text{ m or } 19.37 \text{ mm}$$

Coefficient of drag $\overline{C}_f$:

$$\overline{C}_f = \frac{1.46}{\overline{C}_f} = \frac{1.46}{\sqrt{180000}} = \mathbf{0.00344} \text{ (Ans.)}$$



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Example *A plate of length 750 mm and width 250 mm has been placed longitudinally in a stream of crude oil which flows with a velocity of 5 m/s. If the oil has a specific gravity of 0.8 and kinematic viscosity of 1 stoke, calculate :*

(i) Boundary layer thickness at the middle of plate,

(ii) Shear stress at the middle of plate, and

(iii) Friction drag on one side of the plate.

Solution. Length of the plate, $L = 750 \text{ mm} = 0.75 \text{ m}$

Width of the plate, $B = 250 \text{ mm} = 0.25 \text{ m}$

Velocity of oil, $U = 5 \text{ m/s}$

Specific gravity of oil $= 0.8$

Kinematic viscosity of oil, $\nu = 1 \text{ stoke} = 1 \times 10^{-4} \text{ m}^2/\text{s}$

(i) Boundary layer thickness at middle of the plate δ :

Reynold number, $$Re_x = \frac{\rho U x}{\mu} = \frac{U x}{\nu} = \frac{5 \times (0.375)}{1 \times 10^{-4}} = 18750$$

($\because$ At the middle of plate, $x = 0.75/2 = 0.375$ m)

$\times 10^5$, therefore, boundary layer is of laminar character and Blasius solution gives

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times (0.375)}{18750} = 0.01369 \text{ m} = \mathbf{13.69 \text{ mm}}$$

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(ii) Shear stress at the middle of plate, τ_0 :

According to Blasius, the local coefficient of drag (C_{fx}) is given by

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} = \frac{0.664}{\sqrt{18750}} = 4.849 \times 10^{-3}$$

By definition, $$C_{fx} = \frac{\tau_0}{\frac{1}{2}\rho U^2}$$

$$\tau_0 = C_{fx} = \frac{1}{2}\rho U^2$$

$$= 4.849 \times 10^{-3} \times \frac{1}{2} \times (0.8 \times 1000) \times 5^2 = \mathbf{48.49\ N/m^2\ (Ans.)}$$

(iii) Friction drag on one side of the plate, F_D :

$$Re_L = \frac{UL}{\nu} = \frac{5 \times 0.75}{1 \times 10^{-4}} = 37500$$

As the boundary layer is laminar even at the trailing edge, therefore, the *average drag (friction) coefficient*,

$$\overline{C}_f = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{37500}} = 6.858 \times 10^{-3}$$

∴ Friction drag, $$F_D = \overline{C}_f \times \frac{1}{2}\rho U^2 \times \text{area of plate on one side}$$

$$= 6.858 \times 10^{-3} \times \frac{1}{2} \times (0.8 \times 1000) \times 5^2 \times 0.75 \times 0.25$$

$$= \mathbf{12.86\ N\ (Ans.)}$$

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Example *Air at 20°C and at a pressure of 1 bar is flowing over a flat plate at a velocity of 3 m/s. If the plate is 280 mm wide and at 56°C, calculate the following quantities at x = 280 mm, given that properties of air at the bulk mean temperature* $\left(\frac{20+56}{2}\right) = 38°C$ *are :*

$\rho = 1.1374\ kg/m^3$; $k = 0.02732\ W/m°C$; $c_p = 1.005\ kJ/kgK$; $\nu = 16.768 \times 10^{-6}\ m^2/s$; $Pr = 0.7$.

(i) *Boundary layer thickness,*
(ii) *Local friction coefficient,*
(iii) *Average friction coefficient,*
(iv) *Shearing stress due to friction,*
(v) *Thickness of the boundary layer,*
(vi) *Local convective heat transfer coefficient,*
(vii) *Average convective heat transfer coefficient,*
(viii) *Rate of heat transfer by convection,*
(ix) *Total drag force on the plate, and*
(x) *Total mass flow rate through the boundary.*

ution. *Given :* $U = 3$ m/s, $x = 280$ mm $= 0.28$ m, $\rho = 1.1374$ kg/m^3, $k = 0.02732$ W/m°C, $c_p =$ /kgK, $\nu = 16.768 \times 10^{-6}$ m^2/s.

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Let us first ascertain the type of the flow, whether laminar or turbulent.

$$Re_x = \frac{Ux}{\nu} = \frac{3 \times 0.28}{16.768 \times 10^{-6}} = 5.0 \times 10^4$$

Since $Re_x < 5 \times 10^5$, hence flow in *laminar*.

(i) Boundary layer thickness at $x = 0.28$ m, δ :

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

$$\delta = \frac{5 \times 0.28}{\sqrt{5 \times 10^4}} = 0.00626 \text{ m or } \mathbf{6.26 \text{ mm (Ans.)}}$$

(ii) Local friction coefficient, C_{fx} :

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$$

$$C_{fx} = \frac{0.664}{\sqrt{5 \times 10^4}} = \mathbf{0.002969 \text{ (Ans.)}}$$

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(iii) **Average friction coefficient, C_f :**

$$\overline{C}_f = \frac{1.328}{\sqrt{Re_L}}$$

$$\overline{C}_f = \frac{1.328}{\sqrt{5 \times 10^4}} = 0.005939 \text{ (Ans.)}$$

(iv) **Shearing stress due to friction, τ_0 :**

$$\tau_0 = C_{fx} \times \frac{\rho U^2}{2}$$

$$= 0.002969 \times \frac{1.1374 \times 3^2}{2} = 0.01519 \text{ N/m}^2 \text{ (Ans.)}$$

(v) **Thickness of thermal boundary layer, δ_t :**

$$\delta_t = \frac{\delta}{(Pr)^{1/3}}$$

$$= \frac{0.00626}{(0.7)^{1/3}} = 0.00705 \text{ m or } 7.05 \text{ mm (Ans.)}$$

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(vi) Local convective heat transfer coefficient, h_x :

$$h_x = 0.332 \frac{k}{x} (Re_x)^{1/2} (Pr)^{1/3}$$

$$= 0.332 \times \frac{0.02732}{0.28} \times (5 \times 10^5)^{1/2} \times (0.7)^{1/3}$$

$$= \mathbf{6.43\ W/m^2{}^\circ C\ (Ans.)}$$

(vii) Average convective heat transfer coefficient, $\bar{h}$:

$$\bar{h} = 0.664 \left(\frac{k}{L}\right) (Re_L)^{1/2} (Pr)^{1/3}$$

$$= 0.664 \left(\frac{0.02732}{0.28}\right) (5 \times 10^5)^{1/2} (0.7)^{1/3} = \mathbf{12.86\ W/m^2{}^\circ C\ (Ans.)}$$

(viii) Rate of heat transfer by convection, Q_{conv} :

$$Q_{conv} = \bar{h} A_s (t_s - t_\infty)$$

$$= 12.85 \times (0.28 \times 0.28)\ (56 - 20) = \mathbf{36.29\ W\ (Ans.)}$$

(ix) Total drag force on the plate, F_D :

$$F_D = \tau_0 \times \text{ area of plate on one side upto } 0.28 \text{ m}$$

$$= 0.01519 \times 0.28 \times 0.28 = \mathbf{0.00119\ N\ (Ans.)}$$

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(x) Total mass flow rate through the boundary, m :

$$m = \frac{5}{8} \rho U (\delta_2 - \delta_1)$$

(where $\delta_1 = 0$ at $x = 0$ and $\delta_2 = \delta$ at $x = 0.28$ m)

$$= \frac{5}{8} \times 1.1374 \times 3 (0.00626 - 0) = \mathbf{0.01335\ kg/s\ (Ans.)}$$



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Example *A barge with a rectangular surface 30 m long × 10 m wide is travelling down a river with a velocity of 0.6 m/s. A laminar boundary layer exists upto a Reynolds number equivalent to 5×10^5 and subsequently abrupt transition occurs to turbulent boundary layer. Calculate :*

(i) The maximum distance from the leading edge upto which laminar boundary layer persists and the maximum boundary layer thickness at that point.

(ii) The total drag force on the flat bottom surface of the barge, and

(iii) The power required to push the bottom surface through water at the given velocity.

For water $\rho = 998$ kg/m^3 and $\nu = 1 \times 10^{-6}$ m^2/s.

Solution. Length of the bottom surface, L = 30 m

Width of the bottom surface, $B = 10$ m

$\therefore$ Area, $A = L \times B = 30 \times 10 = 300$ m^2

Velocity, $U = 0.6$ m/s

Density of water, $\rho = 998$ kg/m^3

viscosity, $\nu = 1 \times 10^{-6}$ m^2/s

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(i) The maximum distance up to which laminar boundary layer persists, x_c :

$$(Re)_{x_c} = \frac{Ux_c}{\nu} = 5 \times 10^5$$

$$x_c = \frac{5 \times 10^5 \times \nu}{U} = \frac{5 \times 10^5 \times 1 \times 10^{-6}}{0.6} = 0.833 \text{ m} \quad \text{(Ans.)}$$

Maximum boundary layer (luminar) thickness, δ :

$$\delta = \frac{5x_c}{\sqrt{(Re)_{x_c}}} = \frac{5 \times 0.833}{\sqrt{5 \times 10^5}} = 5.89 \times 10^{-3} \text{ m} \quad \text{or,} \quad \textbf{5.89 mm (Ans.)}$$

(ii) The total drag force F_D :

$$Re_L = \frac{UL}{\nu} = \frac{0.6 \times 30}{1 \times 10^{-6}} = 1.8 \times 10^7$$

The average coefficient of drag,

$$\bar{C}_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} - \frac{1670}{Re_L}$$

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The average coefficient of drag,

$$\bar{C}_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} - \frac{1670}{Re_L}$$

$$= \frac{0.455}{[\log_{10} (1.8 \times 10^7)]^{2.58}} - \frac{1670}{1.8 \times 10^7} = 0.002646$$

$\therefore$ Drag force on the bottom surface of the barge,

$$F_D = \bar{C}_f \times \frac{1}{2} \rho A U^2 = 0.002646 \times \frac{1}{2} \times 998 \times 300 \times (0.6)^2$$

$$= \mathbf{142.6\ N} \quad \textbf{(Ans.)}$$

(iii) **The power required, *P* :**

The power required to push the bottom surface through water at the given velocity,

$$P = F_D \times U = 142.6 \times 0.6 = \mathbf{85.56\ W} \quad \textbf{(Ans.)}$$



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EMPIRICAL CORRELATIONS FOR FORCED CONVECTION

The following dimensionless numbers are used for the usual forced convection problems :

(*i*) *Nusselt number,* $Nu = \frac{hL}{k}$; (*ii*) *Reynolds number,* $Re = \frac{\rho LU}{\mu}$

(*iii*) *Prandtl number,* $Pr = \frac{\mu c_p}{k}$; (*iv*) *Stanton number,* $St = \frac{h}{\rho c_p U}$

In order to determine the value of convection coefficient h, the following conventional generalised basic equations are used :

$$Nu = f_1 (Re, Pr) = C_1 (Re)^m (Pr)^n \; ; \; St = f_2 (Re, Pr) = C_2 (Re)^a (Pr)^b$$

The values (numerical) of the constants and exponents are determined through experiments. The properties of the fluid are evaluated on the basis of bulk temperature (unless stated otherwise).

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EMPIRICAL CORRELATIONS FOR FORCED CONVECTION

Laminar Flow

LAMINAR FLOW OVER FLAT PLATES AND WALLS

(*a*) The *local value of heat transfer coefficient* is given by

$$Nu_x = \frac{h_x x}{k} = 0.332\ (Re_x)^{0.5}\ (Pr)^{0.333} \qquad \text{...Blasius equation}$$

The *average value of heat transfer coefficient* is given by

$$Nu = \frac{\bar{h}L}{k} = 0.664\ (Re_L)^{0.5}\ (Pr)^{0.333}$$

where, $Re_x = \frac{Ux}{\nu}$, $Re_L = \frac{UL}{\nu}$ and $Pr = \frac{\mu\, c_p}{k}$

The above equations are valid for the following :

(*i*) All fluids ($Pr \geq 0.6$) except liquid metals

(*ii*) Reynolds number $Re \geq 40000$

(*iii*) The fluid properties are evaluated at the mean film temperature, $t_f = \frac{t_s + t_\infty}{2}$

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Laminar Flow

(*b*) For *liquid metals*, the following correlation has been proposed

$$Nu_x = 0.565\ (Pe_x)$$

where $Pe_x = Re_x \cdot Pr$

The above equation is valid for the following :

(*i*) $Pr \leq 0.05$

(*ii*) The fluid properties are evaluated at the film temperature.

Example *Air at a temperature of 15°C flows at a velocity of 6.5 m/s across a flat plate maintained at a temperature of 605°C. Calculate the amount of heat transferred per metre width from both sides of the plate over a distance of 350 mm from the leading edge. The following relation holds good in the case of large temperature difference between the plate and the fluid :*

$$Nu_x = 0.332\ (Pr)^{1/3}\ (Re)^{1/2} \left(\frac{T_s}{T_\infty}\right)^{0.117}$$

and T_∞ are the absolute temperatures of the plate surface and free stream respectively and properties are evaluated at the mean film temperature.

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Laminar Flow

Solution. *Given :* $T_\infty = 15 + 273 = 288$ K, $T_s = 605 + 273 = 878$ K, $x = 350$ mm $= 0.35$ m.

Amount of heat transferred per unit width :

The mean film temperature, $t_f = \dfrac{605 + 15}{2} = 310°C$

The thermo-physical properties, at 310°C are : $\rho = 0.614$ kg/m^3; $c_p = 1.046$ kJ/kg°C

$k = 0.04593$ W/m°C; $\mu = 29.7 \times 10^{-6}$ kg/ms; $Pr = 0.675$

Reynolds number, $Re_x = \dfrac{\rho\, U x}{\mu} = \dfrac{0.614 \times 6.5 \times 0.35}{29.7 \times 10^{-6}} = 47032$

The Nusselt number is given as:

$$Nu_x = 0.332\,(Pr)^{1/3}\,(Re)^{1/2}\left(\frac{T_s}{T_\infty}\right)^{0.117}$$

$$= 0.332\,(0.675)^{0.333}\,(47032)^{0.5}\left(\frac{878}{288}\right)^{0.117} = 71.96$$

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Laminar Flow

or, $$\frac{h_x\, x}{k} = 71.96$$

or, $$h_x = \frac{k}{x} \times 71.96 = \frac{0.04593}{0.35} \times 71.96 = 9.44 \text{ W/m}^2\text{°C}$$

The average heat transfer coefficient is given by

$$\bar{h} = \frac{1}{L} \int_0^L h_x\, dx = 2h_x = 2 \times 9.44 = 18.88 \text{ W/m}^2\text{°C}$$

Heat transfer from both sides of the plate, per metre width,

$$Q = 2\,[\bar{h}\, A_s (t_s - t_\infty)] = 2[18.88 \times (0.35 \times 1)\,(605 - 15)]$$

$$= \mathbf{7797.44\ W} \qquad \textbf{(Ans.)}$$



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EMPIRICAL CORRELATIONS FOR FORCED CONVECTION

LAMINAR FLOW INSIDE TUBES

(*a*) For *fully developed flow* : $f = \frac{64}{Re}$

(*b*) For *uniform heat flux* : $Nu = 4.36$

(*c*) For *constant wall temperature* :

Slug flow : $Nu = 5.78$

Fully developed flow : $Nu = 3.66$

(*d*) For average Nusselt number for flow inside tubes, the following correlations, have been developed :

1. $$Nu = \frac{hD}{k} = 3.66 + \frac{0.0668\,(D/L)\,Re.Pr}{1 + 0.04\,[(D/L)\,Re.Pr]^{2/3}} \qquad ...(7.160)$$

This equation is valid

(*i*) for constant wall temperature and fully developed flow,

when properties are evaluated at the bulk temperature,

tube length is much greater than diameter,

when *Re* and *Nu* are calculated on the basis of pipe diameter as the length parameter.

when $0.5 < Pr < 10$).

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LAMINAR FLOW INSIDE TUBES

2. $$Nu = \frac{hD}{k} = 1.86 \left[\frac{D}{L} Re \,.\, Pr\right]^{1/3} \left[\frac{\mu}{\mu_s}\right]^{0.14}$$

This equation is valid for

(*i*) short tubes $\left(\frac{L}{D} > 2\right)$ (*ii*) $Re < 2100$

(*iii*) $0.48 < Pr < 16700$ (*iv*) $0.0044 < (\mu/\mu_s) < 9.75$

(*v*) In this equation all properties except μ_s are evaluated at bulk temperature; μ_s is evaluated at surface temperature.



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EMPIRICAL CORRELATIONS FOR FORCED CONVECTION

LAMINAR FLOW INSIDE TUBES

Example *Gasoline at the mean bulk temperature of 27°C flows inside a circular tube of 19 mm inside diameter. The average bulk velocity is 0.061 m/s and the tube is 1.5 m long. The flow starts at the heated tube inlet (no upstream developing section) and the tube surface temperature is constant at 38°C.*

Determine the average heat transfer co-efficient over 1.5 m length of the tube. Are the temperature and velocity profiles developing or, developed in the 1.5 m length of the tube ?

Take the following fluid properties :

$$\mu_s = 5.223 \times 10^{-4}\ Pa\ s;\ \mu_b = 5.892 \times 10^{-4}\ Pa\ s;\ k_b = 0.1591\ W/m°C;$$

$$\rho_b = 876.6\ kg/m^3;\ c_{pb} = 1757\ J/kg\ K;\ Pr_b = 6.5$$

Use the following formula :

$$Nu = 1.86 \left[\frac{Re_b \cdot Pr_b}{L/D}\right]^{0.33} \left[\frac{\mu_b}{\mu_s}\right]^{0.14}$$

x_v (entry length for fully developed velocity profile) $= 0.05\ Re.D$, and

x_t (entry length for fully developed temperature profile) $= x_v \cdot Pr_b$

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Solution. *Given :* $D = 19 \text{ mm} = 0.019 \text{ m}$; $U = 0.061 \text{ m/s}$; $L = 1.5 \text{ m}$

$$Re_b = \frac{\rho_b U D}{\mu_b} = \frac{874.6 \times 0.061 \times 0.019}{5.892 \times 10^{-4}} = 1720.4 \qquad (1 \text{ Pa s} = 1 \text{ N-s/m}^2)$$

$$Nu = 1.86 \left[\frac{Re_b \,.\, Pr_b}{L/D}\right]^{0.33} \left[\frac{\mu_b}{\mu_s}\right]^{0.14}$$

$$= 1.86 \left[\frac{1720.4 \times 6.5}{1.5/0.019}\right]^{0.33} \left[\frac{5.892 \times 10^{-4}}{5.223 \times 10^{-4}}\right]^{0.14} = 1.86 \times 5.127 \times 1.017 = 9.698$$

$$Nu = \frac{\bar{h}D}{k} = 9.698 \qquad \text{or,} \qquad \bar{h} = \frac{k}{D} \times 9.698 = \frac{0.1591}{0.019} \times 9.698 = \mathbf{81.21 \text{ W/m}^2\text{°C}} \qquad \textbf{(Ans.)}$$

x_v (velocity depth) $= 0.05\, ReD = 0.05 \times 1720.4 \times 0.019 = \mathbf{1.634 \text{ m}}$

x_t (temperature depth) $= x_v \times Pr_b = 1.634 \times 6.5 = \mathbf{10.62 \text{ m}}$

The tube length L is less than x_v as well as x_t, *so both depths are developing.*

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EMPIRICAL CORRELATIONS FOR FORCED CONVECTION

TURBULENT FLOW OVER FLAT PLATE

The local and average Nusselt numbers, from Colburn analogy, are given by :

$$Nu_x = 0.029\ (Re_x)^{0.8}\ (Pr)^{1/3}$$

$$\overline{Nu} = 0.036\ (Re_L)^{0.8}\ (Pr)^{1/3}$$

where the properties are evaluated at the mean film temperature.

When the flow lies in the *transition range*,

$$\overline{Nu} = 0.036\ [(Re_L)^{0.8} - A]\ (Pr)^{1/3}$$

where $A = 18700$ when $Re_c = 4 \times 10^5$

$A = 23100$ when $Re_c = 5 \times 10^5$

TURBULENT FLOW IN TUBES

The following correlations for *fully developed turbulent flow in circular tubes* have been proposed :

. Mc Adams has proposed the following general correlation on the basis of $Nu = \phi\ (Re, Pr)$ for heating and cooling of all fluids with some *limitations* as given below :

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EMPIRICAL CORRELATIONS FOR FORCED CONVECTION

TURBULENT FLOW IN TUBES

$$\overline{Nu} = 0.023\,(Re)^{0.8}\,(Pr)^{n}$$

where $n = 0.4$ ... for heating

$n = 0.3$ ... for cooling

(i) $\frac{L}{D} \geq 60$

(ii) $1 \times 10^4 < Re < 12 \times 10^4$

(iii) $0.7 \leq Pr \leq 160$

(iv) Fluid properties are evaluated at the mean bulk temperature.

2. Colburn suggested the following correlation :

$$Nu = 0.023\,(Re)^{0.8}\,(Pr)^{1/3}$$

$$St = 0.023\,(Re)^{-0.2}\,(Pr)^{2/3}$$

It is valid for the following :

) St is evaluated at the mean bulk temperature.

) Re and Pr are evaluated at the mean film temperature.

) $\frac{L}{D} \geq 60$, $Re \geq 10^4$, $0.7 < Pr < 160$.

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TURBULENT FLOW IN TUBES

3. The Mc Adams and Colburn correlations are fairly accurate for small to moderate temperature difference (10°C in case of liquids and 50°C for gases). For larger temperature differences, Sieder and Tate proposed the following correlation :

$$\overline{Nu} = 0.023\,(Re)^{0.8}\,(Pr)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

This equation is valid for the following :

(i) $\frac{L}{D} > 60$, $Re \geq 10^4$, $0.7 \leq Pr \leq 16700$

(ii) All fluid properties except μ_s are evaluated at the mean bulk temperature. μ_s is evaluated at the surface temperature.

4. Desman and Sams suggested the following equation for very large temperature difference $(t_s - t_b)$ with air :

$$0.026\,(Re)^{0.8}\,(Pr)^{0.4}$$

The equation is valid for the following :

(i) $\frac{t_s}{t_b}$ upto 3.55

(ii) $Re \geq 10^4$

(iii) Re is evaluated at mean film temperature

(iv) Nu and Pr are evaluated at mean bulk temperature.

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Example *A surface condenser consists of two hundred thin walled circular tubes (each tube is 22.5 mm in diameter and 5 m long) arranged in parallel, through which water flows. If the mass flow rate of water through the tube bank is 160 kg/s and its inlet and outlet temperatures are known to be 21°C and 29°C respectively, calculate the average heat transfer coefficient associated with flow of water.*

Solution. *Given :* $D = 22.5 \text{ mm} = 0.0225 \text{ m}$, $L = 5 \text{ m}$, $t_i = 21°C$, $t_o = 29°C$.

Average heat transfer coefficient, $\bar{h}$:

The mean bulk temperature, $$t_b = \frac{t_i + t_o}{2} = \frac{21 + 29}{2} = 25°C$$

The thermo-physical properties of water at 25°C are :

$\rho = 996.65 \text{ kg/m}^3$; $\mu = 0.862 \times 10^{-3} \text{ kg/ms}$;

$k = 0.6079 \text{ W/m°C}$; $c_p = 4.178 \text{ kJ/kg°C}$

Prandtl number, $$Pr = \frac{\mu c_p}{k} = \frac{0.862 \times 10^{-3} \times (4.178 \times 10^3)}{0.6079} = 5.924$$

The mass flow rate of water through each tube,

$$m = \frac{160}{200} = 0.8 \text{ kg/s}$$

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Reynolds number, $Re = \frac{\rho UD}{\mu} = \frac{\rho AUD}{A\mu} = \frac{mD}{A\mu} = \frac{mD}{\frac{\pi}{4}D^2 \times \mu} = \frac{4m}{\pi D\mu}$

$$= \frac{4 \times 0.8}{\pi \times 0.0225 \times 0.862 \times 10^{-3}} = 52518.26$$

As $Re >> 2500$, therefore, the nature of flow is *turbulent*. Since the water is being heated in condenser, hence we may use McAdams correlation

$$\overline{Nu} = 0.023\ (Re)^{0.8}\ (Pr)^{0.4}$$

$$= 0.023\ (52518.26)^{0.8}\ (5.924)^{0.4} = 279.9$$

or, $$\overline{Nu} = \frac{\bar{h}D}{k} = 279.9$$

$\therefore$ $$\bar{h} = \frac{k}{D} \times 279.9 = \frac{0.6079}{0.0225} \times 279.9 = \mathbf{7562.27\ W/m^2{}^\circ C}$$

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TURBULENT FLOW OVER CYLINDERS

The case of single cylinder in cross flow is identical to:

(*i*) The case of cooling of an electrical cable by forced convection by air flowing across it;

(*ii*) The determination of local velocities in a flow by hot wire anemometer which involves the heat transfer from a single platinum wire maintained at a constant temperature (or by passing a constant current through it) and correlating the change in current (or change in resistance) to flow velocity.

In order to calculate the Reynolds number (Re), the characteristic length taken in case of cylinder and sphere as well, is its external diameter D.

As usual, $$Re_D = \frac{UD}{\nu}$$

where, U = Uniform velocity of flow as it approaches the cylinder (or sphere).

In case of flow across cylinder (or sphere) the critical Reynolds number is $Re_{cr} = 2 \times 10^5$, *upto* this value *laminar* boundary layer exists and *beyond* this boundary layer becomes *turbulent*.

Empirical correlations used for turbulent flow (cross flow) over "Cylinders":

1. The following *empirical correlation* is widely used for turbulent flow over cylinders :

$$\overline{Nu} = \frac{hD}{k} = C\,(Re)^n\,(Pr)^{1/3}$$

C and n are constants and have the values as given in the table

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C and n are constants and have the values as given in the table

S.No.	Re	C	n
1.	0.4 to 4	0.989	0.330
2.	4 to 40	0.911	0.385
3.	40 to 4×10^3	0.683	0.466
4.	4×10^3 to 4×10^4	0.193	0.618
5.	4×10^4 to 4×10^5	0.026	0.805

2. *Churchill and Bernstein* have suggested the following comprehensive *empirical correlation* which covers the entire range of Re and wide range of Pr :

$$\overline{Nu} = 0.3 + \frac{0.62\,(Re)^{0.5}\,(Pr)^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{0.25}} \left[1 + \left(\frac{Re}{28200}\right)^{5/8}\right]^{0.8}$$

Valid for $100 < Re < 10^7$, and $Re.Pr > 0.2$ and correlates very well all available

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3. The following equation may be used in the *mid-range of Reynolds members*, i.e. $20{,}000 < Re < 400{,}000$:

$$\overline{Nu} = \frac{h.D}{k} = 0.3 + \frac{0.62(Re)^{0.5}(Pr)^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{0.25}}\left[1+\left(\frac{Re}{28200}\right)^{0.5}\right]$$

for $20{,}000 < Re < 400{,}000$, and $Re.Pr > 0.2$

4. The following relation is recommended by Nakai and Okazaki for $Pe\,(= Re.Pr) < 0.2$:

$$\overline{Nu} = \left[0.8237 - \ln(Pe)^{0.5}\right]^{-1}$$

- All properties, for all the above Equations, are evaluated at the "*film temperature*" (i.e. $t_f = \frac{t_s + t_\infty}{2}$, where t_f, t_s and t_∞ are film, surface and free solution temperature (in °C) respectively.

5. For **liquid metals**, Ishiguro *et. al.* recommended the following relation for *heat transfer from a single cylinder in cross flow*:

$$\overline{Nu} = 1.125(Re.Pr)^{0.415} \text{ for } 1 < Re.Pr < 100$$

It is worth noting that Eqn. being quite comprehensive, is also valid for *liquid metals*.

6. For **gases**, the following relation is widely used for *circular cylinder in crossflow*:

$$\overline{Nu} = C.(Re)^n.(Pr)^{1/3}$$

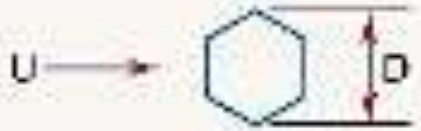
where, values of C and n are given in table

All fluid properties are taken at *film temperature*.

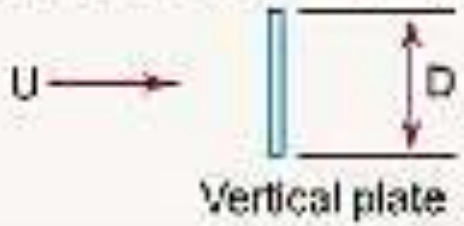
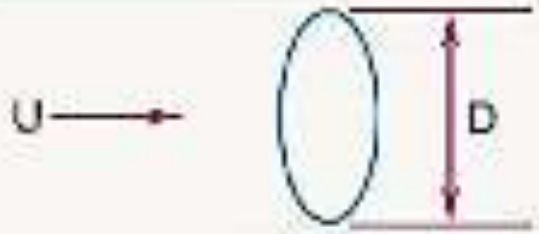
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Non-circular cylinders :

For non-circular cylinder, is applicable (for a *vertical plate as well*) by defining a characteristic length, D, and taking the values of C and n from table 7.3. All properties to be evaluated at the *film temperature*.

S.No.	*Geometry*	*Re*	*C*	*n*
1.	 Square	$5 \times 10^3 - 10^5$	0.246	0.588
2.	 Square	$5 \times 10^3 - 10^5$	0.102	0.675
3.	 Hexagon	$5 \times 10^3 - 1.95 \times 10^4$ $1.95 \times 10^4 - 10^5$	0.16 0.0385	0.638 0.782
4.	 Hexagon	$5 \times 10^3 - 10^5$	0.153	0.638

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5.	 Vertical plate	$4 \times 10^3 - 1.5 \times 10^4$	0.228	0.731
6.	 Ellipse	$2.5 \times 10^3 - 1.5 \times 10^4$	0.224	0.612
7.	 Ellipse	$3 \times 10^3 - 1.5 \times 10^4$	0.085	0.804

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Example *A refrigerated truck is moving on a high way at 90 km/h in a desert area where the ambient air temperature is 50°C. The body of the truck may be considered as a rectangular box measuring 10 m (length) × 4 m (width) × 3 m (height). Assume that the boundary layer on the four walls is turbulent, the heat transfer takes place only from the four surfaces and the wall surface of the truck is maintained at 10°C. Neglecting heat transfer from the front and back and assuming the flow to be parallel to 10 m long side, calculate the following :*

(i) *The heat loss from the four surfaces;*

(ii) *The tonnage of refrigeration;*

(iii) *The power required to overcome the resistance acting on the four surfaces.*

The properties of air (at $t_f = \dfrac{50 + 10}{2} = 30°C$) *are :*

$\rho = 1.165\ kg/m^3$, $c_p = 1.005\ kJ/kg°C$, $k = 0.02673\ W/m^2°C$, $\nu = 16 \times 10^{-6}\ m^2/s$, $Pr = 0.701$.



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Solution. *Given :* $U = 90 \text{ km/h} = \dfrac{90 \times 1000}{3600} = 25 \text{ m/s}$, $t_m = 50°C$, $t_s = 10°C$, $L = 10 \text{ m}$, $B = 4\text{m}$, $H = 3\text{m}$.

(i) The heat loss from the four surfaces :

The Reynolds number at the end of the side,

$$Re_L = \frac{UL}{\nu} = \frac{25 \times 10}{16 \times 10^{-6}} = 15.625 \times 10^6$$

Evidently the boundary layer is *turbulent* for which

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.036\ (Re)^{0.8}\ (Pr)^{0.333}$$

or,
$$\frac{\bar{h}L}{k} = 0.036\ (15.625 \times 10^6)^{0.8}\ (0.701)^{0.333} = 18196.28$$

$$\bar{h} = \frac{k}{L} \times 18196.28 = \frac{0.02673}{10} \times 18196.28 = 48.64 \text{ W/m}^2\text{°C}$$

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Heat loss from the four surfaces,

$$Q = \bar{h} A \Delta t = 48.64 [2 (4 + 3) \times 10] \times (50 - 10) = 272384 \text{ W}$$

$$= \mathbf{272.384 \text{ kW}} \quad \textbf{(Ans.)}$$

(ii) The tonnage of refrigeration (*TR*) :

$$\text{The cooling capacity required} = \frac{272.384 \times 3600}{14000} = \mathbf{70 \text{ TR}} \qquad (\because 1 \text{ TR} = 14000 \text{ kJ/h})$$

(iii) The power required to overcome the resistance, *P* :

Average skin friction coefficient

$$\bar{C}_f = \frac{0.072}{(Re_L)^{0.2}} = \frac{0.072}{(15.625 \times 10^6)^{0.2}} = 2.62 \times 10^{-3}$$

Drag force, $$F_D = \bar{C}_f \times \frac{1}{2} \rho A U^2$$

$$= 2.62 \times 10^{-3} \times \frac{1}{2} \times 1.165 \times [2 (4 + 3) \times 10] \times 25^2$$

$$= 133.54 \text{ N}$$

$$P = F_D \times U = 133.54 \times 25 = 3338.5 \text{ W}$$

$$= \mathbf{3.3385 \text{ kW}} \quad \textbf{(Ans.)}$$

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THEORETICAL QUESTIONS

1. Define the following terms :

   (*i*) Boundary layer thickness (*ii*) Displacement thickness

   (*iii*) Momentum thickness (*iv*) Energy thickness.

2. Derive momentum equation for hydrodynamic boundary layer over a flat plate.

3. Derive expressions for boundary layer thickness and local skin friction coefficient, following the Blasius method of solving laminar boundary layer equations, for flat plate.

4. Define Nusselt number and Prandtl number.

5. Derive an energy equation for thermal boundary layer over a flat plate.

6. Define the local and average skin friction (drag) coefficients for a flat plate at zero incidence, for laminar flow.

   Establish the following relations for laminar boundary layer over a flat plate :

   (*i*) Local skin friction coefficient. $C_{fx} = \dfrac{0.664}{\sqrt{Re_x}}$

   (*ii*) Average skin friction (drag) coefficient. $\bar{C}_f = \dfrac{1.328}{\sqrt{Re_L}}$

   Take $\left(\dfrac{df}{d\eta}\right)_{\eta=0} = 0.332$

7. Derive the Von-Karman momentum integral equation for the flow past a flat plate in the form :

$$\tau_0 = \rho U^2 \frac{d}{dx}\left[\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy\right]$$

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- Explain boundary layer concept and define clearly hydrodynamic and thermal boundary layer thickness when fluid is flowing over the plate.

- Write the significance of following numbers.
  i) Reynold Number
  ii) Prandtl Number
  iii) Peclet Number.

- Air (Ta = 30ºC) flowing over the flat square plate 300mm x 300mm is size, at a velocity of 4m/s. The plate surface is maintained at a temperature of 56ºC.

Calculate :
i) Boundary layer thickness at 300mm from leading edge.
ii) Local and average heat transfer coefficient.
iii) Local and average coefficient of friction.
iv) Total heat transported from the plate.
v) Total drag force on the plate.

- Air at a pressure of 6 kN/m2 and a temperature of 300ºC flows with a velocity of 10m/s over a flat plate 0.6m long. Estimate the cooling rate per unit width of the plate needed to maintain it at a surface ɛrature of 0ºC.

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Unit No.4

Natural convection

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HEAT TRANSFER BY CONVECTION

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CONDUCTION

Mechanism of heat transfer through a solid or fluid in the absence any fluid motion.

CONVECTION

Mechanism of heat transfer through a fluid in the presence of bulk fluid motion

- **Natural (free) Convection**
- **Forced Convection**

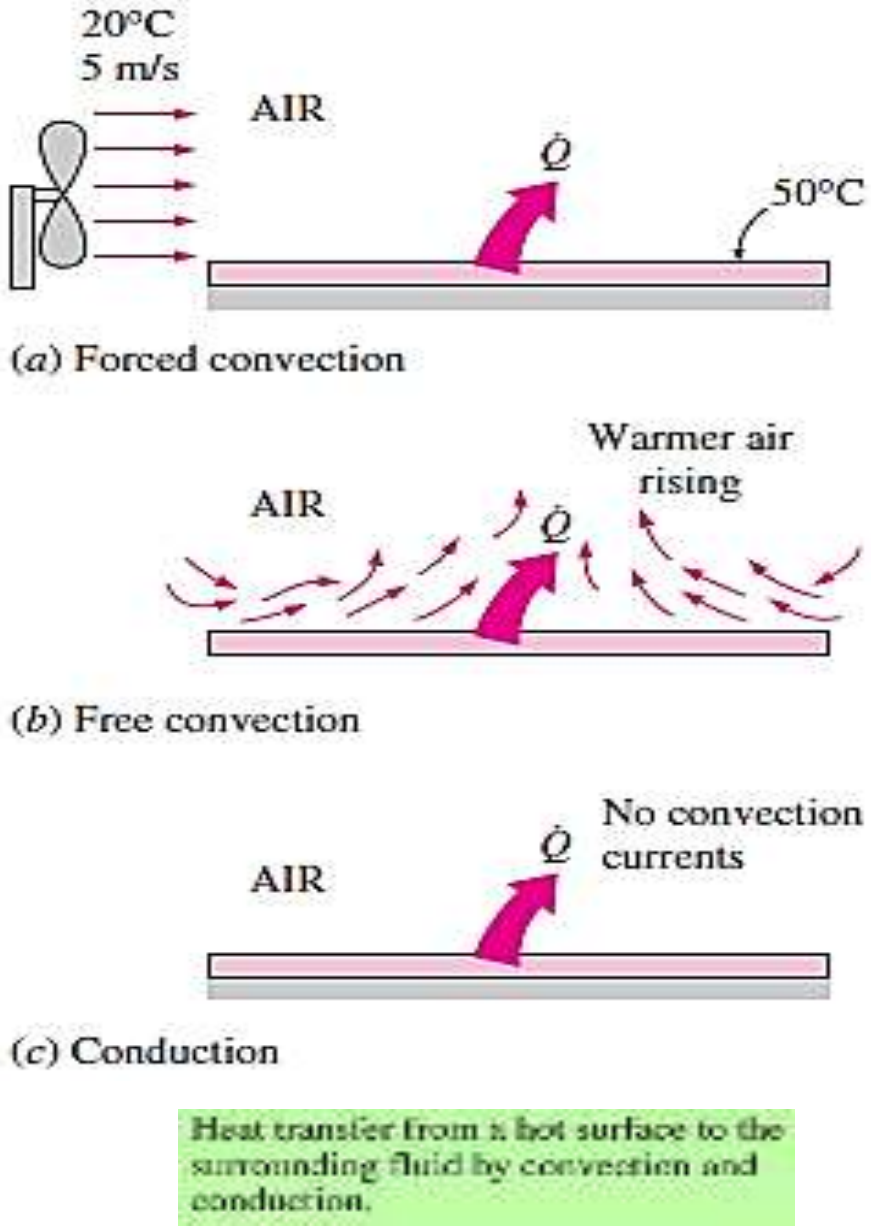
(depending on how the fluid motion is initiated)

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HEAT TRANSFER BY CONVECTION

We mentioned earlier that there are three basic mechanisms of heat transfer: conduction, convection, and radiation. Conduction and convection are similar in that both mechanisms require the presence of a material medium. But they are different in that convection requires the presence of a material medium. But they are different in that convection requires the presence of fluid motion. Heat transfer through a solid is always by conduction, since the molecules of a solid remain at relatively fixed positions. Heat transfer through a liquid or gas, however, can be by conduction or convection, depending on the presence of any bulk fluid motion. Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it. Therefore, conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid

Convection is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion. Convection is classified as *natural (or free*) and *forced* convection depending on how the fluid motion is initiated. In natural convection, any fluid motion is caused by natural h as the buoyancy effect, i.e. the rise of warmer fluid and er fluid. Whereas in forced convection, the fluid is forced to surface or in a tube by external means such as a pump or



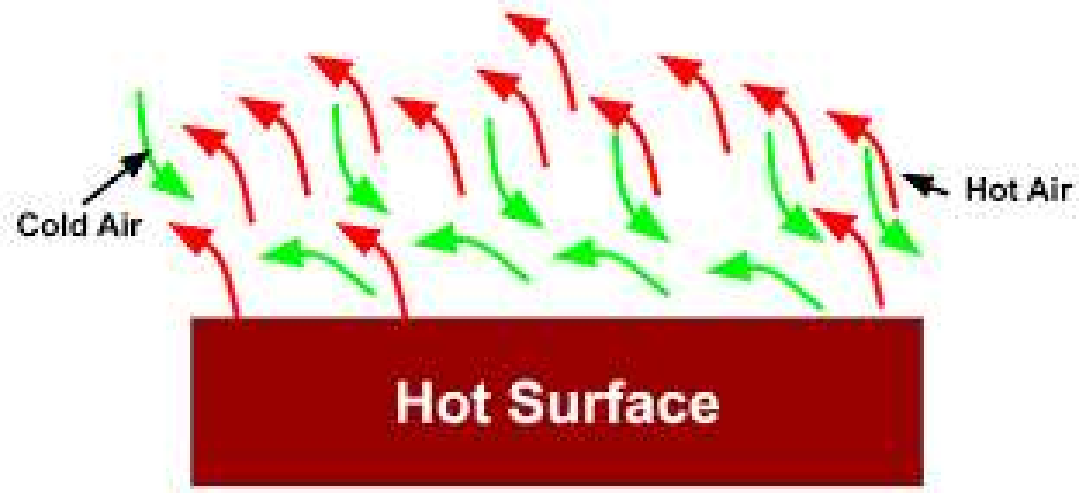
Heat transfer from a hot surface to the surrounding fluid by convection and conduction.

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NATURAL CONVECTION

What is natural and forced convection?

In **natural convection**, any fluid motion is caused by **natural** means such as the buoyancy effect, i.e. the rise of warmer fluid and fall the cooler fluid. Whereas in **forced convection**, the fluid is **forced** to flow over a surface or in a tube by external means such as a pump or fan.



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CONVECTIVE HEAT TRANSFER COEFFICIENT

Coefficient of volume expansion

Grashof number

$$Gr_L = \frac{g\beta(T_\infty - T_C)L^3}{\upsilon^2}$$

viscosity

Rayleigh number

$$Ra_L = Gr_L \Pr$$

Prandtl number

Nusselt number

$$Nu = \frac{hL_C}{k} = CRa^n{}_L$$

Table 20-1

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Objectives of Natural Convection Heat Transfer

- Understand the physical mechanism of natural convection
- Derive the governing equations of natural convection, and obtain the dimensionless Grashof number by nondimensionalizing them
- Evaluate the Nusselt number for natural convection associated with vertical, horizontal, and inclined plates as well as cylinders and spheres
- alyze natural convection inside enclosures such as double-pane windows

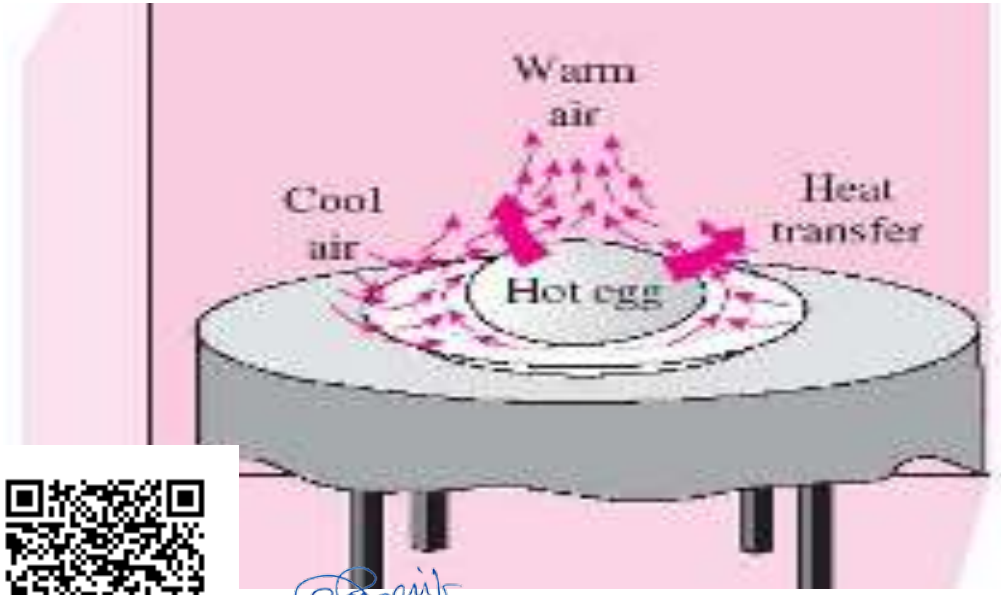
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PHYSICAL MECHANISM OF NATURAL CONVECTION

Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer.
Examples?

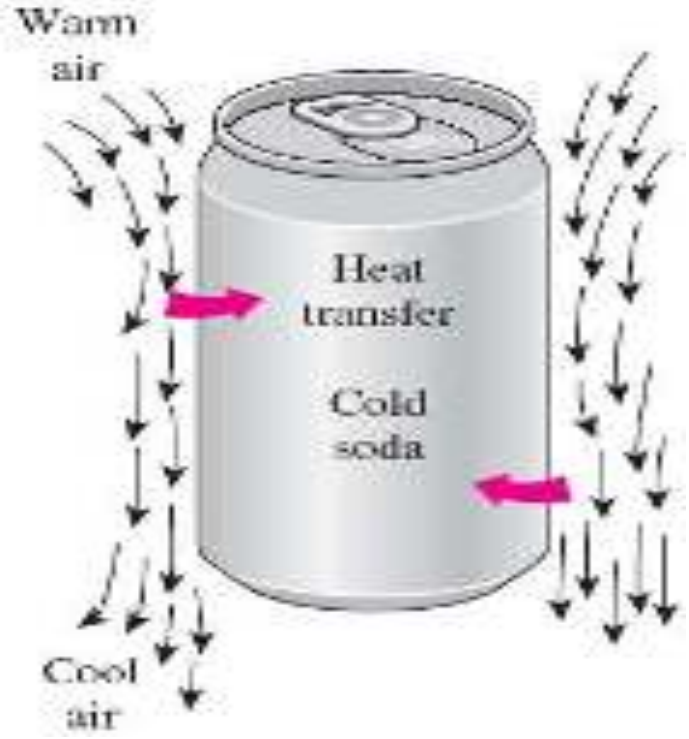
Natural convection in gases is usually accompanied by radiation of comparable magnitude except for low-emissivity surfaces.

The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a natural convection current, and the heat transfer that is enhanced as a result of this current is called natural convection heat transfer.



g of a boiled egg in a cooler environment
convection.

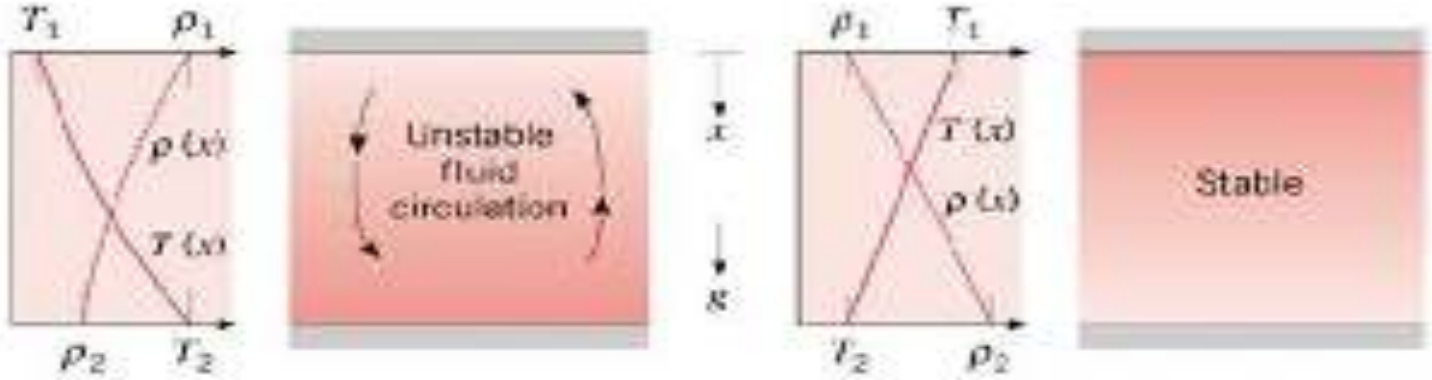
The warming up of a cold drink in a warmer environment by natural convection.



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General Considerations

- Free convection refers to fluid motion induced by buoyancy forces.
- Buoyancy forces may arise in a fluid for which there are density gradients and a body force that is proportional to density.
- In heat transfer, density gradients are due to temperature gradients and the body force is gravitational.
- Stable and Unstable Temperature Gradients



$\frac{dT}{dx} > 0, \frac{d\rho}{dx} < 0$

(a)

$\frac{dT}{dx} < 0, \frac{d\rho}{dx} > 0$

(b)

What is buoyancy force ?

- The upward force exerted by a fluid on a body completely or partially immersed in it in a gravitational field.
- The magnitude of the buoyancy force is equal to the weight of the *fluid displaced* by the body.

$$F_{\text{buoyancy}} = \rho_{\text{fluid}}\, g\, V_{\text{body}}$$

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Buoyancy force: The upward force exerted by a fluid on a body completely or partially immersed in it in a gravitational field. The magnitude of the buoyancy force is equal to the weight of the *fluid displaced* by the body.

$$F_{\text{buoyancy}} = \rho_{\text{fluid}}\, g\, V_{\text{body}}$$

The net vertical force acting on a body

$$\begin{aligned} F_{\text{net}} &= W - F_{\text{buoyancy}} \\ &= \rho_{\text{body}}\, g\, V_{\text{body}} - \rho_{\text{fluid}}\, g\, V_{\text{body}} \\ &= (\rho_{\text{body}} - \rho_{\text{fluid}})\, g\, V_{\text{body}} \end{aligned}$$

Archimedes' principle: A body immersed in a fluid will experience a "weight loss" in an amount equal to the weight of the fluid it displaces.

The "chimney effect" that induces the upward ɔt combustion gases through a is due to the buoyancy effect.

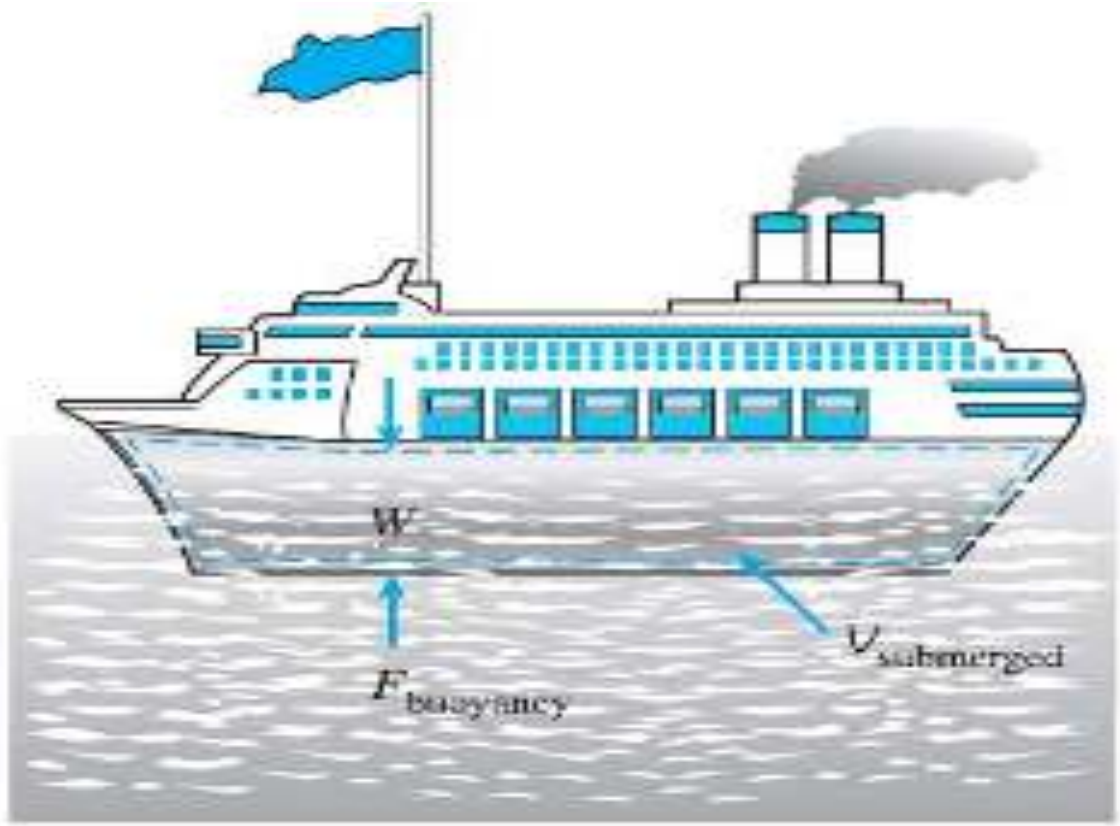


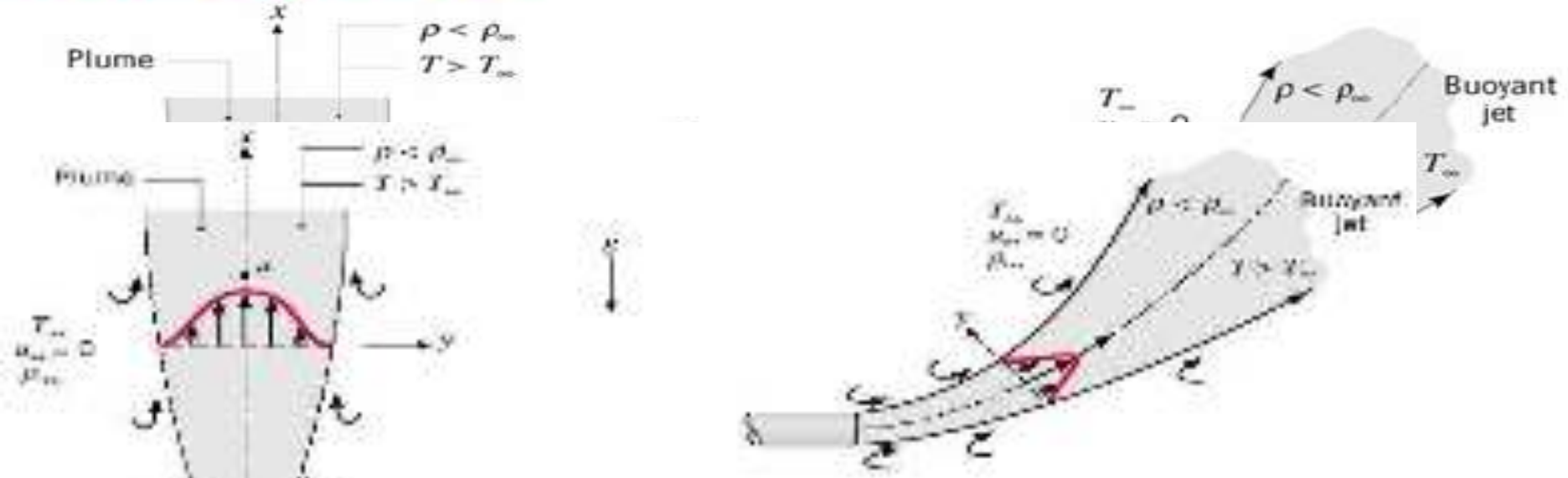
FIGURE 20–3
It is the buoyancy force that keeps the ships afloat in water ($W = F_{\text{buoyancy}}$ for floating objects).

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Free Boundary Flows

- Occur in an extensive (in principle, infinite), quiescent (motionless at locations far from the source of buoyancy) fluid.

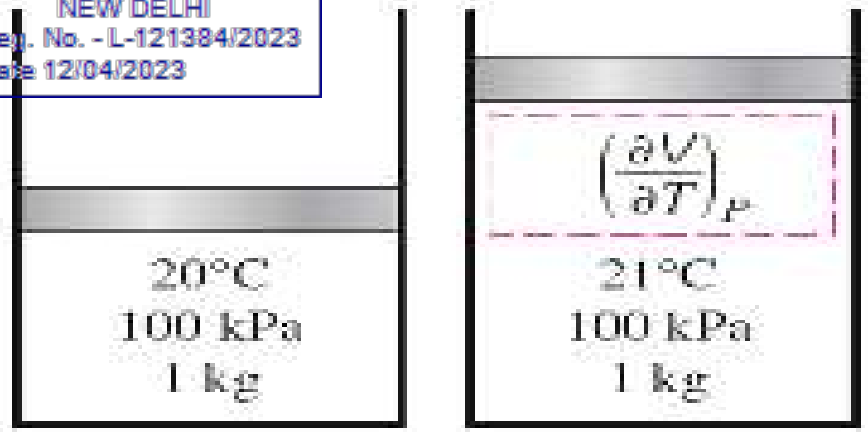
- Plumes and Buoyant Jets:



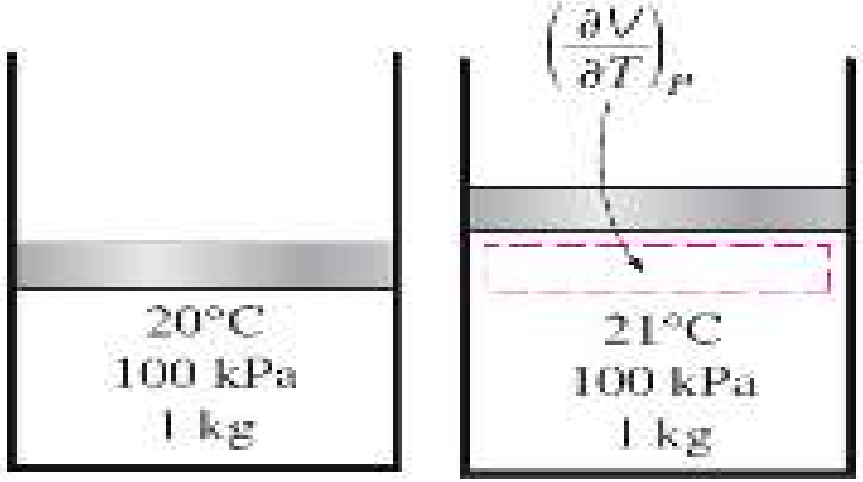
- Free C

Boundary layer flow on a heated or cooled surface $(T_s \neq T_\infty)$ induced by buoyancy forces.

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(a) A substance with a large β



stance with a small β

ient of volume expansion is a
the change in volume of a substance
rature at constant pressure.

Volume expansion coefficient: Variation of the density of a fluid with temperature at constant pressure.

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \quad (1/\mathrm{K})$$

$$\beta \approx -\frac{1}{\rho}\frac{\Delta \rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \quad \text{(at constant } P\text{)}$$

$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \quad \text{(at constant } P\text{)}$$

$$\beta_{\text{ideal gas}} = \frac{1}{T} \quad (1/\mathrm{K})$$

ideal gas $(P = \rho RT)$

The larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the *larger* the buoyancy force and the *stronger* the natural convection currents, and thus the *higher* the heat transfer rate.

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In natural convection, no blowers are used, and therefore the flow rate cannot be controlled externally.

The flow rate in this case is established by the dynamic balance of *buoyancy* and *friction*.

An interferometer produces a map of interference fringes, which can be interpreted as lines of *constant temperature.*

The smooth and parallel lines in (*a*) indicate that the flow is *laminar,* whereas the eddies and irregularities in (*b*) indicate that the flow is *turbulent.*

The lines are closest near the surface, indicating a *higher temperature gradient.*

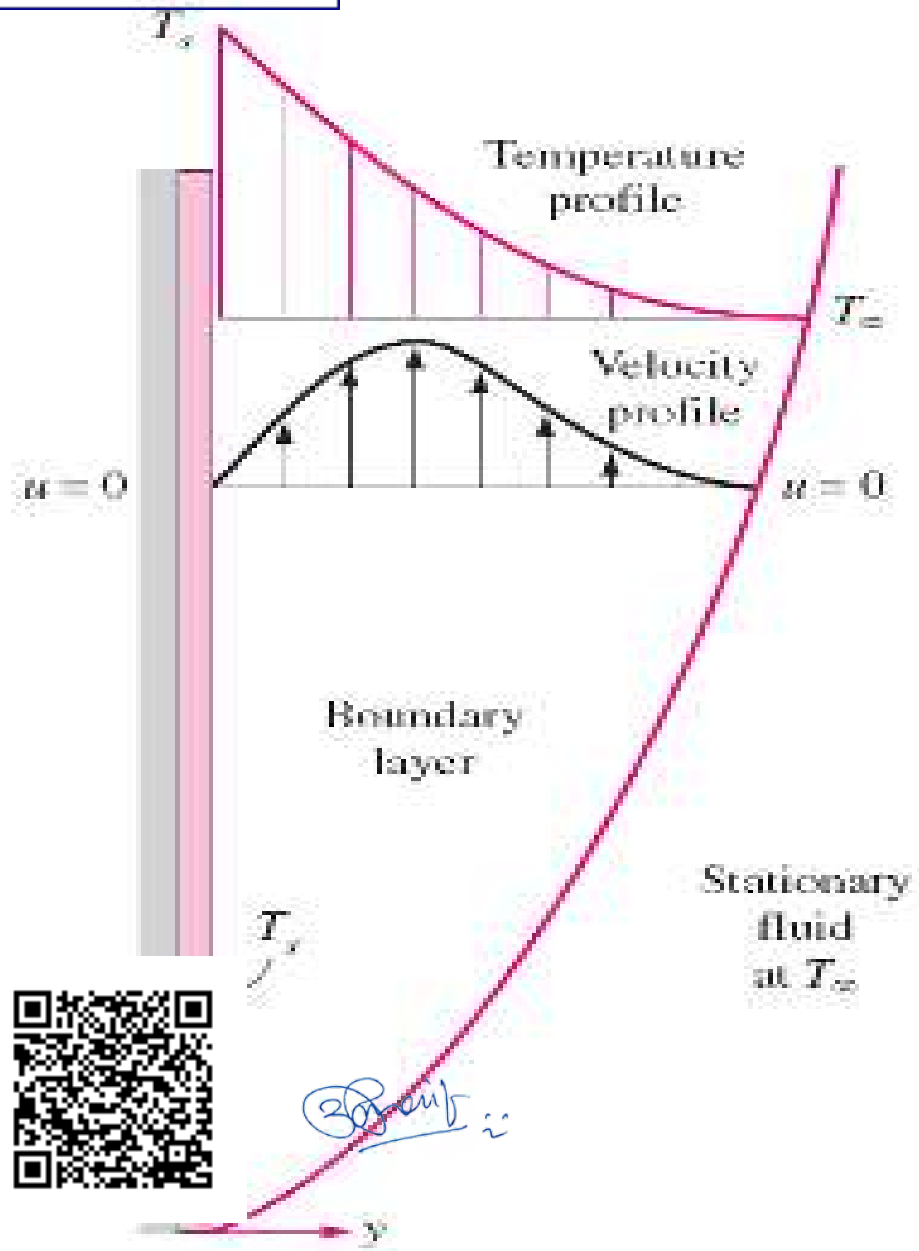


(*a*) Laminar flow (*b*) Turbulent flow

Isotherms in natural convection over a hot plate in air.

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EQUATION OF MOTION AND THE GRASHOF NUMBER



The thickness of the boundary layer increases in the flow direction.

Unlike forced convection, the fluid velocity is *zero* at the outer edge of the velocity boundary layer as well as at the surface of the plate.

At the surface, the fluid temperature is equal to the plate temperature, and gradually decreases to the temperature of the surrounding fluid at a distance sufficiently far from the surface.

In the case of *cold surfaces,* the shape of the velocity and temperature profiles remains the same but their direction is reversed.

Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature T_s inserted in a fluid at temperature T_∞.

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Derivation of the equation of motion that governs the natural convection flow in laminar boundary layer

$$\delta m \cdot a_x = F_x$$

$$\delta m = \rho(dx \cdot dy \cdot 1)$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$F_x = \left(\frac{\partial \tau}{\partial y}dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x}dx\right)(dy \cdot 1) - \rho g(dx \cdot dy \cdot 1)$$

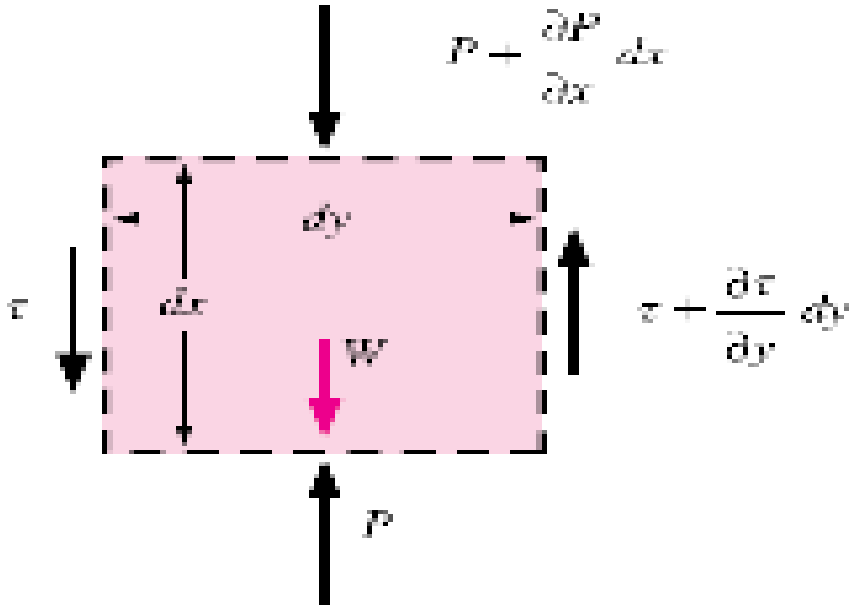
$$= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g\right)(dx \cdot dy \cdot 1)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g$$

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g \qquad \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g$$

$$\rho_\infty - \rho = \rho\beta(T - T_\infty)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$



ting on a differential volume
the natural convection
layer over a vertical flat plate.

This is the equation that governs the fluid motion in the boundary layer due to the effect of buoyancy. The momentum equation involves the temperature, and thus the momentum and energy equations must be solved simultaneously.

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The complete set of conservation equations, continuity (Eq. 6–39), momentum (Eq. 9–13), and energy (Eq. 6–41) that govern natural convection flow over vertical isothermal plates are:

$$\text{Continuity:} \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\text{Momentum:} \qquad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

$$\text{Energy:} \qquad u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

with the following boundary conditions (see Fig. 9–6):

$$\text{At } y = 0: \qquad u(x, 0) = 0,\ v(x, 0) = 0, \qquad T(x, 0) = T_s$$

$$\text{At } y \to \infty: \qquad u(x, \infty) \to 0,\ v(x, \infty) \to 0, \qquad T(x, \infty) \to T_\infty$$

ve set of three partial differential equations can be reduced to a set of two ordinary
ır differential equations by the introduction of a similarity variable. But the resulting
ıs must still be solved along with their transformed boundary conditions numerically.

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The Grashof Number

The governing equations of natural convection and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by suitable constant quantities:

$$x^* = \frac{x}{L_c} \qquad y^* = \frac{y}{L_c} \qquad u^* = \frac{u}{V} \qquad v^* = \frac{v}{V} \qquad \text{and} \qquad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

Substituting them into the momentum equation and simplifying give

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right] \frac{T^*}{\mathrm{Re}_L^2} + \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

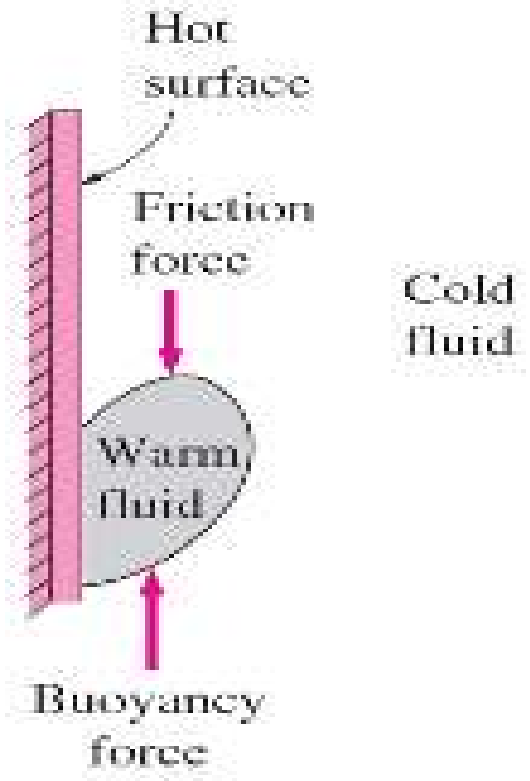
$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

Grashof number: Represents the natural convection effects in momentum equation

g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, m^2/s

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- The Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection.
- For vertical plates, the critical Grashof number is observed to be about 10^9.



When a surface is subjected to external flow, the problem involves both natural and forced convection.

The relative importance of each mode of heat transfer is determined by the value of the coefficient Gr/Re^2:

- Natural convection effects are negligible if $Gr/Re^2 << 1$.
- Free convection dominates and the forced convection effects are negligible if $Gr/Re^2 >> 1$.
- Both effects are significant and must be considered if $Gr/Re^2 \approx 1$ (mixed convection).

The Grashof number Gr is a measure of the agnitudes of the *buoyancy force* pposing *viscous force* acting on

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Pertinent Dimensionless Parameters

- **Grashof Number:**

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \sim \frac{\text{Buoyancy Force}}{\text{Viscous Force}}$$

$L \rightarrow$ characteristic length of surface

$\beta \rightarrow$ thermal expansion coefficient (a thermodynamic property of the fluid)

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

Liquids: $\beta \rightarrow$ Tables A.5, A.6

Perfect Gas: $\beta = 1/T(\mathrm{K})$

- Ratio of buoyancy forces and thermal and momentum diffusivities.

Rayleigh Number:

$$Ra_L = Gr_L \,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$$

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Rayleigh Number, Ra=Gr.Pr

- In fluid mechanics, the Rayleigh number for a fluid is a dimensionless number associated with buoyancy driven flow (also known as free convection or natural convection).
- When the Rayleigh number is below the critical value for that fluid, heat transfer is primarily in the form of conduction; when it exceeds the critical value, heat transfer is primarily in the form of convection.
- The Rayleigh number is defined as the product of the Grashof number, which describes the relationship between buoyancy and viscosity within a fluid, and the Prandtl number, which describes the relationship between momentum and thermal diffusivity.



ce the Rayleigh number itself may also be viewed as the ratio of buoyancy and viscosity forces times the ratio of momentum and thermal diffusivities.

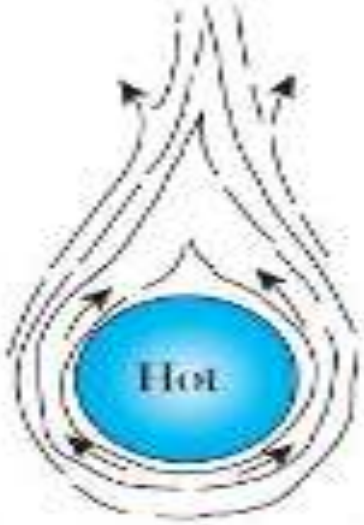
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Forced vs Natural Convection

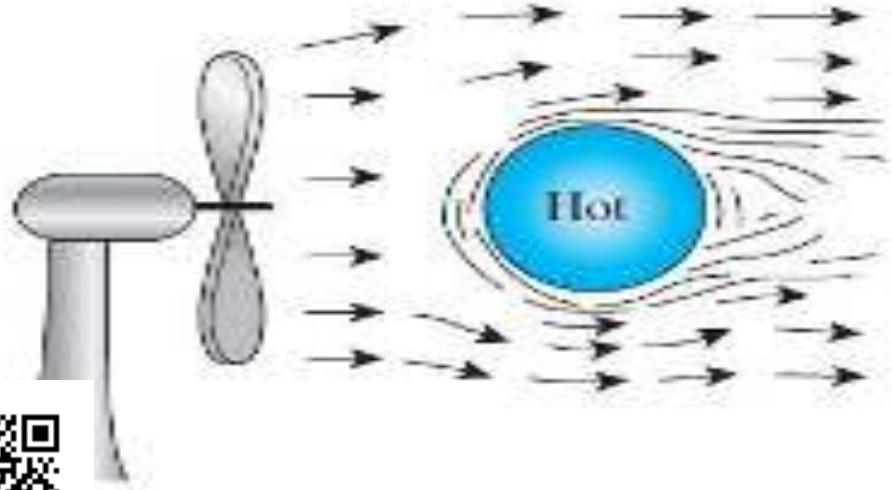
- When analyzing potentially mixed convection, a parameter called the Archimedes number(Ar) parametrizes the relative strength of free and forced convection.
- The Archimedes number is the ratio of Grashof number and the square of Reynolds number, which represents the ratio of buoyancy force and inertia force, and which stands in for the contribution of natural convection.
- When Ar >> 1, natural convection dominates and when Ar << 1, forced convection dominates.

$$Ar = \frac{Gr}{Re^2}$$

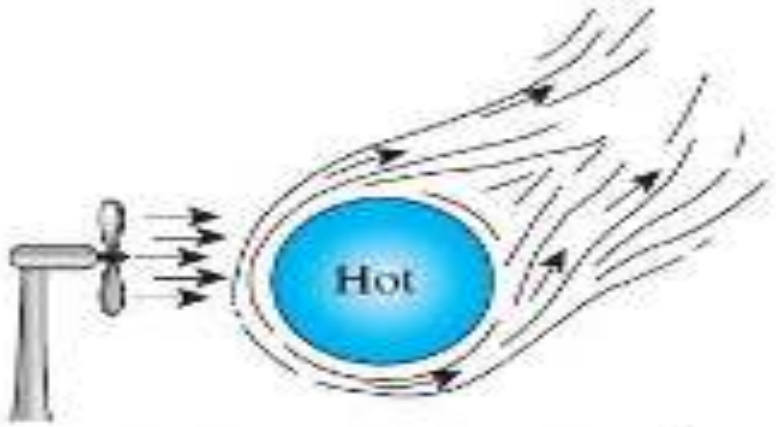
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(a) Natural convection ($Gr_L / Re_L^2 >> 1$)



(b) Forced convection ($Gr_L / Re_L^2 << 1$)



(c) Mixed convection ($Gr_L / Re_L^2 = 1$)

FIGURE 20–9
The relative importance of convection heat transfer regimes for flow near a hot sphere.

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The role of Grashoff number is the same in free convection as that of Reynolds number in forced convection.

The *critical Grashoff number for the flow of air over a flat plate* has been observed to be 4×10^8 (approximately).

In general, $Nu = f(Gr, Pr) = C\,(Gr)^a\,(Pr)^b$

In several cases, the above relation simplifies to the form

$$Nu = C\,(Gr.\,Pr)^m$$

Hence, a new dimensionless group is often used called *Rayleigh number* viz.,

$$Ra = Gr\,.\,Pr$$

The product is also a criterion of laminar or turbulent character of the flow as determined by its values. Thus

$10^4 < Gr\,Pr < 10^9$ ... *for laminar flow*

$Gr\,Pr > 10^9$ ... *for turbulent flow*

alues of Gr and Pr are *evaluated at the mean film temperature* $\left(t_f = \frac{t_w + t_\infty}{2}\right)$.

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TRANSITION AND TURBULENCE IN FREE CONVECTION

A transition from laminar to turbulent condition in both forced and free convection is caused by thermal and hydrodynamic instabilities. It has been observed that the transition in free convection boundary layer occurs when the value of Rayleigh number approaches 10^9. Thus

$$Ra_{x,c} = Gr_{x,c}\ Pr = \frac{g\beta\,(t_s - t_\infty)\,x^3}{\nu\,\alpha} = 10^9$$

The analysis of turbulent free convection is very complex. However, the results obtained from various experiments have been used to obtain the pertinent empirical relations.

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EMPIRICAL CORRELATIONS FOR FREE CONVECTION

The following dimensionless numbers apply for the usual free convection problems:

(*i*) Nusselt number $Nu = \frac{hL}{k}$; (*ii*) Grashoff number $Gr = \frac{L^3 \beta g \Delta t}{\nu^2}$

(*iii*) Prandtl number $Pr = \frac{\mu c_p}{k}$

VERTICAL PLATES AND CYLINDERS

The commonly used correlations are:

Laminar flow: $\overline{Nu}_L = 0.59\ (Gr \,.\, Pr)^{1/4}$ for $(10^4 < Gr \,.\, Pr < 10^9)$

rbulent flow: $\overline{Nu}_L = 0.10\ (Gr \,.\, Pr)^{1/3}$ for $(10^9 < Gr \,.\, Pr < 10^{12})$

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EMPIRICAL CORRELATIONS FOR FREE CONVECTION

VERTICAL PLATES AND CYLINDERS

All the fluid properties are evaluated at the mean film temperature $\left(t_f = \frac{t_s + t_\infty}{2}\right)$.

Churchill and Chu have recommended the following correlations:

$$\overline{Nu_L} = 0.68 + \frac{0.67\,(Gr\,.\,Pr)^{1/4}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9}} \qquad \text{for } (Gr\,.\,Pr < 10^9)$$

$$\overline{Nu_L} = \left[0.825 + \frac{0.387\,(Gr\,.\,Pr)^{1/6}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right]^2 \qquad \text{for } (Gr\,.\,Pr > 10^9)$$

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EMPIRICAL CORRELATIONS FOR FREE CONVECTION

HORIZONTAL PLATES

In case of an irregular plate, the *characteristic length* is defined as the *surface area divided by the perimeter of the plate.*

(i) The upper surface heated or the lower surface cooled:

Laminar flow: $\overline{Nu}_L = 0.54\,(Gr \,.\, Pr)^{1/4}$ for $(10^5 < Gr \,.\, Pr \leq 2 \times 10^7)$

Turbulent flow: $\overline{Nu}_L = 0.14\,(Gr \,.\, Pr)^{1/3}$ for $(2 \times 10^7 < Gr \,.\, Pr \leq 3 \times 10^{10})$

(ii) The lower surface heated or upper surface cooled:

Laminar flow: $\overline{Nu}_L = 0.27\,(Gr \,.\, Pr)^{1/4}$ for $(3 \times 10^5 < Gr \,.\, Pr \leq 3 \times 10^{10})$

urbulent flow: $\overline{Nu}_L = 0.107\,(Gr \,.\, Pr)^{1/3}$ for $(7 \times 10^6 < Gr \,.\, Pr \leq 11 \times 10^{10})$

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EMPIRICAL CORRELATIONS FOR FREE CONVECTION

HORIZONTAL CYLINDERS

For such a case, the *outside diameter is used as the characteristic dimension.*

Mc Adams has recommended the following correlations:

$$\textit{Laminar flow: } \overline{Nu} = 0.53\,(Gr \,.\, Pr)^{1/4} \qquad \text{for } (10^4 < Gr \,.\, Pr < 10^9)$$

$$\textit{Turbulent flow: } \overline{Nu} = 0.13\,(Gr \,.\, Pr)^{1/3} \qquad \text{for } (10^9 < Gr \,.\, Pr < 10^{12})$$

The following general correlation has been suggested by Churchill and Chu for use over a wide range of $Gr \,.\, Pr$.

$$\overline{Nu} = \left[0.60 + \frac{0.387\,(Gr \,.\, Pr)^{1/6}}{\{1 + (0.559/Pr)^{9/16}\}^{8/27}}\right] \qquad \text{for } (10^{-5} < Gr \,.\, Pr < 10^{12})$$

luid properties, in all preceding equations, are determined at the mean film temperature

$$t_f = \left(\frac{t_s + t_\infty}{2}\right).$$

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EMPIRICAL CORRELATIONS FOR FREE CONVECTION

INCLINED PLATES

For this case multiply Grashoff number by cos θ, where θ is the angle of inclination from the vertical and use vertical plate constants.

SPHERES

Yuge (1959) has recommended the following correlation for free convection from a sphere of diameter D :

$$\overline{Nu} = 2 + 0.43\,(Gr \,.\, Pr)^{1/4}$$

$$\text{for } (1 < Gr \,.\, Pr < 10^5) \text{ and } Pr = 1)$$

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EMPIRICAL CORRELATIONS FOR FREE CONVECTION

ENCLOSED SPACES

In the literature, several correlations for heat transfer between surfaces at different temperatures separated by enclosed fluids are available. In this section, some correlations are presented that are pertinent to the most common geometries.

(a) Vertical spaces

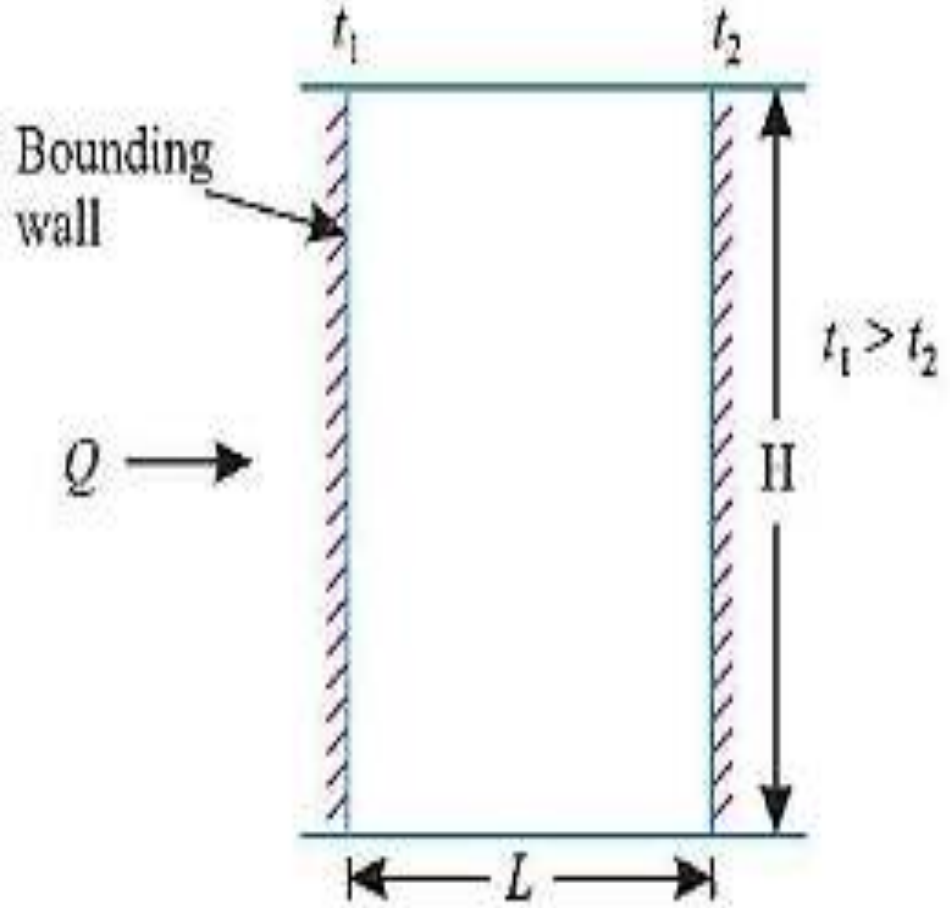
Jacob has suggested the following correlations for *vertical enclosed air spaces* shown in Fig. 8.2.

(i) $$\frac{k_e}{k} = \overline{Nu} = \frac{\bar{h}\,L}{k} = 0.18\,(Gr_L)^{1/4}\left(\frac{H}{L}\right)^{-1/9}$$

for $(2000 < Gr_L < 2 \times 10^4)$ ...(8.49)

$$= \overline{Nu} = \frac{\bar{h}\,L}{k} = 0.064\,(Gr_L)^{1/3}\left(\frac{H}{L}\right)^{-1/9}$$

for $(2 \times 10^4 < Gr_L < 11 \times 10^6)$



Free convection in vertical enclosure.

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EMPIRICAL CORRELATIONS FOR FREE CONVECTION

ENCLOSED SPACES

where, k_e = Effective thermal conductivity, $\frac{Q}{h} = k_e \left(\frac{t_1 - t_2}{L} \right)$

H = Height of the air space, and

L = Thickness of the air space.

Gr is based on thickness of air space, L.

(b) Horizontal spaces

The following correlation has been suggested by Jacob for *horizontal enclosed air spaces*:

(i) $$\frac{k_e}{k} = \overline{Nu} = \frac{\bar{h} L}{k} = 0.195 \, (Gr_L)^{1/4} \qquad \text{for } (10^4 < Gr_L < 4 \times 10^5)$$

(ii) $$\frac{k_e}{k} = \overline{Nu} = \frac{\bar{h} L}{k} = 0.068 \, (Gr_L)^{1/3} \qquad \text{for } (4 \times 10^5 < Gr_L)$$

where $\overline{Nu} = \frac{q \cdot L}{k \, (t_1 - t_2)}$

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EMPIRICAL CORRELATIONS FOR FREE CONVECTION

ENCLOSED SPACES

In the case of *liquids* contained in horizontal spaces, Globe and Dropkin have proposed the following correlation:

$$\overline{Nu} = \frac{\bar{h} L}{k} = 0.069\ (Gr_L)^{1/3}\ (Pr)^{0.407}$$

for $(3 \times 10^5 < Gr \cdot Pr < 7 \times 10^9)$

CONCENTRIC CYLINDERS SPACES

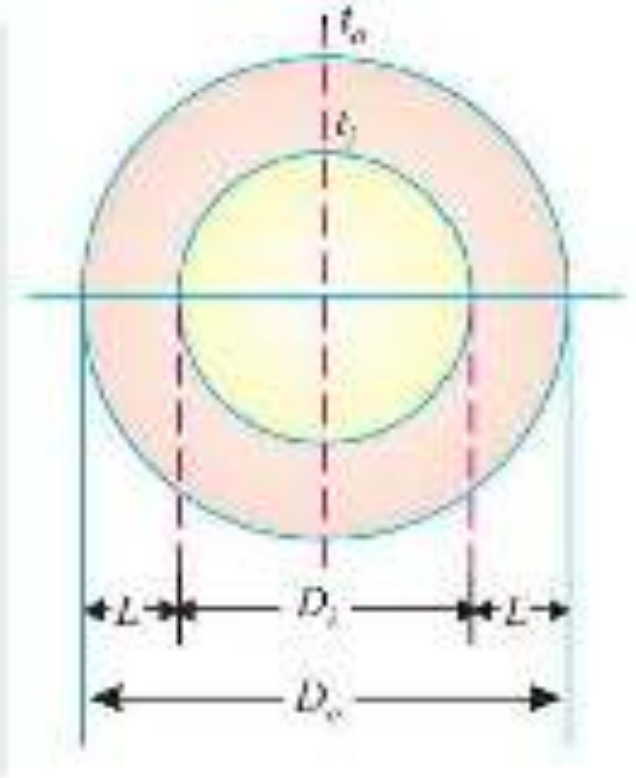
Raithby and Hollands have recommended the following correlations for *long horizontal concentric cylinders*:

$$\frac{k_e}{k} = 0.386 \left(\frac{Pr}{0.861 + Pr} \right)^{1/4} (Ra_c)^{1/4}$$

$10^2 \le Ra_c \le 10^7$ where, $Ra_c = (Gr \cdot Pr)_c = \dfrac{[\ln (D_o / D_i)]^4}{L^3 \left[D_i^{-3/5} + D_o^{-3/5} \right]^5} Ra_L$

and k_e is related as

$$Q = \frac{2\pi k_e}{\ln (D_o / D_i)} (t_i - t_o)$$



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EMPIRICAL CORRELATIONS FOR FREE CONVECTION

CONCENTRIC SPHERES SPACES

Raithby and Hollands have recommended the following correlations:

$$Q = k_e \pi (D_i D_o / L)(t_i - t_o)$$

and, k_e is expressed as

$$\frac{k_e}{k} = 0.74 \left[\frac{Pr}{0.861 + Pr}\right]^{1/4} (Ra_s)^{1/4}$$

where, $$Ra_s = (Gr \,.\, Pr)_s = \left[\frac{L\, Ra_L}{(D_o D_i)^4 \,(D_i^{-7/15} + D_o^{-7/15})^5}\right]^{1/4}$$

The above equations are valid for $(10^2 \le Ra_s \le 10^4)$.

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SIMPLIFIED FREE CONVECTION RELATIONS FOR AIR

The simplified relation for the heat transfer coefficient from different surfaces to air at atmospheric pressure, in general, is given by

$$h = C\left(\frac{\Delta t}{L}\right)^m$$

where C and m are constants depending on geometry and flow conditions; the significant length L is also a function of geometry and flow. Table 8.1 lists the values suggested by Mc Adams for various geometries, orientations and flow conditions indicated by the magnitude of product (Gr . Pr).

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SIMPLIFIED FREE CONVECTION RELATIONS FOR AIR

S.No.	Surface and its orientation	Laminar	Turbulent
1.	Vertical plate or cylinder	$\bar{h} = 1.42 \left(\frac{\Delta T}{L}\right)^{1/4}$ $(10^4 < Gr \cdot Pr < 10^9)$	$\bar{h} = 1.32 (\Delta T)^{1/3}$ $(10^9 < Gr \cdot Pr < 10^{13})$
2.	Horizontal cylinder	$\bar{h} = 1.32 \left(\frac{\Delta T}{D}\right)^{1/4}$ $(10^4 < Gr \cdot Pr < 10^9)$	$\bar{h} = 1.25 (\Delta T)^{1/3}$ $(10^9 < Gr \cdot Pr < 10^{12})$
3.	Horizontal plate:		
	Heated surface facing up	$\bar{h} = 1.32 \left(\frac{\Delta T}{L}\right)^{1/4}$ $(10^5 < Gr \cdot Pr < 2 \times 10^7)$	$h = 1.87 (\Delta T)^{1/3}$ $(2 \times 10^7 < Gr \cdot Pr < 3 \times 10^{10})$
	Heated surface facing down	$\bar{h} = 0.59 \left(\frac{\Delta T}{L}\right)^{1/4}$ $(3 \times 10^5 < Gr \cdot Pr < 3 \times 10^{10})$	
4.	Spheres	$\bar{h} = [2 + 0.392 (Gr)^{1/4}] \frac{k}{D}$ $(1 < Gr < 10^5)$	

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COMBINED FREE AND FORCED CONVECTION

Free convection effects are negligible when $Gr << R^2_e$, at the other extreme, free convection dominates when $Gr >> R^2_e$. There are certain situations where the free as well as the forced convection are of comparable magnitude. One such case is *when air is flowing over a heated surface at a low velocity*. A dimensionless group (Gr/R_e^2) is used to delineate the convection regimes. The various flow regimes are characterised as follows:

$(Gr/R_e^2) \geq 1$ ... Pure free convection

$(Gr/R_e^2) \simeq 1$ ... Mixed (free and forced) convection

$(Gr/R_e^2) \leq 1$ ... Pure forced convection.

EXTERNAL FLOWS

The local Nusselt number Nu_x for *mixed convection on vertical plates* is given by

$$Nu_x = 0.332\,(Re_x)^{1/2}\,(Pr)^{1/3} \text{ if } (Gr_x/R_{e_x}^2) \leq A$$

and,
$$Nu_x = 0.508\,(Pr)^{1/2}\,(0.952 + Pr)^{-1/4}\,(Gr_x)^{1/4} \text{ if } (Gr_x/R_{e_x}^2) > A$$

where, $A = 0.6$ for $Pr \leq 10$

and, $A = 1.0$ for $Pr = 100$



r *horizontal plates* when $(Gr_x/Re_x^{2.5}) \leq 0.083$ the following equation for forced convection used

$$Nu_x = 0.332\,Re_x^{1/2}\,Pr^{1/3}$$

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COMBINED FREE AND FORCED CONVECTION

(*i*) For mixed convection in laminar flow, Brown and Gauvin recommended a correlation of the form as

$$\overline{Nu} = 1.75 \left(\frac{\mu_b}{\mu_s}\right)^{0.14} [Gz + 0.012\,(Gz \,.\, Gr^{1/3})^{4/3}]^{1/3}$$

where, Gz = Graetz number = $Re\ Pr\ (D/L)$

and, μ_b, μ_s = Viscosities of the fluid at the bulk mean temperature and surface temperature respectively.

(*ii*) For mixed convection with turbulent flow in horizontal tubes, Metais and Eckert (1966) suggest

$$\overline{Nu} = 4.69\ (Re)^{0.27}\ (Pr)^{0.21}\ (Gr)^{0.07}\ (D/L)^{0.36}$$



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Example *A vertical cylinder 1.5 m high and 180 mm in diameter is maintained at 100°C in atmosphere environment of 20°C. Calculate heat loss by free convection from the surface of the cylinder. Assume properties of air at mean temperature as, $\rho = 1.06\ kg/m^3$, $\nu = 18.97 \times 10^{-6}\ m^2/s$, $c_p = 1.004\ kJ/kg°C$ and $k = 0.1042\ kJ/mh°C$.*

Solution. *Given :* $L = 1.5$ m; $D = 180$ mm $= 0.18$ m, $t_s = 100°C$;

$t_\infty = 20°C$; $\rho = 1.06\ kg/m^3$; $\nu = 18.97 \times 10^{-6}\ m^2/s$;

$c_p = 1.004$ kJ/kg°C; $k = 0.1042$ kJ/m h°C.

Heat loss by free convection, Q :

$$\mu = \rho\nu = 1.06 \times (18.97 \times 10^{-6} \times 3600) = 0.07239 \text{ kg/mh}$$

$$\beta = \frac{1}{T} = \frac{1}{273 + t_f} = \frac{1}{273 + \left(\frac{100 + 20}{2}\right)} = 0.003\ K^{-1}$$

$$Gr = \frac{L^3 g \beta \Delta t}{\nu^2}$$

$$= \frac{(1.5)^3 \times 9.81 \times 0.003 \times (100 - 20)}{(18.97 \times 10^{-6})^2} = 2.208 \times 10^{10}$$

$$Pr = \frac{\mu c_p}{k} = \frac{0.07239 \times 1.004}{0.1042} = 0.6975$$

$$Gr\ Pr = 2.208 \times 10^{10} \times 0.6975 = 1.54 \times 10^{10}$$

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For this value of $Gr\,Pr$ (tubulent range),

$$\overline{Nu}_L = \frac{\bar{h}\,L}{k} = 0.10\,(Gr.Pr)^{1/3} \quad (\text{for } 10^9 < Gr.Pr < 10^{12})$$

$$= 0.10\,(1.54 \times 10^{10})^{1/3} = 248.79$$

$$\therefore \quad \bar{h} = \frac{k}{L} \times 248.79 = \frac{0.1042}{1.5} \times 248.79 = 17.283 \text{ kJ/h m}^2\text{°C}$$

$\therefore$ Rate of heat loss, $Q = \bar{h}A\,(t_s - t_\infty)$

$$= 17.283 \times (\pi \times 0.18 \times 1.5) \times (100 - 20)$$

$$= \mathbf{1172.8\ kJ/h} \quad \textbf{(Ans.)}$$

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Example *A cylindrical body of 300 mm diameter and 1.6 m height is maintained at a constant temperature of 36.5°C. The surrounding temperature is 13.5°C. Find out the amount of heat to be generated by the body per hour if* $\rho = 1.025\ kg/m^3$; $c_p = 0.96\ kJ/kg°C$; $\nu = 15.06 \times 10^{-6}\ m^2/s$; $k = 0.0892\ kJ/m\text{-}h\text{-}°C$ *and* $\beta = \frac{1}{298}\ K^{-1}$. *Assume* $Nu = 0.12\ (Gr \cdot Pr)^{1/3}$ *(the symbols have their usual meanings).*

Solution. *Given :* $D = 300\ mm = 0.3\ m$; $L = 1.6\ m$; $t_s = 36.5°C$; $t_\infty = 13.5°C$;

$\rho = 1.025\ kg/m^3$; $c_p = 0.96\ kJ/kg°C$; $\nu = 15.06 \times 10^{-6}\ m^2/s$;

$k = 0.0892\ kJ/m\text{-}h\text{-}°C$; $\beta = \frac{1}{298}\ K^{-1}$; $Nu = 0.12\ (Gr \cdot Pr)^{1/3}$

The amount of heat to be generated :

Grashoff number, $$Gr = \frac{L^3 g\beta\ (t_s - t_\infty)}{\nu^2}$$

$$= \frac{(1.6)^3 \times 9.81 \times \left(\frac{1}{298}\right) \times (36.5 - 13.5)}{(15.06 \times 10^{-6})^2} = 1.3674 \times 10^{10}$$

Prandtl number, $$Pr = \frac{\mu c_p}{k} = \frac{\rho \nu c_p}{k} = \frac{1.025 \times (15.06 \times 10^{-6} \times 3600) \times 0.96}{0.0892} \qquad \left(\because \nu = \frac{\mu}{\rho}\right)$$

Nusselt number, $$Nu = \frac{hL}{k} = 0.12\ (Gr \cdot Pr)^{1/3}$$

$$= 0.12\ (1.3674 \times 10^{10} \times 0.598)^{1/3} = 241.75$$

$$\therefore \qquad h = \frac{k}{L} \times 241.75 = \frac{0.0892}{1.6} \times 241.75 = 13.478\ kJ/h\ m^2°C$$

Heat lost from the surface by natural convection,

$$Q = hA\ (t_s - t_\infty)$$

$$= 13.478 \times (\pi \times 0.3 \times 1.6)\ (36.5 - 13.5) = \mathbf{*467.5\ kJ/h} \qquad \textbf{(Ans.)}$$

* This is the amount of heat to be generated.

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Example *A hot plate 1.2 m wide, 0.35 m high and at 115°C is exposed to the ambient still air at 25°C. Calculate the following:*

(i) Maximum velocity at 180 mm from the leading edge of the plate;

(ii) The boundary layer thickness at 180 mm from the leading edge of the plate;

(iii) Local heat transfer coefficient at 180 mm from the leading edge of the plate;

(iv) Average heat transfer coefficient over the surface of the plate;

(v) Total mass flow through the boundary;

(vi) Heat loss from the plate;

(vii) Rise in temperature of the air passing through the boundary.

Use the approximate solution.

Solution. *Given* : $t_s = 115°C$, $t_\infty = 25°C$, $x = 180 \text{ mm} = 0.18 \text{ m}$

The mean film temperature, $$t_f = \frac{t_s + t_\infty}{2} = \frac{115 + 25}{2} = 70°C$$

The thermo-physical properties of air at 70°C are :

$$\rho = 1.029 \text{ kg/m}^3;\ k = 0.02964 \text{ w/m°C};\ \nu = 20.02 \times 10^{-6} \text{ m}^2/\text{s}$$

$$Pr = 0.694,\ \beta = \frac{1}{T} = \frac{1}{(273 + t_f)} = \frac{1}{(273 + 70)} = \frac{1}{343} = 0.002915$$

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Solution. Grashoff number $Gr_x = \dfrac{x^3 \, g\beta \, (t_s - t_\infty)}{\nu^2}$

$$= \frac{(0.18)^3 \times 9.81 \times 0.002915 \, (115 - 25)}{(20.02 \times 10^{-6})^2} = 3.745 \times 10^7$$

and, $$Gr_L = 3.745 \times 10^7 \times \left(\frac{0.35}{0.18}\right)^3 = 27.532 \times 10^7$$

(i) Maximum velocity at 180 mm from the leading edge of the plate, u_{max}:

$$u_{max} = 0.766 \, \nu \left(0.952 + \frac{\nu}{\alpha}\right)^{-1/2} \left[\frac{g\beta \, (t_s - t_\infty)}{\nu^2}\right]^{1/2} (x)^{1/2}$$

$$= 0.766 \times 20.02 \times 10^{-6} \, (0.952 + 0.694)^{-1/2} \times \left[\frac{9.81 \times 0.002915 \, (115 - 25)}{(20.02 \times 10^{-6})^2}\right]^{1/2} (0.18)^{1/2}$$

06 m/s (Ans.)

$\left(\because Pr = \dfrac{\nu}{\alpha} = 0.694 \text{ ...as given above}\right)$

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Solution

(ii) The boundary layer thickness at 180 mm from the leading edge of the plate, δ:

$$\frac{\delta}{x} = 3.93\,(0.952 + Pr)^{1/4}\,(Gr_x)^{-1/4}\,(Pr)^{-1/2}$$

$$= 3.93\,(0.952 + 0.694)^{1/4}\,(3.745 \times 10^7)^{-1/4}\,(0.694)^{-1/2} = 0.0683$$

$$\delta = 0.0683 \times 0.18 = 0.01229 \text{ m} = \mathbf{12.29 \text{ mm}} \quad \textbf{(Ans.)}$$

(iii) Local heat transfer coefficient at 180 mm from the leading edge of the plate, h_x:

$$Nu_x = \frac{h_x\,x}{k} = 0.508\,(Pr)^{1/2}\,(0.952 + Pr)^{-1/4}\,(Gr_x)^{1/4} = \frac{2x}{\delta}$$

$$h_x = \frac{k}{x} \times \frac{2x}{\delta} = \frac{2k}{\delta} = \frac{2 \times 0.02964}{0.01229} = \mathbf{4.823 \text{ W/m}^2\,^\circ\text{C}} \quad \textbf{(Ans.)}$$

(iv) Average heat transfer coefficient over the surface of the plate, $\bar{h}$:

$$\bar{h} = \frac{k}{L}\,[0.677\,(Pr)^{1/2}\,(0.952 + Pr)^{-1/4}\,(Gr_L)^{1/4}]$$

$$= \frac{0.02964}{0.35}\,[0.677\,(0.694)^{1/2}\,(0.952 + 0.694)^{-1/4}\,(27.532 \times 10^7)^{1/4}]$$

5.43 W/m^2°C (Ans.)

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Solution

(v) Total mass flow through the boundary, m_f :

Total mass flow through the boundary can be calculated from the following formula :

$$m_f = 1.7\ \rho.\nu \left[\frac{Gr_L}{(P_r^2)\ (Pr + 0.952)}\right]^{1/4}$$

$$= 1.7 \times 1.029 \times 20.02 \times 10^{-6} \left[\frac{27.532 \times 10^7}{(0.694)^2\ (0.694 + 0.952)}\right]$$

or, $m_f = \mathbf{0.00478\ kg/s}$ **(Ans.)**

(vi) Heat loss from the plate, Q :

Heat lost from the plate will be from both the sides, hence

$$Q = 2\ \bar{h}\ A_s\ (t_s - t_\infty)$$

$$= 2 \times 5.43 \times (0.35 \times 1.2)\ (115 - 25) = \mathbf{410.5\ W} \qquad \textbf{(Ans.)}$$

(vii) Rise in temperature of the air passing through the boundary, Δt:

Heat lost, $Q = m_f\ c_p\ \Delta t$

or, $410.5 = 0.00478 \times (1.005 \times 10^3) \times \Delta t$

or, $\Delta t = \dfrac{410.5}{0.00478 \times (1.005 \times 10^3)} = \mathbf{85.45°C}$ **(Ans.)**

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NATURAL CONVECTION OVER SURFACES

Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation, the variation of temperature on the surface and the thermophysical properties of the fluid involved.

With the exception of some simple cases, heat transfer relations in natural convection are based on experimental studies.

$$\mathrm{Nu} = \frac{hL_c}{k} = C(\mathrm{Gr}_L\,\mathrm{Pr})^n = C\,\mathrm{Ra}_L^n$$

$$\mathrm{Ra}_L = \mathrm{Gr}_L\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,\mathrm{Pr}$$

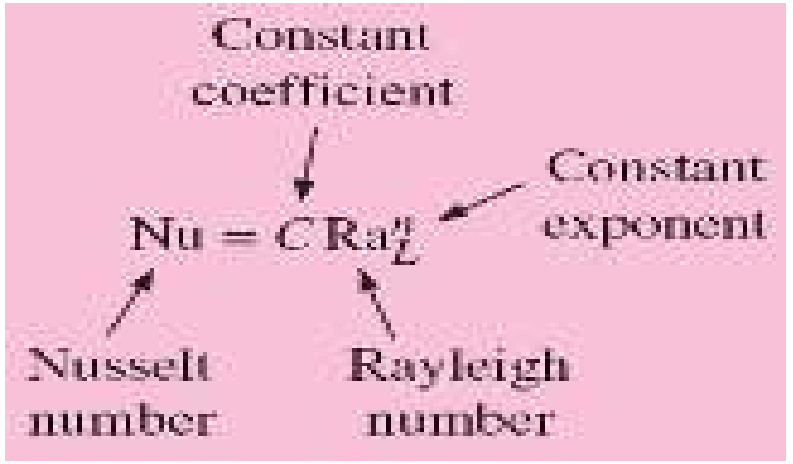
Rayleigh number

The constants C and n depend on the *geometry* of the surface and the *flow regime*, which is characterized by the range of the Rayleigh number.

The value of n is usually 1/4 for laminar flow and 1/3 for turbulent flow.

All fluid properties are to be evaluated at the film re $T_f = (T_s + T_\infty)/2$.

$$= hA_s(T_s - T_\infty) \qquad (\mathrm{W})$$



Natural convection heat transfer correlations are usually expressed in terms of the Rayleigh number raised to a constant n multiplied by another constant C, both of which are determined experimentally.

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- **General correlations for vertical plate**

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = C Ra_L^n$$ → **Eq. (9.24)**

where,

Laminar	$10^4 \le Ra_L \le 10^9$	C = 0.59	n = 1/4
Turbulent	$10^9 \le Ra_L \le 10^{13}$	C = 0.10	n = 1/3

- **For wide range and more accurate solution, use correlation Churchill and Chu**

→ **Eq. (20.21)**

➢ All Conditions:

$$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387\ Ra_L^{1/6}}{\left[1 + (0.492/\mathrm{Pr})^{9/16}\right]^{8/27}} \right\}^2$$

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Natural convection over surfaces (Summary)

1) $$\mathrm{Nu} = \frac{hL_c}{k} = C(\mathrm{Gr}_L\,\mathrm{Pr})^n = C\,\mathrm{Ra}_L^n$$

*C & n is depend on the geometry of the surface and flow regime.
n=1/4 → laminar flow
n-=1/3 → turbulent flow

2) $$\mathrm{Ra}_L = \mathrm{Gr}_L\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha}$$

1. What is the difference between Re_L and Ra_L ?
2. What is the transition range in a free convection boundary ?

$10^4 \leq Ra \leq 10^9$ (Laminar)
$10^9 \leq Ra \leq 10^{13}$ (Turbulent)

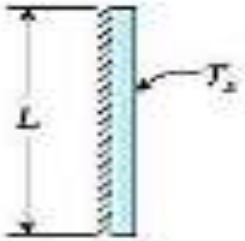
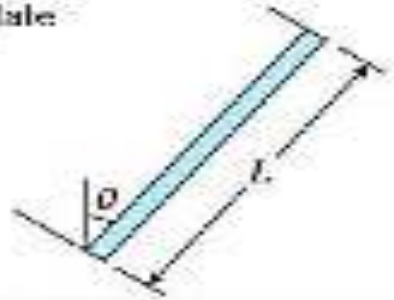
3) $$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

*All the properties are evaluated at the film temperature, $T_f=(T_s+T_\infty)/2$

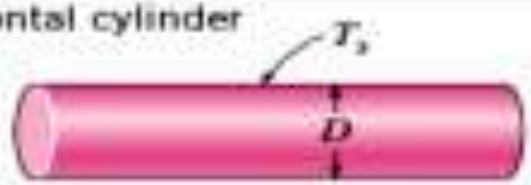
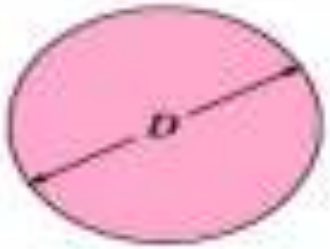
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TABLE 20–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu	
Vertical plate 	L	10^4–10^9 10^{10}–10^{12} Entire range	$\overline{Nu} = 0.59\overline{Ra}_L^{1/4}$ $\overline{Nu} = 0.1\overline{Ra}_L^{1/3}$ $\overline{Nu} = \left\{0.825 + \frac{0.387\overline{Ra}_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)	(20–19) (20–20) (20–21)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $0 < \theta < 60°$	
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11}	$\overline{Nu} = 0.54\overline{Ra}_L^{1/4}$ $\overline{Nu} = 0.15\overline{Ra}_L^{1/3}$	(20–22) (20–23)
		10^5–10^{11}	$\overline{Nu} = 0.27\overline{Ra}_L^{1/4}$	(20–24)

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Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$
Sphere 	D	$Ra_D \leq 10^{11}$ ($Pr \geq 0.7$)	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$

Vertical Plates (T_s = constant)

For a vertical flat plate, the characteristic length is the plate height L. In Table give three relations for the average Nusselt number for an isothermal plate. The first two relations are very simple. Despite its complexity, gest using the third one (Eq. 9–21) recommended by Churchill and 975) since it is applicable over the entire range of Rayleigh number. relation is most accurate in the range of $10^{-1} < Ra_L < 10^9$.

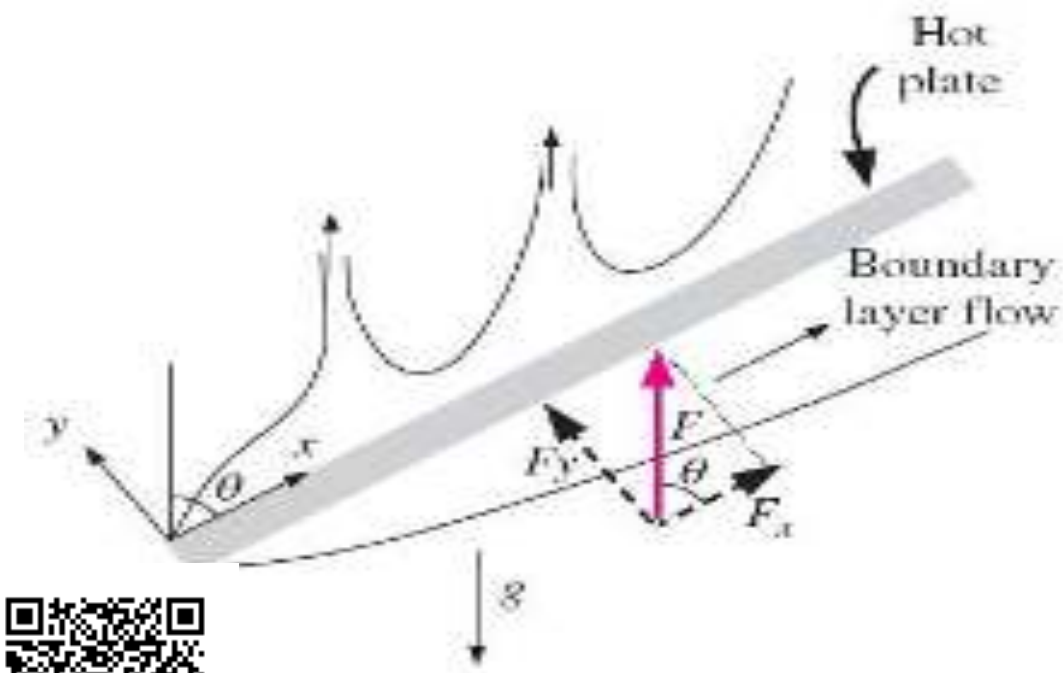
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Vertical Plates (q_s = constant)

The relations for isothermal plates in the table can also be used for plates subjected to uniform heat flux, provided that the plate midpoint temperature $T_{L/2}$ is used for T_s in the evaluation of the film temperature, Rayleigh number, and the Nusselt number.

$$\mathrm{Nu} = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)} \qquad \dot{Q} = \dot{q}_s A_s$$

Inclined Plates



nvection flows on the upper and
aces of an inclined hot plate.

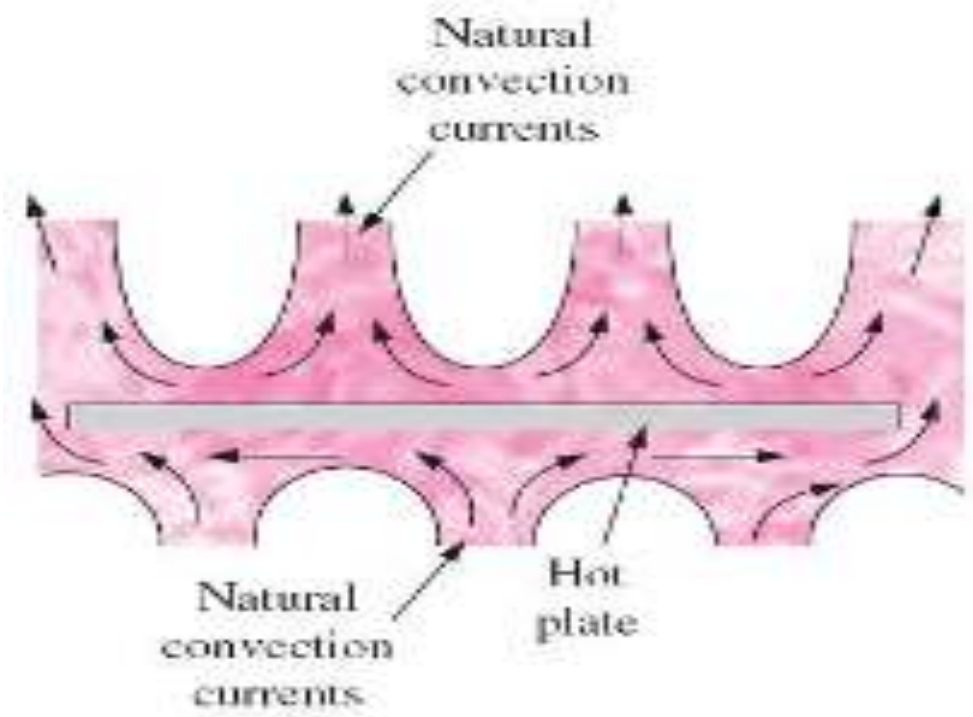
In a hot plate in a cooler environment for the lower surface of a hot plate, the convection currents are weaker, and the rate of heat transfer is lower relative to the vertical plate case.

On the upper surface of a hot plate, the thickness of the boundary layer and thus the resistance to heat transfer decreases, and the rate of heat transfer increases relative to the vertical orientation.

In the case of a cold plate in a warmer environment, the opposite occurs.

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Horizontal Plates



Natural convection flows on the upper and lower surfaces of a horizontal hot plate.

For a hot surface in a cooler environment, the net force acts upward, forcing the heated fluid to rise.

If the hot surface is facing upward, the heated fluid rises freely, inducing strong natural convection currents and thus effective heat transfer.

But if the hot surface is facing downward, the plate blocks the heated fluid that tends to rise, impeding heat transfer.

The opposite is true for a cold plate in a warmer environment since the net force (weight minus buoyancy force) in this case acts downward, and the cooled fluid near the plate tends to descend.

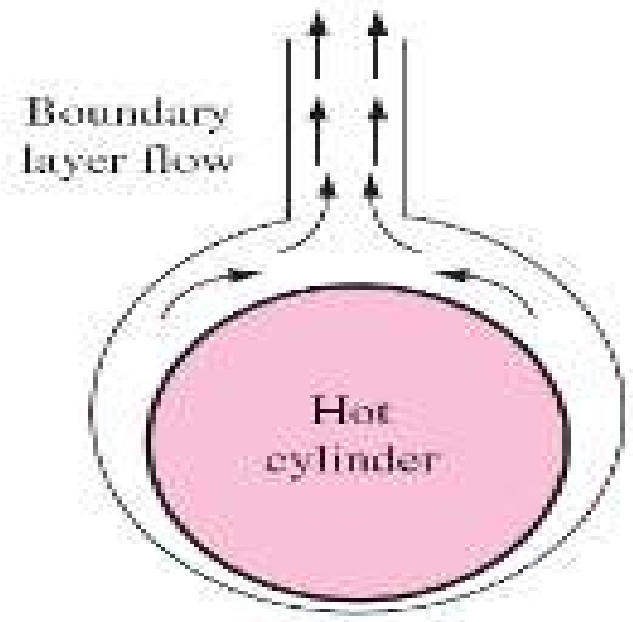
$$L_c = \frac{A_s}{p}$$ Characteristic length

$L_c = a/4$ for a horizontal square surface of length a

$L_c = D/4$ for a horizontal circular surface of diameter D

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Horizontal Cylinders and Spheres



Natural convection flow over a horizontal hot cylinder.

The boundary layer over a hot horizontal cylinder starts to develop at the bottom, increasing in thickness along the circumference, and forming a rising plume at the top.

Therefore, the local Nusselt number is highest at the bottom, and lowest at the top of the cylinder when the boundary layer flow remains laminar.

The opposite is true in the case of a cold horizontal cylinder in a warmer medium, and the boundary layer in this case starts to develop at the top of the cylinder and ending with a descending plume at the bottom.

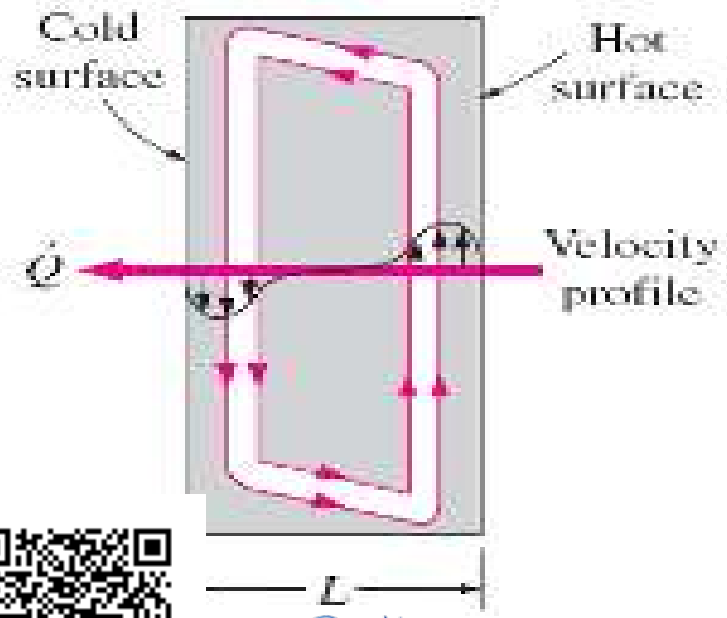
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NATURAL CONVECTION INSIDE ENCLOSURES

Enclosures are frequently encountered in practice, and heat transfer through them is of practical interest. In a vertical enclosure, the fluid adjacent to the hotter surface rises and the fluid adjacent to the cooler one falls, setting off a rotationary motion within the enclosure that enhances heat transfer through the enclosure.

$$\mathrm{Ra}_{L_c} = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\,\mathrm{Pr}$$

L_c charecteristic length: the distance between the hot and cold surfaces
T_1 and T_2: the temperatures of the hot and cold surfaces



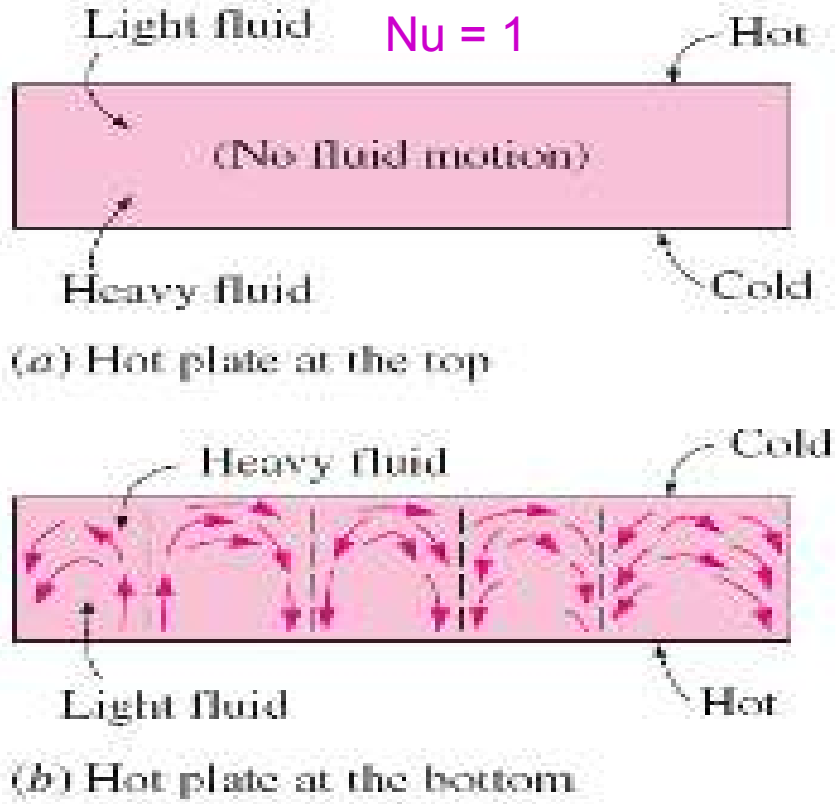
e currents in a vertical
ır enclosure.

Ra > 1708, natural convection currents
Ra > 3×10^5, turbulent fluid motion

Fluid properties at

$$T_{avg} = (T_1 + T_2)/2$$

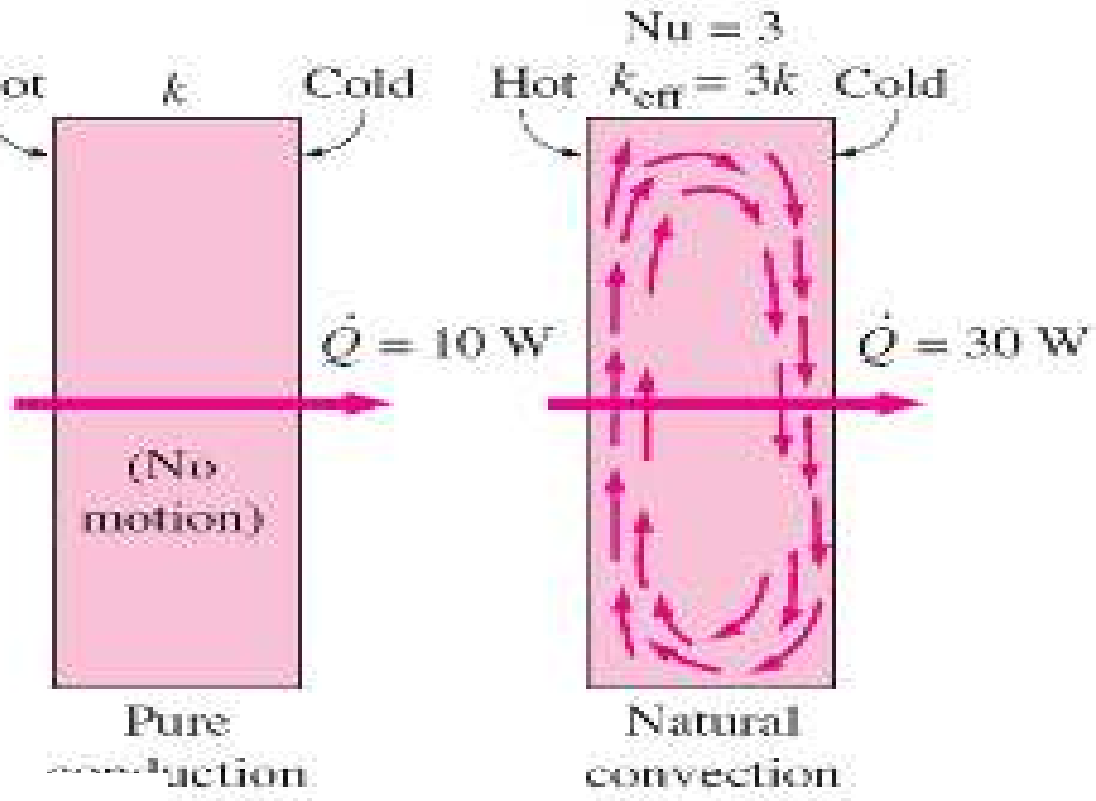
Convective currents in a horizontal enclosure with (*a*) hot plate at the top and (*b*) hot plate at the bottom.



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Effective Thermal Conductivity

$$\dot{Q} = hA_s(T_1 - T_2) = kNuA_s\frac{T_1 - T_2}{L_c} \qquad h = kNu/L \qquad \dot{Q}_{cond} = kA_s\frac{T_1 - T_2}{L_c}$$



$k_{eff} = kNu$ effective thermal conductivity

The fluid in an enclosure behaves like a fluid whose thermal conductivity is kNu as a result of convection currents.

Nu = 1, the effective thermal conductivity of the enclosure is equal to the conductivity of the fluid. This case corresponds to pure conduction.

umber of 3 for an enclosure indicates that
er through the enclosure by *natural*
is three times that by *pure conduction.*

Numerous correlations for the Nusselt number exist. Simple power-law type relations in the form of $Nu = CRa^n$, where C and n are constants, are sufficiently accurate, but they are usually applicable to a narrow range of Prandtl and Rayleigh numbers and aspect ratios.

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Horizontal Rectangular Enclosures

$$\mathrm{Nu} = 0.195\mathrm{Ra}_L^{1/4} \qquad 10^4 < \mathrm{Ra}_L < 4 \times 10^5$$

$$\mathrm{Nu} = 0.068\mathrm{Ra}_L^{1/3} \qquad 4 \times 10^5 < \mathrm{Ra}_L < 10^7$$

For horizontal enclosures that contain air, These relations can also be used for other gases with 0.5 < Pr < 2.

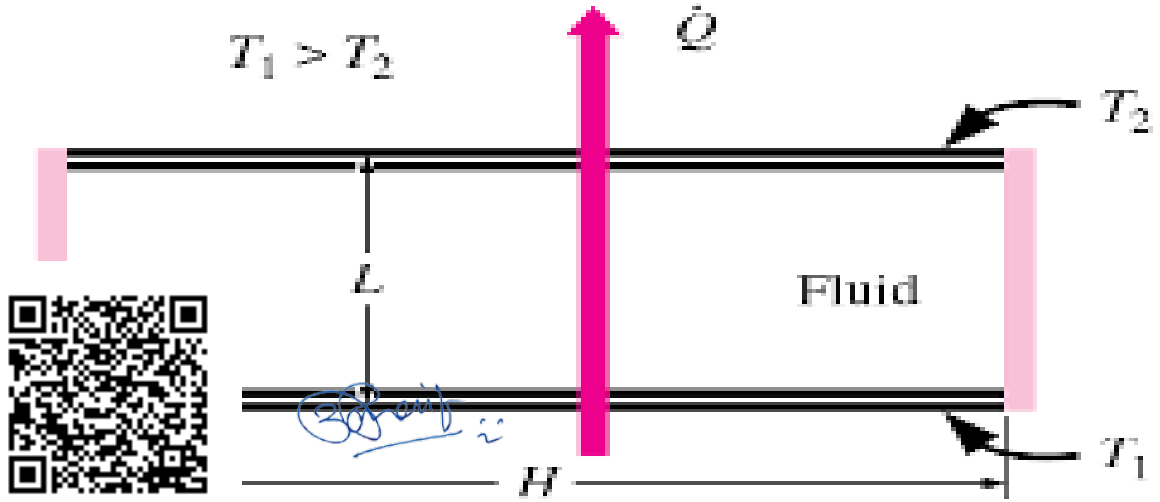
$$\mathrm{Nu} = 0.069\mathrm{Ra}_L^{1/3}\,\mathrm{Pr}^{0.074} \qquad 3 \times 10^5 < \mathrm{Ra}_L < 7 \times 10^9$$

For water, silicone oil, and mercury

$$\mathrm{Nu} = 1 + 1.44\left[1 - \frac{1708}{\mathrm{Ra}_L}\right]^+ + \left[\frac{\mathrm{Ra}_L^{1/3}}{18} - 1\right]^+ \qquad \mathrm{Ra}_L < 10^8$$

$[\,]^+$ only positive values to be used

Based on experiments with air. It may be used for liquids with moderate Prandtl numbers for $\mathrm{Ra}_L < 10^5$.



When the hotter plate is at the top, Nu = 1.

A horizontal rectangular enclosure with isothermal surfaces.

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Inclined Rectangular Enclosures

$$\mathrm{Nu} = 1 + 1.44\left[1 - \frac{1708}{\mathrm{Ra}_L \cos\theta}\right]^{+}\left(1 - \frac{1708(\sin 1.8\theta)^{1.6}}{\mathrm{Ra}_L \cos\theta}\right) + \left[\frac{(\mathrm{Ra}_L \cos\theta)^{1/3}}{18} - 1\right]^{+}$$

$$\mathrm{Ra}_L < 10^5,\ 0 < \theta < 70°,\ \text{and}\ H/L \geq 12$$

$$\mathrm{Nu} = \mathrm{Nu}_{\theta=0°}\left(\frac{\mathrm{Nu}_{\theta=90°}}{\mathrm{Nu}_{\theta=0°}}\right)^{\theta/\theta_{cr}} (\sin\theta_{cr})^{\theta/(4\theta_{cr})} \qquad 0° < \theta < \theta_{cr} \qquad H/L < 12$$

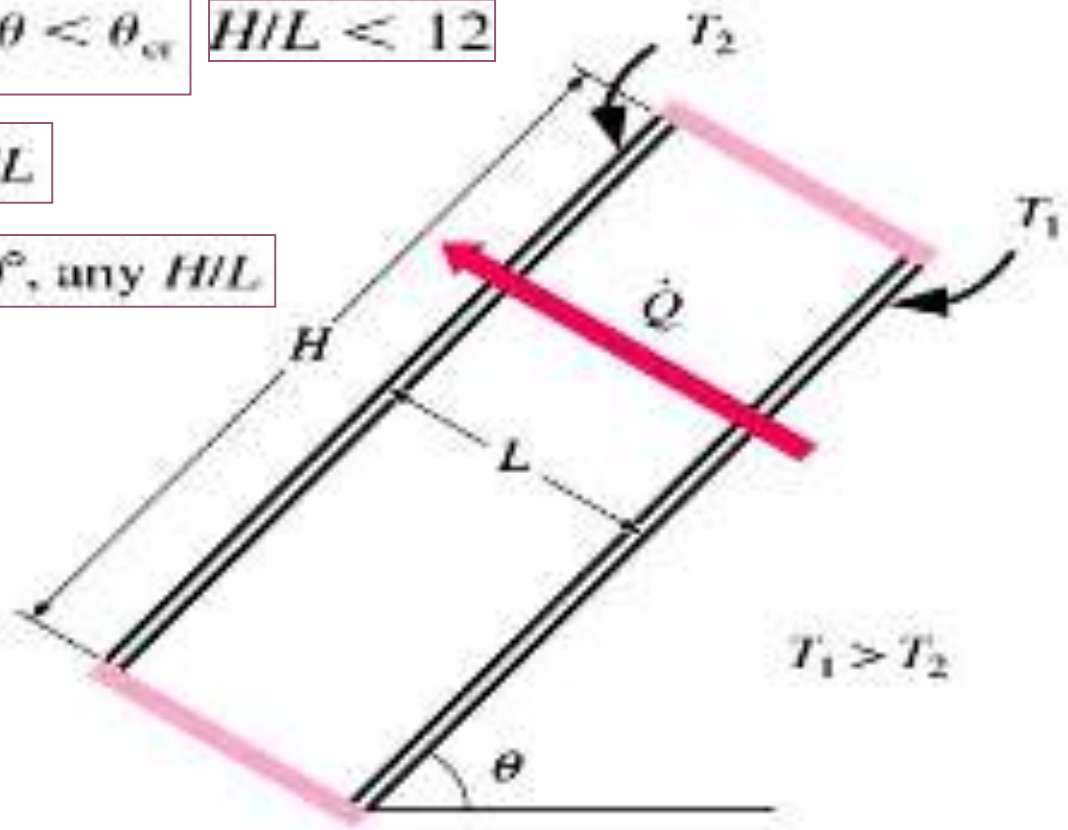
$$\mathrm{Nu} = \mathrm{Nu}_{\theta=90°}(\sin\theta)^{1/4} \qquad \theta_{cr} < \theta < 90°,\ \text{any } H/L$$

$$\mathrm{Nu} = 1 + (\mathrm{Nu}_{\theta=90°} - 1)\sin\theta \qquad 90° < \theta < 180°,\ \text{any } H/L$$

TABLE 20–2

Critical angles for inclined rectangular enclosures

Aspect ratio, H/L	Critical angle, θ_{cr}
	25°
	53°
	60°
2	67°
12	70°



An inclined rectangular enclosure with isothermal surfaces.

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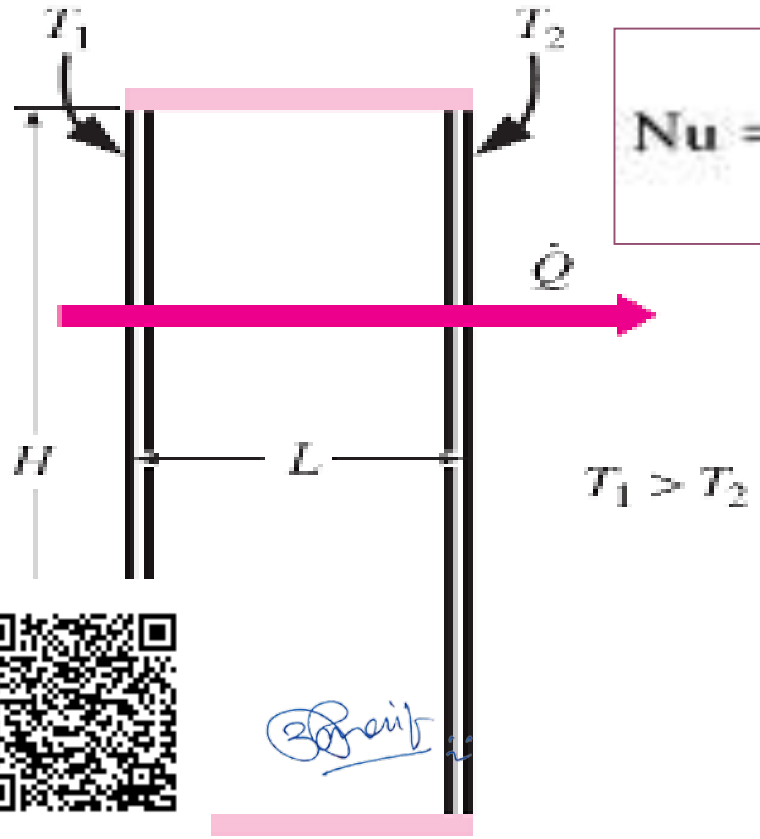
Vertical Rectangular Enclosures

$$\mathrm{Nu} = 0.18\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.29}$$

$1 < H/L < 2$
any Prandtl number
$\mathrm{Ra}_L\,\mathrm{Pr}/(0.2+\mathrm{Pr}) > 10^3$

$$\mathrm{Nu} = 0.22\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4}$$

$2 < H/L < 10$
any Prandtl number
$\mathrm{Ra}_L < 10^{10}$



$$\mathrm{Nu} = 0.42\mathrm{Ra}_L^{1/4}\,\mathrm{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3}$$

$10 < H/L < 40$
$1 < \mathrm{Pr} < 2 \times 10^4$
$10^4 < \mathrm{Ra}_L < 10^7$

$$\mathrm{Nu} = 0.46\mathrm{Ra}_L^{1/3}$$

$1 < H/L < 40$
$1 < \mathrm{Pr} < 20$
$10^6 < \mathrm{Ra}_L < 10^9$

A vertical rectangular enclosure with isothermal surfaces.

Again, all fluid properties are to be evaluated at the average temperature $(T_1+T_2)/2$.

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Concentric Cylinders

The rate of heat transfer through the annular space between the cylinders by natural convection per unit length

$$\dot{Q} = \frac{2\pi k_{eff}}{\ln(D_o/D_i)}(T_i - T_o) \qquad \text{(W/m)}$$

Characteristic length

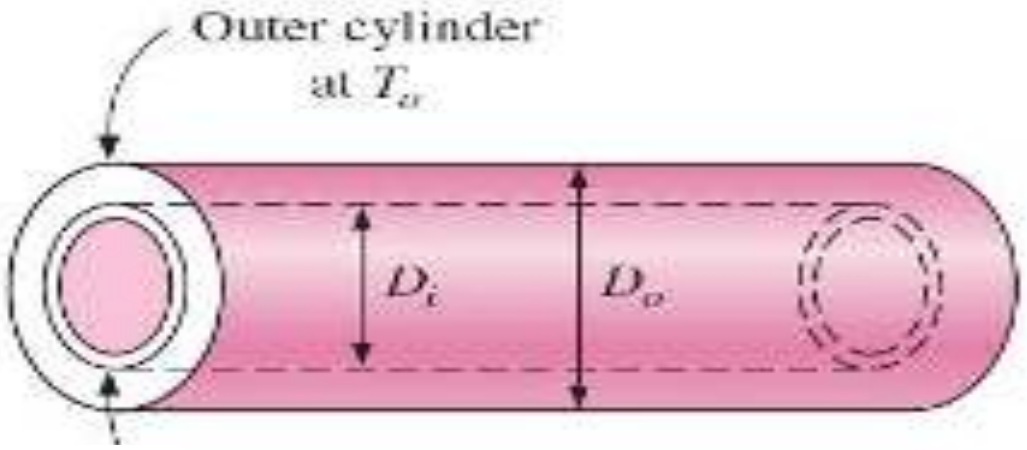
$$L_c = (D_o - D_i)/2$$

$$\frac{k_{eff}}{k} = 0.386\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{cyl}\text{Ra}_L)^{1/4}$$

$$0.70 \le \text{Pr} \le 6000 \text{ and } 10^2 \le F_{cyl}\text{Ra}_L \le 10^7$$

$$F_{cyl} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$$

the geometric factor for concentric cylinders



centric horizontal isothermal cylinders.

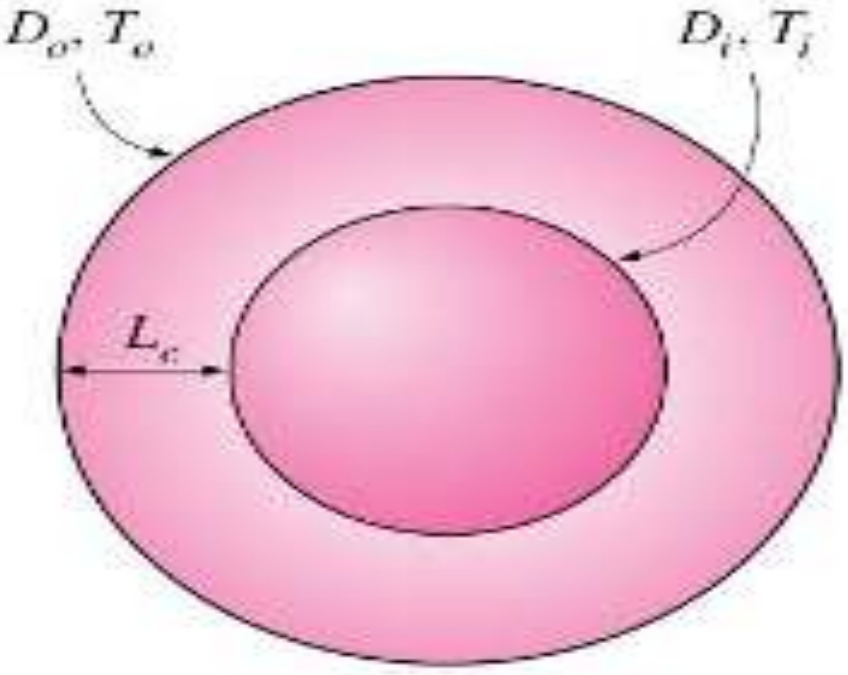
For $F_{cyl}\text{Ra}_L < 100$, natural convection currents are negligible and thus $k_{eff} = k$.

Note that k_{eff} cannot be less than k, and thus we should set $k_{eff} = k$ if $k_{eff}/k < 1$.

The fluid properties are evaluated at the average temperature of $(T_i + T_o)/2$.

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Concentric Spheres



oncentric isothermal
es.

Characteristic length

$$L_c = (D_o - D_i)/2$$

$$\dot{Q} = k_{\text{eff}} \frac{\pi D_i D_o}{L_c} (T_i - T_o) \qquad (\text{W})$$

$$\frac{k_{\text{eff}}}{k} = 0.74 \left(\frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{sph}} \text{Ra}_L)^{1/4}$$

$$0.70 \le \text{Pr} \le 4200 \text{ and } 10^2 \le F_{\text{sph}} \text{Ra}_L \le 10^4$$

$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5}$$

If $k_{\text{eff}}/k < 1$, we should set $k_{\text{eff}} = k$.

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Example *A sheet metal air duct carries air-conditioned air at an average temperature of 10°C. The duct size is 320 mm × 200 mm and length of the duct exposed to the surrounding air at 30°C is 15 m long. Find the heat gain by the air in the duct. Assume 200 mm side is vertical and top surface of the duct is insulated. Use the following properties :*

$Nu = 0.6\ (Gr.Pr)^{0.25}$ *for vertical surface.*

$Nu = 0.27\ (Gr.Pr)^{0.25}$ *for horizontal surface.*

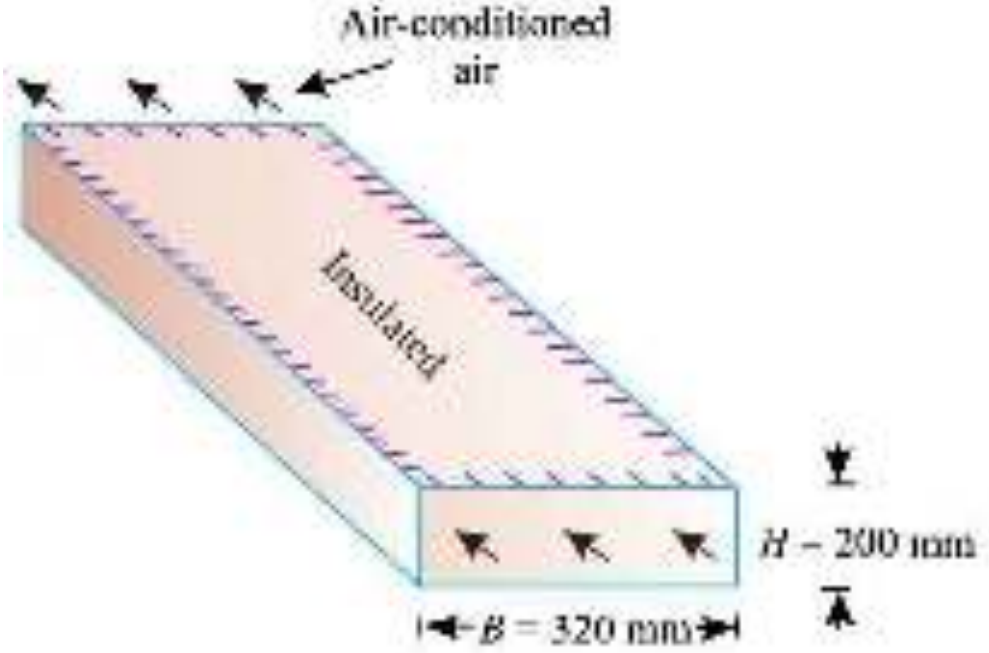
Take the properties of the air at mean temperature of (30 + 10)/2 = 20°C as given below :

$c_p = 100$ *J/kg K;* $\rho = 1.204$ *kg/m³;* $\mu = 18.2 \times 10^{-6}$ *N-s/m²;*

$\nu = 15.1 \times 10^{-6}$ *m²/s;* $k = 0.256$ *W/m K and* $Pr = 0.71$.

Solution.

There will be two vertical surfaces and one horizontal surface from which the heat will be gained by *natural convection*. There is no heat transfer from the top surface as it is insulated.



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Solution.

$$\text{Now.} \quad (Gr)_v = \frac{g\beta H^3 (\Delta t)}{\nu^2}$$

$$= \frac{9.81 \times \left(\frac{1}{273 + 20}\right) \times 0.2^3 \times (30 - 10)}{(15.1 \times 10^{-6})^2}$$

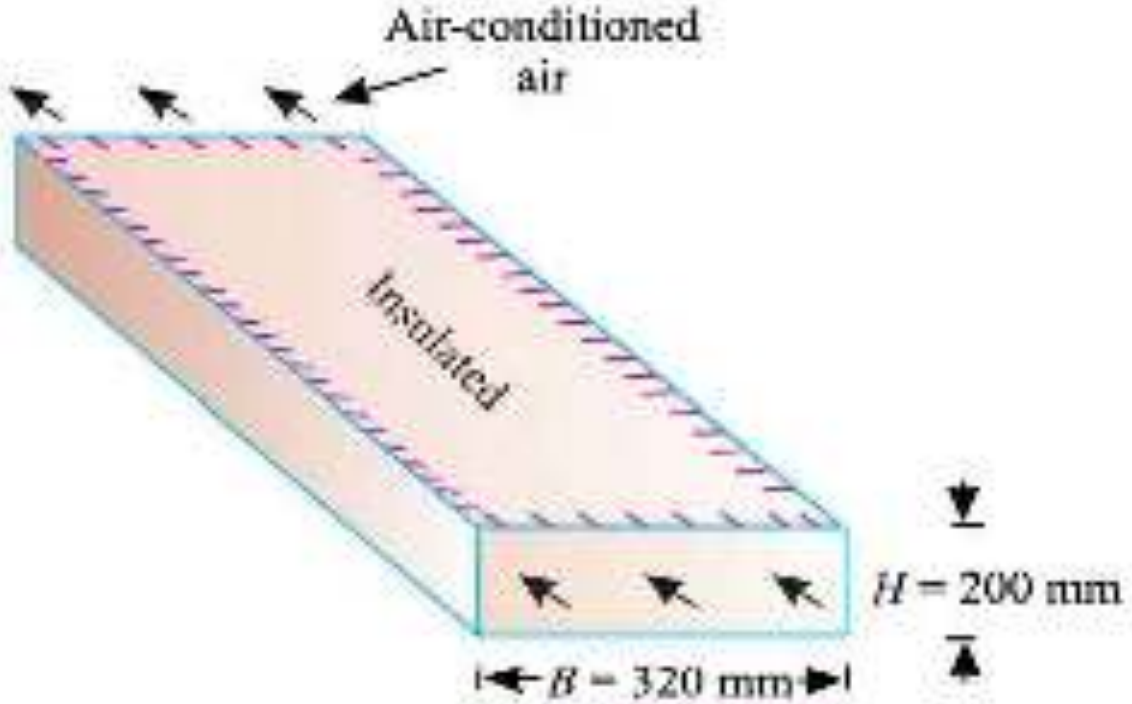
$$= 2.35 \times 10^7$$

$$(Gr.Pr)_v = 2.35 \times 10^7 \times 0.71 = 16.68 \times 10^6$$

$$(Nu)_v = \frac{h_v . H}{k} = 0.6\,(16.68 \times 10^6)^{0.25} = 38.34$$

$$h_v = \frac{k}{H} \times 38.34 = \frac{0.256}{0.2} \times 38.34 = 49.07 \text{ W/m}^2\text{°C}$$

$_v$ x (Δt) = 49.07 x (2 x 15 x 0.2) x (30 – 10) = 5888.4 W



$$(Gr)_h = \frac{9.81 \times \frac{1}{(273 + 20)} \times (0.32)^3 \times (30 - 10)}{(15.1 \times 10^{-6})^2} = 9.62 \times 10^7$$

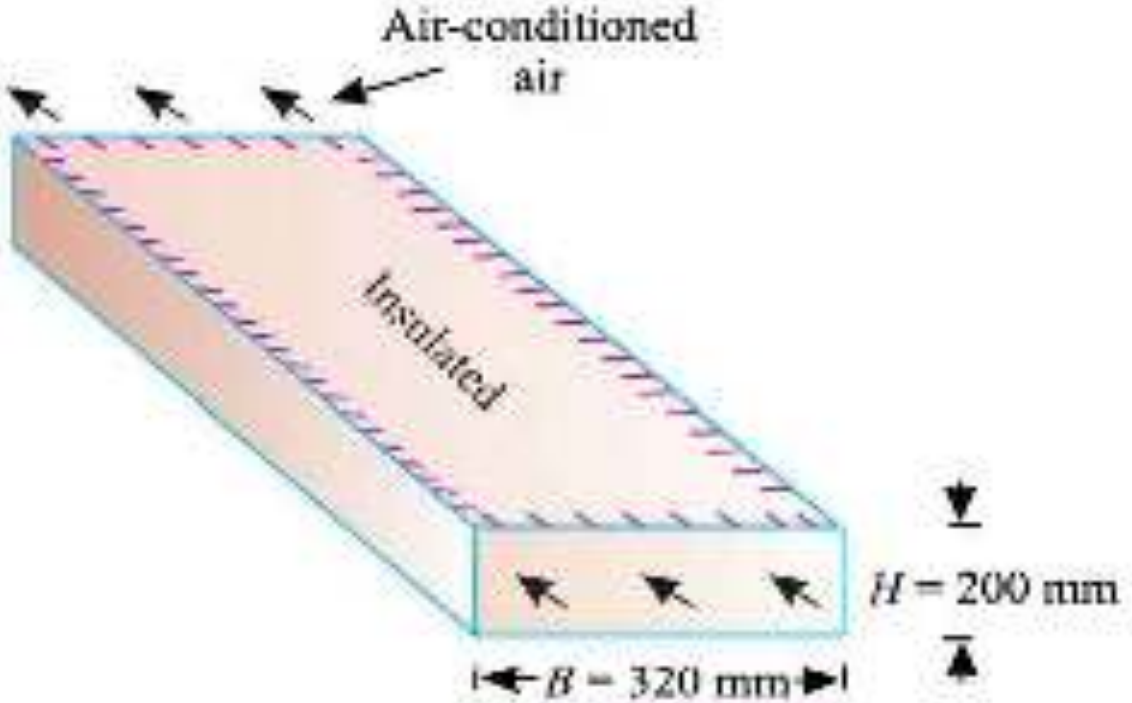
$$(Gr.Pr)_h = 9.62 \times 10^7 \times 0.71 = 6.83 \times 10^7$$

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Solution.

$$(Nu)_L = \frac{h_v \times B}{k} = 0.27\,(6.83 \times 10^7)^{0.25} = 24.54$$

$$h_v = \frac{k}{\beta} = 24.54 = \frac{0.256}{0.32} = 24.54 = 19.63 \text{ W/m}^2\text{°C}$$



$$Q_h = h_h \times A_h \times (\Delta t) = 19.63 \times (15 \times 0.32) \times (30 - 10) = 1884.5 \text{ W}$$

$\therefore$ Total heat gain, $Q = Q_v + Q_h = 5888.4 + 1884.5 = \mathbf{7772.9\ W}$ **(Ans.)**

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Example *Air flows through long rectangular heating duct of width and height of 0.75 m and 0.3 m respectively. The outer surface temperature of the duct is maintained at 45°C. If the duct is exposed to air at 15°C in a cramp-space beneath a home, what is the heat loss from the duct per metre length ?*

Use the following correlations :

(i) Top surface :

$$\text{Take} \qquad L_c = \frac{B}{2} = 0.375\ m$$

$$\overline{Nu}_L = 0.54\ (Ra_L)^{0.25} \text{ if } 10^4 \le Ra_L \le 10^7$$

$$= 0.15\ (Ra_L)^{0.33} \text{ if } 10^7 \le Ra_L \le 10^{11}$$

(ii) Bottom surface :

$$\text{Take} \qquad L_c = \frac{B}{2} = 0.375\ m$$

$$\overline{Nu}_L = 0.27\ (Ra_L)^{0.25},\ 10^5 \le Ra_L \le 10^{10}$$

(iii) Sides of the duct :

$$\overline{Nu}_L = 0.68 + \frac{0.67\ (Ra_L)^{0.25}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9}}$$

Use the following properties of air :

$$\nu = 16.2 \times 10^{-6}\ m^2/s,\ \alpha = 22.9 \times 10^{-6}\ m^2/s$$

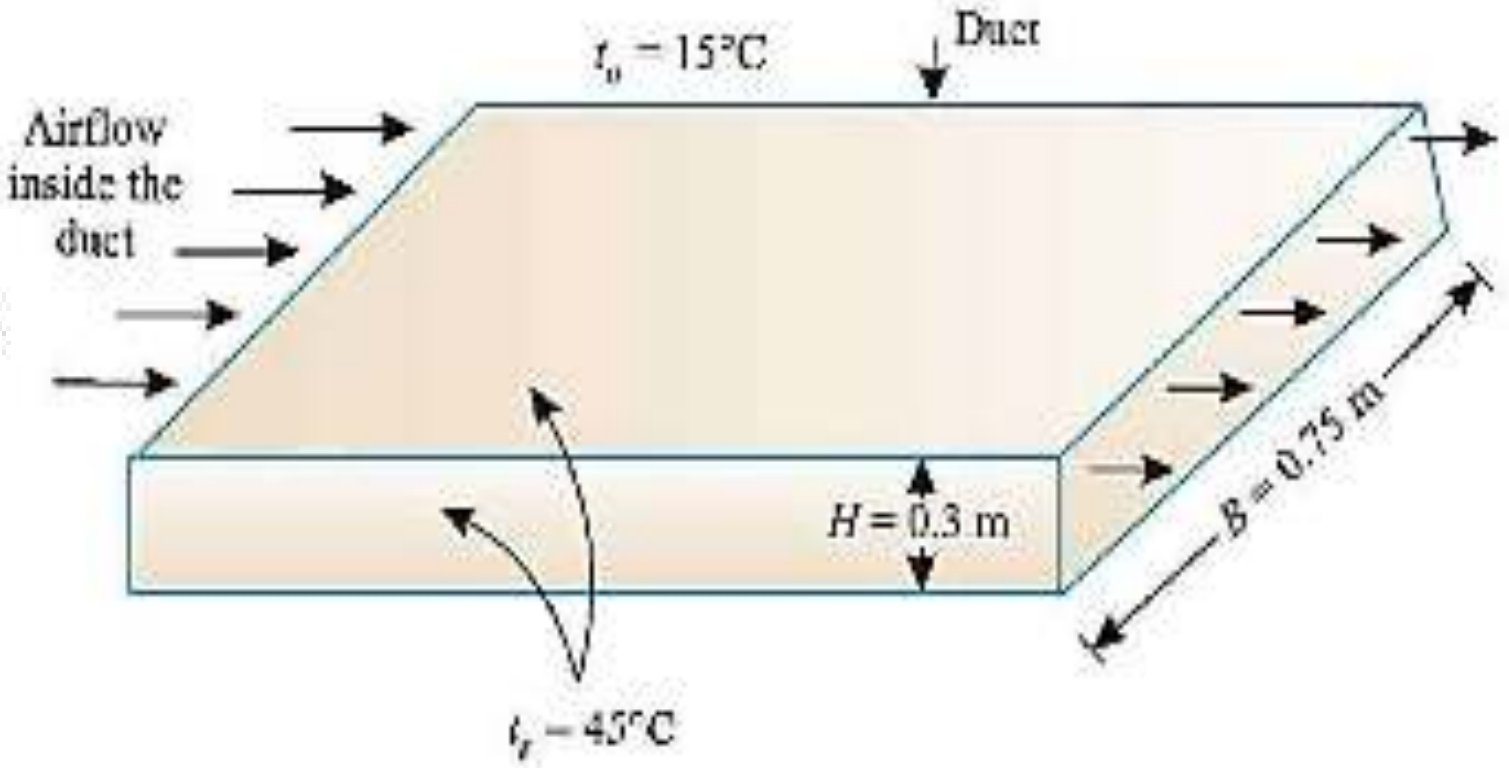
$$k = 0.0265\ W/m\ K,\ \beta = 0.0033\ K^{-1} \text{ and } Pr = 0.71.$$

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Solution. The configuration of the duct is shown in Fig.

$$Ra_{L_c} = \frac{g\beta\,(t_s - t_a)\,L_c^3}{\alpha\nu}$$

$$= \frac{9.81 \times 0.0033 \times (45 - 15)\,L_c^3}{(22.9 \times 10^{-6}) \times (16.2 \times 10^{-6})} = 2.62 \times 10^8\,(L_c^3)$$



For two sides :

$$L_c = H = 0.3 \text{ m}$$

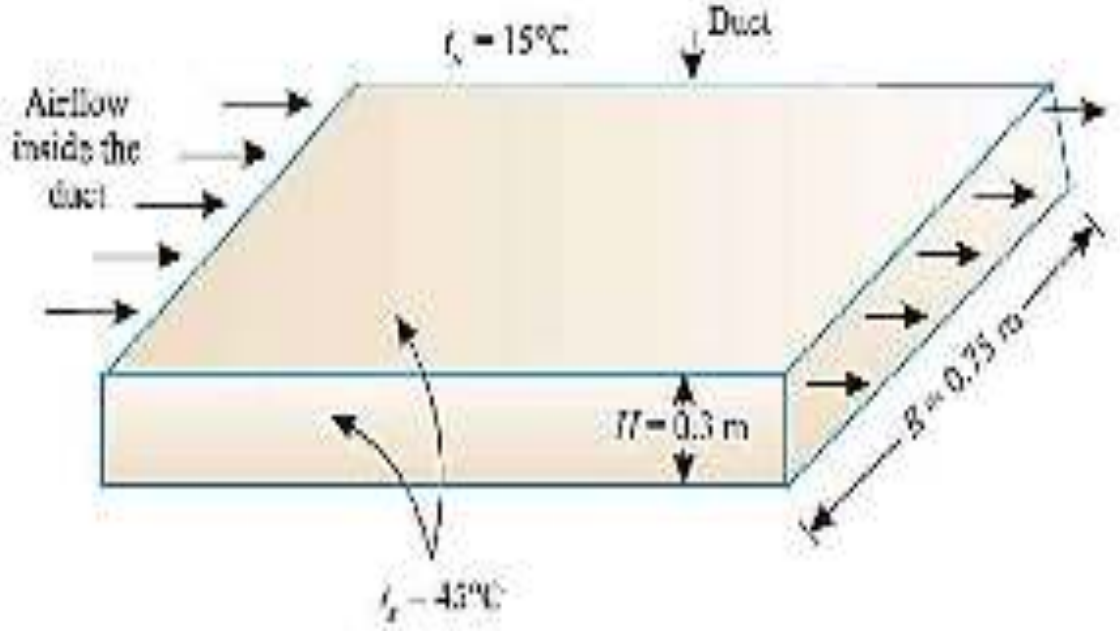
$$Ra_{L_c} = 2.62 \times 10^8\,(0.3)^3 = 7.07 \times 10^7$$

$$Nu_L = 0.68 + \frac{0.67\,(7.07 \times 10^7)^{0.25}}{\left[1 + \left(\frac{0.492}{0.71}\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{61.44}{1.3} = 47.94$$

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$$\therefore \quad \frac{h_s L_c}{k} = 47.94$$

or,
$$h_s = \frac{k}{L_c} \times 47.94 = \frac{0.0265 \times 47.94}{0.3} = 4.23 \text{ W/m}^2\text{°C}$$



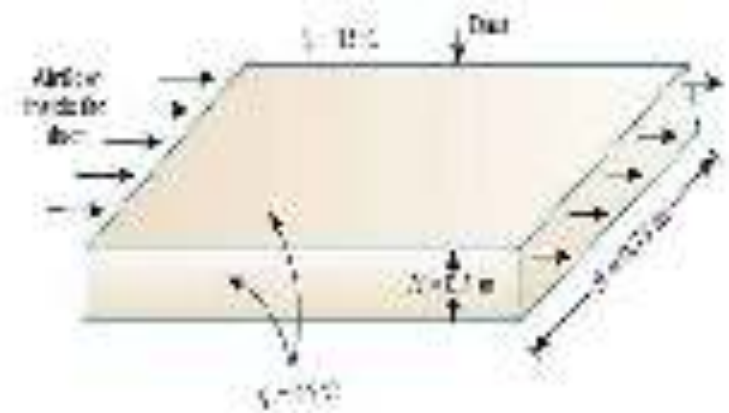
For top surface :

$$\overline{Nu}_L = \frac{h_t L_c}{k} = 0.15 (Ra_L)^{0.33} = 0.15 (7.07 \times 10^7)^{0.33} = 58.4$$

$$\therefore \quad h_t = \frac{k}{L_c} \times 58.4 = \frac{0.0265 \times 58.4}{0.375} = 4.127 \text{ W/m}^2\text{°C}$$

$$\left(\because L_c = \frac{B}{2} = \frac{0.75}{2} = 0.375 \text{ m} \right)$$

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For bottom surface :

$$\overline{Nu}_L = \frac{h_b L_c}{k} = 0.27\,(Ra_L)^{0.25} = 0.27\,(7.07 \times 10^7)^{0.25} = 24.76$$

$$\therefore \qquad h_b = \frac{k}{L_c} \times 24.76 = \frac{0.0265}{0.375} \times 24.76 = 1.75 \text{ W/m}^2\text{°C}$$

The rate of heat loss per unit length of the duct is given by,

$$Q = Q_s + Q_t + Q_b$$

$$= [2 \times h_s \times (0.3 \times 1) + h_t \times (0.75 \times 1) + h_b \times (0.75 \times 1)]\,(t_s - t_a)$$

$$= [2 \times 4.23 \times 0.3 + 4.127 \times 0.75 \times 1 + 1.75 \times 0.75 \times 1]\,(45 - 15)$$

$$= \mathbf{208.4 \text{ W/m}} \qquad \textbf{(Ans.)}$$

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Condensation





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Condensation

What is Condensation?

Condensation is the process where water vapor becomes liquid. It is the reverse of evaporation, where liquid water becomes a vapor. **Condensation** happens one of two ways: Either the air is cooled to its dew point or it becomes so saturated with water vapor that it cannot hold any more water.





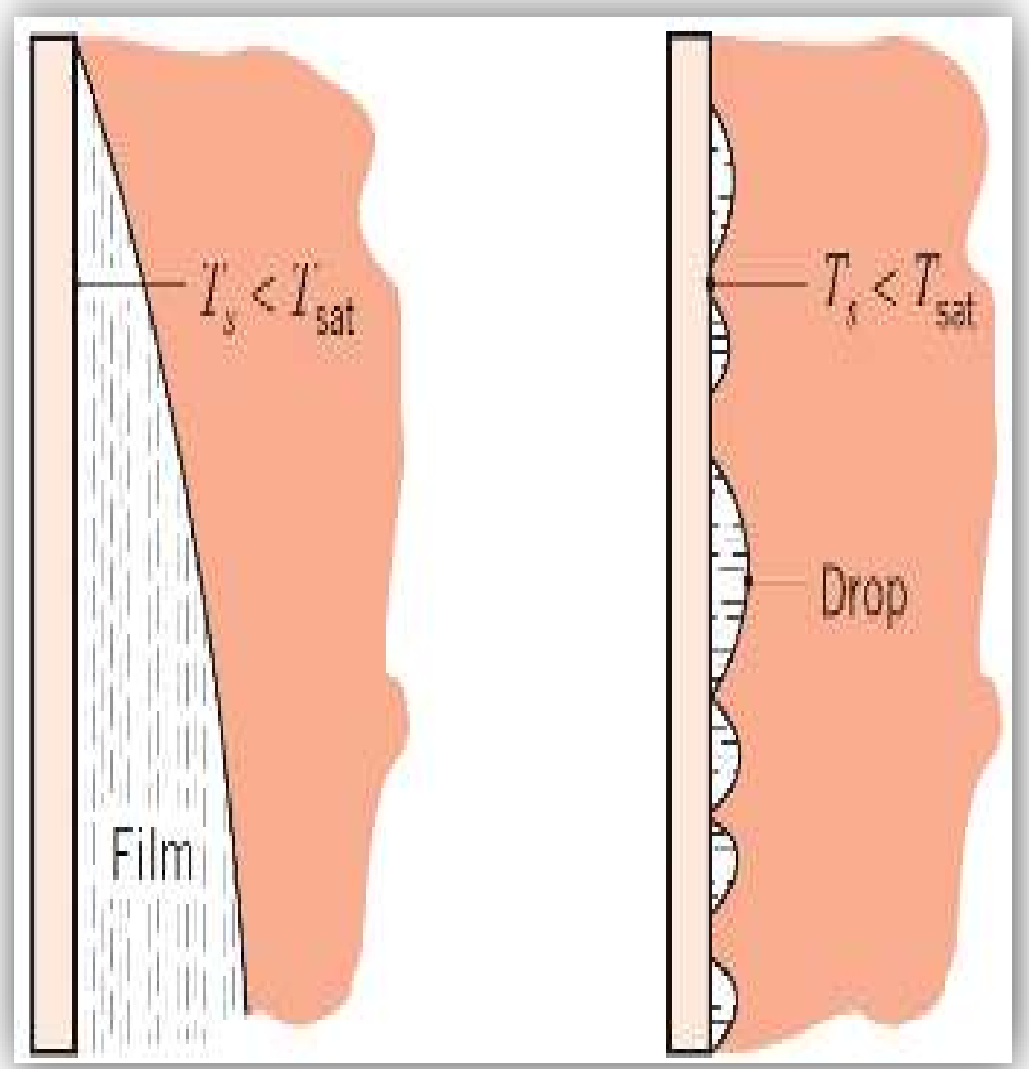


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Film wise and Drop wise condensation

Condensation occurs when the temperature of a vapor is reduced below its saturation temperature corresponding to the vapor pressure."

- This is usually done by bringing the vapor into contact with a solid surface whose temperature, Ts is below the saturation temperature Tsat of the vapor.
- The latent energy of the vapor is released, heat is transferred to the surface, and the condensate is formed.
- The condensation can also occur on the free surface of a liquid or even in a gas when the temperature of the liquid or the gas to which the vapor is exposed is below Tsat.
- In this study we will consider surface condensation only.
- Depending upon the behavior of condensate upon the cooled surface, the condensation process has been ed into two distinct modes: (A) Film wise ation and (B) Drop wise condensation.

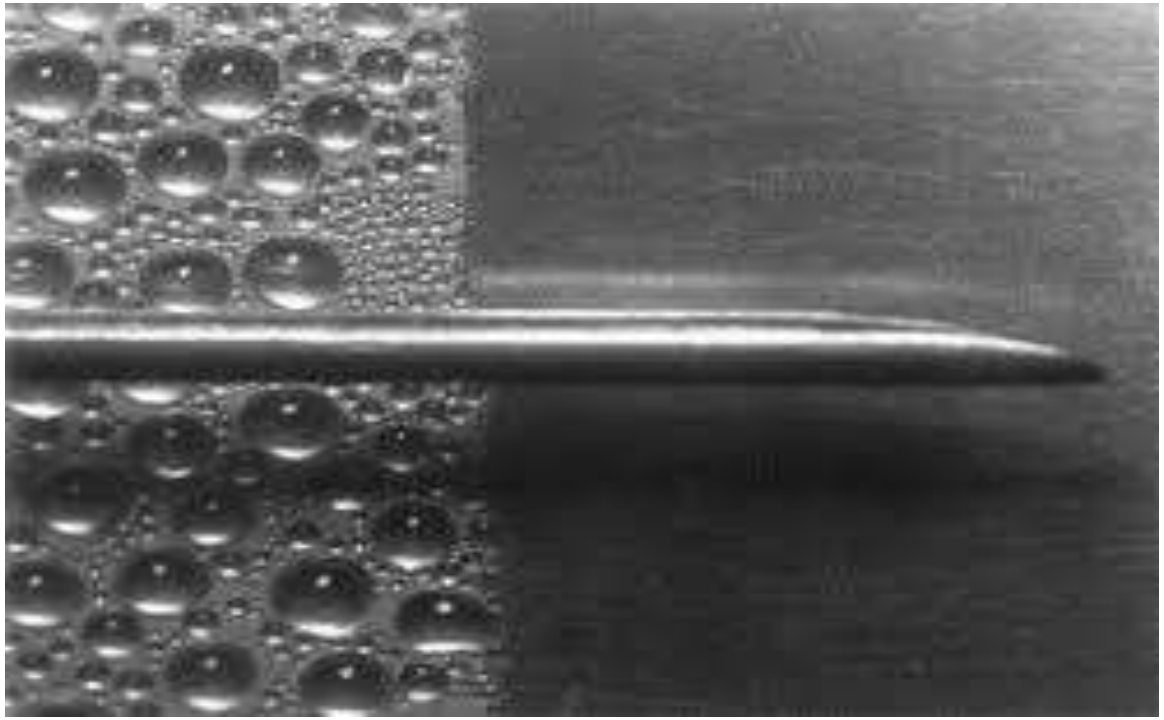


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Film wise condensation

- The liquid condensate wets the solid surface, spread out and forms a continuous film over the entire surface.
- The liquid flows down the cooling surface under the action of gravity and the layer continuously grows in thickness because of newly condensing vapors.
- The continuous film offers resistance and restricts further transfer of heat between the vapor and the surface.
- Film condensation only occurs when a vapor relatively free from impurities, is allowed to condense on a clean
- ensation is generally a characteristic of clean, uncontaminated surfaces

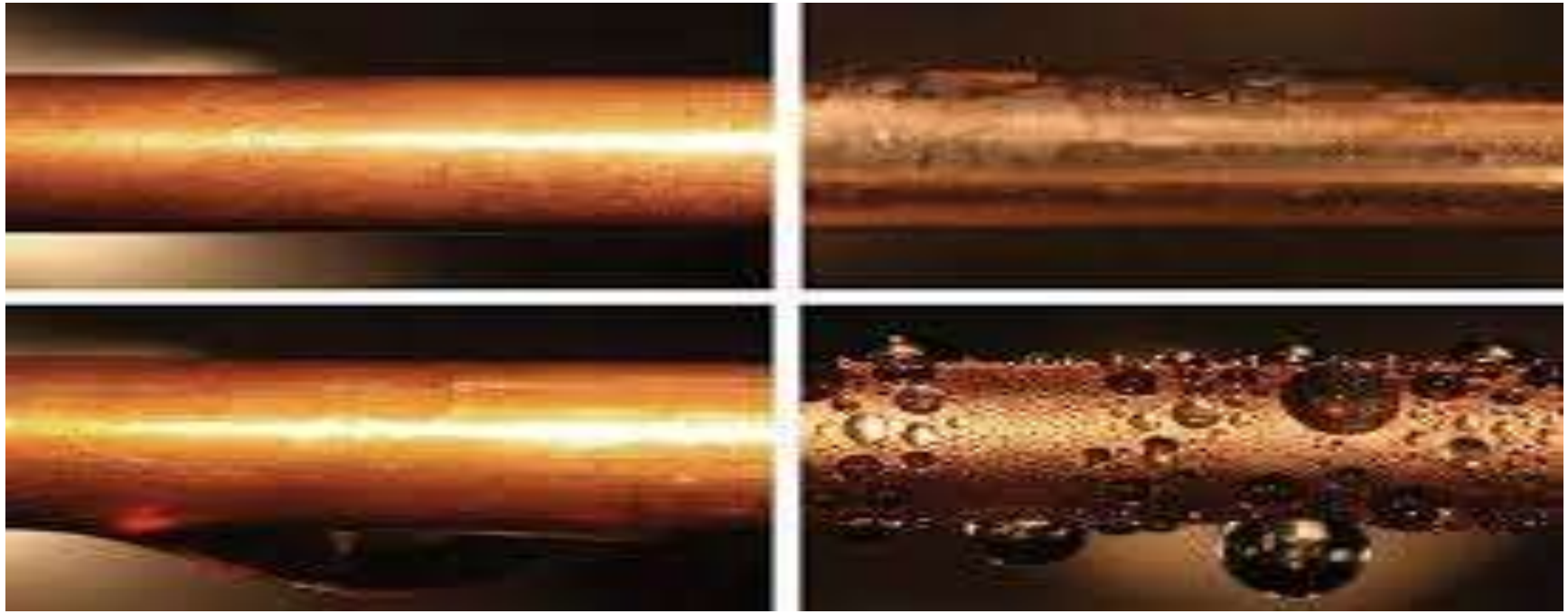
In film wise condensation, the surface over which the steam condenses is wet-able and hence, as the steam condenses, a film of condensate is formed. Generally, film wise condensation results in low heat transfer rates as the film of condensate impedes the heat transfer.



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Drop wise condensation

- The condensed vapor forms droplet on the surface instead of continuous **film**. ... **FILMWISE CONDENSATION**
- The **condensate** tends to wet the surface and thereby forms a liquid **film**.
- The liquid flows down the cooling surface under the action of gravity.



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Drop wise condensation

1. **Dropwise condensation** occurs when a vapor condenses on a surface not wetted by the **condensate**. For nonmetal vapors, **drop wise** The liquid condensate collects in droplets and does not wet the solid cooling surface.
2. **The droplets develop in cracks**, pits and cavities on the surface, grow in size, break away from the surface, knock-off other droplets and eventually run-off the surface without forming a film.
3. A part of the condensation surface is directly exposed to the vapor without an insulating film of condensate liquid.
4. Evidently there is no film barrier to heat flow and higher heat transfer rates are experienced.
5. **Drop wise condensation** has been observed to occur either on highly polished surfaces, or on surfaces contaminated with impurities like fatty acids and organic compounds.
6. Drop wise condensation gives co-efficient of heat transfer generally 5 to 10 times larger than with film condensation.
7. It is therefore common practice to use surface coatings that inhibit wetting, and hence simulate drop wise condensation.
8. Silicon, Teflon and an assortment of waxes and fatty acids are often used for this purpose.
9. However such coatings gradually lose their effectiveness due to oxidation, fouling or outright removal and film condensation eventually occurs.
10. Although it is desirable to achieve drop wise condensation in industrial applications, it is often difficult to maintain
    on.
11. design calculations are often based on the assumption of film condensation.
12. **se condensation** gives much **higher heat transfer coefficients** than those found with **nsation**.

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BOILING HEAT TRANSFER

Boiling is *the convective heat transfer process that involves a phase change from liquid to vapour state. Boiling* is also defined as *evaporation at a solid-liquid surface*. This is possible only when the temperature of the surface (t_s) *exceeds* the saturation temperature corresponding to the liquid pressure (t_{sat}). Heat is transferred from the solid surface to the liquid according to the law

$$Q = h\, A_s(t_s - t_{sat}) = h\, A_s\, \Delta t_e$$

where, $\Delta t_e = (t_s - t_{sat})$ is known as *excess temperature*.

The boiling process finds *applications* in the following cases:

(*i*) Steam production (for generation of power and for industrial processes and space heating) in steam and nuclear power plants;

(*ii*) Heat absorption in refrigeration and air conditioning systems;

(*iii*) Distillation and refining of liquids;

(*iv*) Concentration, dehydration and drying of foods and materials.

(*v*) Cooling the machines like nuclear reactors and rocket motors where the large quantities of : are released in relatively small volume (dissipation rates are as high as 10^8 W/m^2; the dimum heat transfer rate in modern boiler is about 2×10^5 W/m^2).



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BOILING HEAT TRANSFER

The boiling heat transfer phenomenon may occur in the following forms:

1. *Pool boiling* :

In this case the liquid above the hot surface is essentially stagnant and its motion near the surface is due to free convection and mixing induced by bubble growth and detachment.

The pool boiling occurs in steam boilers *involving natural convection.*

2. *Forced convection boiling* :

This refers to a situation where the fluid motion is induced by external means (and also by free convection and bubble induced mixing). The



is pumped and forced to flow. pe of boiling occurs in water oilers involving forced con-

3. *Sub-cooled or local boiling* :

In this case the liquid temperature is below the saturation temperature and

bubbles are formed in the vicinity of heat surface. These bubbles after travelling a short path get condensed in the liquid which has a temperature less than the boiling point.

4. *Saturated boiling* :

Here, the liquid temperature exceeds the saturation temperature. The vapour bubbles formed at the solid surface (liquid-solid interface) are then propelled through the liquid by buoyancy effects and eventually escape from a free surface (liquid-vapour interface).

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Unit No.5

Radiation Heat Transfer

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HEAT TRANSFER BY RADIATION



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Subject - Heat Transfer

What is HEAT TRANSFER BY RADIATION

Heat transfer by **thermal radiation** is **transfer** of **heat** by electromagnetic waves. It is different from **conduction** and convection as it requires no matter or medium to be present. The radiative energy will pass perfectly through vacuum as well as clear air.

- **Heat transfer by radiation** occurs when microwaves, infrared **radiation**, visible light, or another form of electromagnetic **radiation** is emitted or absorbed. An obvious **example** is the warming of the Earth by the Sun. A less obvious **example** is thermal **radiation** from the human body.

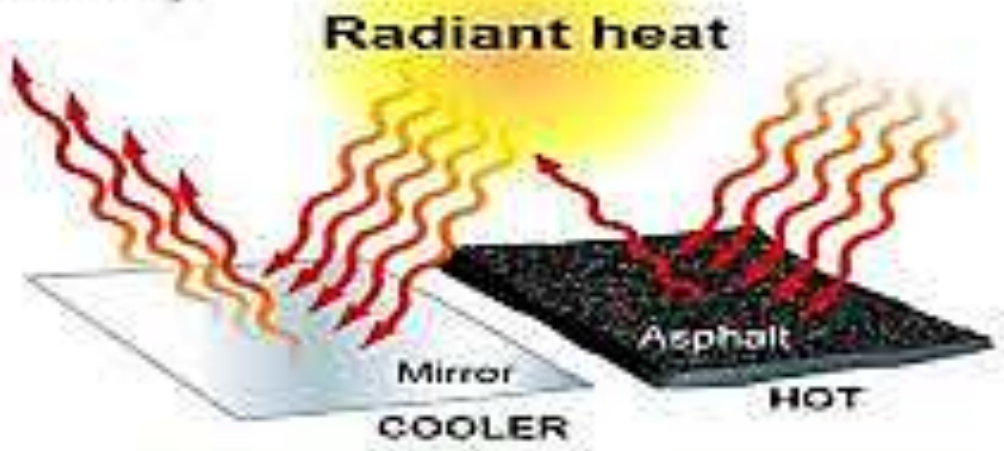
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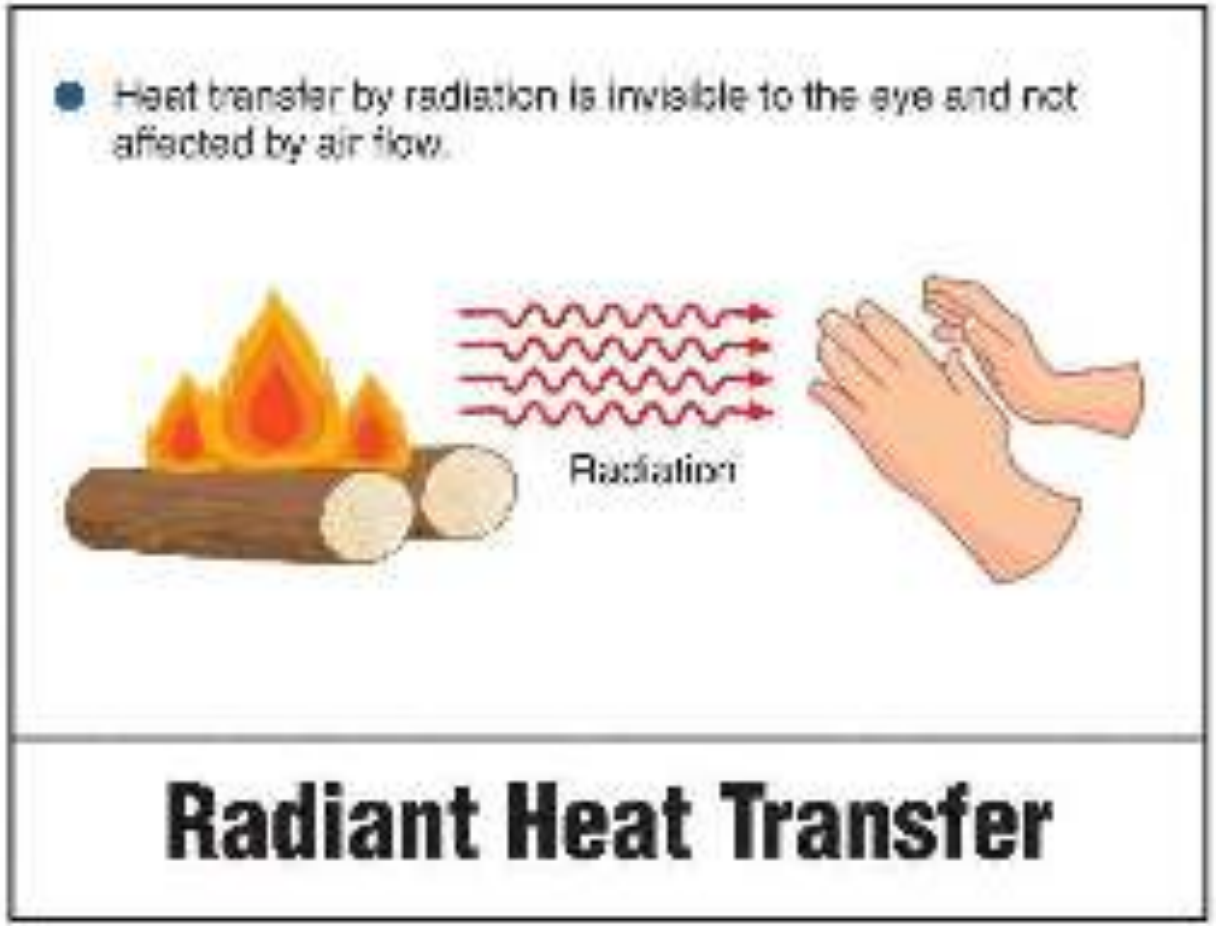
Subject - Heat Transfer

HEAT TRANSFER BY RADIATION

Radiation: heat transfer in the form of electromagnetic waves, including light.

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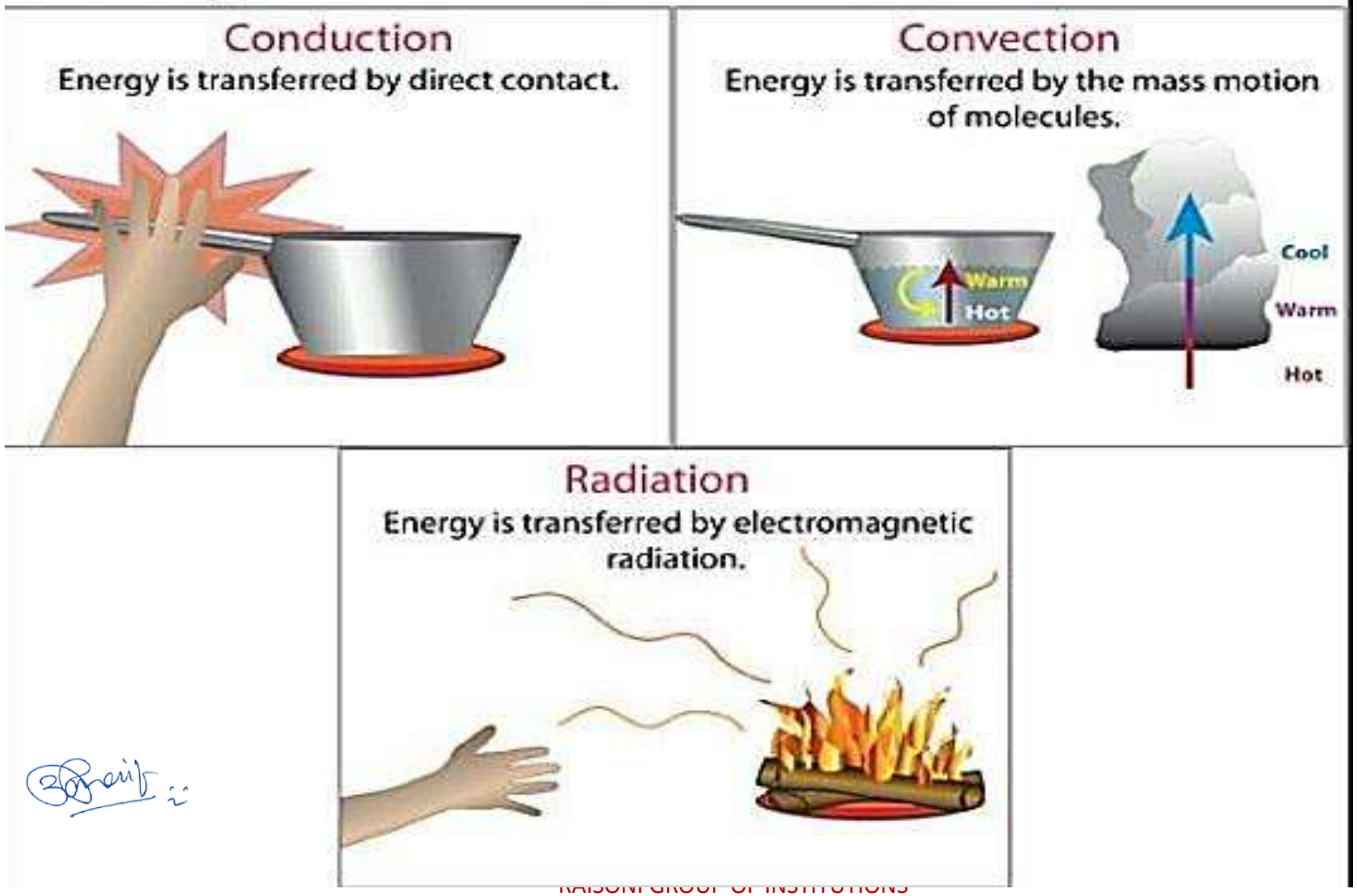




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Subject - Heat Transfer

Figure 2--Conduction, Convection, and Radiation



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RADIATIVE HEAT TRANSFER

Thermal radiation is the electromagnetic radiation emitted by a body as a result of its temperature.

There are many types of electromagnetic radiation; thermal is only one of them.

It propagated at the speed of light, 3×10^8 m/s.

The wavelength of thermal radiation lies in the range from 0.1 to 100 μm,

ɔle light has wavelength from 0.4 to 0.7 μm.

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RADIATIVE HEAT TRANSFER (2)

The sun with an effective surface temperature of 5760 K

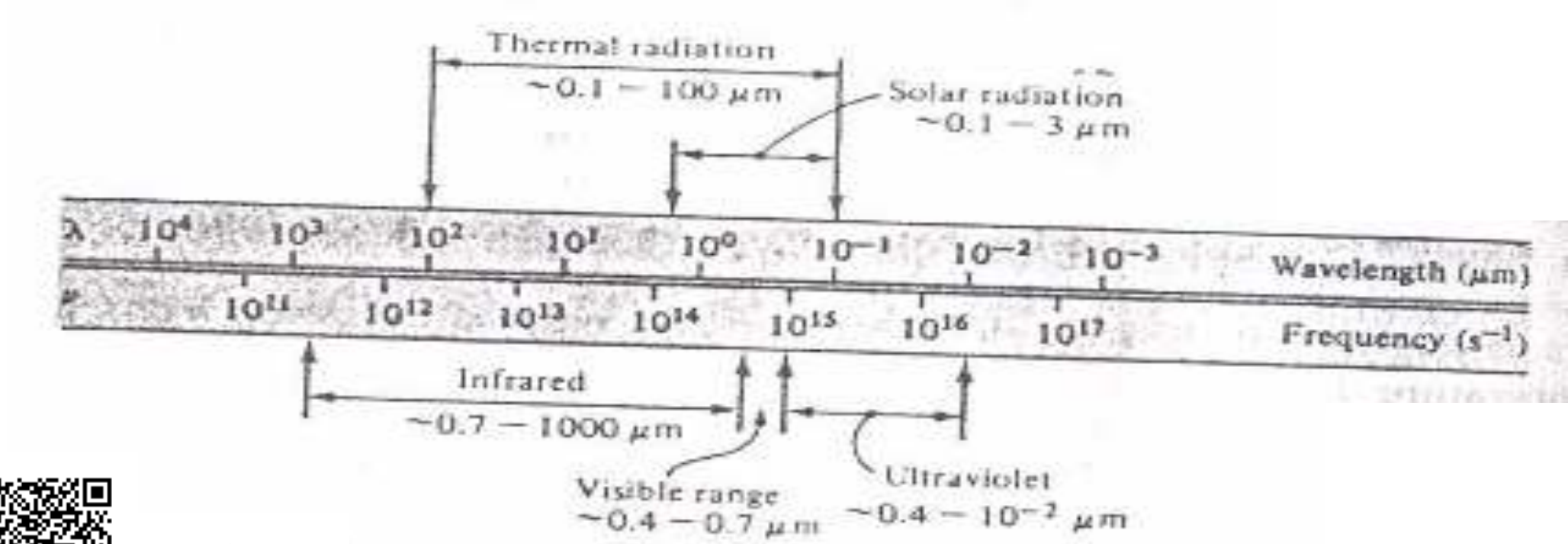
emits most of its at the extreme lower end of the spectrum 0.1 to 4 μm (μm = 10^{-6} m).

The radiations from a lamp filament are in the range of 1 to 10 μm.

Most solids and liquids have a continuous spectrum; they emit radiations pf all wavelengths.

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Spectrum of Electromagnetic Wave



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RADIATIVE HEAT TRANSFER (3)

Gases and vapours radiate energy only at certain bands of wavelength and hence are called selective emitters.

The emission of thermal radiation depends upon the nature, temperature and state of the emitting surface.

However with gases the dependence is also upon the thickness of the emitting layer and the gas pressure.

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Absorptivity, Reflectivity and Transmissivity

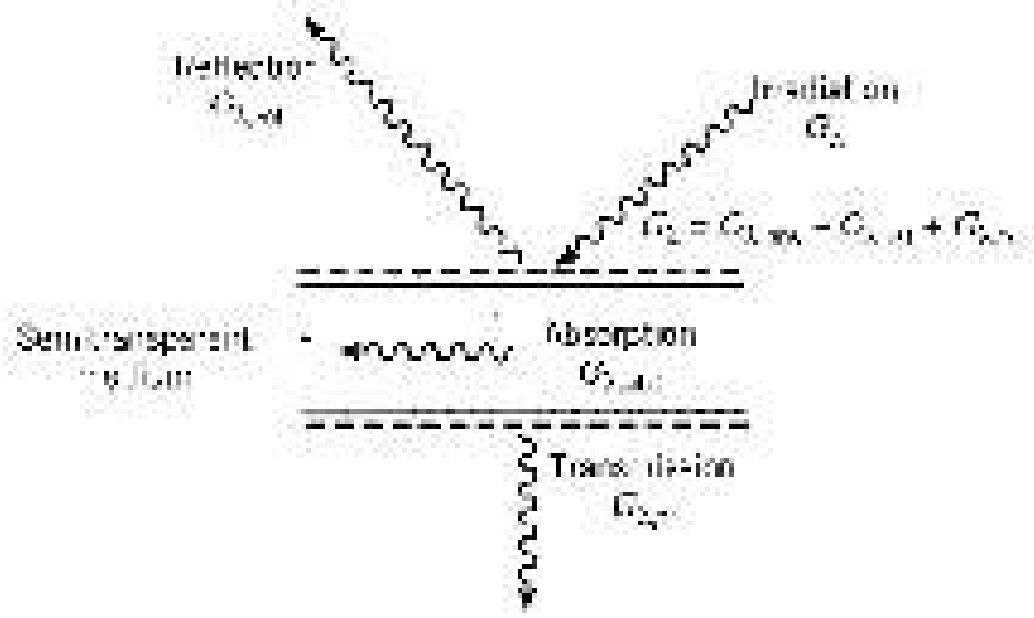
The total radiant energy (Q_0) impinging upon a body be

(1) partially or totally absorbed by it (Q_a),

(2) reflected from its surface (Q_r) or

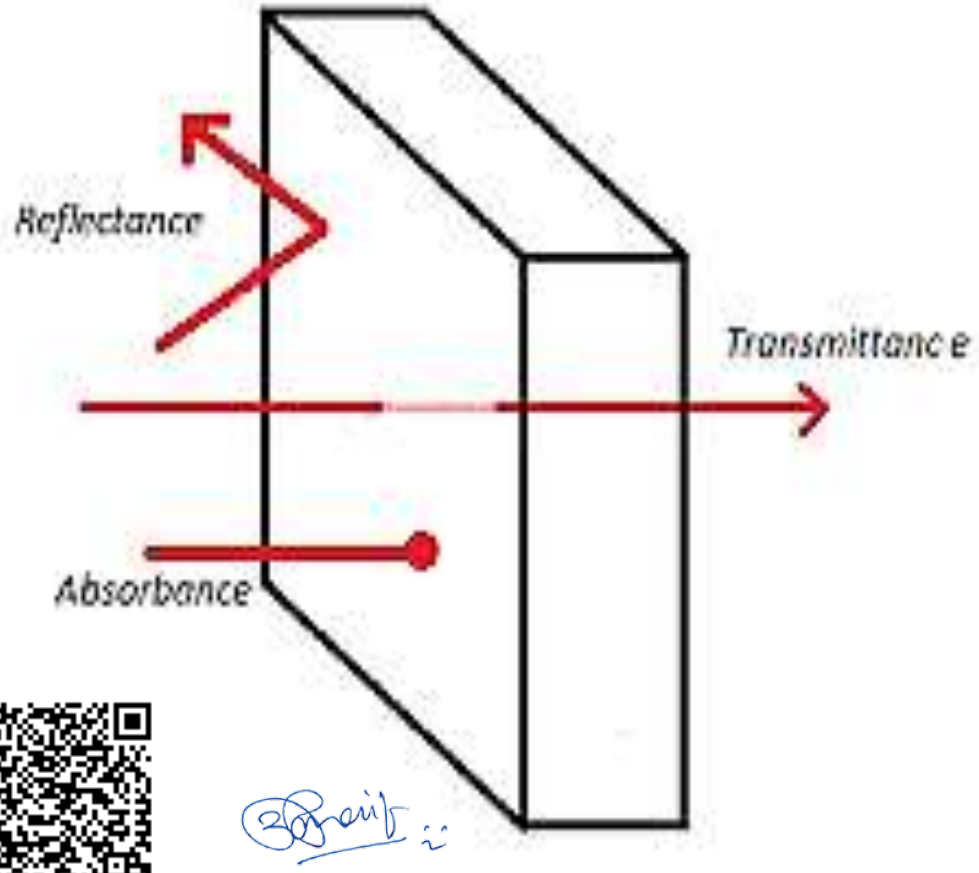
(3) transmitted through it (Q_t)

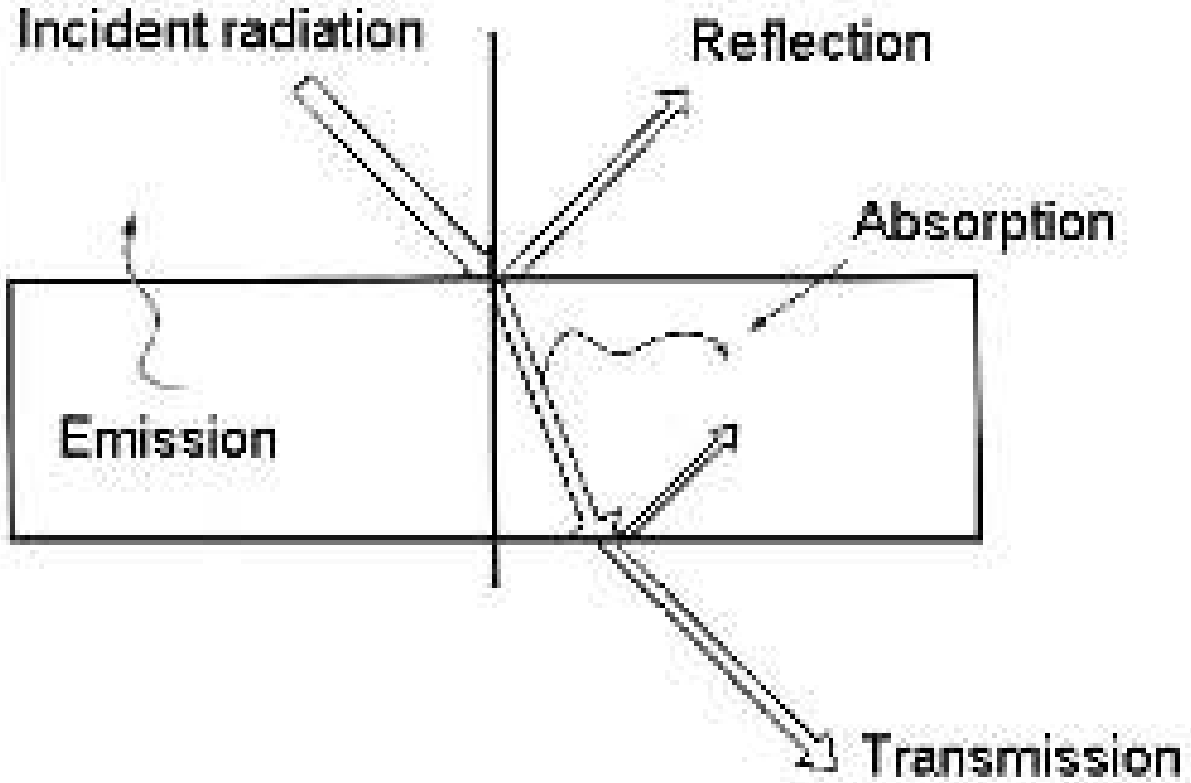
in accordance with the characteristics of the body.



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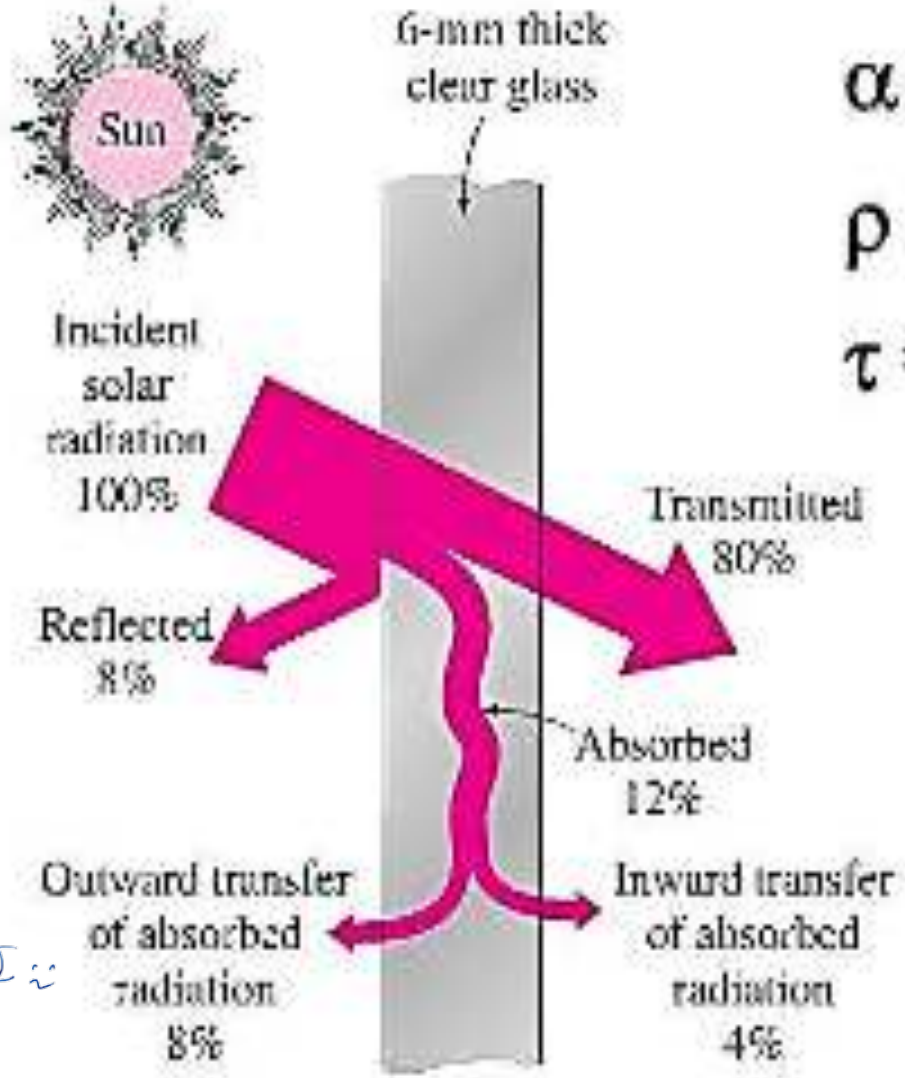
Absorptivity, Reflectivity and Transmissivity





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Absorptivity, Reflectivity and Transmissivity



α = absorptivity

ρ = reflectivity

τ = transmissivity

In general:

$$\alpha + \rho + \tau = 1$$

Opaque material:

$$\alpha + \rho = 1$$

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Absorptivity, Reflectivity and Transmissivity (contd.)

$$Q_a + Q_r + Q_t = Q_0$$

$$\frac{Q_a}{Q_0} + \frac{Q_r}{Q_0} + \frac{Q_t}{Q_0} = 1$$

$$\alpha + \rho + \tau = 1$$

α	absorptivity
ρ	reflectivity
τ	transmissivity

...es of these quantities depend upon the nature of the surface of the bodies, its ...re and wavelength of incident rays.

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BLACK BODY

For black body,

$\alpha = 1, \quad \rho = 0, \quad \tau = 0$

Snow is nearly black to thermal radiations. $\alpha = 0.985$

The absorptivity of surfaces can be increased to 90-95% by coating their surfaces with lamp black or dark range paint.

In actual practice, there does not exist a perfectly black body that will absorb all the incident radiations.

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GRAY BODY

A gray body has the absorptivity less than unity,

Absorptivity remains constant over the range of temperature and wavelength of incident radiation.

For a real body, it does not satisfy the condition of constant.

So Gray body is a concept only.

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Specular body and absolutely white body

A body that reflects all the incident thermal radiations is called a specular body (if reflection is regular)

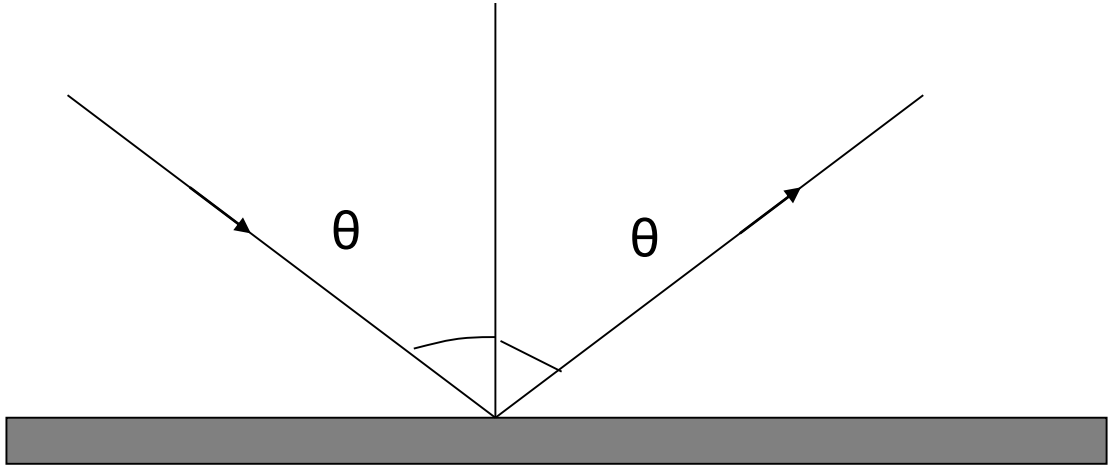
or an absolutely white body (if the reflection is diffused).

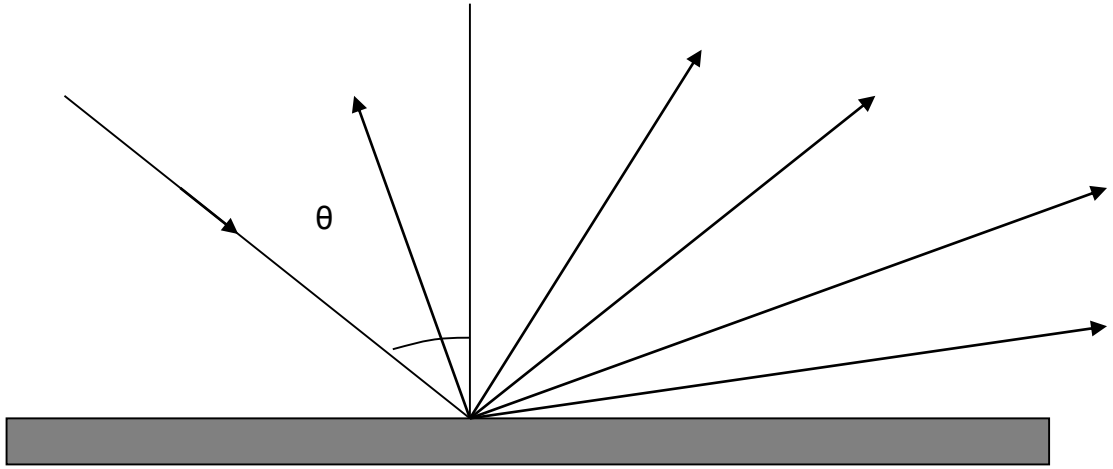
For such bodies,

$\rho = 1, \quad \alpha = 0, \quad \tau = 0$

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Reflections





ecular Reflection

Diffuse Reflection

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Transparent or Diathermaneous.

A body that allows all the incident radiations to pass through it is called transparent or diathermaneous.

For such bodies, $\rho = 0$, $\alpha = 0$, $\tau = 1$

Transmissivity varies with wavelength of incident radiation.

A material may be transparent for certain wavelengths and non-transparent for other wavelengths.

A thin glass plate transmits most of the thermal radiations from sun, but absorbs in equally great measure the thermal radiations emitted from the low temperature interior of a building.

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SURFACE EMISSION PROPERTIES

The rate of emission of radiation by a body depends upon the following factors:

(*i*) The temperature of the surface.

(*ii*) The nature of the surface, and

(*iii*) The wavelength or frequency of radiation.

The *parameters* which deal with the surface emission properties are given below :

(*i*) **Total emissive power (E).** The "*emissive power*" is defined as the *total amount of radiation emitted by a body per unit area and time*. It is expressed in W/m^2. The *emissive power of a black body*, according to Stefan-Boltzmann, is *proportional to absolute temperature to the fourth power.*

$$E_b = \sigma T^4 \ W/m^2$$

$$E_b = \sigma A T^4 \ W$$

where, σ = Stefan-Boltzmann constant = 5.67×10^{-8} $W/m^2 K^4$

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SURFACE EMISSION PROPERTIES

(*ii*) **Monochromatic (spectral) emissive power (E_λ).** It is often necessary to determine the spectral distribution of the energy radiated by a surface. At any given temperature the amount of radiation emitted per unit wavelength varies at different wavelengths. For this purpose the *monochromatic emissive power* E_λ of the surface is used. It is defined as *the rate of energy radiated per unit area of the surface per unit wavelength.*

The total emissive power is given by,

$$E = \int_0^\infty E_\lambda \, d\lambda \ \text{W/m}^2$$

wavelength (λ)

(*iii*) **Emission from real surface-emissivity.** The emissive power from a real surface is given by

$$E = \varepsilon \sigma A T^4 \ \text{W}$$

where, ε = Emissivity of the material.

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SURFACE EMISSION PROPERTIES

Emissivity (ε). It is defined as the *ability of the surface of a body to radiate heat.* It is also defined as the *ratio of the emissive power of any body to the emissive power of a black body of equal temperature* (*i.e.*, $\varepsilon = \frac{E}{E_b}$). Its values varies for different substances ranging from 0 to 1. For a black body $\varepsilon = 1$, for a white body surface $\varepsilon = 0$ and for gray bodies it lies between 0 and 1. It may vary with temperature or wavelength.

(*iv*) **Intensity of radiation.**

(*v*) **Radiation density and pressure.**

(*vi*) **Radiosity (J).** It refers to all of the radiant energy leaving a surface.

Interrelationship between surface emission and irradiation properties.

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BLACK BODY RADIATION

The energy emitted by a black surface varies with

(i) wavelength,

(ii) temperature and

(iii) surface characteristics of a body.

For a given wavelength, the body radiates more energy at elevated temperatures.

Based on experimental evidence, Planck suggested the following law for the spectral distribution of emissive power for a fixed temperature.

- Blackbody – a perfect emitter & absorber of radiation
- Emits radiation uniformly in all directions – no directional distribution – it's <u>diffuse</u>
- Joseph Stefan (1879)– total radiation emission per unit time & area over all lengths and in all directions:

$$E_b = \sigma T^4 \left(W/m^2\right)$$

σ–Stefan-Boltzmann constant =5.67 x10^{-8} W/m^2K^4

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Stefan- Boltzman Law

The total emissive power E of a surface is defined as the total radiant energy emitted by the surface in all directions over the entire wavelength range per unit surface area per unit time.

The amount of radiant energy emitted per unit time from unit area of black surface is proportional to the fourth power of its absolute temperature.

$$E_b = \sigma_b T^4$$

where

E_b = Emissive power of a black body, and

σ = Stefan-Boltzmann constant

$= 5.67 \times 10^{-8}\ \mathrm{W/m^2\,K^4}$.

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Planck 's Law

$$(E\lambda)_b = 2\pi C^2 h \times \frac{\lambda^{-5}}{\exp\left(\frac{Ch}{k\lambda T}\right) - 1} \qquad (1)$$

where

h = Planck's constant, 6.6256×10^{-34} J-s

C = Velocity of light in vacuum, 2.998×10^{8} m/s

nan constant, 1.3805×10^{-23} J/K

ength of radiation waves, m

ute temperature of black body, K

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Simplification

Equation (1) may be written as

$$(E\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1}$$

$$C_1 = 2\pi c^2 h = 3.742 \times 10^8 \text{ W.}\mu\text{m}^4/\text{m}^2;$$

$$C_2 = \frac{ch}{k} = 1.4388 \times 10^4 \ \mu m K$$

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SPECTRAL ENERGY DISTRIBUTION

$(E_\lambda)_b$ denotes monochromatic (single wavelength) emissive power and is defined as the energy emitted by the black surface (in all directions) at a given wavelength per unit wavelength interval around λ.

The rate of energy emission in the interval

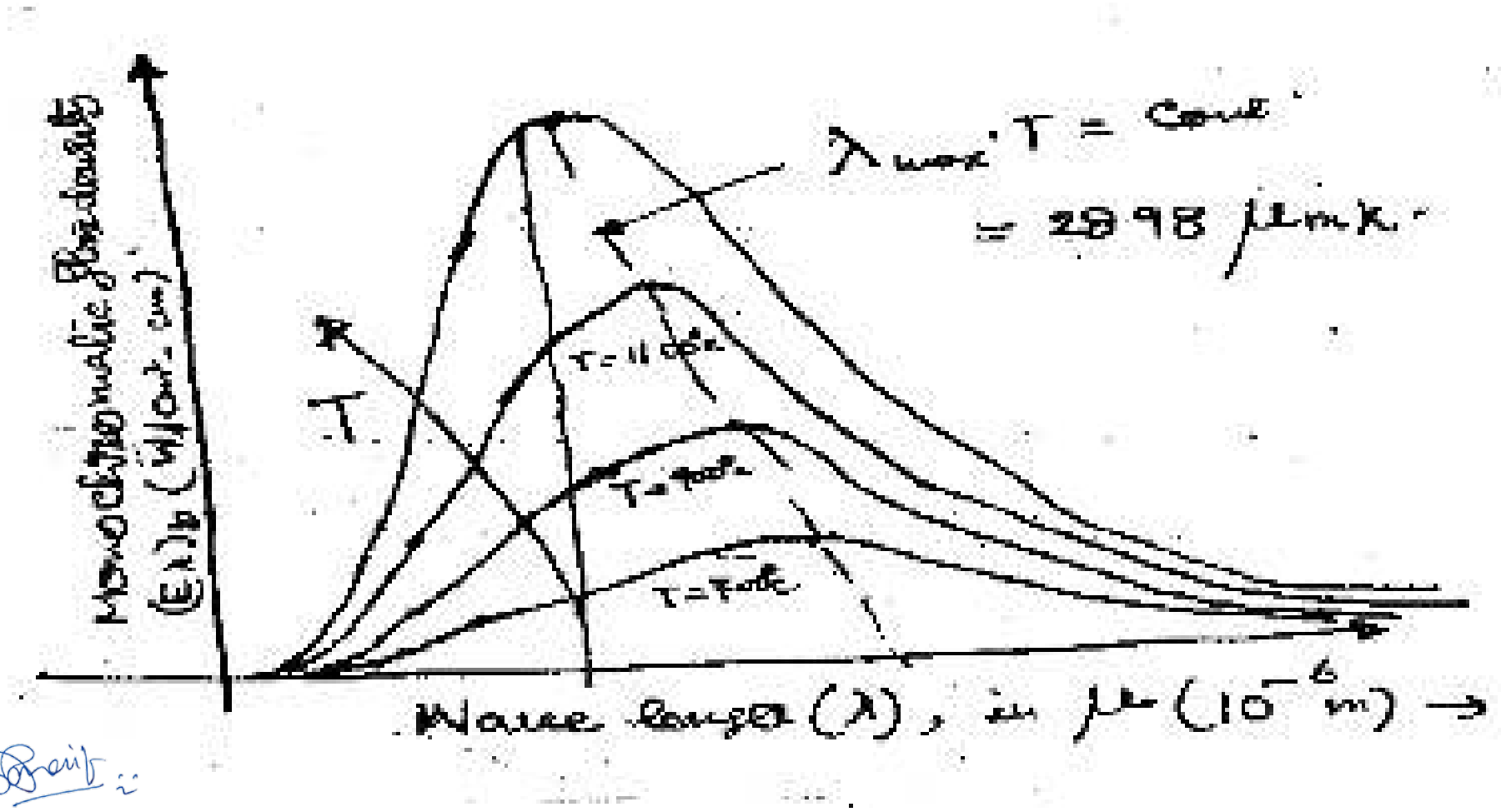
$$E_b = \int_0^\infty (E_\lambda)_b \, d\lambda$$

The variation of distribution of monochromatic emission power with
ngth is called the spectral energy distribution.

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SPECTRAL ENERGY DISTRIBUTION

Graph



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Features of Spectral Energy Distribution

The monochromatic emissive power varies across the wavelength spectrum, the distribution is continuous, but non-uniform.

The emitted radiation is practically zero at zero wavelength. With increase in wavelength, the monochromatic emissive power increases and attains a certain maximum value.

With further increase in wavelength, the emissive power drops again to almost zero value at infinite wavelength.

At any wavelength the magnitude of the emitted radiation increases with increasing temperature

The wavelength at which the monochromatic emissive power is maximum shifts in the direction of shorter wavelengths as the temperature increases.

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TOTAL EMISSIVE POWER

At any temperature, the rate of total radiant energy emitted by a black body is given by

$$E_b = \int_0^\infty (E_\lambda)_b d\lambda$$

The above integral measures the total area under the monochromatic emissive power versus ngth curve for the black body, and it represents the total emissive power per unit area t energy flux density) radiated from a black body.

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Wien's Displacement Law.

In 1893 Wien established a relationship between the temperature of a black body and the wavelength at which the maximum value of monochromatic emissive power occurs. A peak monochromatic emissive power occurs at a particular wavelength. **Wien's displacement law** *states that the product of* λ_{max} *and* T *is constant, i.e.,*

For shorter wavelength,

$\frac{C_2}{\lambda T}$ is very large and $\exp\left[\frac{C_2}{\lambda T}\right] >> 1$

Then Planck's law reduces to

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left(C_2/\lambda T\right)}$$

which is called Wien's law.

$$\lambda_{max} T = 2898\ \mu m K$$

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For Maximum Emission

$$\therefore \lambda_{max} T = \frac{C_2}{4.965} = \frac{1.438 \times 10^{-2}}{4.965} = 2.898 \times 10^{-3} \approx 0.002$$ mK

λ_{max} ⟶ denotes the wavelength at which emissive power is maximum

Statement of Wein's Displacement law

The product of the absolute temperature and the wavelength, at which the emissive power is maximum, is constant.

displacement law finds application in the prediction of a very high
:ature through measurement of wavelength.

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Rayleigh-Jean's Law

For longer wavelengths

$\frac{C_2}{\lambda T}$ is very small and hence we can write

$$\exp\left(\frac{C_2}{\lambda T}\right)=1+\frac{C_2}{\lambda T}+\frac{1}{2!}\left(\frac{C_2}{\lambda T}\right)^2+\text{----------}\cong 1+\frac{C_2}{\lambda T}$$

So, Planck's distribution law becomes

$$(E_\lambda)_b=\frac{C_1\lambda^{-5}}{1+C_2/\lambda T-1}=\frac{C_1}{C_2}\ \frac{T}{\lambda^4}$$

lentity is called **Rayleigh-Jean's Law**.

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SOME DERIVATION

$$E_b = \int_0^\infty (E_\lambda)_b d\lambda \quad = \int_0^\infty \frac{C_1 \lambda^{-5}}{\exp\left[C_2 / \lambda T\right] - 1} d\lambda$$

Let

$$\frac{C_2}{\lambda T} = y \qquad \therefore d\lambda = -\frac{C_2}{y^2 T} dy$$

$\lambda \to 0,\ y \to \alpha$ and as $\lambda \to \alpha,\ y \to 0$

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SOME DERIVATION, contd.

$$E_b = -C_1 \int_{\alpha}^{0} \frac{y^5 T^5 C_2}{C_2^5 [\exp(y) - 1] y^2 T} d\lambda$$

$$= \frac{C_1 T^4}{C_2^4} \int_0^{\infty} y^3 [\exp(y) - 1]^{-1} dy$$

$$= \frac{C_1 T^4}{C_2^4} \int_0^{\infty} y^3 [\exp(-y) + \exp(-2y) + \exp(-3y) + \ldots\ldots\ldots\ldots] dy$$

expanding

$[\exp(y) - 1]^{-1}$

SOME DERIVATION, contd. 2

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We have

$$\int_0^\infty y^n [\exp(-ay)]dy = \frac{n!}{a^{n+1}}$$

$$\therefore E_b = \frac{C_1 T^4}{C_2^4}\left[\frac{3!}{1^4} + \frac{3!}{2^4} + \frac{3!}{3^4} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\right]$$

$$= \frac{C_1 T^4}{C_2^4} \times 6.48 = \frac{3.742 \times 10^{-16}}{(1.4389 \times 10^{-2})^4} \times 6.48 \times T^4$$

or, $$E_b = \sigma_b T^4$$

w $$\sigma_b = 5.67 \times 10^{-8}$$ W/m²K², Stefan-Boltzman constant

re two bodies, the net radiant heat flux is given by

$$Q_{net} = \sigma_b (T_1^4 - T_2^4)$$

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Maximum Monochromatic Emissive Power for a Black Body

Combining Planck's law and Wien's displacement law

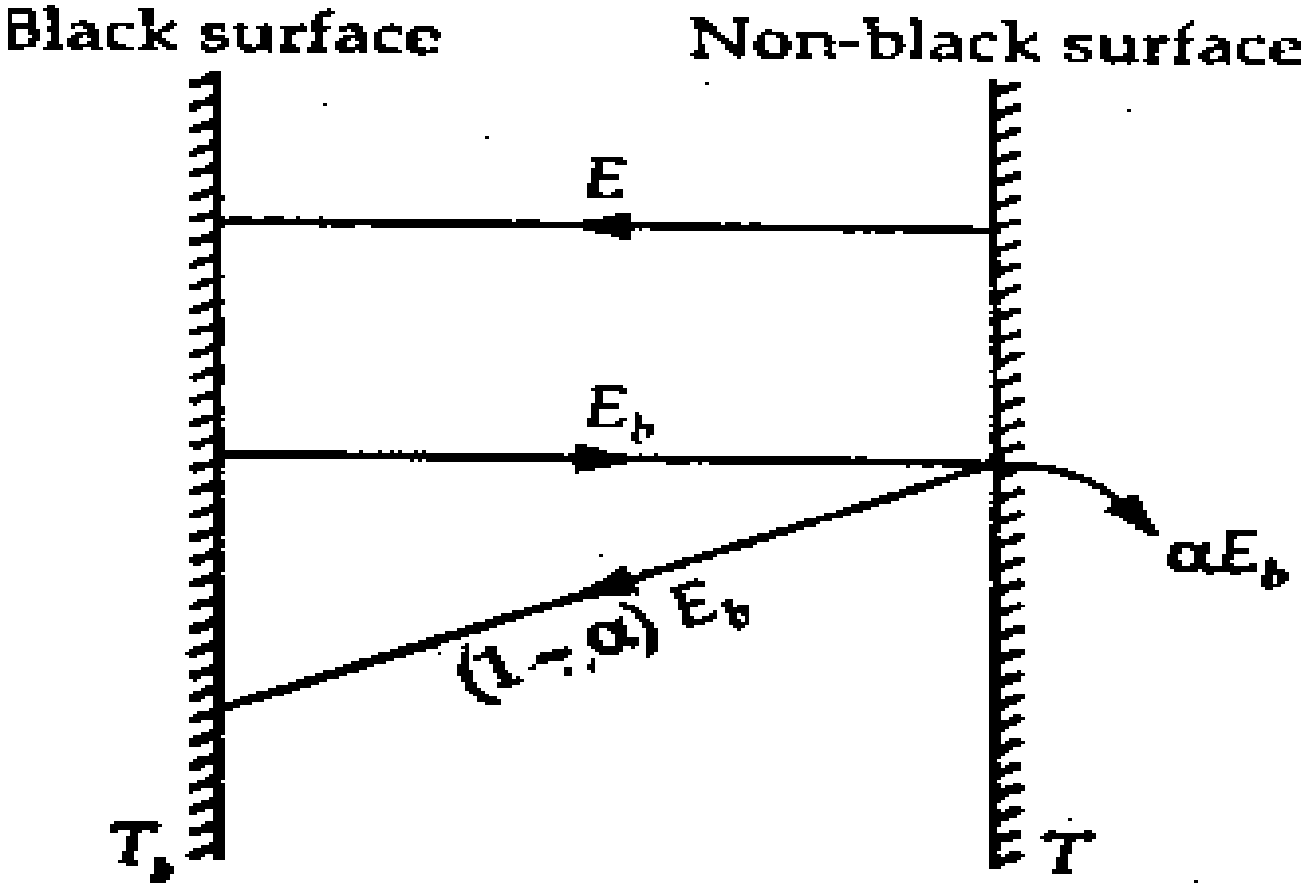
$$(E_\lambda)_{max} = \frac{C_1 \lambda_{max}^{-5}}{\exp\left[\frac{C_2}{\lambda_{max} T}\right] - 1}$$

$$= \frac{0.374 \times 10^{-15} \times \left(2.898 \times 10^{-3}/T\right)^{-5}}{\exp\left[1.438 \times 10^{-2}/2.898 \times 10^{-3}\right] - 1} = 1.285 \times 10^{-5} T^5$$

W/m^2 per metre wavelength

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Kirchoff's Law Fig



Heat exchange between black and non- black surfaces

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Kirchoff's Law

The surfaces are arranged parallel and so close to each other so that the radiations from one fall totally on the other.

Let E be the radiant emitted by non-black surface and gets fully absorbed.

E_b is emitted by the black surface and strikes non-black surface.

If the non-black surface has absorptivity α, it will absorb αE_b

and the remainder $(1-\alpha)E_b$ will be reflected back for full absorption at the black surface.

Radiant interchange for the non-black surface equals $(E - \alpha E_b)$.

the surfaces are at the same temperature, $T = T_b$, then the resultant interchange of heat

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Kirchoff's Law contd.

Then, $E - \alpha E_b = 0$ or,

$$\frac{E}{\alpha} = E_b$$

The relationship can be extended by considering different surfaces in turn as

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3} = \ldots\ldots\ldots\ldots\ldots\ldots = \frac{E_b}{\alpha_b} = E_b = f(T)$$

α_b (absorptivity for black surface is unity.

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Emissivity

The ratio of the emissive power E to absorptivity α is same for all bodies and is equal to the emissive power of a black body at the same temperature.

The ratio of the emissive power of a certain non-black body E to the emissive power black body E_b, both being at the same temperature, is called the emissivity of the body.

Emissivity of a body is a function of its physical and chemical properties and the state of its surface, rough or smooth.

$$\therefore \frac{E}{E_b} = \varepsilon$$ (emissivity)

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Statement of Kirchoff's Law

Also, we have,

$$\frac{E}{E_b} = \alpha$$

$$\therefore \varepsilon = \alpha$$

The emissivity and absorptivity of a real surface are equal for radiation with identical temperatures and wavelengths.

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Numerical on Various Laws of Radiation

Example *The effective temperature of a body having an area of 0.12 m^2 is 527°C. Calculate the following:*

(i) The total rate of energy emission,
(ii) The intensity of normal radiation, and
(iii) The wavelength of maximum monochromatic emissive power.

Solution. *Given:* $A = 0.12\ m^2$; $T = 527 + 273 = 800\ K$

(i) The total rate of energy emission, E_b:

$$E_b = \sigma A T^4 \text{ W (watts)} \qquad \text{...[Eqn. (11.2}a\text{)]}$$

$$= 5.67 \times 10^{-8} \times 0.12 \times (800)^4 = 5.67 \times 0.12 \times \left(\frac{800}{100}\right)^4 = \mathbf{2786.9\ W\ (Ans.)}$$

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(ii) The intensity of normal radiation, I_{bn} :

$$I_{bn} = \frac{E_b}{\pi}, \qquad \text{where } E_b \text{ is in W/m}^2 K^4 \qquad \text{...(Eqn. 11.23)}$$

$$= \frac{\sigma T^4}{\pi} = \frac{5.67 \times \left(\frac{800}{100}\right)^4}{\pi} = \mathbf{7392.5\ W/m^2.sr\ (Ans.)}$$

(iii) The wavelength of maximum monochromatic emissive power, I_{max} :

From Wien's displacement law,

$$\lambda_{max} T = 2898\ \mu m K \qquad \text{...[Eqn. (11.18)]}$$

or,
$$\lambda_{max} = \frac{2898}{T} = \frac{2898}{800} = \mathbf{3.622\,\mu m\ (Ans.)}$$

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Example *Calculate the following for an industrial furnace in the form of a black body and emitting radiation at 2500°C :*

(i) Monochromatic emissive power at 1.2 μm length,

(ii) Wavelength at which the emission is maximum,

(iii) Maximum emissive power,

(iv) Total emissive power, and

(v) Total emissive power of the furnace if it is assumed as a real surface with emissivity equal to 0.9.

Solution. *Given* : $T = 2500 + 273 = 2773K$; $\lambda = 1.2\ \mu m$, $\varepsilon = 0.9$

(i) Monochromatic emissive power at 1.2 μm length, $(E_\lambda)_b$:

According to Planck's law,

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1}$$

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$C_1 = 3.742 \times 10^8$ W.μm⁴/m² $= 0.3742 \times 10^{-15}$ W.m⁴/m², and

$C_2 = 1.4388 \times 10^{-2}$ mK

Substituting the values, we get

$$(E_\lambda)_b = \frac{0.3742 \times 10^{-15} \times (1.2 \times 10^{-6})^{-5}}{\exp\left(\frac{1.4388 \times 10^{-2}}{1.2 \times 10^{-6} \times 2773}\right) - 1} = \frac{1.5 \times 10^{14}}{74.48} = \mathbf{2.014 \times 10^{12}\ W/m^2\ (Ans.)}$$

(ii) Wavelength at which the emission is maximum, λ_{max}:

Accodrding to Wien's displacement law,

$$\lambda_{max} = \frac{2898}{T} = \frac{2898}{2773} = \mathbf{1.045\ \mu m\ (Ans.)}$$

(iii) Maximum emissive power, $(E_{\lambda b})$max:

$(E_{\lambda b})_{max} = 1.285 \times 10^{-5}\ T^5$ W/m² per metre length [Eqn. (11.19)]

$= 1.285 \times 10^{-5} \times (2773)^5 = \mathbf{2.1 \times 10^{12}\ W/m^2}$ **per metre length (Ans.)**

ote: At high temperature the difference between $(E_\lambda)_b$ and $(E_{\lambda b})_{max}$ is very small].

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(iv) Total emissive power, E_b :

$$E_b = \sigma T^4 = 5.67 \times 10^{-8} (2773)^4 = 5.67 \left(\frac{2773}{100} \right)^4 = \mathbf{3.352 \times 10^6 \ W/m^2.} \ \textbf{(Ans.)}$$

(v) Total emissive power, E with emissivity (ε) = 0.9 :

$$E = \varepsilon \sigma T^4 = 0.9 \times 5.67 \left(\frac{2773}{100} \right)^4 = \mathbf{3.017 \times 10^6 \ W/m^2.} \ \textbf{(Ans.)}$$

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Example

Assuming the sun (diameter = 1.4×10^9 m) as a black body having a surface temperature of 5750 K and at a mean distance of 15×10^{10} m from the earth (diameter = 12.8×10^6 m), estimate the following:

(i) The total energy emitted by the sun,

(ii) The emission received per m^2 just outside the atmosphere of the earth,

(iii) The total energy received by the earth if no radiation is blocked by the atmosphere of the earth, and

(iv) The energy received by a 1.6 m × 1.6 m solar collector whose normal is inclined at 50° to the sun. The energy loss through the atmosphere is 42 percent and diffuse radiation is 22 percent of direct radiation.

Solution: Radius of the sun, $r_s = \dfrac{1.4 \times 10^9}{2} = 0.7 \times 10^9$ m

Mean distance of the sun from the earth,

$$R = 15 \times 10^{10} \text{ m}$$

s of the earth $\qquad r_e = \dfrac{12.8 \times 10^6}{2} = 6.4 \times 10^6$ m

e temperature of the sun, $\qquad T = 5750$ K

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(i) The total energy emitted by the sun, E_s:

$$E_s = \sigma A T^4 = 5.67 \times 10^{-8} \times 4\pi r_s^2 \times (5750)^4$$

$$= 5.67 \times 4\pi \times (0.7 \times 10^9)^2 \times \left(\frac{5750}{100}\right)^4$$

$$= \mathbf{3.816 \times 10^{26}\ W\ (Ans.)}$$

(ii) The emission received per m^2:

The sun may be regarded as a point source at a distance of 15×10^{10} m from the earth. The *mean area* just outside the earth's atmosphere over which the radiation will fall is

$$= 4\pi R^2 = 4\pi \times (15 \times 10^{10})^2\ m^2$$

$\therefore$ The emission received outside the earth's atmosphere

$$= \frac{3.816 \times 10^{26}}{4\pi \times (15 \times 10^{10})^2} = \mathbf{1349.6\ W/m^2\ (Ans.)}$$

(iii) The total energy received by the earth:

Assuming the earth a spherical body, the energy received by it will be proportional to the perpendicular projected area, *i.e.*, that of a circle ($= \pi r_e^2$).

$$\text{Energy received by the earth} = 1349.6 \times \pi \times (6.4 \times 10^6)^2$$

$$= \mathbf{1.736 \times 10^{17}\ W\ (Ans.)}$$

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(iv) The energy received by the solar collector:

The direct energy reaching the earth $= (1 - 0.42) \times 1349.6 = 782.77$ W/m^2

The diffuse radiation $= 0.22 \times 782.77 = 172.21$ W/m^2

$\therefore$ Total radiation reaching the collector $= 782.77 + 172.21 \simeq 955$ W/m^2

The projected area $= A \cos \theta = 1.6 \times 1.6 \times \cos 40^\circ = 1.961$ m^2

$\therefore$ Energy received by the solar collector

$= 955 \times 1.961 =$ **1872.7 W (Ans.)**

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HIGHLIGHTS

1. '*Radiation*' *heat transfer* is defined as "the transfer of energy across a system boundary by means of an electromagnetic mechanism which is caused solely by a temperature difference.
2. The *emissive power* (E) is defined as the total amount of radiation emitted by a body per unit area per unit time; it is expressed in W/m^2.
3. *Emissivity* (ε) is defined as the ability of the surface of a body to radiate heat. It is also defined as the ratio of the emissive power of any body to the emissive power of a black body of equal temperature $\left(i.e., \ \varepsilon = \frac{E}{E_b} \right)$.
4. A *black body* is one which neither reflects nor transmits any part of the incident radiation but absorbs all of it.
5. A *gray body* is one whose absorptivity of a surface does not vary with temperature and wavelength of the incident radiation [$\alpha = (\alpha)_\lambda$ = constant].
6. The *Stefan-Boltzmann* law states that the emissive power of a black body is directly proportional to the fourth power of its absolute temperature.

   i.e., $E_b = \sigma T^4$

   where, E_b = Emissive power of a black body, and

   σ = Stefan-Boltzmann constant = 5.67×10^{-8} $W/m^2 K^4$

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7. *Kirchhoff's law* states that at any temperature the ratio of total emissive power E to the total absorptivity α is constant for all substances which are in thermal equilibrium with their environment.

8. *Planck's law* is given by:

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left[\frac{C_2}{\lambda T}\right] - 1}$$

where, $C_1 = 2\pi c^2 h = 3.742 \times 10^8$ W.$\mu m^4/m^2$;

$$C_2 = \frac{ch}{k} = 1.4388 \times 10^4 \ \mu m K$$

[C = velocity of light in vacuum $= 3 \times 10^8$ m/s; h = Planck's constant = 6.625×10^{-34} Js; k = Boltzmann constant = 1.3805×10^{-23} J/K]

9. *Wien's displacement law* states that the product of λ_{max} and T is constant *i.e.*,

$$\lambda_{max} T = \text{constant } (= 2900 \ \mu m \ K)$$

A combination of Planck's law and Wien's displacement law yields the condition for the maximum monochromatic emissive power for a black body.

$$(E_{\lambda b})_{max} = 1.285 \times 10^{-5} T^5 \ W/m^2 \text{ per metre length.}$$

10. The *intensity of radiation* (I) is defined as the rate of energy leaving a space in a given direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space.

Lambert's cosine law states that the total emissive power E_θ from a radiating plane surface in any direction is directly proportional to the cosine of the angle of emission.

i.e. $E_\theta = E_n \cos\theta$ ... (true only for diffuse radiation surface)

where E_n is the total emissive power of the radiating surface in the direction of its normal.

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THEORETICAL QUESTIONS

1. Define 'radiation heat transfer'.
2. Enumerate the factors on which the rate of emission of radiation by a body depend.
3. Define the following terms:
   (*i*) Total emissive power (E) (*ii*) Monochromatic emissive power (E_λ).
   (*iii*) Emissivity, and (*iv*) Intensity of radiation.
4. Define the terms absorptivity, reflectivity and transmittivity of radiation.
5. What is a 'black body'? How does it differ from a gray body?
6. State and prove Kirchhoff's law of radiation.
7. State Planck's law.
8. State Stefan-Boltzmann law.
9. State and explain Wien's displacement law.
10. What is 'Intensity of radiation'?

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Radiation Exchange Between Surfaces

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Radiation Shields

One way of reducing radiant heat transfer between two particular surfaces is to use materials which are highly reflective.

An alternative method is to use radiation shields between the hear exchange surfaces.

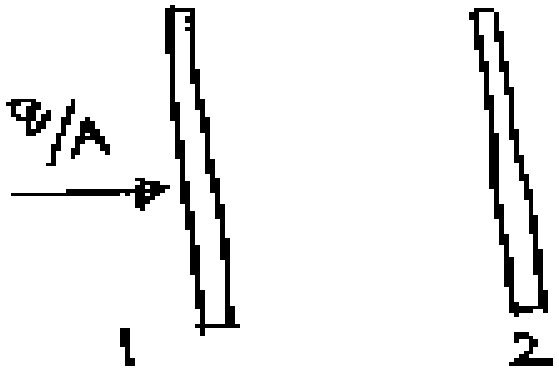
The shields do not deliver or remove any heat from the overall system.

They only place another resistance in the heat flow path, so that the overall heat transfer is retarded.

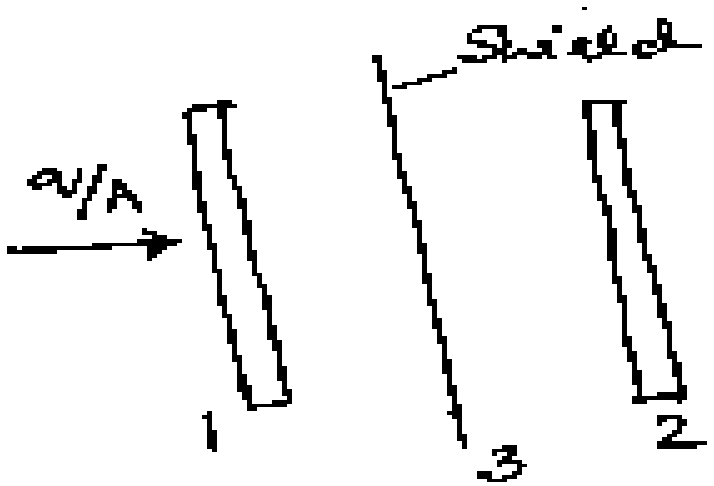
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Single Radiation Shield

Consider two parallel infinite planes with and without shield.



Without Shield



With Shield

Since the shield does not deliver or remove heat from the system, the heat transfer between plate 1 and eld must be precisely the same, as that between the shield and plate 2, and this is the overall heat r.

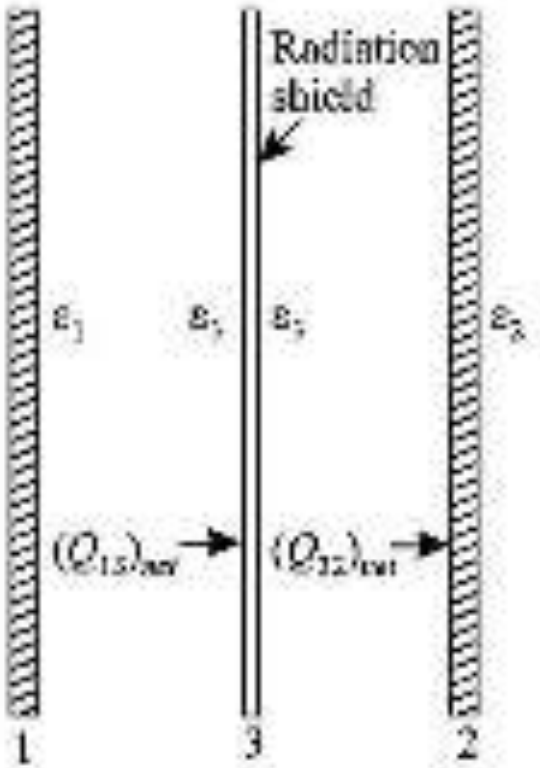
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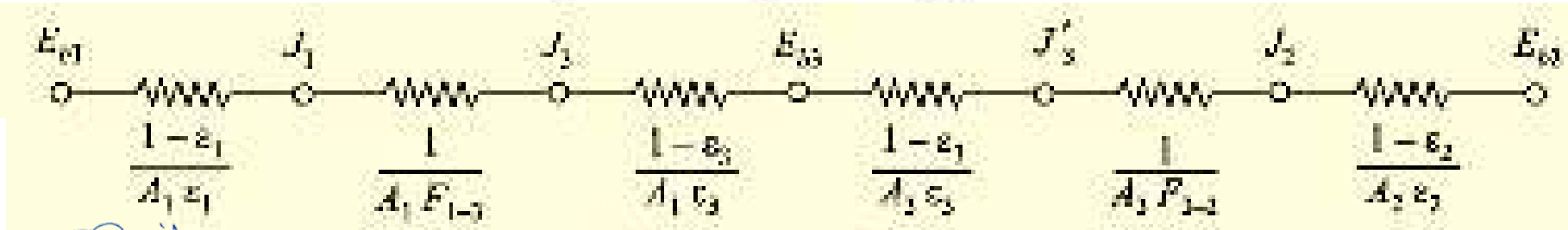
Single Radiation Shield

In certain situations it is required to reduce the overall heat transfer between two radiating surfaces. This is done by either *using materials which are highly reflective or by using radiation shields between the heat exchanging surfaces. The radiation shields reduce the radiation heat transfer by effectively increasing the surface resistances without actually removing any heat from the overall system.* Thin sheets of plastic coated with highly reflecting metallic films on both sides serve as very effective radiation shields. These are used for the insulation of cryogenic storage tanks. A familiar application of radiation shields is *in the measurement of the temperature of a fluid by a thermometer or a thermocouple which is shielded to reduce the effects of radiation.*

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Refer Fig. 12.45. Let us consider two parallel plates, 1 and 2, each of area A ($A_1 = A_2 = A$) at temperatures T_1 and T_2 respectively with a radiation shield placed between them as shown in





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With no radiation shields, the net heat exchange between the parallel plates is given by:

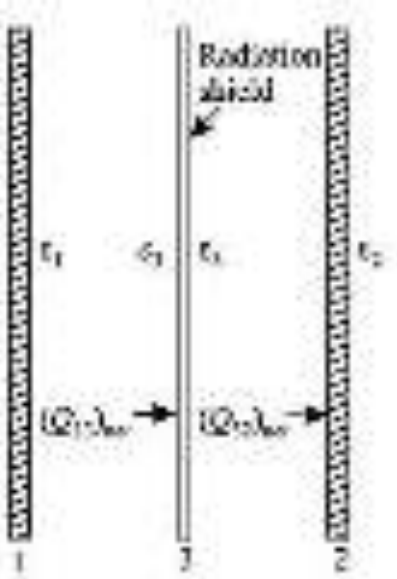
$$(Q_{12})_{net} = \frac{A\,\sigma\,(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

If the emissivity of the radiation shield is ε_3, we can use this equation to find heat exchange between surfaces 1, 3 and 3, 2.

$$(Q_{13})_{net} = \frac{A\,\sigma\,(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$$

and,

$$(Q_{32})_{net} = \frac{A\,\sigma\,(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$



Since the radiation shield does not deliver or remove heat from the system, therefore,

$$(Q_{13})_{net} = (Q_{32})_{net}$$

i.e.,

$$\frac{A\,\sigma\,(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{A\,\sigma\,(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

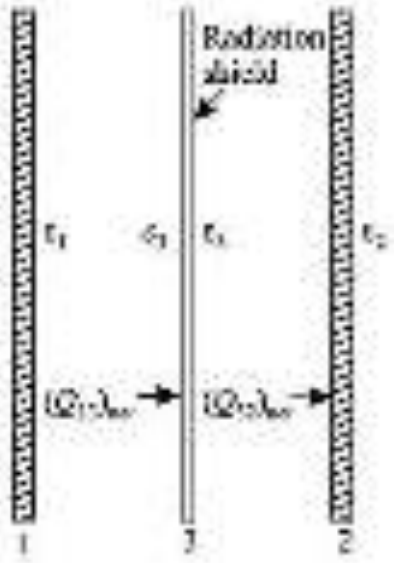
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$$T_3^4 = \frac{T_1^4\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right)+T_2^4\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)}{\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)}$$

Substituting the value of T_3 in the left hand side of eqn.

we get

$$(Q_{12})_{net} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)+\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right)}$$



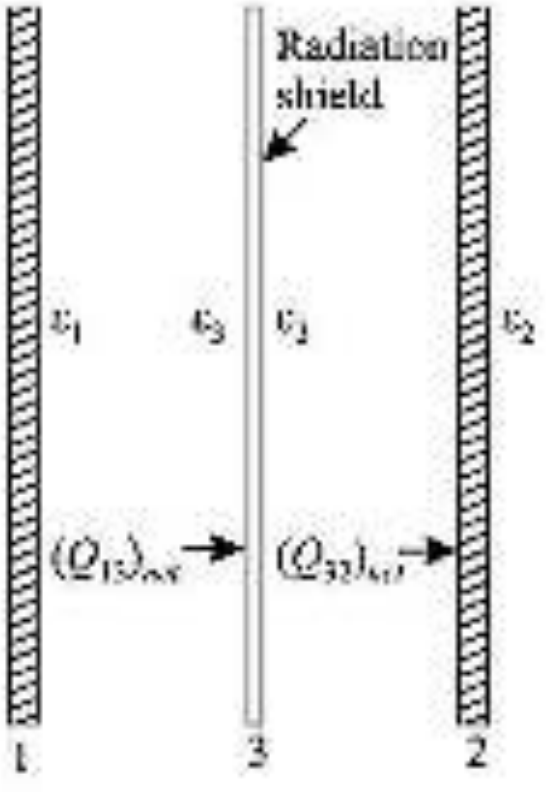
Dividing eqn. we get

$$\frac{[Q_{12})_{net}]_{with\ shield}}{[(Q_{12})_{net}]_{without\ shield}} = \frac{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)+\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right)}$$

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The equation can be generalized for a system of two parallel plates separated by n-shields of emissivity $\varepsilon_{s1}, \varepsilon_{s2} \ldots \varepsilon_{sn}$ as

$$(Q_{12})_{net} = \frac{A\,\sigma\,(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + 2\sum_{i=1}^{n}\frac{1}{\varepsilon_{si}} - (n+1)}$$



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Heat Transfer with shield

$$\therefore \left(\frac{q}{A}\right)_{1-3} = \left(\frac{q}{A}\right)_{3-2} = \frac{q}{A}$$

$$\frac{q}{A} = \frac{\sigma_b\left(T_1^4 - T_3^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{\sigma_b\left(T_3^4 - T_2^4\right)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

The only unknown in equation (9) is the temperature of the shield T_3.

If the emissivity of all three surfaces are same, i.e., $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$, then

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Heat Transfer with shield -2

$$T_3^4 = \frac{1}{2}\left(T_1^4 + T_2^4\right)$$

The heat transfer is given by

$$\frac{q}{A} = \frac{1}{2}\frac{\sigma_b\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$$

As $\varepsilon_3 = \varepsilon_2$, the heat flow is just one-half of that which would be experienced if there is no shield present.

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Multi Radiation shield

Consider n number of shields

Assume the emissivity of all the surfaces are same.

All the surface resistances will be same as the emissivity are same.

There will be two of these resistances for each shield and one for each heat transfer surface.

There will be (n+1) space resistances and these would all be unity since the radiation shape factors are unity for infinite parallel planes.

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Multi Radiation Shield

Therefore, the total resistance in the network is

$$R_{(nshield)} = (2n+2)\frac{1-\varepsilon}{\varepsilon} + (n+1)(1) = \left(n+1\left(\frac{2}{\varepsilon}-1\right)\right)$$

The total resistance with no shield present

$$R_{(noshield)} = \frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1 = \frac{2}{\varepsilon} - 1$$

So, the resistance with shield is (n + 1) times the resistance without shield.

$$\left(\frac{q}{A}\right)_{withshields} = \frac{1}{n+1}\left(\frac{q}{A}\right)_{withoutshoel}$$

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Example *Calculate the net radiant heat exchange per m^2 area for two large parallel plates at temperatures of 427° C and 27° C respectively. ε (hot plate) = 0.9 and ε (cold plate) = 0.6.*

If a polished aluminium shield is placed between them, find the percentage reduction in the heat transfer, ε (shield) = 0.4.

Solution. *Given :* $T_1 = 427 + 273 = 700$ K; $T_2 = 27 + 273 = 300$ K; ε_1 (hot plate) = 0.9, ε_2 (cold plate) = 0.6, ε_3 (shield) = 0.4.

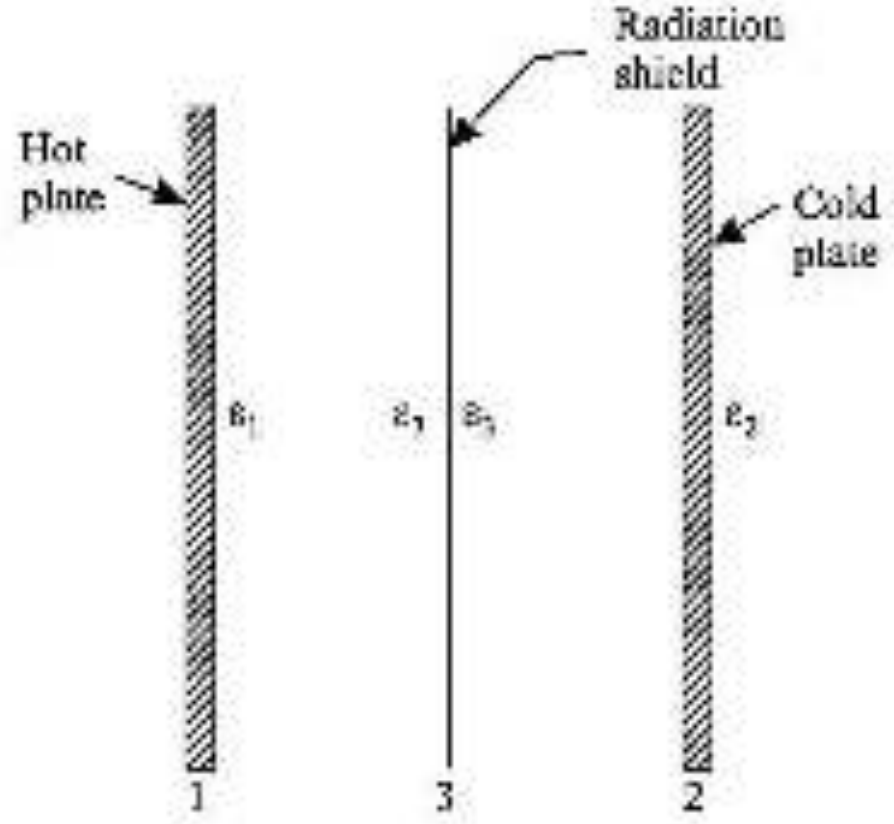
Net radiant heat exchange per m^2 area:

In the absence of radiation shield the heat flow between plates 1 and 2 is given by

$$(Q_{12})_{ow} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \quad \text{...[Eqn. (12.21)]}$$

$$= \frac{5.67 \left[\left(\frac{700}{100}\right)^4 - \left(\frac{300}{100}\right)^4 \right]}{\frac{1}{0.9} + \frac{1}{0.6} - 1}$$

$$= \frac{13154.4}{1.777} = \mathbf{7402.6\ W.\ (Ans.)}$$



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Percentage reduction in the heat transfer flow:

When a shield is placed between the plates 1 and 2, then

$$(Q_{13})_{with} = (Q_{32})_{with}$$

$$\therefore \quad \frac{A\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{A\sigma(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$\text{or,} \quad \frac{\left(\frac{700}{100}\right)^4 - \left(\frac{T_3}{100}\right)^4}{\frac{1}{0.9} + \frac{1}{0.4} - 1} = \frac{\left(\frac{T_3}{100}\right)^4 - \left(\frac{300}{100}\right)^4}{\frac{1}{0.4} + \frac{1}{0.6} - 1}$$

$$\text{or,} \quad \frac{2401 - x^4}{1.11 + 25 - 1} = \frac{x^4 - 81}{25 + 1.67 - 1} \qquad \left[\text{where } x = \frac{T_3}{100}\right]$$

$$\text{or,} \quad 2401 - x^4 = \frac{25.11}{25.67}(x^4 - 81) = 0.9782 = (x^4 - 81)$$

$$1.9782x^4 = 2480.2 \qquad \therefore x^4 = 1253.8$$

$$x = \frac{T_3}{100} = (1253.8)^{1/4} = 5.95 \text{ or } T_3 = 595 \text{ K}$$

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$$(Q_{13})_{net} = \frac{\sigma (T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{5.67\left[\left(\frac{700}{100}\right)^4 - \left(\frac{595}{100}\right)^4\right]}{\frac{1}{0.9} + \frac{1}{0.4} - 1}$$

$$= \frac{6507.2}{25.11} = 259.1 \text{ W}$$

$\therefore$ Reduction in heat flow due to shield

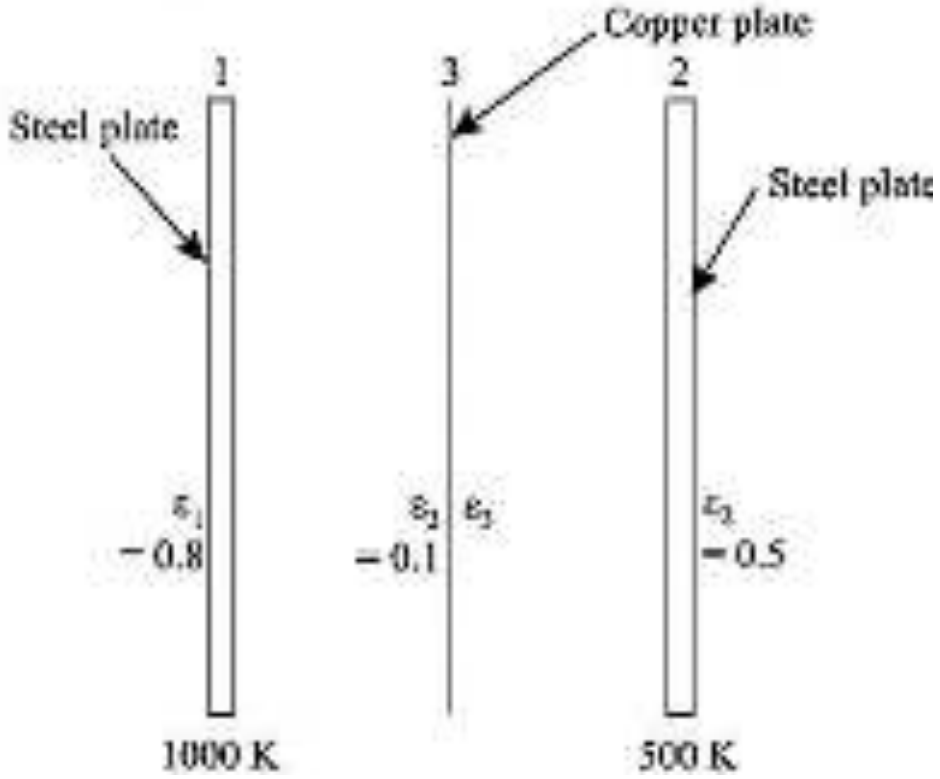
$$= (Q_{12})_{net} - (Q_{13})_{net} = 7402.6 - 259.1 = 7143.5 \text{ W}$$

or, Percentage reduction $= \frac{7143.5}{7402.6} \times 100 = \mathbf{96.5\%}$ **(Ans.)**

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Example *Determine the radiant heat exchanger in W/m² between two large parallel steel plates of emissivities 0.8 and 0.5 held at temperatures of 1000 K and 500 K respectively, if a thin copper plate of emissivity 0.1 is introduced as a radiation shield between the two plates. Use* $\sigma = 5.67 \times 10^{-8}\ W/m^2\ K^4$.

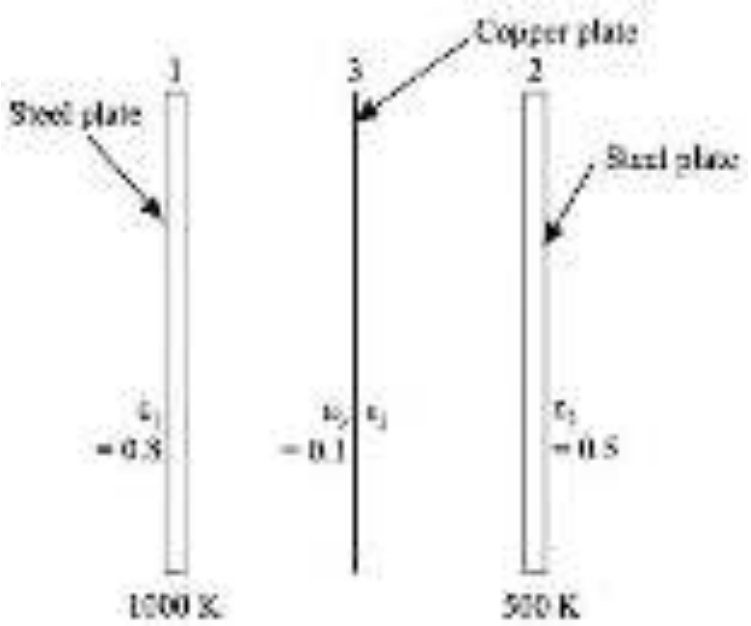
Solution. *Given :* $T_1 = 1000$ K; $\varepsilon_1 = 0.8$, $T_2 = 500$ K; $\varepsilon_2 = 0.5$, $\varepsilon_3 = 0.1$.



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Radiant heat exchange in W/m^2, $(Q_{12})_{net}$:

We know that, $$(Q_{12})_{net} = \frac{A\sigma(T_1^4 - T_2^4)}{\left[\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right] + \left[\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1\right]}$$ ...[Eqn. (



For $A = 1\ m^2$, we have

$$(Q_{12})_{net} = \frac{5.67\left[\left(\frac{1000}{100}\right)^4 - \left(\frac{500}{100}\right)^4\right]}{\left[\frac{1}{0.8} + \frac{1}{0.1} - 1\right] + \left[\frac{1}{0.1} + \frac{1}{0.5} - 1\right]}$$

$$= \frac{5.67\ (10000 - 625)}{(12.5 + 10 - 1) + (10 + 2 - 1)} = \mathbf{2501.5\ W/m^2\ (Ans.)}$$

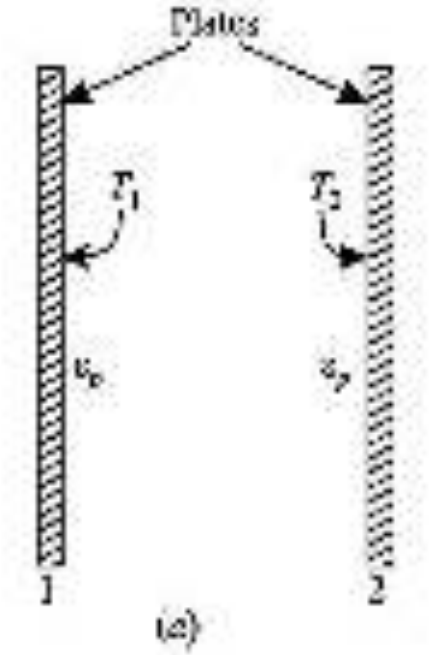
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Example *Two large parallel plates with ε = 0.5 each, are maintained at different temperatures and are exchanging heat only by radiation. Two equally large radiation shields with surface emissivity 0.05 are introduced in parallel to the plates. Find the percentage reduction in net radiative heat transfer.*

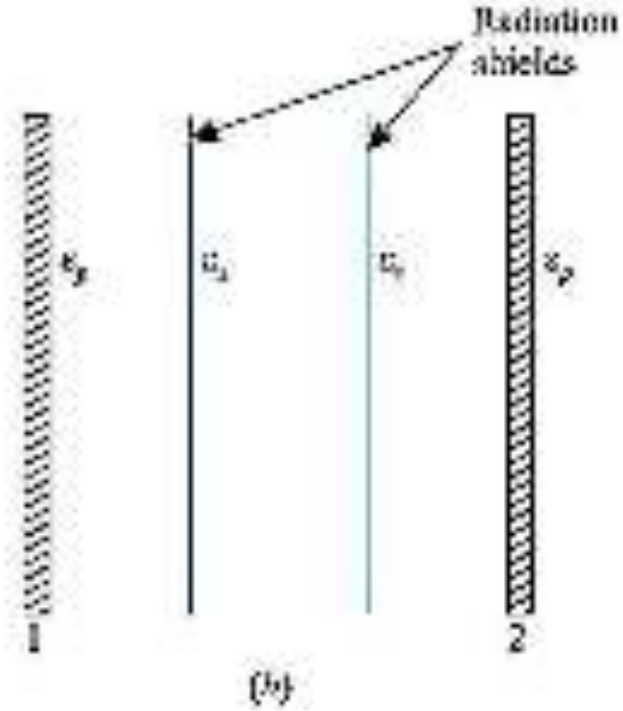
Solution. *Given:* $\varepsilon_p = 0.5; \varepsilon_s = 0.05$

Consider all resistances per unit surface area.

(i) When shields are not used:



(a)



(b)

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$(Q)_{without\ shields}$

$$= \frac{\sigma (T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_p}{\varepsilon_p}\right) + 1 + \left(\frac{1-\varepsilon_p}{\varepsilon_p}\right)} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_p} + \frac{1}{\varepsilon_p} - 1}$$

$$= \frac{C}{\frac{1}{0.5} + \frac{1}{0.5} - 1} = 0.33\,C \text{ [where } C = \sigma (T_1^4 - T_2^4)]$$

(ii) When shields are used :

$$(Q)_{with\ shields} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_p} + \frac{1}{\varepsilon_p} + 2\left[\frac{1}{\varepsilon_s} + \frac{1}{\varepsilon_s}\right] - (2+1)}$$

$$= \frac{\sigma (T_1^4 - T_2^4)}{\frac{2}{\varepsilon_p} + \frac{4}{\varepsilon_s} - 3}$$

$$= \frac{C}{\frac{2}{0.5} + \frac{4}{0.05} - 3} = \frac{C}{81} = 0.012345\,C$$

$$\therefore \quad \text{Percentage reduction in heat flow} = \left(\frac{(Q)_{without\ shields} - (Q)_{with\ shields}}{(Q)_{without\ shields}}\right) \times 100$$

$$= \left[1 - \frac{(Q)_{with\ shields}}{(Q)_{without\ shields}}\right] \times 100 = 1 - \frac{0.012345\,C}{0.33\,C}$$

$$= \mathbf{96.26\%\ (Ans.)}$$

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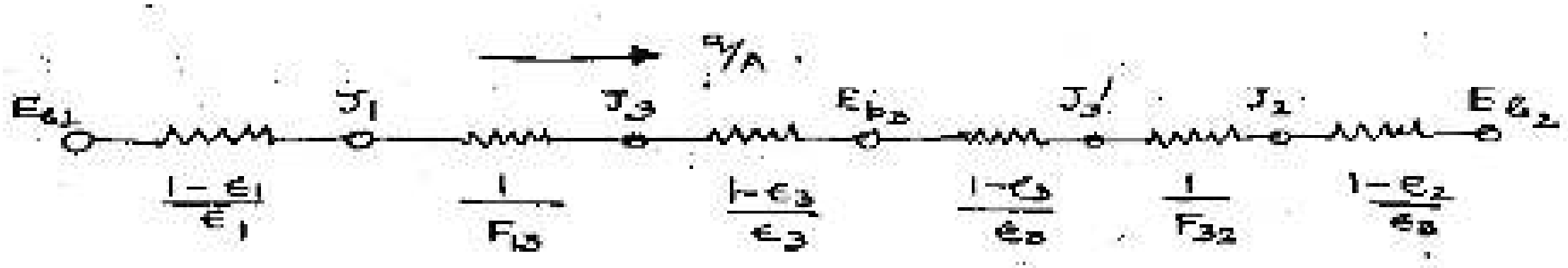
For More Question on Radiation Shield Kindly Refer, Heat Transfer Book by Rajput and Mahesh Rathod





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Equivalent circuit



When the emissivity of all surfaces are different, the overall heat transfer may be calculated most easily by using a series radiation network with appropriate number of elements as shown in the figure.

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RADIATION AMONG SURFACES IN A NON-PARTICIPATING MEDIUM

For any two given surfaces, the orientation between them affects the fraction of radiation energy leaving one surface and that strikes the other.

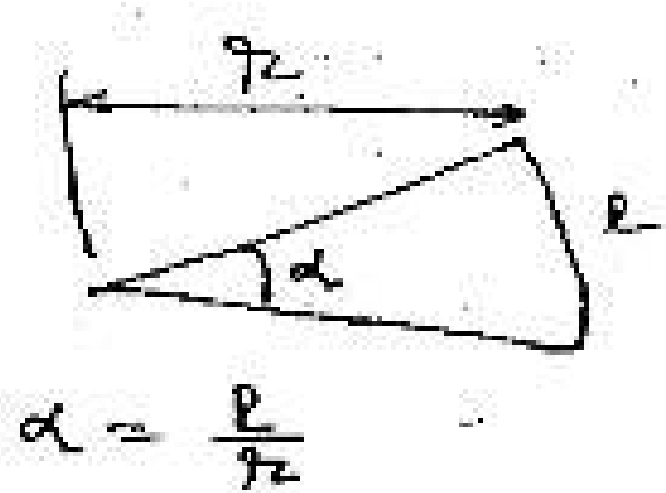
To take into account this, the concept of view factor/ shape factor/ configuration factor is introduced.

The physical significance of the view factor between two surfaces is that it represents the fraction of the radiative energy leaving one surface that strikes the other surface directly.

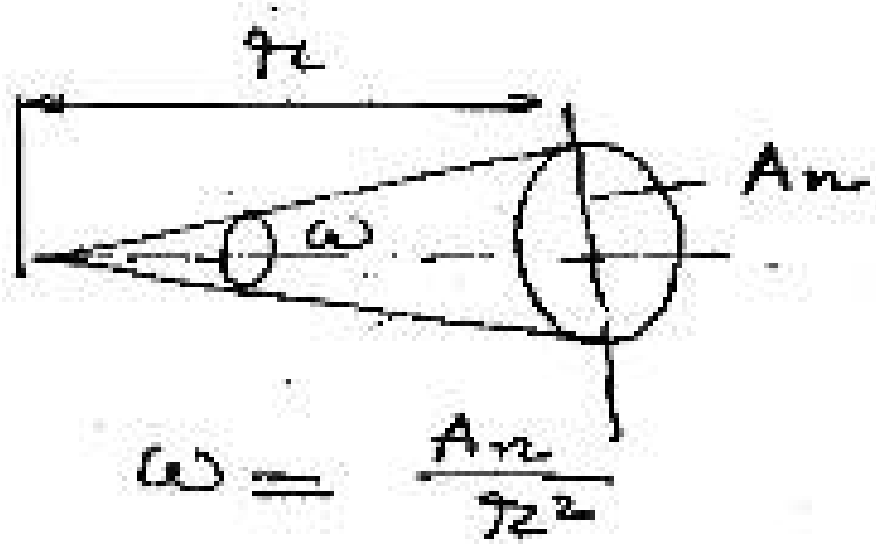
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Plane Angle and Solid Angle

The plane angle (α) is defined by a region by the rays of a circle. The solid angle (ω) is defined by a region by the rays of a sphere.



e Angle



Solid Angle

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Plane Angle and Solid Angle

$$\omega = \frac{A_n}{r^2} = \frac{A\cos\theta}{r^2}$$

A_n: projection of the incident surface normal to the line of projection

θ: angle between the normal to the incident surface and the line of propagation.

r: length of the line of propagation between the radiating and the incident surfaces

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View factor between two elemental surfaces

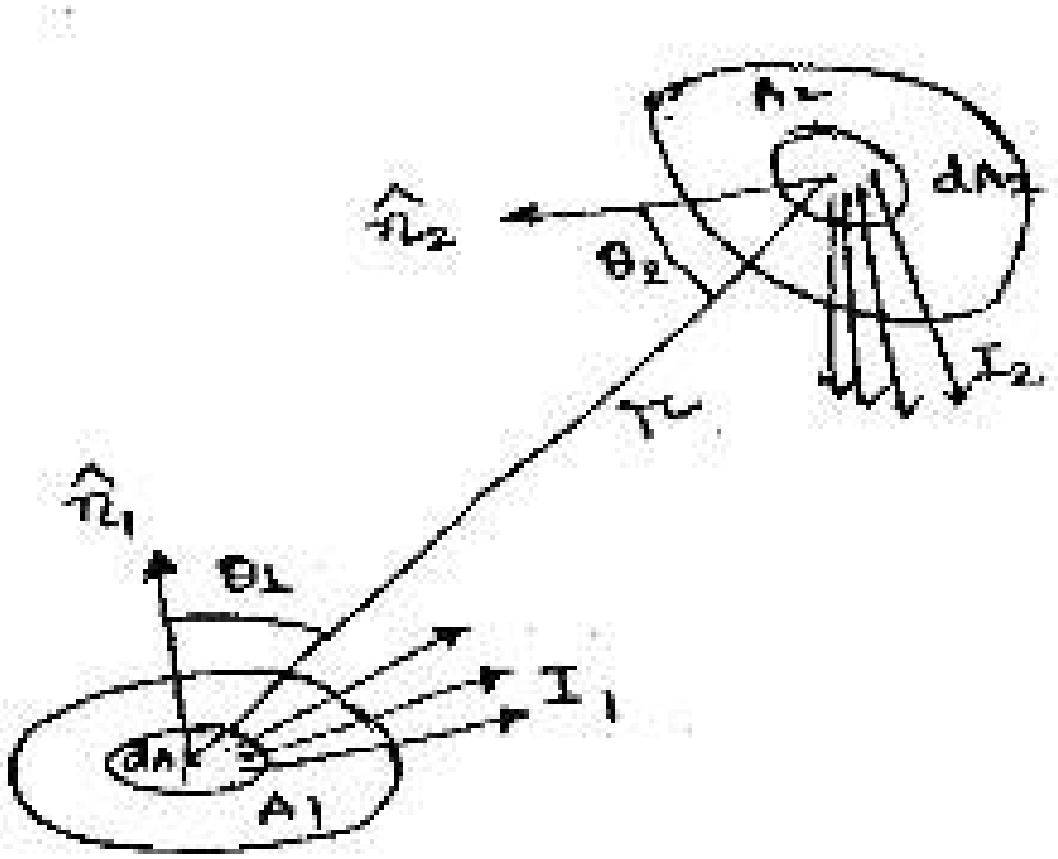
Consider two elemental areas dA_1 and dA_2 on body 1 and 2 respectively.

Let $d\omega_{12}$ be the solid angle under which an observer at dA_1 sees the surface element dA_2 and

I_1 be the intensity of radiation leaving the surface element diffusely in all directions in hemispherical space.

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View Factor Figure



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View factor

Therefore, the rate of radiative energy dQ_1 leaving dA_1 and strikes dA_2 is

$$dQ_{12} = dA_1 I_1 \cos\theta_1 d\omega_{12} \qquad \text{---- (3)}$$

where solid angle $d\omega_{12}$ is given by

$$d\omega_{12} = \frac{dA_2 \cos\theta_2}{r^2} \qquad \text{------ (4)}$$

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View factor

Combining (3) and (4), we get

$$dQ_{12} = dA_1 I_1 \frac{\cos\theta_1 \cos\theta_2 dA_2}{r^2}$$

------ (5)

Now, the intensity of normal radiation is given by

$$I_1 = \frac{E_b}{\pi} = \frac{\sigma_b T_1^4}{\pi}$$

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Shape Factor

$$\therefore dQ_{12} = \frac{\sigma_b T_1^4}{\pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2}$$

Now, we define shape factor, F_{12} as

$$F_{12} = \frac{\text{direct radiation from surface 1 incident on surface 2}}{\text{total radiation from emitting surface 1}} = \frac{Q_{12}}{\sigma_b A_1 T_1^4}$$

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Shape factor

$$= \frac{1}{\sigma_b A_1 T_1^4} \frac{\sigma_b T_1^4}{\pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2}$$

$$= \frac{1}{A_1 \pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2}$$

---- (6)

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Radiant Heat Transfer Between Two Bodies

The amount of radiant energy leaving A_1 and striking A_2 may be written as

$$Q_{12} = A_1 F_{12} \sigma_b T_1^4$$

Similarly, the energy leaving A_2 and arriving A_1 is

$$Q_{21} = A_2 F_{21} \sigma_b T_2^4$$

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Radiant Heat Transfer Between Two Bodies (2)

So, net energy exchange from A_1 to A_2 is

$$(Q_{12})_{net} = A_1 F_{12} \sigma_b T_1^4 - A_2 F_{21} \sigma_b T_2^4$$

When the surfaces are maintained at the same temperatures, $T_1 = T_2$, there cannot be any heat exchange between them.

$$\therefore 0 = A_1 F_{12} \sigma_b T_1^4 - A_2 F_{21} \sigma_b T_2^4$$

$$A_1 F_{12} = A_2 F_{21} \quad \text{--- (7)}$$

Reciprocity theorem

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Net Heat transfer

$$(Q_2)_{net} = A_1 F_{12} \sigma_b \left(T_1^4 - T_2^4\right) = A_2 F_{21} \sigma_b \left(T_1^4 - T_2^4\right)$$

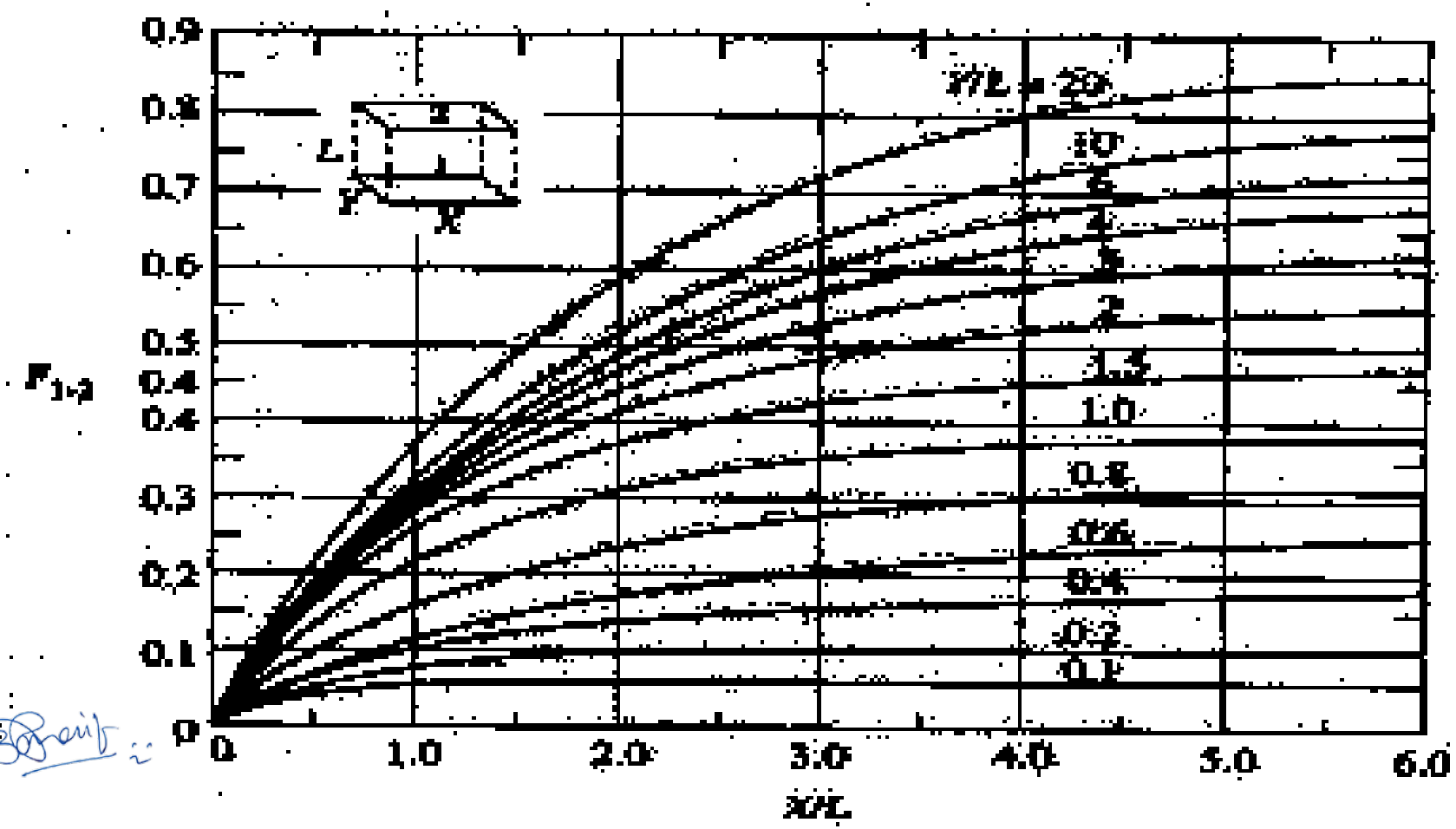
--- (8)

The evaluation of the integral of equation (6) for the determination of shape factor for complex geometries is rather complex and cumbersome.

Results have been obtained and presented in graphical form for the geometries normally encountered in engineering practice.

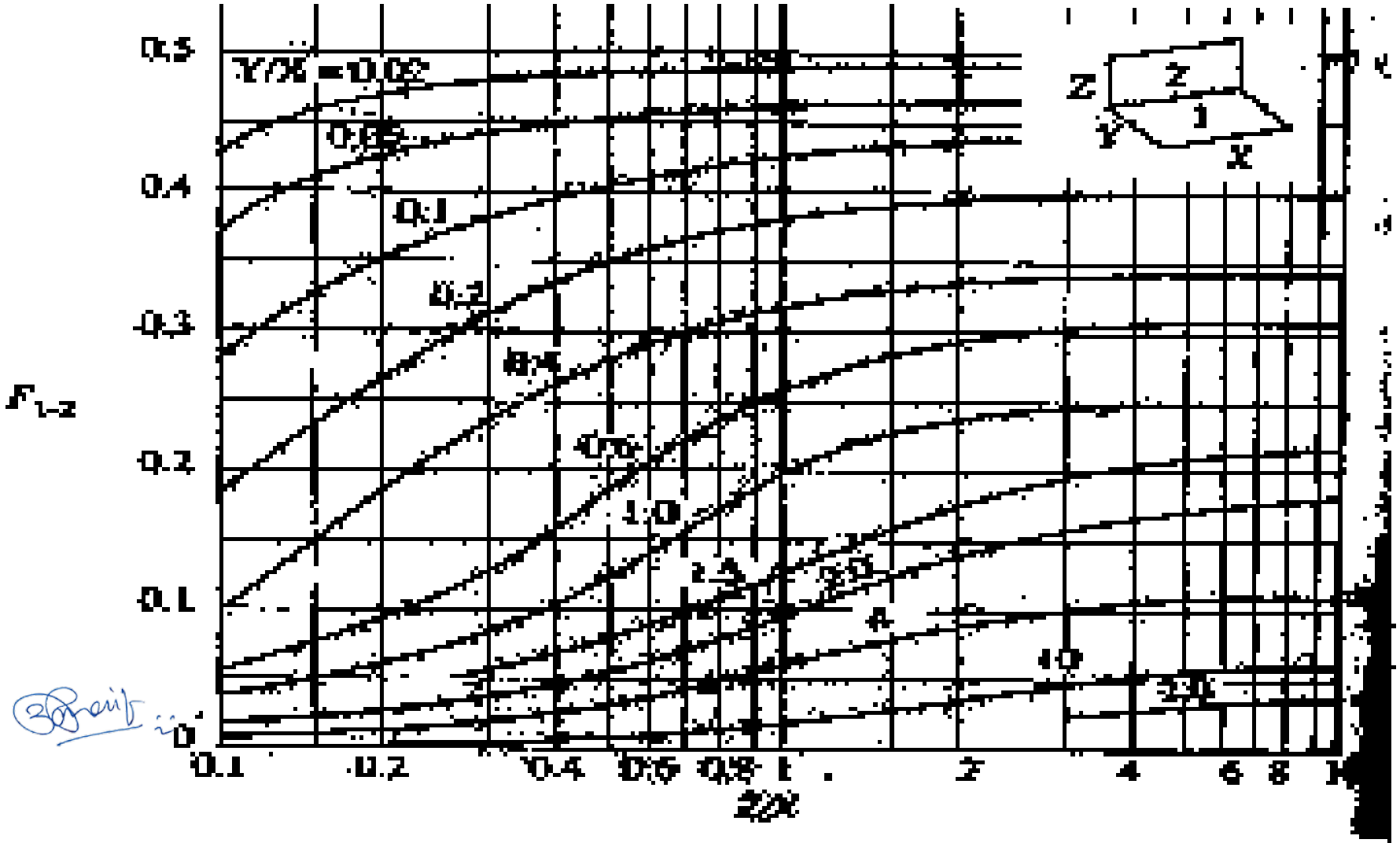
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SHAPE FACTOR FOR ALLIGNED PARALLEL PLATES



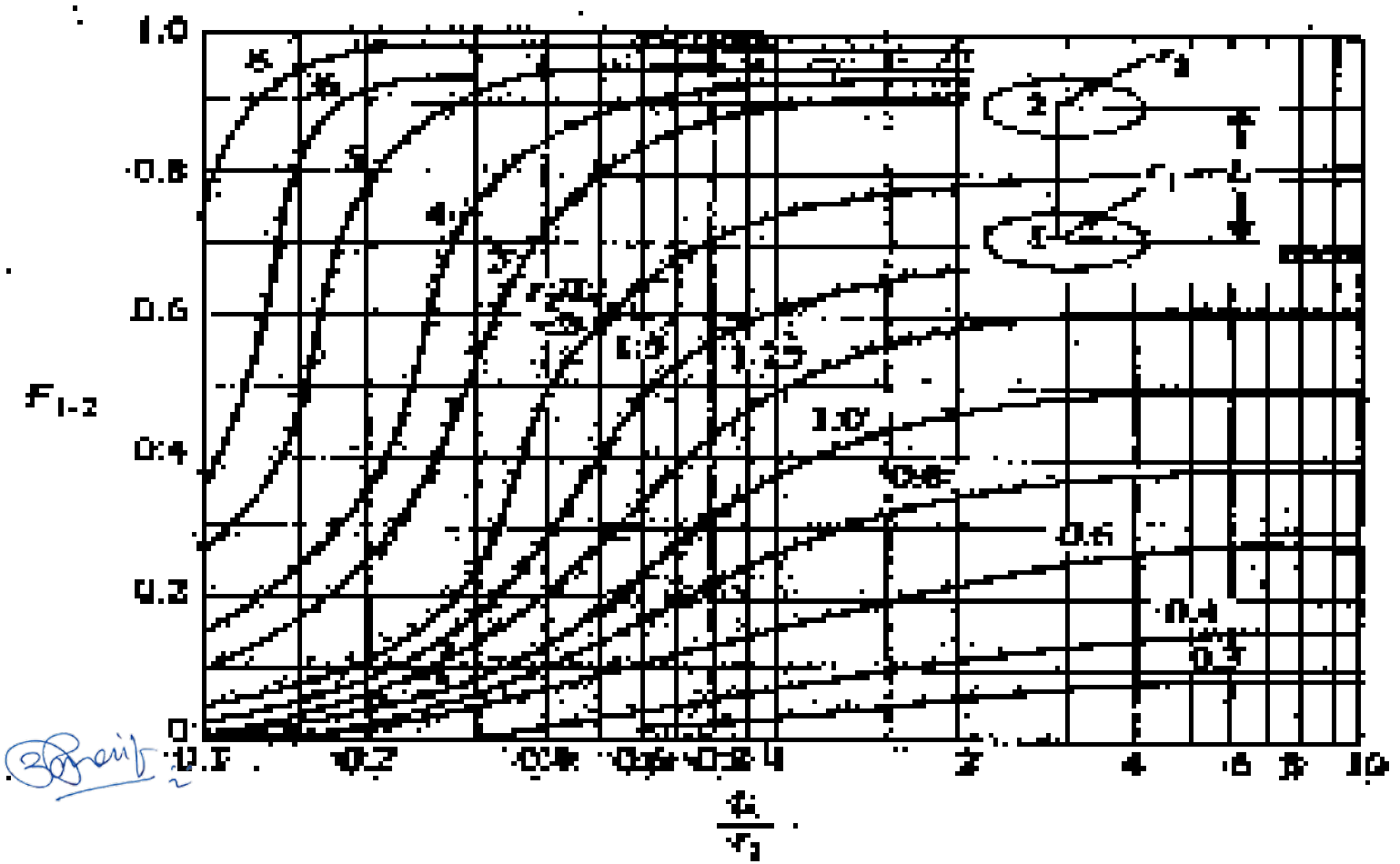
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SHAPE FACTOR FOR PERPENDICULAR RECTANGLES WITH COMMON BASE



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SHAPE FACTOR FOR COAXIAL PARALLEL PLATES



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SHAPE FACTOR ALGEBRA

The shape factors for complex geometries can be derived in terms of known shape factors for other geometries.

For that the complex shape is divided into sections for which the shape factor is either known or can be readily evaluated.

The unknown configuration factor is worked out by adding and subtracting known factors of related geometries.

The method is based on the definition of shape factor, the reciprocity principle and the energy conservation law.

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Some Features of Shape Factor

The value of the shape factor depends only on the geometry and orientation of surfaces with respect to each other. Once the shape factor between two surfaces is known, it can be used for the calculating heat exchange between two surfaces at any temperature.

All the radiation coming out from a convex surface 1 is intercepted by the enclosing surface 2. The shape factor of convex surface with respect to the enclosure (F_{12}) is unity.

The radiant energy emitted by a concave surface is intercepted by another part of the same surface. A concave surface has a shape factor with ect to itself and it is denoted by F_{11}. For a convex and flat surface , F_{11} =

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Features of Shape Factor

If one of the two surfaces (say A_i) is divided into sub-areas A_{i1}, A_{i2}, …., A_{in}, then

$$A_i F_{ij} = \sum A_{in} F_{inj}$$

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Features of Shape Factor

In Fig.1,

$$A_1F_{12} = A_3F_{32} + A_4F_{42}$$

Here,

$$A_1 = A_3 + A_4$$

Hence

$$F_{12} \neq F_{32} + F_{42}$$

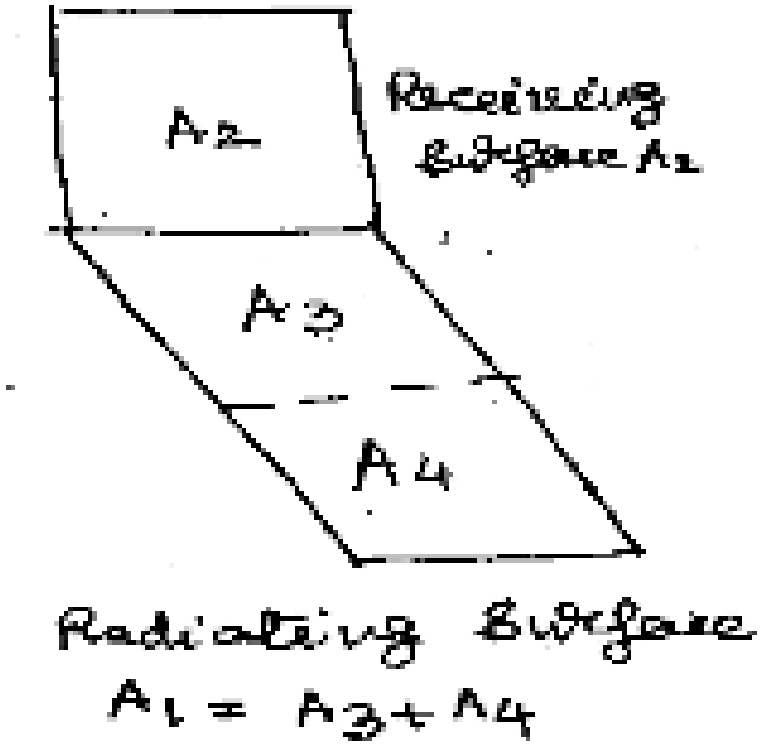


Fig.1

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Features of Shape Factor

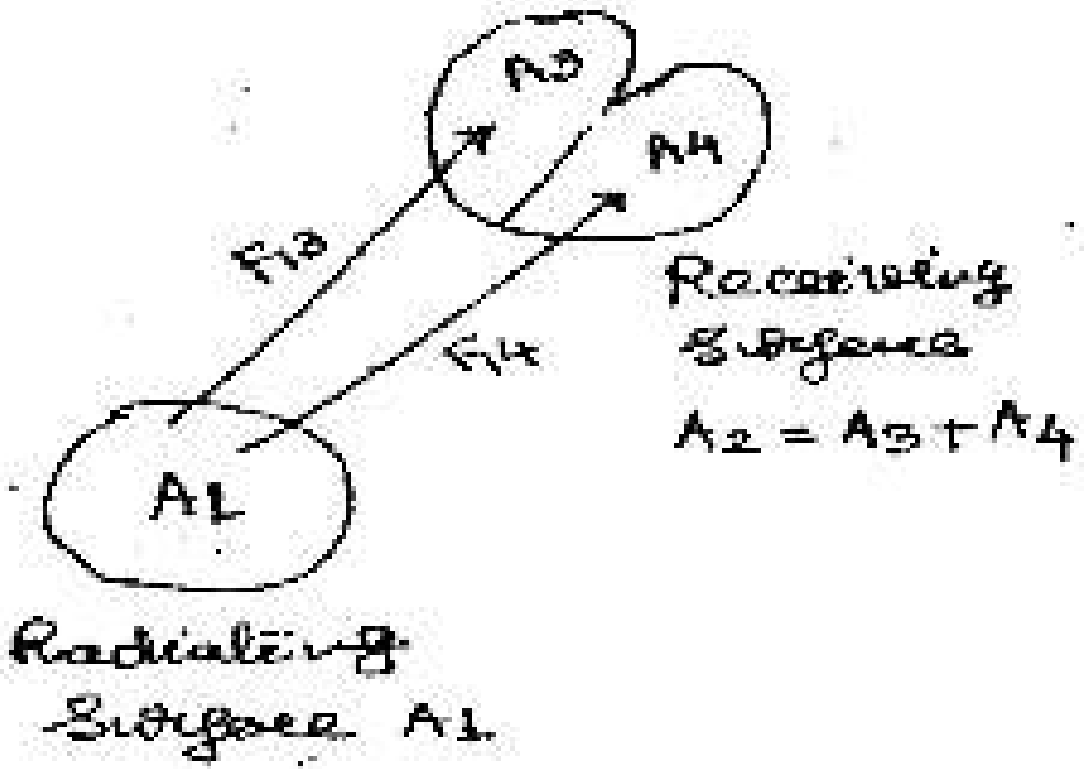
For Fig. 2,

$$A_1F_{12} = A_1F_{13} + A_1F_{14}$$

Here

$$A_2 = A_3 + A_4$$

$$F_{12} = F_{13} + F_{14}$$



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Shape Factor Algebra

Any radiating surface will have finite area and therefore will be enclosed by many surfaces.

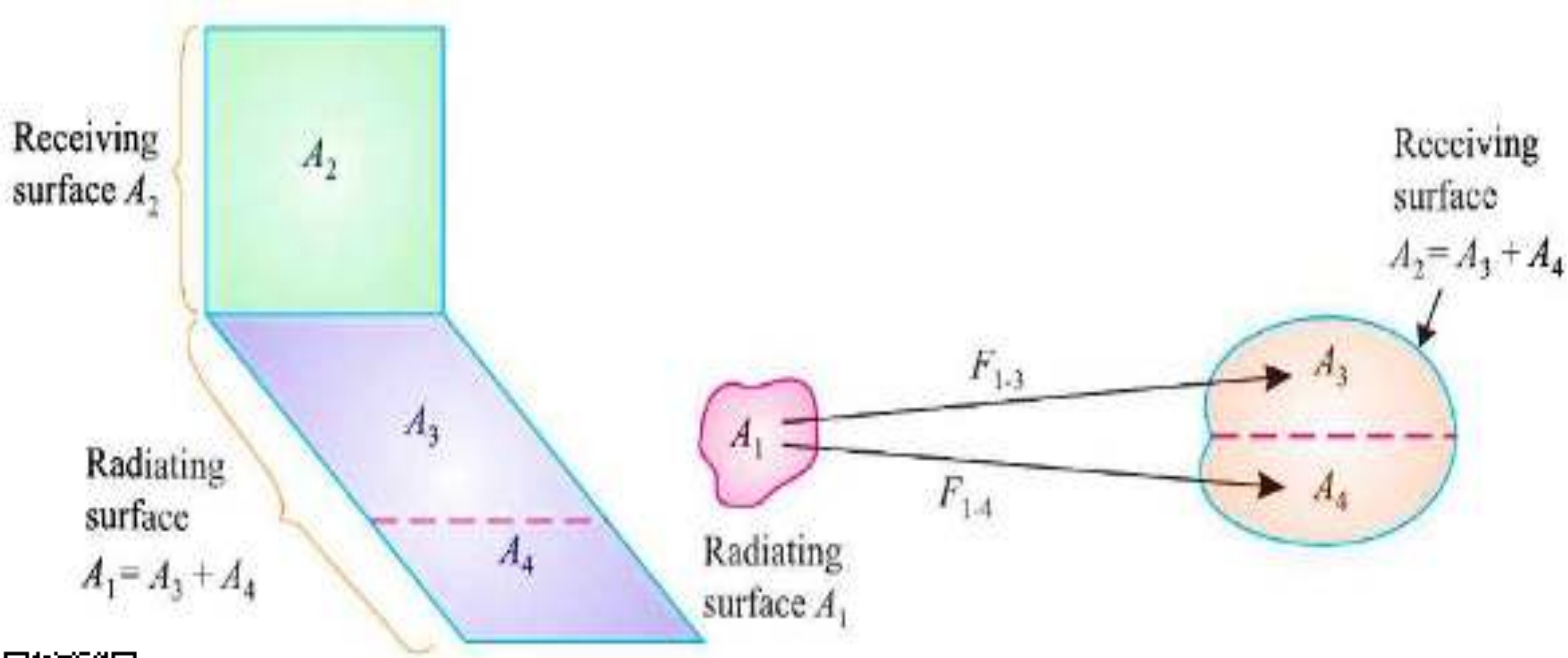
The total radiation being emitted by the radiating surface will be received and absorbed by each of the confining surfaces.

Since shape factor is the fraction of total radiation leaving the radiating surface and falling upon a particular receiving surface

$$\sum_{j=1}^{n} F_{ij} = 1$$

, *i = 1,2, ……, n*

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Shape Factor Algebra

If the interior surface of a complete enclosed space has been subdivided in *n* parts having finite area A_1, A_2, …. A_n, then

$$F_{11}+F_{12}+F_{13}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+F_{1n}=1$$

$$F_{21}+F_{22}+F_{23}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+F_{2n}=1$$

$$F_{31}+F_{32}+F_{33}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+F_{3n}=1$$

$$F_{n1}+F_{n2}+F_{n3}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+F_{nn}=1$$

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HEAT EXCHANGE BETWEEN NON-BLACK BODIES

The black bodies absorb the entire incident radiation and this aspect makes the calculation procedure of heat exchange between black bodies rather simple.

One has to only determine the shape factor.

However, the real surfaces do not absorb the whole of the incident radiation: a part is reflected back to the radiating surface.

Also the absorptivity and emissivity are not uniform in all directions and for all wavelengths.





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Infinite parallel planes

Assumptions

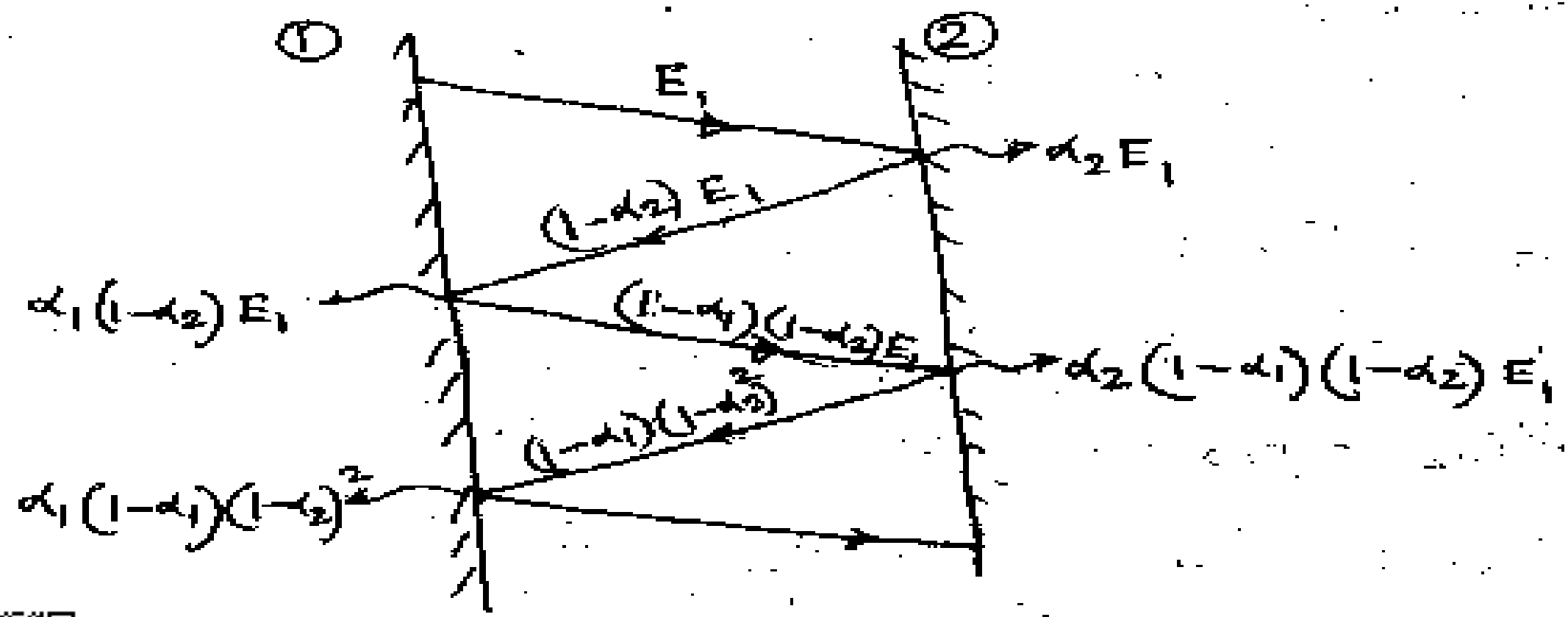
(i) Surfaces are arranged at small distance from each other and are of equal areas so that practically all radiation emitted by one surface falls on the other. The shape factor of either surface is therefore unity.

(ii) Surfaces are diffuse and uniform in temperature, and that the reflected and emissive properties are constant over all the surface.

(iii) The surfaces are separated by a non-absorbing medium as air.

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Infinite parallel planes



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Heat Transfer between Infinite parallel planes

The amount of radiant energy which left surface 1 per unit time is

$$Q_1 = E_1 - \left[\alpha_1(1-\alpha_2)E_1 + \alpha_1(1-\alpha_1)(1-\alpha_2)^2 E_1 + \alpha_1(1-\alpha_1)^2(1-\alpha_2)^3 E_1 + \text{---------}\right]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1\left[1 + (1-\alpha_1)(1-\alpha_2) + (1-\alpha_1)^2(1-\alpha_2)^2 + \text{---------}\right]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1\left[1 + p + p^2 + \text{---------}\right]$$ where $p = (1-\alpha_1)(1-\alpha_2)$

$$1 + p + p^2 + \text{---------------} \text{ upto } \infty = \frac{1}{1-p}$$

p is less than unity

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Calculations

$$Q_1 = E_1 - \frac{\alpha_1(1-\alpha_2)E_1}{1-p}$$

$$= E_1\left[1 - \frac{\alpha_1(1-\alpha_2)}{1-(1-\alpha_1)(1-\alpha_2)}\right]$$

$$= E_1\left[\frac{\alpha_2}{\alpha_1+\alpha_2-\alpha_1\alpha_2}\right]$$

$$= E_1\left[\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}\right] \quad \text{as } \alpha = \varepsilon \text{ from Kirchoff's law}$$

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Surface 2

Similarly, the amount of heat which leaves surface 2

$$Q_2 = E_2\left[\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}\right]$$

Therefore, the net heat flow from surface 1 to surface 2 per unit time is given by

$$Q_{12} = Q_1 - Q_2 = E_1\left[\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}\right] - E_2\left[\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}\right]$$

$$= \frac{E_1\alpha_2 - E_2\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

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Black Surface

Now, for the black surfaces,

$$E_1 = \alpha_1 \sigma_b T_1^4$$

$$E_2 = \alpha_2 \sigma_b T_2^4$$

$$\therefore Q_{12} = \frac{\varepsilon_1 \sigma_b T_1^4 \varepsilon_2 - \varepsilon_2 \sigma_b T_2^4 \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$$= \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \sigma_b \left(T_1^4 - T_2^4 \right)$$

$$= f_{12} \sigma_b \left(T_1^4 - T_2^4 \right)$$

where f_{12} is called the interchange factor for the radiation from surface 1 to surface 2 and is given by.

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Interchange Factor

$$f_{12} = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

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Small Gray Bodies

Small bodies signify that their sizes are very small compared to the distance between them.

The radiant energy emitted by surface 1 would be partly absorbed by surface 2 and the unabsorbed reflected portion will be lost in space.

It will not be reflected back to surface 1 because of its small size and large distance between the two surfaces.

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Calculations for Small Gray Bodies

Energy emitted by body 1 = $A_1 \varepsilon_1 \sigma_b T_1^4$

Energy incident on by body 2 = $F_{12} A_1 \varepsilon_1 \sigma_b T_1^4$

Energy absorbed by surface 2 = $\alpha_2 F_{12} A_1 \varepsilon_1 \sigma_b T_1^4$

$$\therefore Q_1 = \varepsilon_1 \varepsilon_2 A_1 F_{12} \sigma_b T_1^4$$

putting $\alpha_2 = \varepsilon_2$

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Calculations for Small Gray Bodies (2)

Similarly, energy transfer from surface 2 to 1 is

$$\therefore Q_2 = \varepsilon_1 \varepsilon_2 A_2 F_{21} \sigma_b T_2^4$$

Net energy exchange

$$Q_{12} = Q_1 - Q_2 = \varepsilon_1 \varepsilon_2 A_1 F_{12} \sigma_b T_1^4 - \varepsilon_1 \varepsilon_2 A_2 F_{21} \sigma_b T_2^4$$

$$A_1 F_{12} = A_2 F_{21}$$

$$Q_{12} = \varepsilon_1 \varepsilon_2 A_1 F_{12} \sigma_b \left(T_1^4 - T_2^4\right) = f_{12} A_1 F_{12} \sigma_b \left(T_1^4 - T_2^4\right)$$

Interchange factor ,

$$f_{12} = \varepsilon_1 \varepsilon_2$$

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Small Body in large Enclosure

The large gray enclosure acts like a black body.

It absorbs practically all radiation incident upon it and reflects negligibly small energy back to the small gray body.

The entire radiation emitted by the small body would be intercepted by the outer large enclosure.

$$\therefore F_{12} = 1$$

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Radiation calculations

Energy emitted by small body 1 and absorbed by large enclosure 2=

$$A_1\varepsilon_1\sigma_b T_1^4$$

Energy emitted by enclosure =

$$A_2\varepsilon_2\sigma_b T_2^4$$

Energy incident upon small body =

$$F_{21}A_2\varepsilon_2\sigma_b T_2^4$$

Energy absorbed

nall body =

$$\alpha_1 F_{21}A_2\varepsilon_2\sigma_b T_2^4 = \varepsilon_1\varepsilon_2 A_2 F_{21}\sigma_b T_2^4$$

ange of energy =

$$Q_{12} = \alpha_1 A_1\sigma_b T_1^4 - \varepsilon_1\varepsilon_2 A_2 F_{21}\sigma_b T_2^4$$

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Interchange Factor

If $T_1 = T_2$, $Q_{12} = 0$ and we get

$$A_1 = A_2 \varepsilon_2 F_{21}$$

$$\therefore Q_{12} = \varepsilon_1 A_1 \sigma_b \left(T_1^4 - T_2^4 \right)$$

$$= f_{12} A_1 \sigma_b \left(T_1^4 - T_2^4 \right)$$

(so, $f_{12} = \varepsilon_1$)

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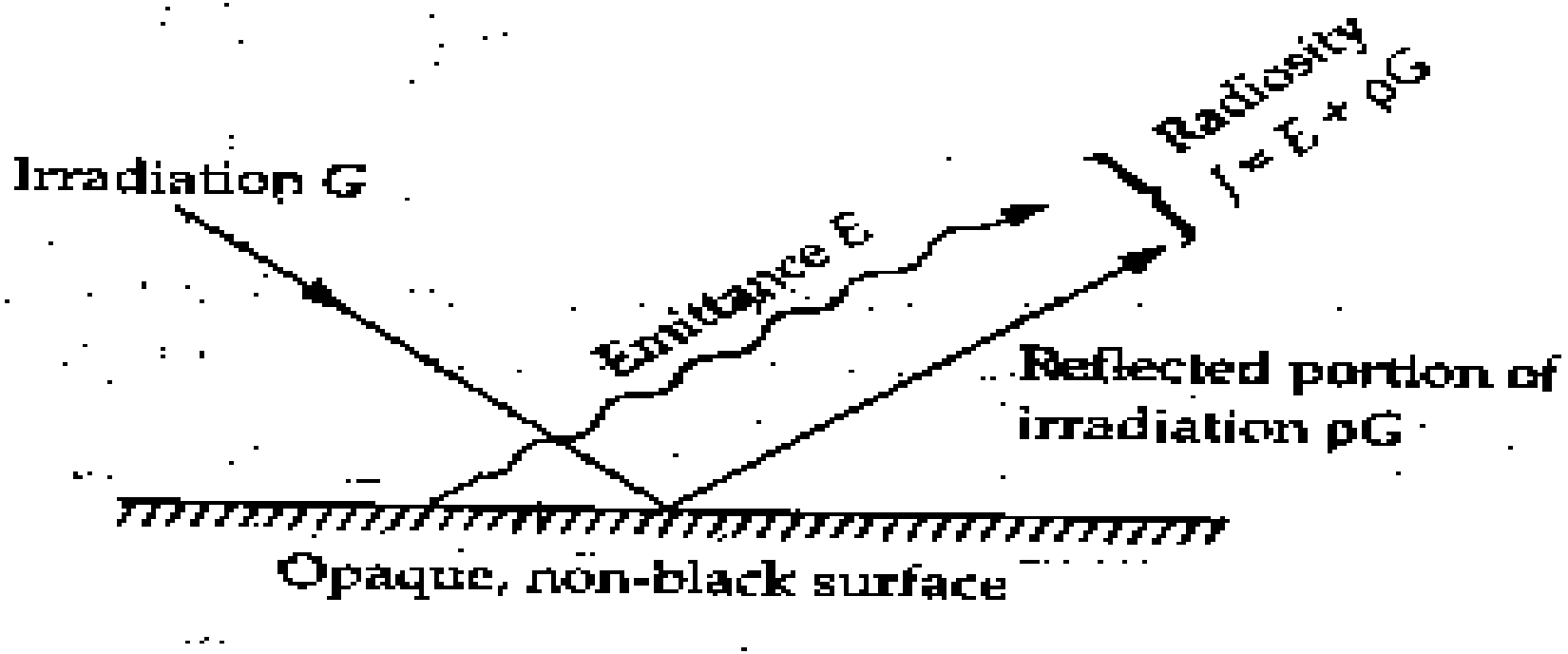
ELECTRICAL NETWORK ANALOGY

Radiosity (J) indicates the total radiant energy leaving a surface per unit time per unit surface area. It comprises the original emittance from the surface plus the reflected portion of any radiation incident upon it.

Irradiation (G) denotes the total radiant energy incident upon a surface per unit time per unit area; some of it may be reflected to become a part of the radiosity of the surface.

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Radiosity and Irradiation Concept



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Radiosity and Irradiation Relation

The total radiant energy (J) leaving the surface is the sum of its original emittance (E) and the energy reflected (ρG) by it out of the irradiation (G) impinging on it.

Hence $J = E + \rho G$

$= \varepsilon E_b + \rho G$ (1)

E_b is the emissive power of black body at the same temperature

$\alpha + \rho = 1$ (opaque body)

$\rho = 1 - \alpha$

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Radiosity and Irradiation Relation

From equ.(1) we get,

$J = \varepsilon E_b + (1- \alpha)\, G$

$$\therefore G = \frac{J - \varepsilon E_b}{1 - \varepsilon}$$

Now

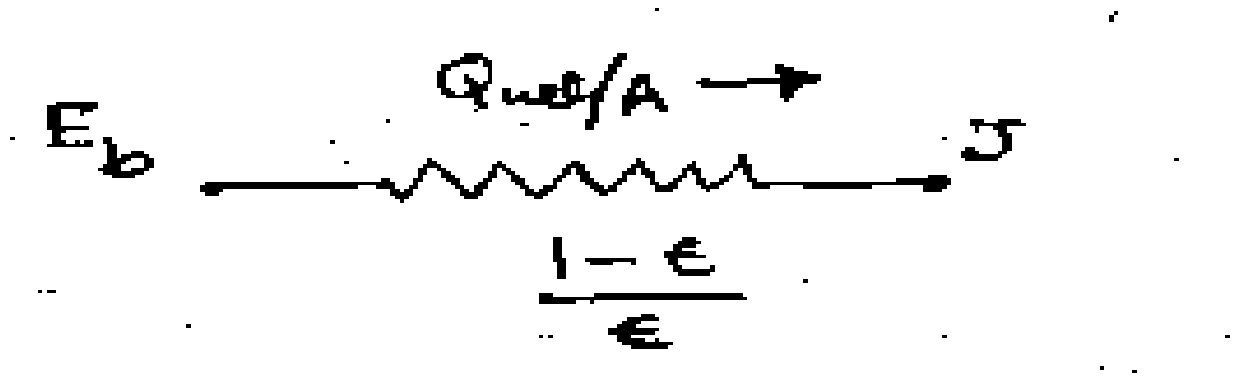
$$\frac{Q_{net}}{A} = J - G = J - \frac{J - \varepsilon E_b}{1 - \varepsilon} = \frac{\varepsilon (E_b - J)}{1 - \varepsilon}$$

$$Q_{net} = \frac{A\varepsilon}{1 - \varepsilon}(E_b - J) = \frac{E_b - J}{(1 - \varepsilon)/A\varepsilon} \quad \text{--- (3)}$$

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Electrical Network Analogy

The above equation (3) can be represented as electrical network as shown below



$\frac{(1-\varepsilon)}{A\varepsilon}$ is called surface resistance to radiation heat transfer.

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Heat Transfer Between Non-Black Bodies

Heat transfer between two non-black surfaces is given by

$(Q_{1\text{-}2})_{net} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$

J_1 and J_2 are the radiosities of surfaces 1 and 2.

Also, $A_1F_{12} = A_2F_{21}$

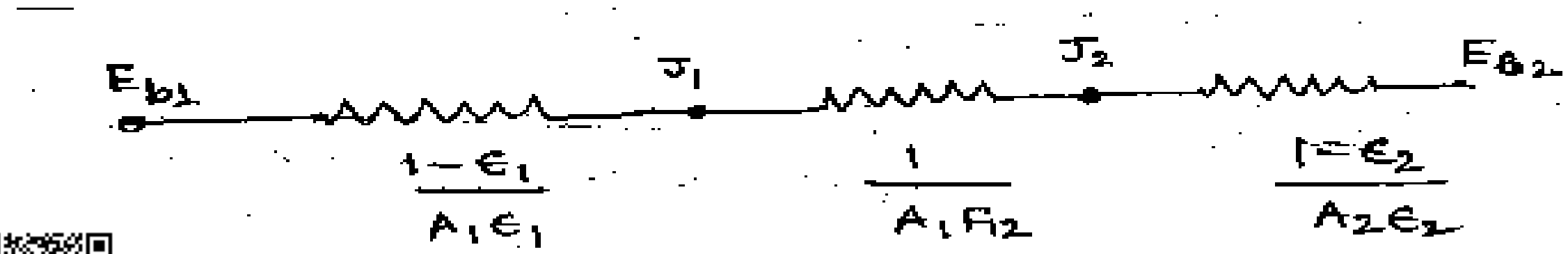
$$\therefore (Q_{1-2})_{net} = (J_1 - J_2) A_1 F_{12} = \frac{(J_1 - J_2)}{\frac{1}{A_1 F_{12}}}$$

$\frac{1}{F_{12}}$ is called space resistance.

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Electrical Analogy Circuit

The final electrical analogy circuit for heat transfer between two non-black surfaces is drawn considering both surface resistance and space resistance as



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Net Heat Transfer

$$\therefore (Q_{12})_{net} = \frac{E_{b1} - E_{b2}}{\frac{(1-\varepsilon_1)}{A_1\varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{(1-\varepsilon_2)}{A_2\varepsilon_2}}$$

$$= \frac{A_1\sigma_b\left(T_1^4 - T_2^4\right)}{\frac{(1-\varepsilon_1)}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{(1-\varepsilon_2)}{\varepsilon_2} \times \frac{A_1}{A_2}}$$

$$= \left(F_g\right)_{12} A_1\sigma_b\left(T_1^4 - T_2^4\right)$$

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Gray body factor

$$(F_g)_{12} = \frac{1}{\frac{(1-\varepsilon_1)}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{(1-\varepsilon_2)}{\varepsilon_2} \times \frac{A_1}{A_2}}$$

Called Gray body factor

For radiant heat exchange between two black surfaces, the surface resistance becomes zero as

$$\varepsilon_1 = \varepsilon_2 = 1$$

And F_g becomes F_{12}, the shape factor only.So for black surfaces

$$\therefore Q_{12} = A_1 F_{12} \sigma_b (T_1^4 - T_2^4)$$

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Special Cases

Two Infinite Parallel Planes:

Here, $F_{12} = F_{21}=1$ and also $A_1 = A_2$

$$\left(F_g\right)_{12}=\frac{1}{\frac{(1-\varepsilon_1)}{\varepsilon_1}+\frac{1}{F_{12}}+\frac{(1-\varepsilon_2)}{\varepsilon_2}\times\frac{A_1}{A_2}}$$

$$=\frac{1}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}$$

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Two Concentric Cylinders or Spheres

If the inner surface is surface 1, then $F_{12} = 1$

$$\left(F_g\right)_{12} = \frac{1}{\frac{\left(1-\varepsilon_1\right)}{\varepsilon_1} + 1 + \frac{\left(1-\varepsilon_2\right)}{\varepsilon_2} \times \frac{A_1}{A_2}}$$

Now, for concentric cylinders of equal length,

$$\frac{A_1}{A_2} = \frac{\pi d_1 l}{\pi d_2 l} = \frac{d_1}{d_2}$$

oncentric spheres,

$$\frac{A_1}{A_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

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A small body in a large enclosure:

$F_{12} = 1$ $A_1 << A_2$ so A_1/A_2 0 ⟶

$$\left(F_g\right)_{12} = \frac{1}{\frac{\left(1-\varepsilon_1\right)}{\varepsilon_1}+1}$$

ical example of this kind:

pe carrying steam in a large room

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Unit No.6

Introduction to Heat Exchangers

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Heat Exchanger

- Heat exchangers can be described as:
- It include all devices that are designed for exchanging heat.
- Only heat exchangers that are meant to exchange heat between two fluids are taken into account.
- These fluids can be gasses as well as liquids. It is still difficult to have an overview, and a classification needs to be made. It is possible to classify heat exchangers in a number of ways.

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What are heat exchangers for?

- Heat exchangers are practical devices used to transfer energy from one fluid to another
- To get fluid streams to the right temperature for the next process reactions often require feeds at high temp.
- To condense vapours
- To evaporate liquids
- To recover heat to use elsewhere
- To reject low-grade heat

drive a power cycle

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Classification Heat Exchangers

Classification of heat exchangers depending on the basic of the fluid paths through the heat exchanger.

Difference is made between :

- Parallel flow,
- Counter flow
- Cross flow.

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Classification



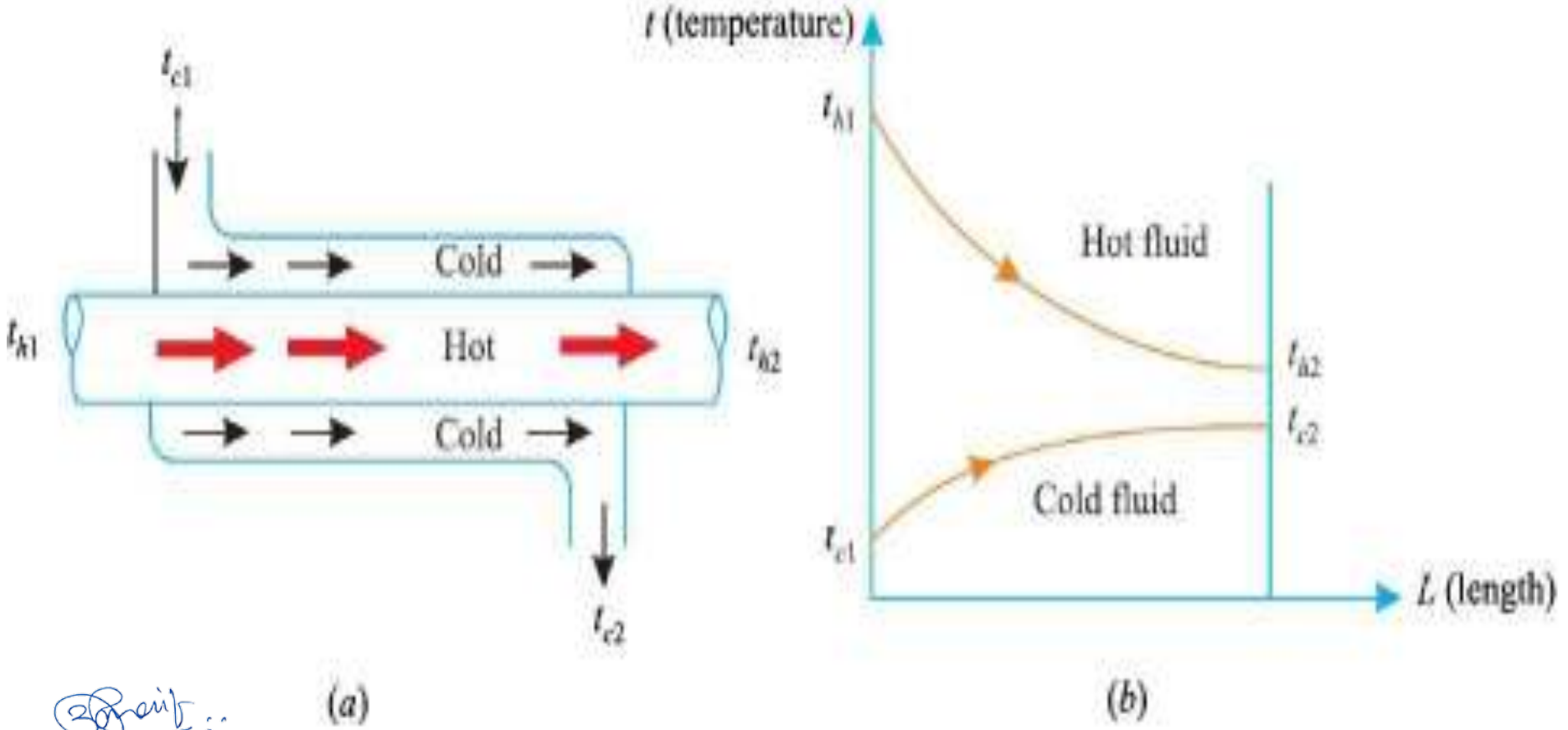
cross flow air to liquid heat exchanger

This is a heat exchangers that is found on the top buildings and is for instance needed for air conditioning inside. Here a fluid is leaded between the plates
at the top of the heat exchanger and flows horizontally. Air is blown vertically against the plates to cool the fluid inside.

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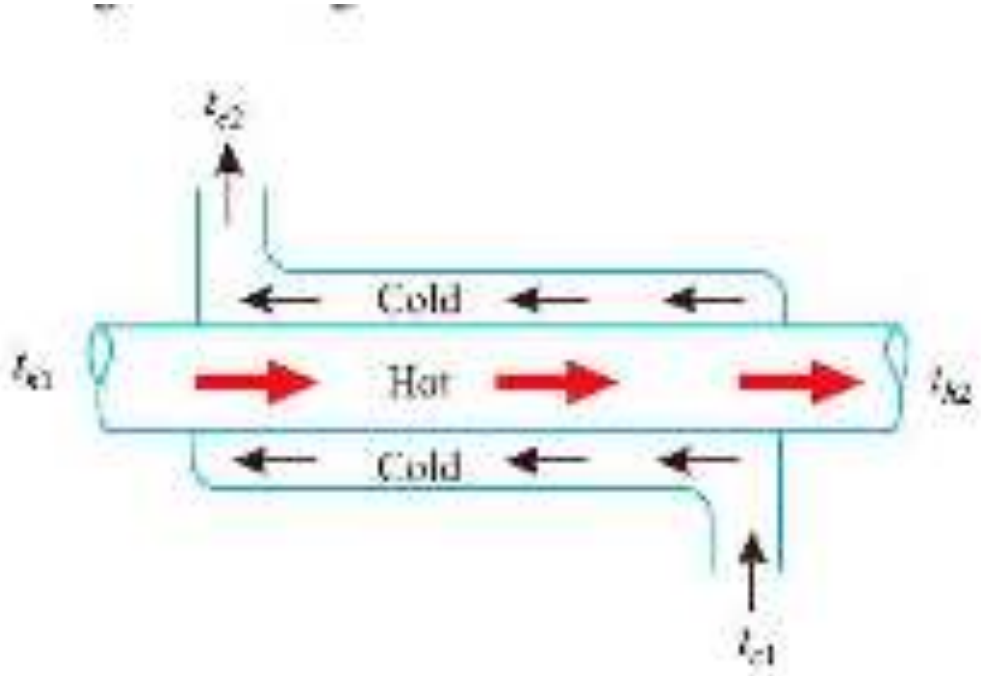
Parallel flow heat exchanger.



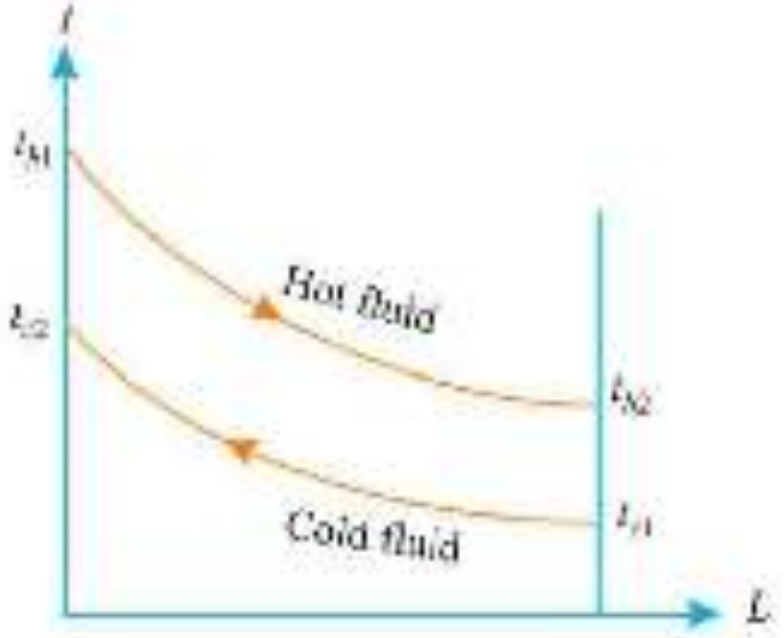
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Classification

Counter-flow heat exchanger



(a)

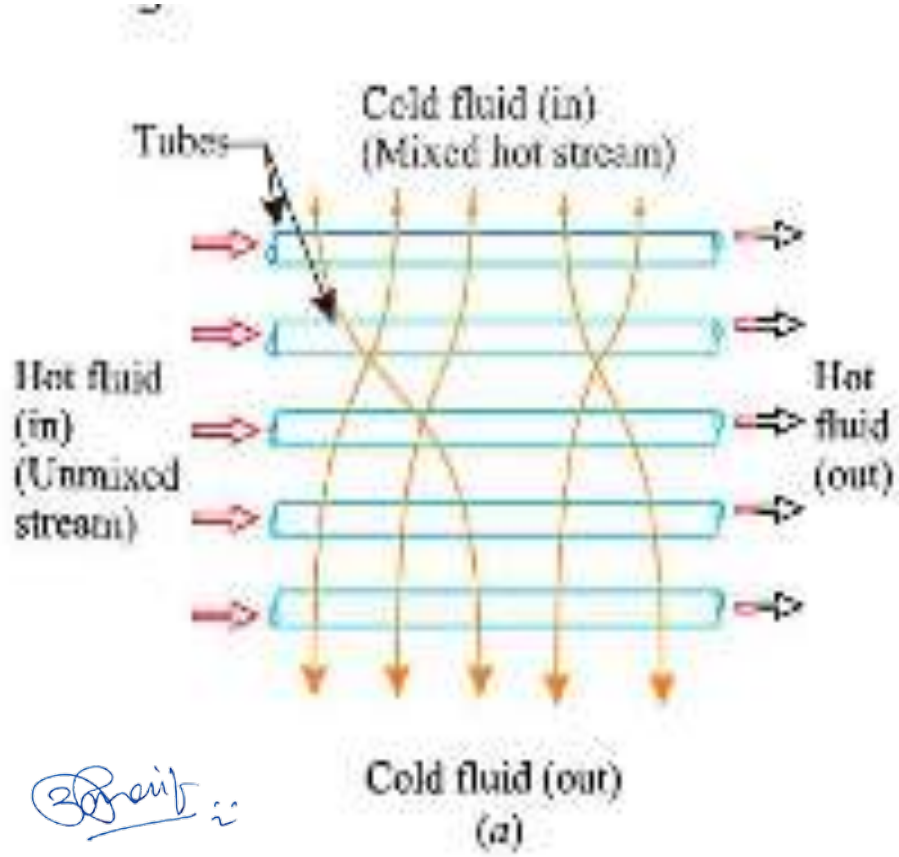


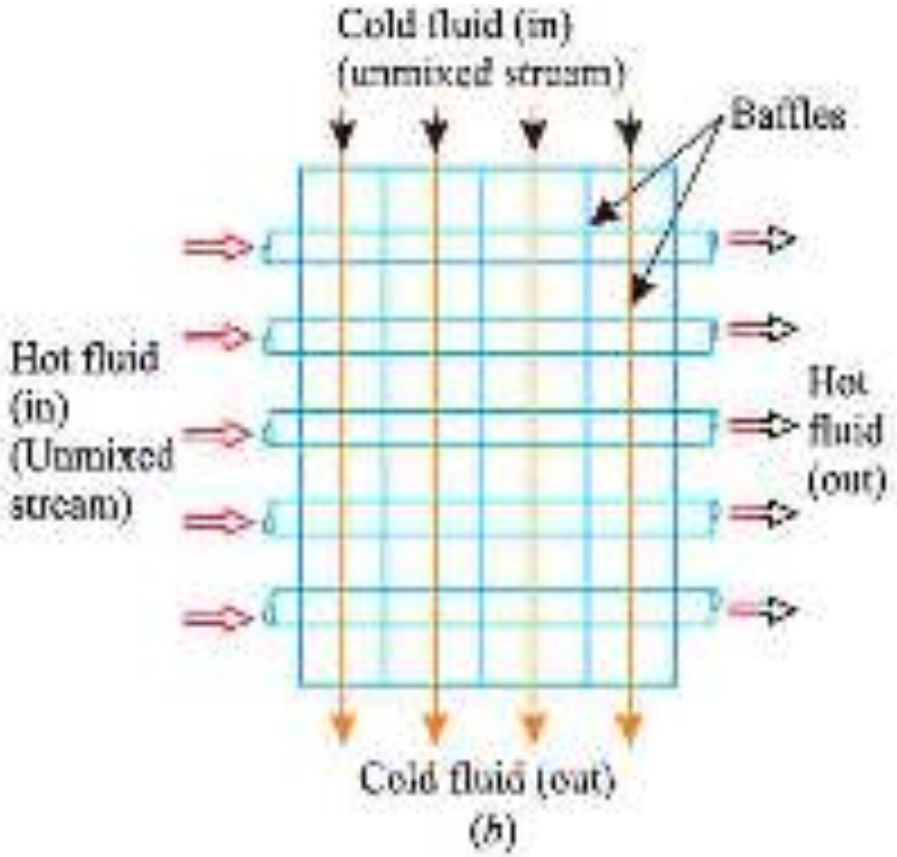
(b)

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Classification

Cross-flow heat exchangers





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Classification

The second classification made, is depending on the state of the media in the heat exchanger.

Liquid-to-liquid exchangers are those in which two liquids interact. Also gas-to-gas heat exchangers like air preheaters in steam plants and helium-cooled reactor gas turbine plants have to be mentioned. These devices operate with heat transfer coefficients that are between ten and one hundred times lower than the coefficients of liquid-to-liquid exchangers. Gas to gas exchangers are general much larger and heavier if a same amount of transferred heat is demanded.

The third type is the liquid-to-gas heat exchanger (or vice versa), usually water and air are used, for instance in automotive radiators. Because of the lower heat transfer coefficients on the gas-side there are usually fines placed on the exchanging surfaces.

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Classification

The third classification method is based the purpose of heat exchanger.

The last classification is actually the most important choice of the designer of a heat exchanging system. This is the choice what kind of construction he is going to use.

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Classification

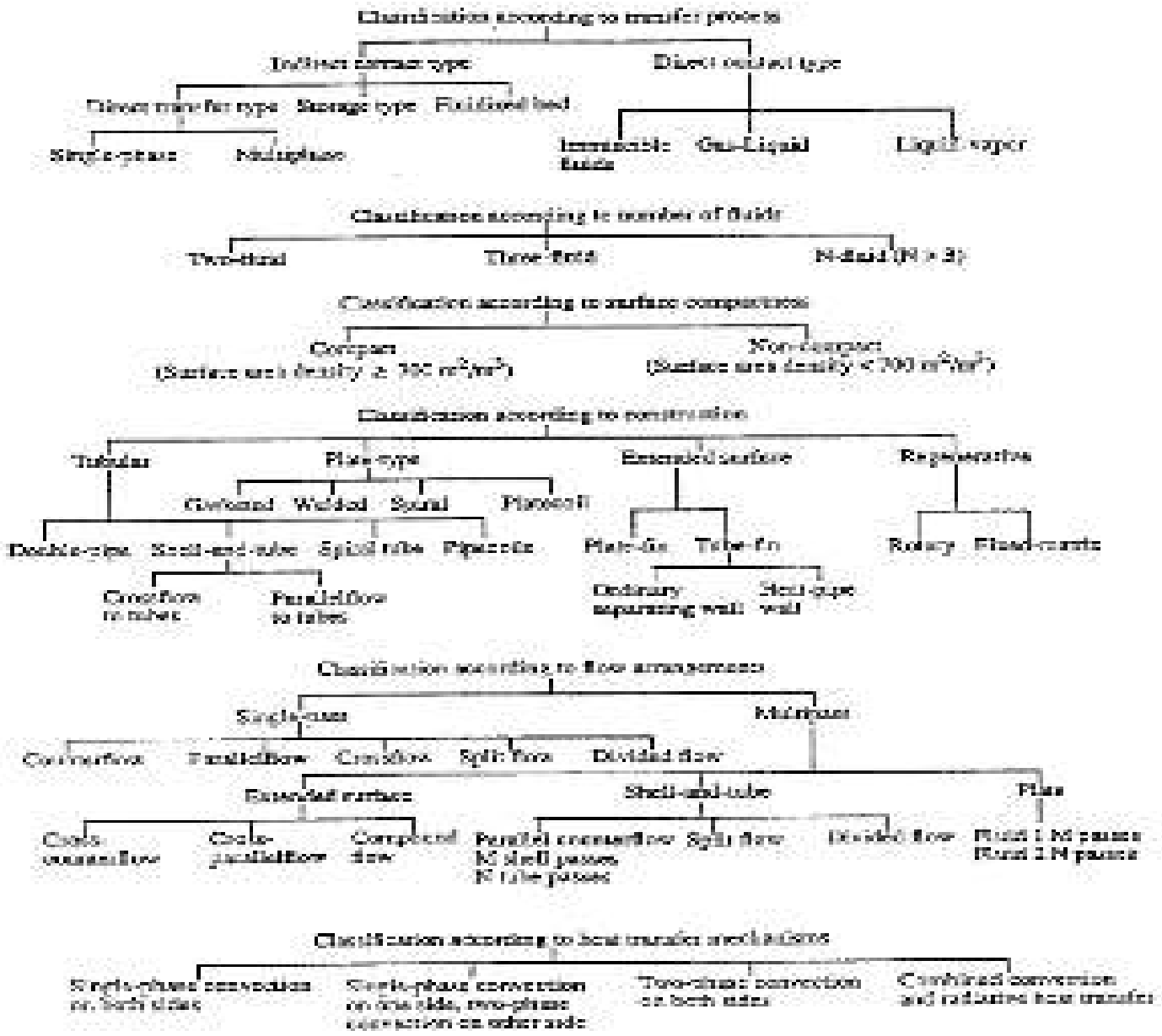


FIGURE 4.3.1 Classification of heat exchangers.

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Regenerative Heat Exchangers

- The regenerator represents class of heat exchangers in which heat is alternately stored and removed from a surface
- This heat transfer surface is usually referred to as the matrix of the regenerator.
- For continuous operation, the matrix must be moved into and out of the fixed hot and cold fluid streams. In this case, the regenerator is called a rotary regenerator.
- If, on the other hand, the hot and cold fluid streams are switched into and out of the matrix, the the regenerator is referred to as a fixed matrix regenerator.

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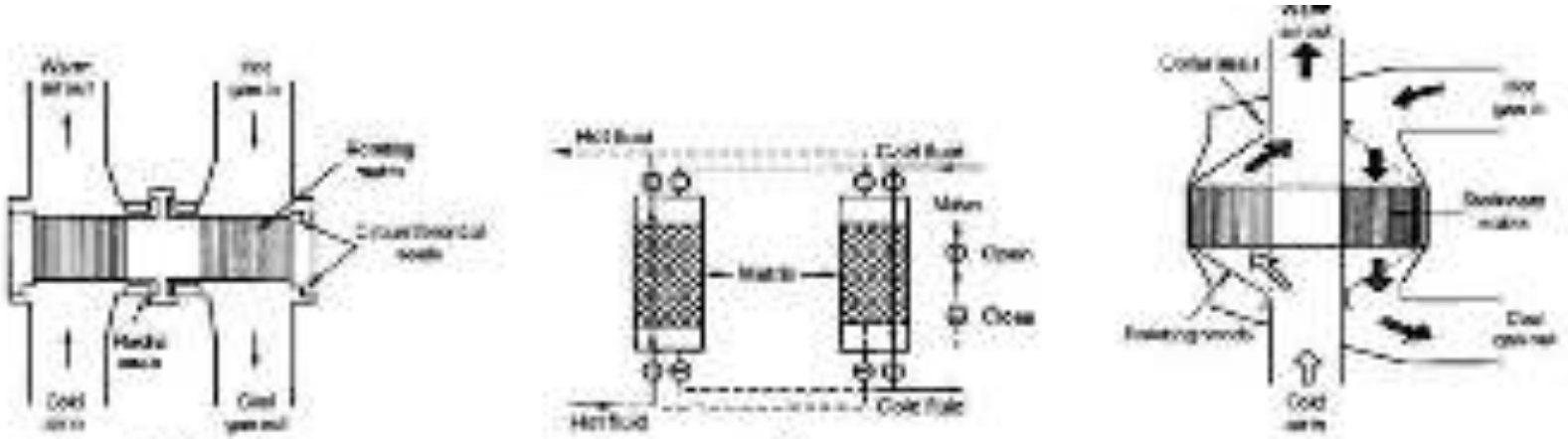


Figure 11. Regenerators: (a) rotary, (b) fixed-matrix, and (c) rotating heads.

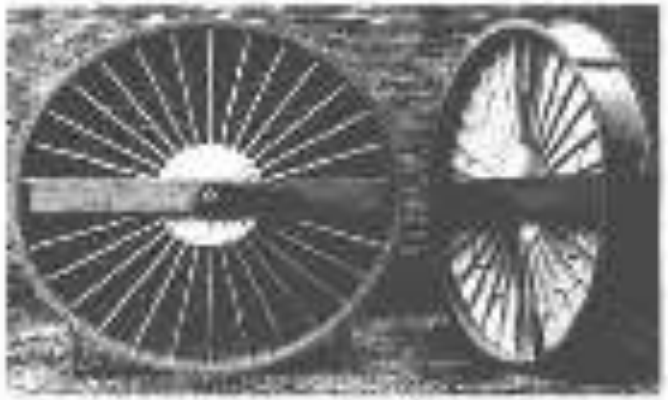
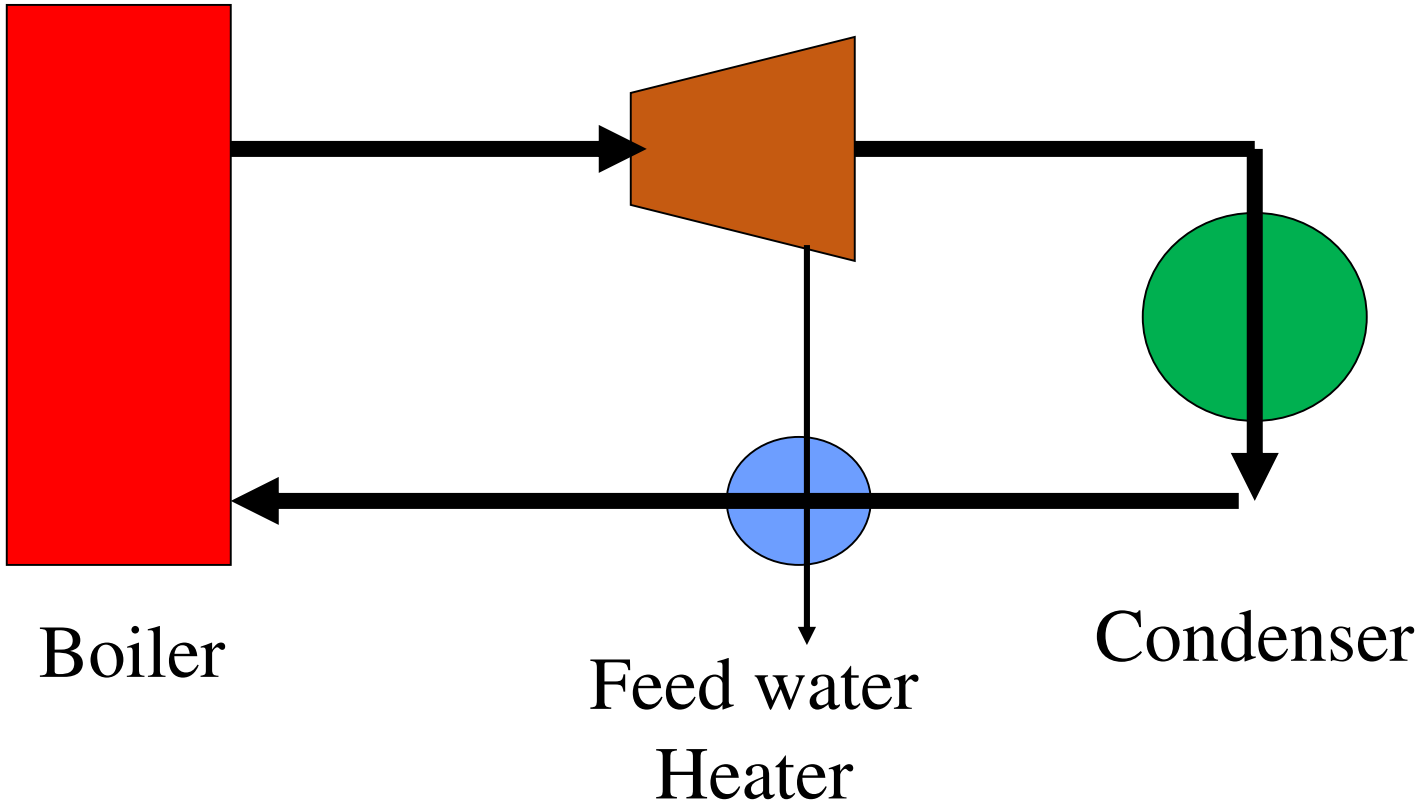


Figure 13. typical rotary regenerators or heat wheels.

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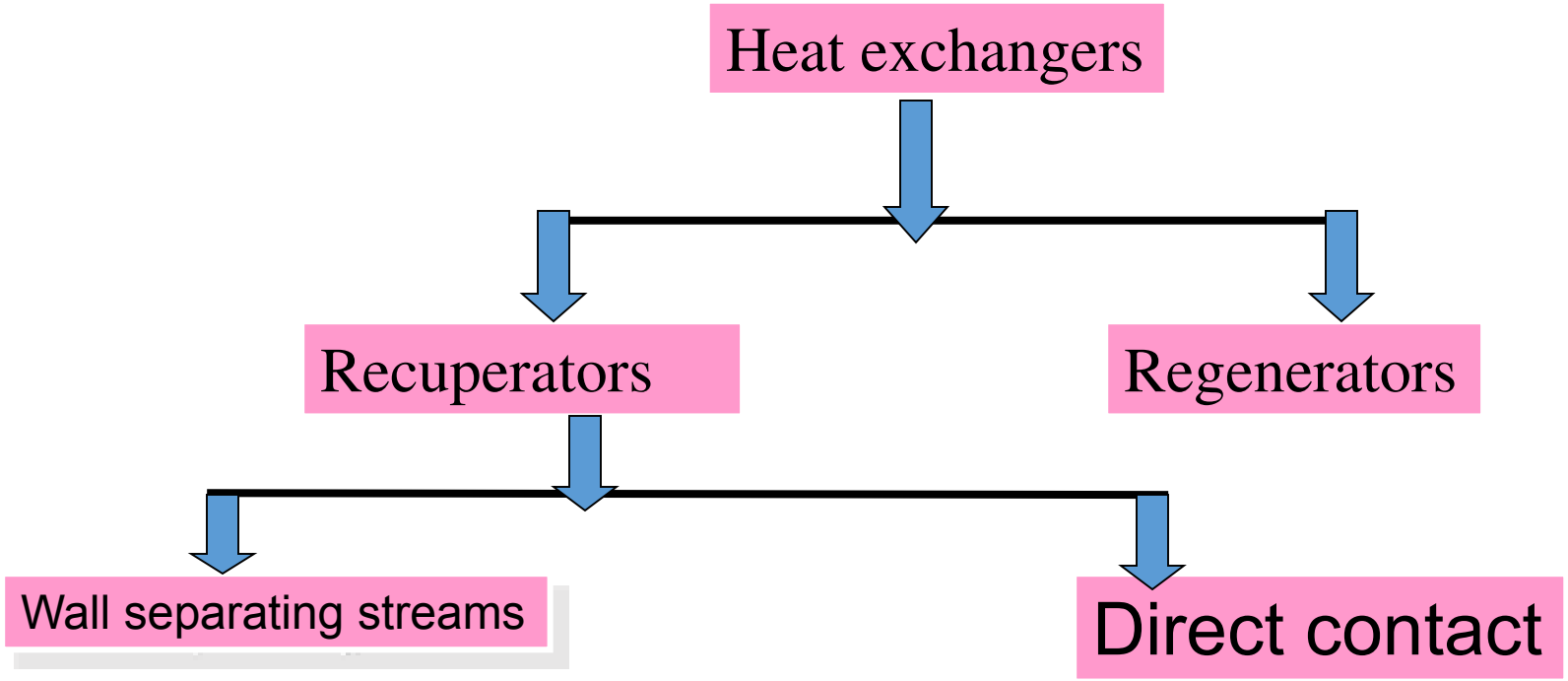
Application: Power cycle

Steam Turbine



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Main Categories Of Exchanger



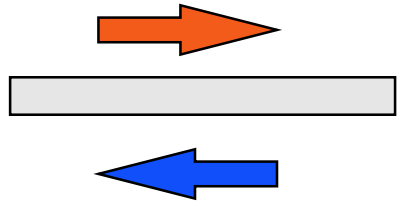
- Most heat exchangers have two streams, *hot* and *cold*, but some have more than two

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Recuperators/Regenerators

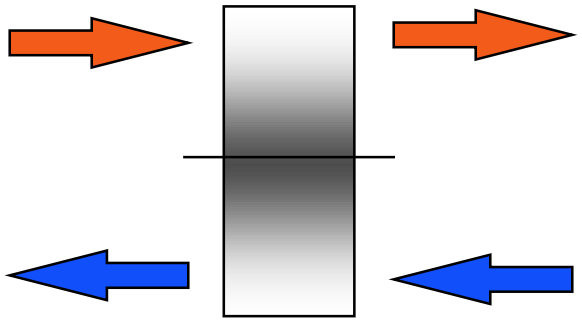
- **Recuperative:**

Has separate flow paths for each fluid which flow simultaneously through the exchanger transferring heat between the streams



- **Regenerative**

Has a single flow path which the hot and cold fluids alternately pass through.



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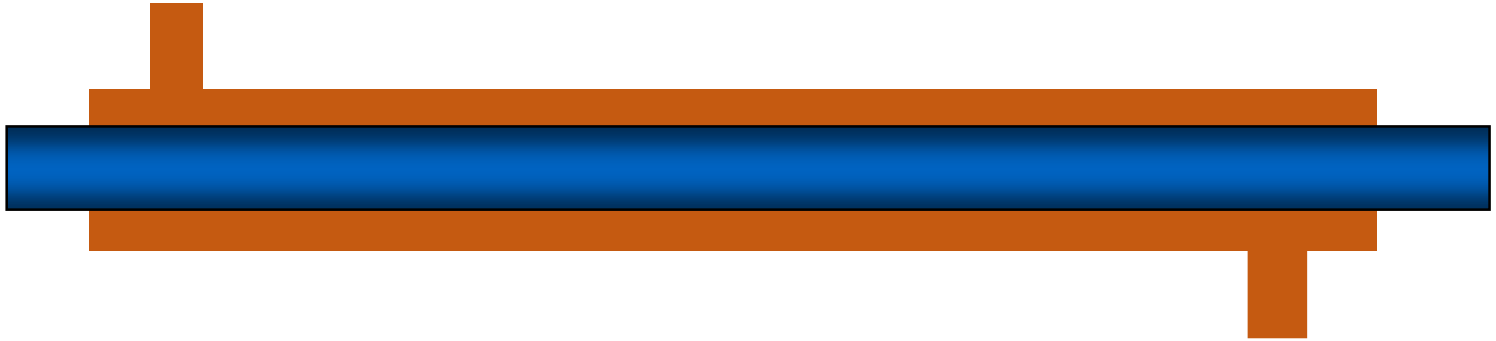
Compactness

- Can be measured by the heat-transfer area per unit volume or by channel size
- Conventional exchangers (shell and tube) have channel size of 10 to 30 mm giving about 100m^2/m^3
- Plate-type exchangers have typically 5mm channel size with more than 200m^2/m^3
- More compact types available

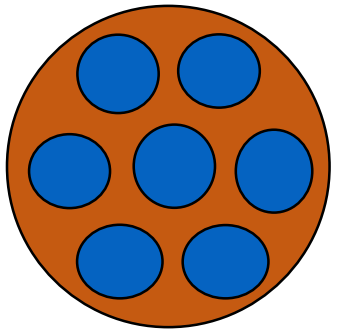
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Double Pipe

- Simplest type has one tube inside another - inner tube may have longitudinal fins on the outside



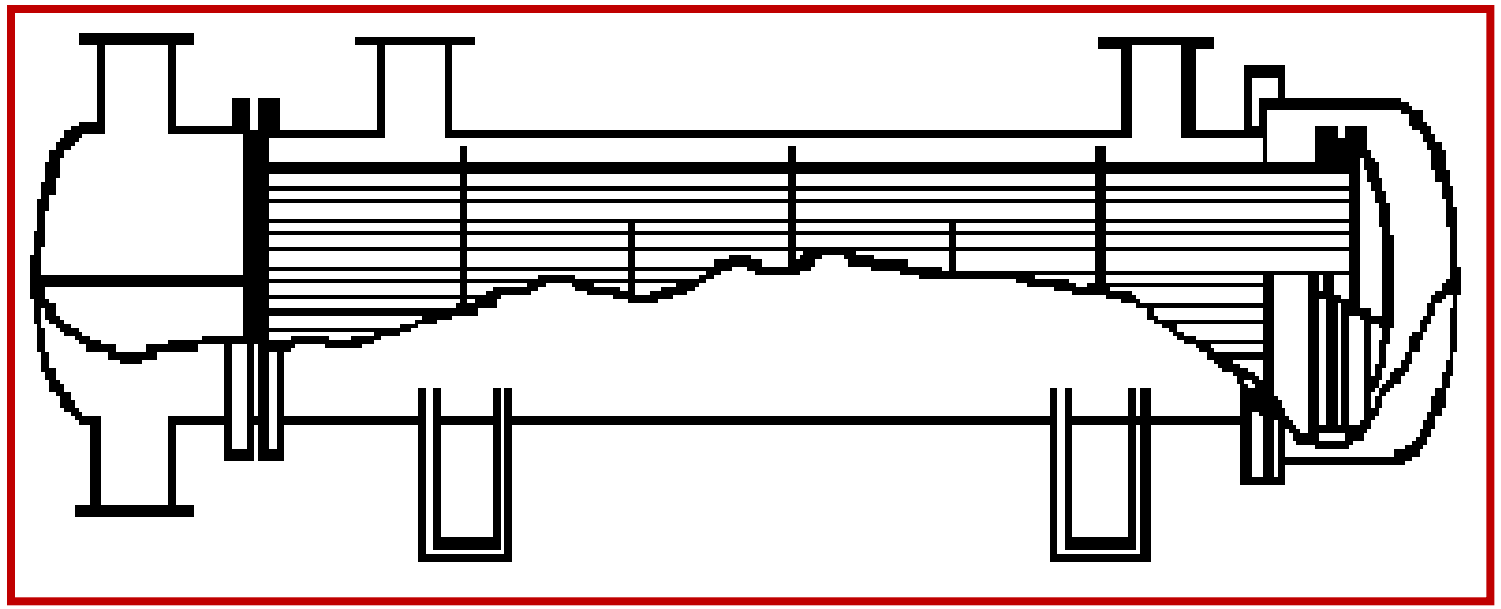
- However, most have a number of tubes in the outer tube - can have very many tubes thus becoming a shell-and-tube



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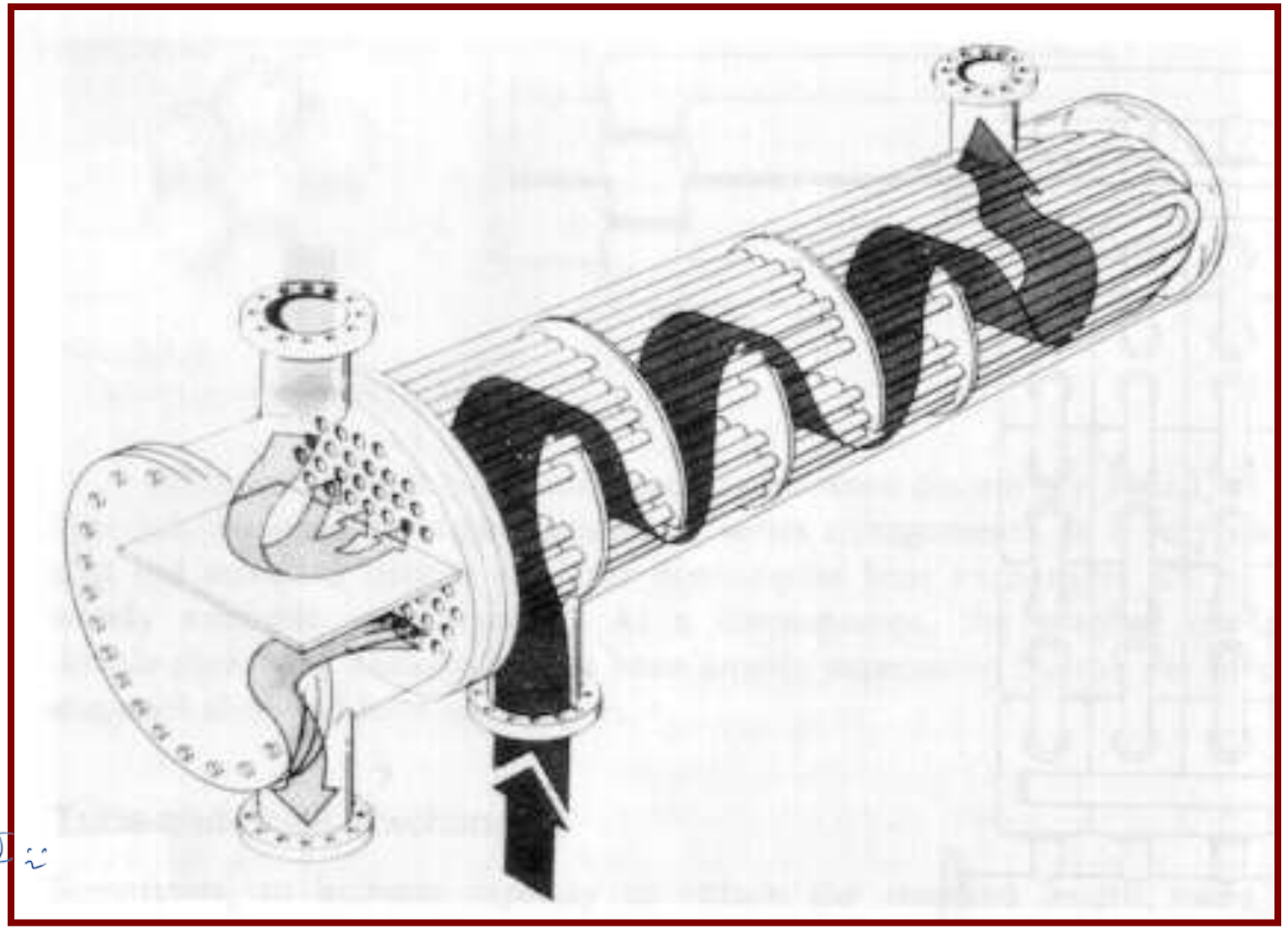
Shell and Tube

- Typical shell and tube exchanger as used in the process industry



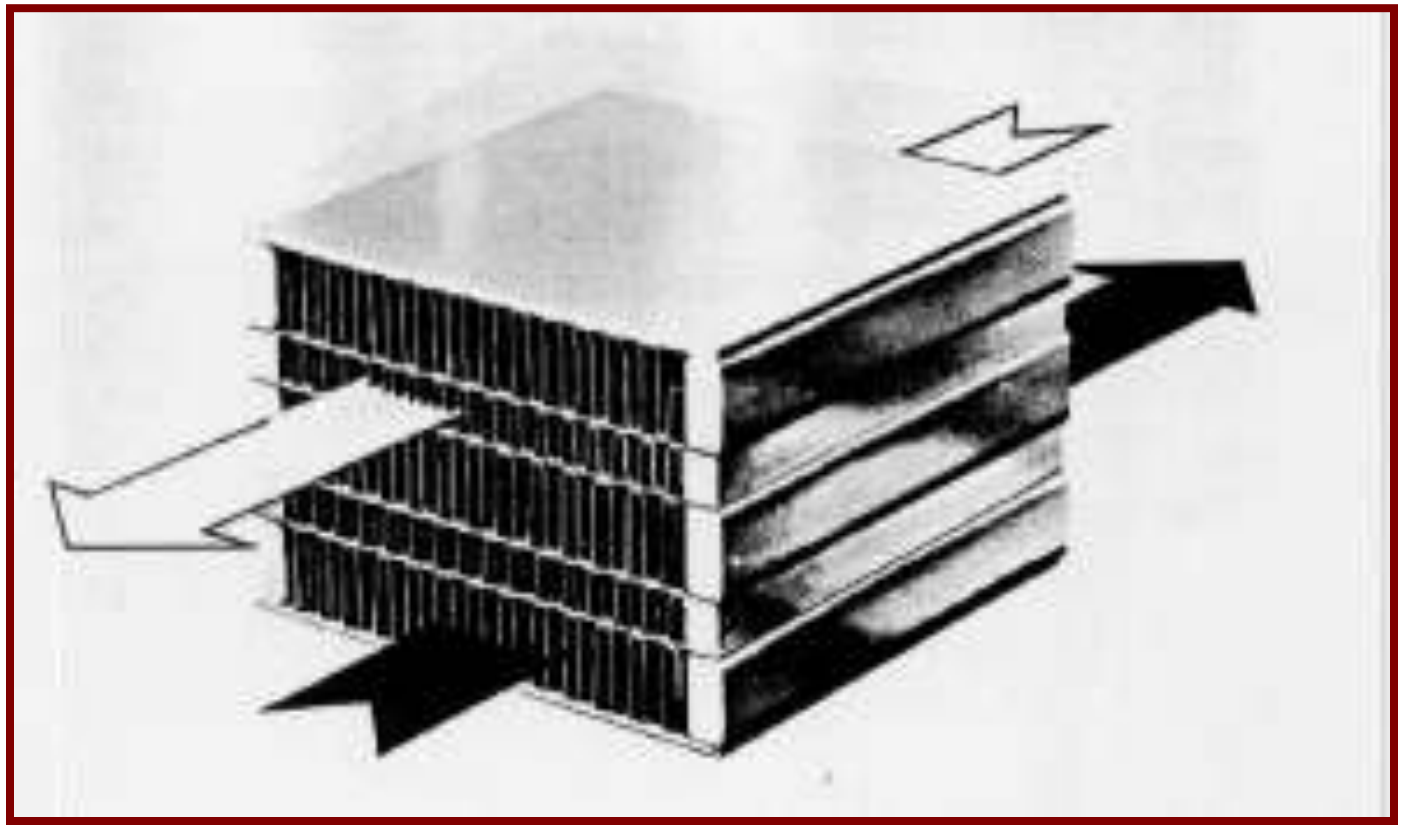
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Shell-Side Flow



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Plate-Fin Exchanger



- Made up of flat plates (parting sheets) and corrugated sheets which form fins
- Brazed by heating in vacuum furnace

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Heat Transfer of Heat Exchanger

- How is the heat transfer?
- Mechanism of Convection
- Applications .
- Mean fluid Velocity and Boundary and their effect on the rate of heat transfer.
- Fundamental equation of heat transfer
- Logarithmic-mean temperature difference.
- Heat transfer Coefficients.
- Heat flux and Nusselt correlation

lation program for Heat Exchanger

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How is the heat transfer?

- Heat can transfer between the surface of a solid conductor and the surrounding medium whenever temperature gradient exists.

  Conduction

  Convection

    Natural convection

    Forced Convection

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Natural and forced Convection

- Natural convection occurs whenever heat flows between a solid and fluid, or between fluid layers.
- As a result of heat exchange

  Change in density of effective fluid layers taken place, which causes upward flow of heated fluid.

If this motion is associated with heat transfer mechanism only, then it is called Natural Convection

Forced Convection

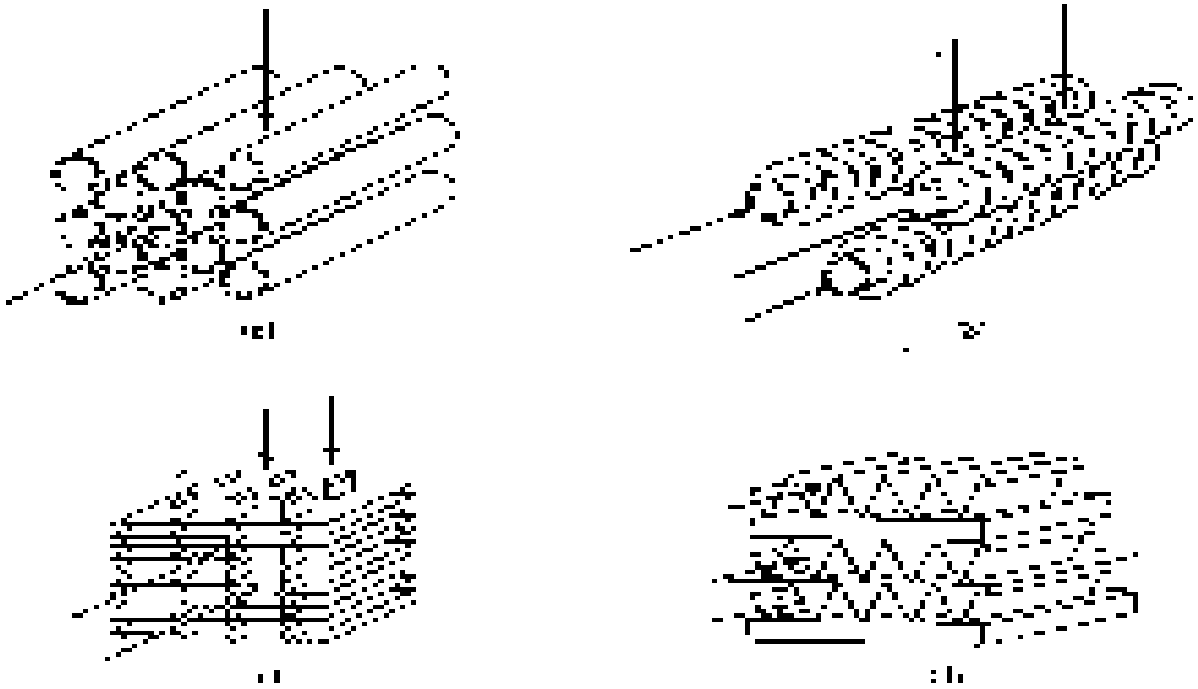
- If this motion is associated by mechanical means such as pumps, gravity or fans, the movement of the fluid is enforced.

n this case, we then speak of Forced convection.

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Heat Exchangers

- A device whose primary purpose is the transfer of energy between two fluids is named a Heat Exchanger.



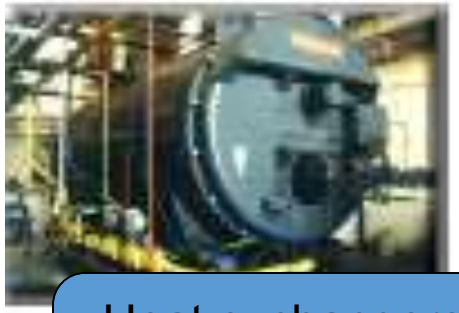


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Applications of Heat Exchangers



Heat Exchangers prevent car engine overheating and increase efficiency



Heat exchangers are used in Industry for heat transfer



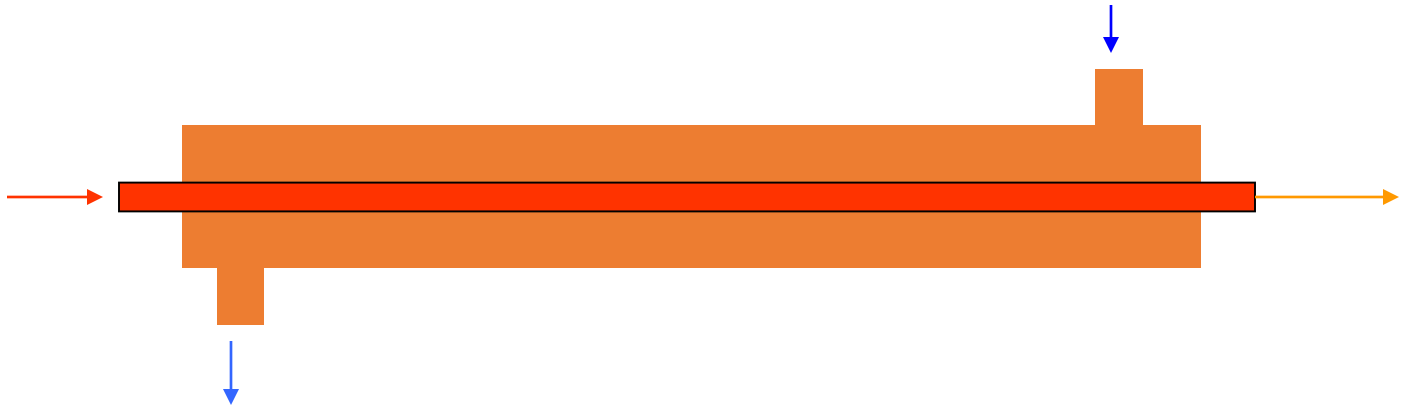
Heat exchangers are used in AC and furnaces





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- The closed-type exchanger is the most popular one.
  - One example of this type is the Double pipe exchanger.



- In this type, the hot and cold fluid streams do not come into direct contact with each other. They are separated by a tube wall or flat plate.

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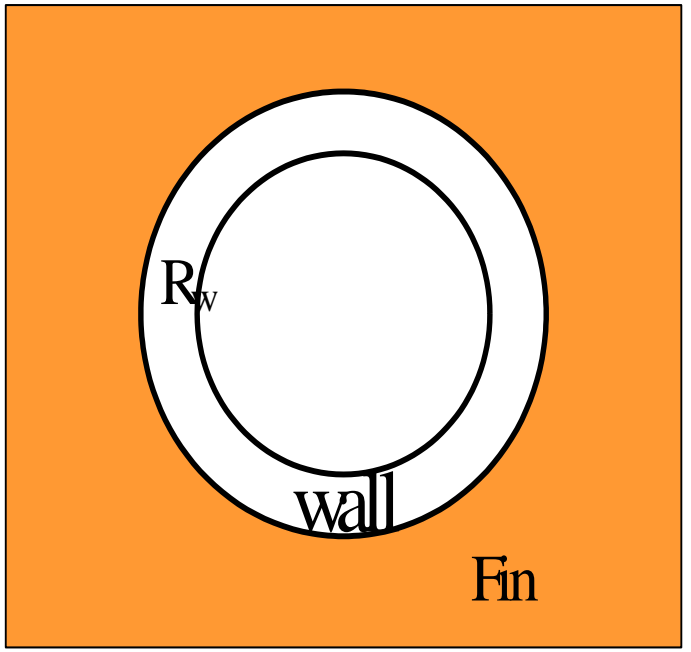
Heat Transfer Considerations:

Overall heat transfer coefficient

- Internal and external thermal resistances in series

$$\frac{1}{UA} = \frac{1}{(UA)_c} = \frac{1}{(UA)_h}$$

$$\frac{1}{UA} = \frac{1}{(h\eta_o A)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{1}{(h\eta_o A)_h} + \frac{R''_{f,h}}{(\eta_o A)_h}$$



- A is wall total surface area on hot or cold side
- R''_f is fouling factor (m^2K/W)
- η_o is overall surface efficiency (if finned)

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Heat Transfer Considerations (contd…):

❑Fouling factor

Material deposits on the surfaces of the heat exchanger tube may add further resistance to heat transfer in addition to those listed above. Such deposits are termed fouling and may significantly affect heat exchanger performance.

❑ Scaling is the most common form of fouling and is associated with inverse solubility salts. Examples of such salts are $CaCO_3$, $CaSO_4$, $Ca_3(PO_4)_2$, $CaSiO_3$, $Ca(OH)_2$, $Mg(OH)_2$, $MgSiO_3$, Na_2SO_4, $LiSO_4$, and Li_2CO_3.

❑ Corrosion fouling is classified as a chemical reaction which involves the heat exchanger tubes. Many metals, copper and aluminum being specific examples, form adherent oxide coatings which serve to passivity the surface and prevent further corrosion.

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Heat Transfer Considerations (contd…):

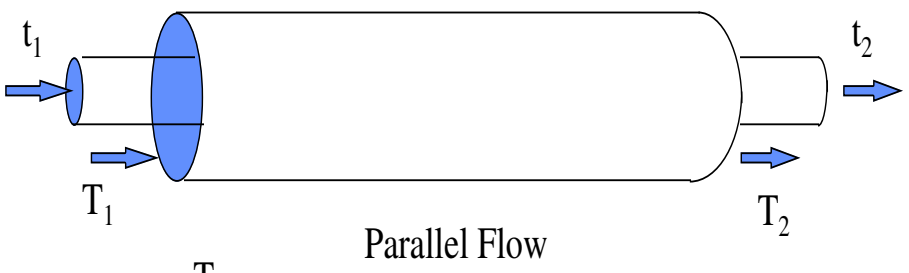
❑ Chemical reaction fouling involves chemical reactions in the process stream which results in deposition of material on the heat exchanger tubes. When food products are involved this may be termed scorching but a wide range of organic materials are subject to similar problems.

❑ Freezing fouling is said to occur when a portion of the hot stream is cooled to near the freezing point for one of its components. This is most notable in refineries where paraffin frequently solidifies from petroleum products at various stages in the refining process, obstructing both flow and heat transfer.

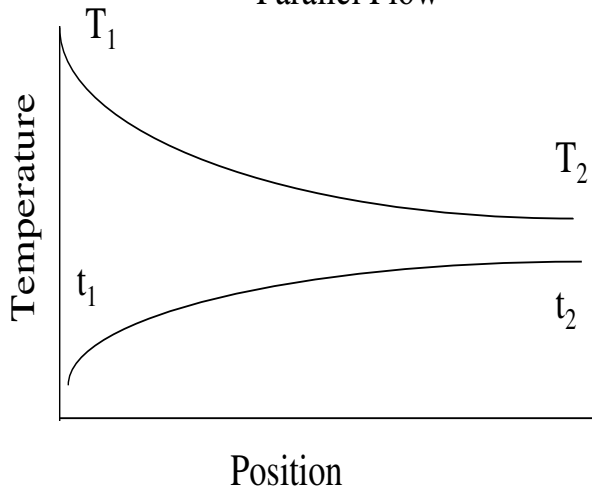
❑ Biological fouling is common where untreated water is used as a coolant stream. Problems range from algae or other microbes to barnacles.

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Basic flow arrangement in tube in tube flow

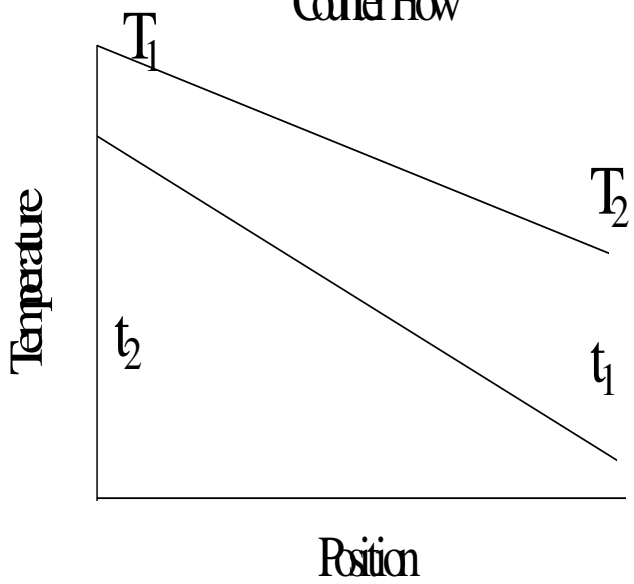


Parallel Flow





Counter Flow



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Heat Exchanger Analysis

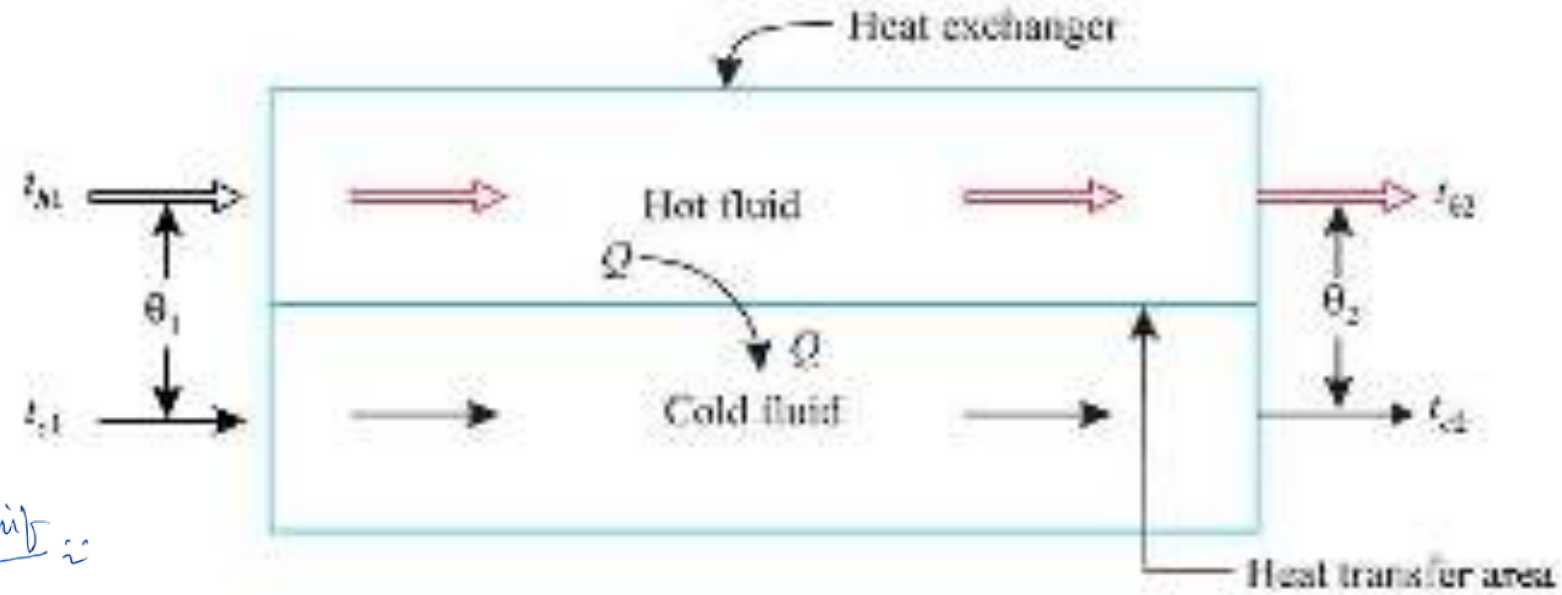
For designing or predicting the performance of a heat exchanger it is necessary that the total heat transfer may be related with its governing parameters : (*i*) U (overall heat transfer coefficient) due to various modes of heat transfer, (*ii*) A total surface area of the heat transfer, and (*iii*) t_1, t_2 (the inlet and outlet fluid temperatures). Fig. 10.8 shows the overall energy balance in a heat exchanger.

Let, $\dot{m}$ = Mass flow rate, kg/s,

c_p = Specific heat of fluid at constant pressure, J/kg°C,

t = Temperature of fluid, °C, and

Δt = Temperature drop or rise of a fluid across the heat exchanger.



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Heat Exchanger Analysis

Subscripts *h* and *c* refer to the *hot* and *cold* fluids respectively; subscripts 1 and 2 correspond to the *inlet* and *outlet* conditions respectively.

Assuming that there is no heat loss to the surroundings and potential and kinetic energy changes are negligible, from the energy balance in a heat exchanger, we have :

Heat given up by the hot fluid, $Q = m_h c_{ph} (t_{h1} - t_{h2})$

Heat picked up by the cold fluid, $Q = m_c c_{pc} (t_{c2} - t_{c1})$

Total heat transfer rate in the heat exchanger, $Q = U A \theta_m$

where, U = Overall heat transfer coefficient between the two fluids,

A = Effective heat transfer area, and

θ_m = *Appropriate mean value of temperature difference or logarithmic mean temperature difference (LMTD).*

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Log mean temperature difference (LMTD) method

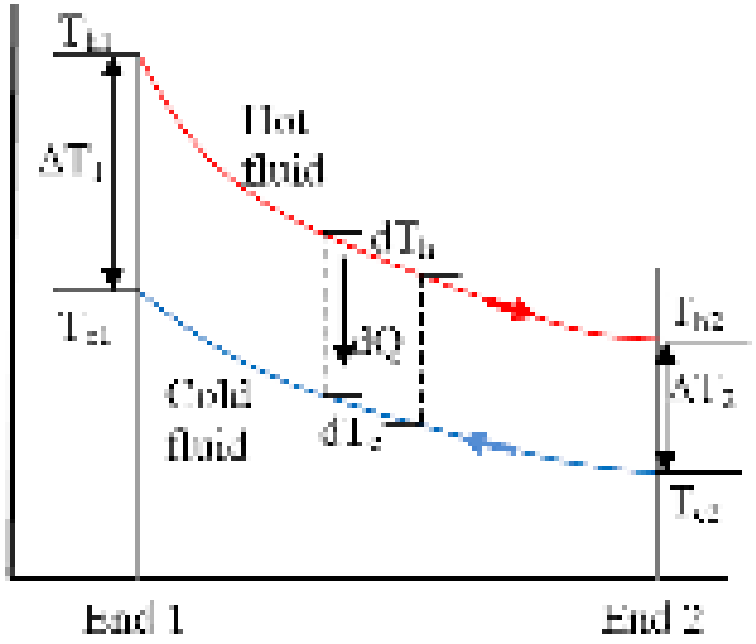
Logarithmic mean temperature difference (LMTD) *is defined as* that *temperature difference which, if constant, would give the same rate of heat transfer as actually occurs under variable conditions of temperature difference.*

In order to derive expression for *LMTD* for various types of heat exchangers, the following **assumptions** are made :

1. The overall heat transfer coefficient U is constant.
2. The flow conditions are steady.
3. The specific heats and mass flow rates of both fluids are constant.
4. There is no loss of heat to the surroundings, due to the heat exchanger being perfectly insulated.
5. There is no change of phase either of the fluid during the heat transfer.
6. The changes in potential and kinetic energies are negligible.
7. Axial conduction along the tubes of the heat exchanger is negligible.

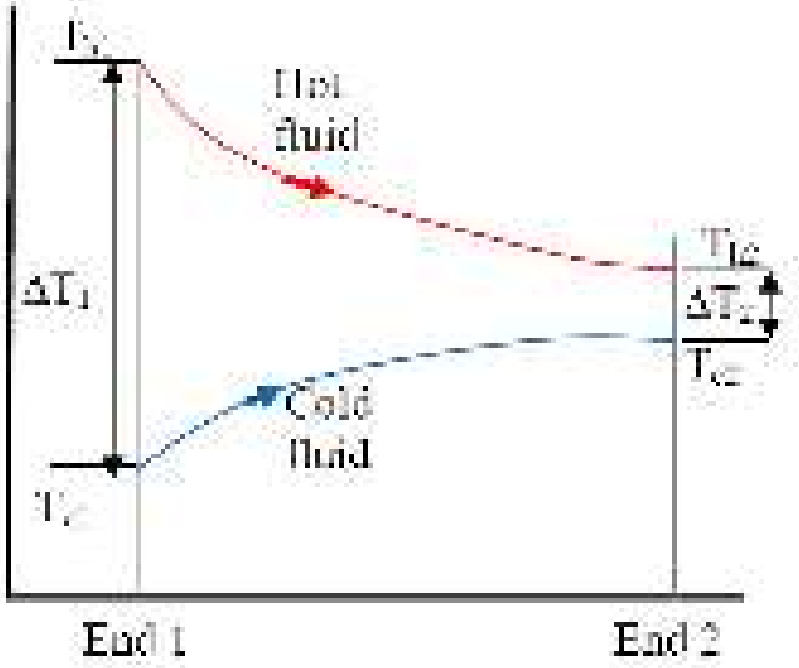
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Log mean temperature difference (LMTD) method



Counterflow

Note $T_{h,out}$ can be $< T_{c,out}$



Parallel flow

Ts cannot cross

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Log mean temperature difference (LMTD) method

Energy balance (**counter flow**) on element shown

$$d\dot{Q} = -\dot{m}_h c_h dT_h = -\dot{m}_c c_c dT_c \qquad (1)$$

$\dot{m}$ = mass flow rate of fluid

c = specific heat

Rate Equation

$$d\dot{Q} = U dA (T_h - T_c) \qquad (2)$$

Now from (1) $\quad dT_h = \dfrac{-d\dot{Q}}{\dot{m}_h c_h} \qquad dT_c = \dfrac{-d\dot{Q}}{\dot{m}_c c_c}$

$$\therefore d(T_h - T_c) = d\dot{Q}\left(\frac{1}{\dot{m}_c c_c} - \frac{1}{\dot{m}_h c_h}\right)$$

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Log mean temperature difference (LMTD) method

Subtract $d\dot{Q}$ from (2),

$$\frac{d(T_h - T_c)}{T_h - T_c} = U\left(\frac{1}{\dot{m}_c c_c} - \frac{1}{\dot{m}_h c_h}\right) dA$$

Integrate 1 → 2

$$\ln\left(\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}\right) = UA\left(\frac{1}{\dot{m}_c c_c} - \frac{1}{\dot{m}_h c_h}\right)$$

Total heat transfer rate

$$\dot{Q} = \dot{m}_h c_h (T_{h1} - T_{h2}) \text{ and } \dot{Q} = \dot{m}_c c_c (T_{c1} - T_{c2})$$

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Log mean temperature difference (LMTD) method

Substitute for ΔT_e and ΔT_1

$$\Delta T_1 = T_{h1} - T_{c1} \qquad END1$$

$$\Delta T_2 = T_{h2} - T_{c2} \qquad END2$$

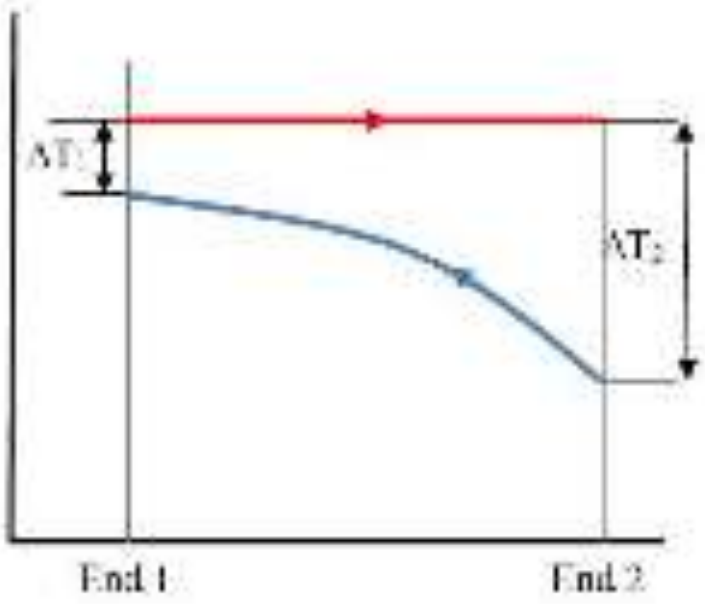
$$\dot{Q} = UA\left[\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}\right]$$

$$\dot{Q} = UA(LMTD)$$

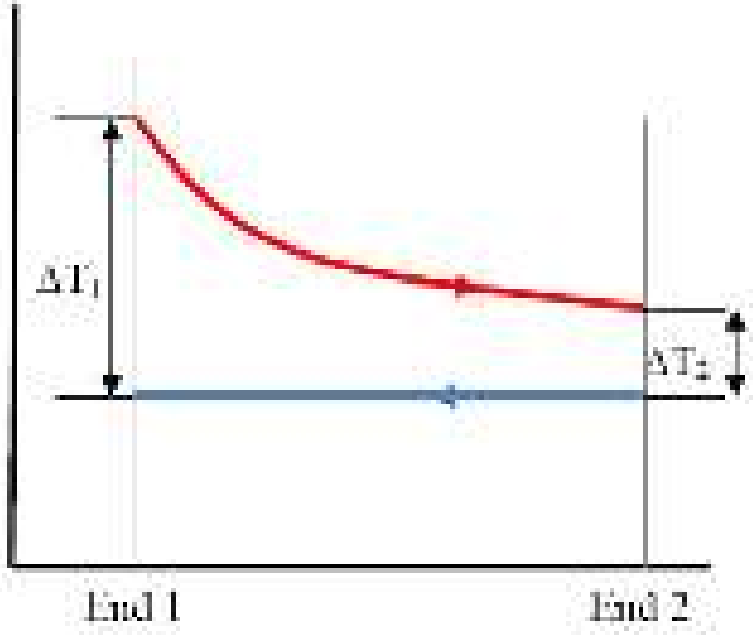
LMTD is Log Mean Temperature Difference

- Remember – 1 and 2 are ends, not fluids
- Same formula for parallel flow (but ΔT's are different)
- Counterflow has highest LMTD, for given T's therefore smallest area for Q.

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Condenser



Evaporator

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1. A '*heat exchanger*' may be defined as an equipment which transfers the energy from a hot fluid to a cold fluid, with maximum rate and minimum investment and running costs.

2. Logarithmic mean temperature difference (LMTD) is defined as "*that temperature difference which, if constant, would give the same rate of heat transfer as actually occurs under variable conditions of temperature difference*".

   LMTD (θ_m) for parallel flow as well as counter-flow is given by :

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln (\theta_1/\theta_2)}$$

   θ_m for a *counter-flow unit is always greater than that for parallel flow unit*, hence counter-flow heat exchanger can transfer more heat than parallel flow one; in other words a counter-flow heat exchanger needs a *smaller heating surface* for the same rate of heat transfer. For this reason, the counter-flow arrangement is *usually used*.

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3. *Fouling or Scaling* : The phenomenon of rust formation and deposition of fluid impurities in the tubes of a heat exchanger, during its normal operation, is called **fouling.**

The reciprocal of scale heat transfer coefficient, h_s is called the *fouling factor*, R_f.

$$R_f = \frac{1}{h_s} \text{ m}^2\text{°C/W} \qquad ...(i)$$

$$R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \qquad ...(ii)$$

The heat transfer, considering the thermal resistance due to scale formation, is given by

$$Q = \frac{(t_i - t_o)}{\frac{1}{A_i h_i} + \frac{1}{A_i h_{si}} + \frac{1}{2\pi L k} \ln (r_o/r_i) + \frac{1}{A_o h_{so}} + \frac{1}{A_o h_o}} \qquad ...(iii)$$

The overall heat transfer coefficients, U based on the inner and outer surfaces of the inner tube are given by

$$U_i = \frac{1}{\frac{1}{h_i} + R_{fi} + \frac{r_i}{k} \ln (r_o/r_i) + (r_i/r_o) R_{fo} + (r_i/r_o) \frac{1}{h_o}} \qquad ...(iv)$$

$$U_o = \frac{1}{(r_o/r_i) \frac{1}{h_i} (r_o/r_i) R_{fi} + \frac{r_o}{k} \ln (r_o/r_i) + R_{fo} + \frac{1}{h_o}} \qquad ...(v)$$

$$U_o = \frac{1}{(r_o/r_i) \frac{1}{h_i} + \frac{r_o}{k} \ln (r_o/r_i) + \frac{1}{h_o}} \quad \text{... neglecting fouling factor} \qquad ...(vi)$$

$$U_o = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}}$$

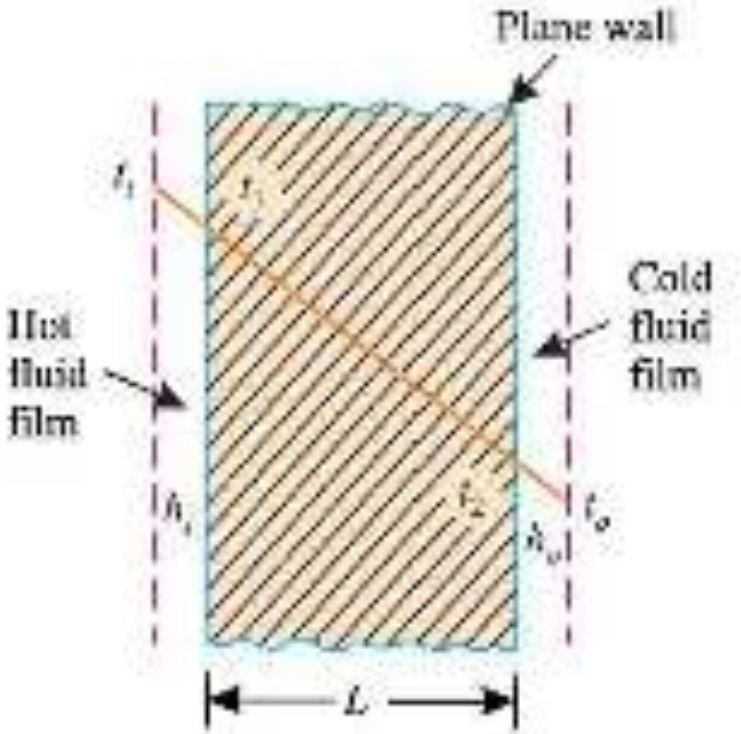
...When the tube is thin walled and thermal resistance due to tube wall thickness and scale formed are neglected.

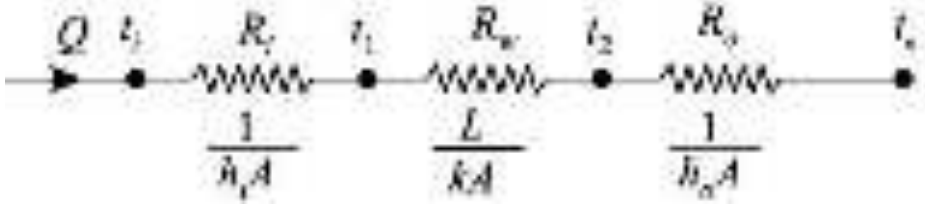
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OVERALL HEAT TRANSFER COEFFICIENT

In a heat exchanger in which two fluids are separated by a **plane wall** as shown in the Fig. the overall heat transfer coefficient is given by

$$U = \frac{1}{\frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_o}}$$





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Effectiveness-NTU Method

How will existing H.Ex. perform for given inlet conditions?

Define effectiveness: $\varepsilon = \dfrac{\dot{Q}_{actual}}{\dot{Q}_{\max}}$

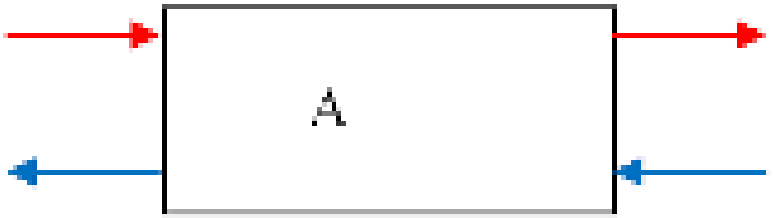
where $\dot{Q}_{\max}$ is for an infinitely long H.Ex.

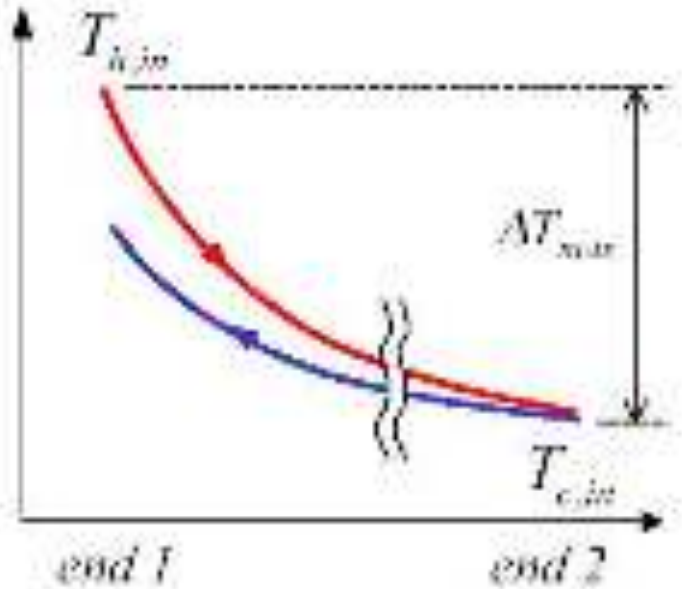
One fluid $\Delta T \rightarrow \Delta T_{\max} = T_{h,in} - T_{c,in}$

and since $\dot{Q} = (\dot{m}c_A)\Delta T_A = (\dot{m}c_B)\Delta T_B$

$$= C_A \Delta T_A = C_B \Delta T_B$$

then only the fluid with less of C_A, C_B heat capacity rate can have $\Delta T_{\max}$





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Effectiveness-NTU Method(contd…)

$$\text{i.e. } \dot{Q}_{max} = C_{min}\Delta T_{max} \text{ and } \varepsilon = \frac{\dot{Q}}{C_{min}\left(T_{h.in} - T_{c.in}\right)}$$

$$\text{or, } \dot{Q} = \varepsilon C_{min}\left(T_{h.in} - T_{c.in}\right)$$

Want expression for ε which does not contain outlet T's

Substitute back into $\dot{Q} = UA(LMTD)$.....

$$\varepsilon = \frac{1 - \exp\left[\frac{-UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right]}{1 - \frac{C_{min}}{C_{max}}\exp\left[\frac{-UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right]}$$

$$\therefore \varepsilon = \varepsilon\left(NTU, \frac{C_{min}}{C_{max}}\right)$$

and No. of transfer units (size of HEx.) $NTU = \frac{UA}{C_{min}}$

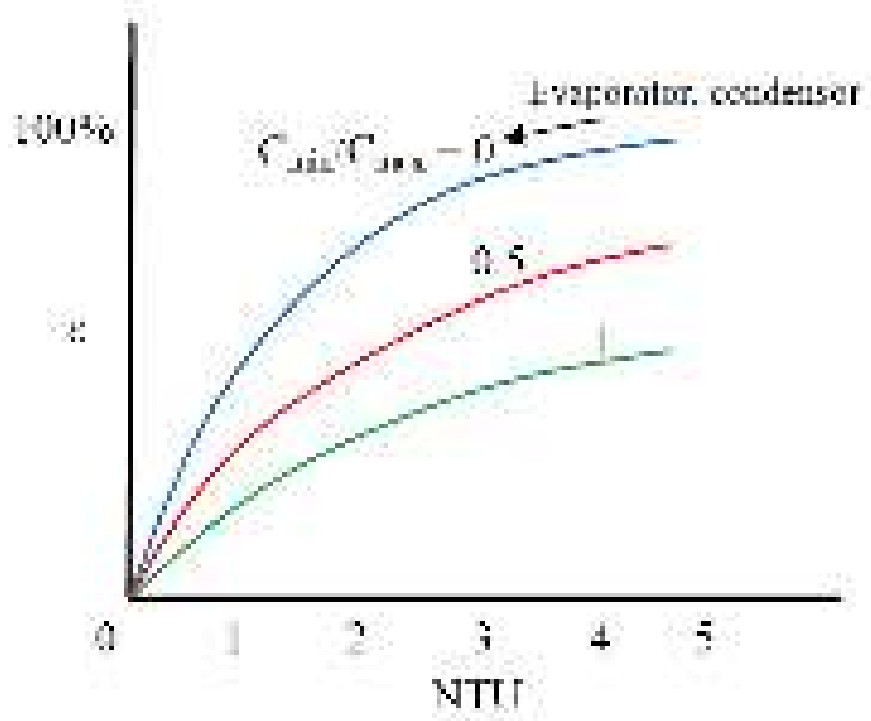
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Charts for each Configuration

Procedure:

Determine C_{max}, C_{min}/C_{max}

Get $UA/C_{min,} \rightarrow$ ε from chart



$$\dot{Q} = \varepsilon C_{\min}\left(T_{h.in} - T_{c.in}\right)$$

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Effectiveness-NTU Method(contd…)

$$NTU_{max} = \frac{UA}{C_{min}} \quad \Rightarrow \quad A = \frac{NTU_{max} C_{min}}{U}$$

- NTU_{max} can be obtained from figures in textbooks/handbooks

  First, however, we must determine which fluid has C_{min}



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The heat exchanger effectiveness (ε) is defined as the ratio of actual heat transfer to the maximum possible heat transfer. Thus

$$\varepsilon = \frac{\text{Actual heat transfer } (Q)}{\text{Maximum possible heat transfer } (Q_{max})} \quad \ldots(i)$$

$$(\varepsilon)_{parallel\ flow} = \frac{1 - \exp[-NTU\ (1 + R)]}{1 + R} \quad \ldots(ii)$$

$$(\varepsilon)_{counter-flow} = \frac{1 - \exp[-NTU\ (1 - R)]}{1 - R \exp[-NTU\ (1 - R)]}$$

where, R = (capacity ratio) = $\frac{C_{min}}{C_{max}}$, and

NTU = Number of transfer units.

NTU is a measure of effectiveness of the heat exchanger.

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Thank you !