INFORMATION CODING TECHNIQUES

Name of Subject Teacher : Prof.Sadaf Zama

SEMESTER : VI Sem

UNIT : I

Course Title : : Information Coding Technique
Semester : IV
Course Code :
Teaching Mode : Offline
TAE :10
CAE :15
TOTAL : 50Marks
Duration of ESE : 3 Hrs.

Course Objectives

1.**To introduce the fundamental principles of information theory** by explaining the concepts of information, entropy, mutual information, and channel capacity, and to help students understand how information is measured and transmitted efficiently.

2. **To develop knowledge of coding theory** by studying different types of codes, including Huffman codes and convolutional codes, and enabling students to apply these codes through numerical problem-solving

3. **To provide a strong understanding of cyclic codes** by explaining their generation, algebraic structure, and matrix representation, and to train students in solving problems related to cyclic encoding and decoding

4. **To familiarize students with error detection and correction techniques** and enable them to analyze, design, and evaluate basic error control mechanisms through appropriate numerical examples

5. **To impart knowledge of linear block codes** by teaching systematic block code encoding, syndrome computation, and decoding methods, along with developing proficiency in solving related numerical problems.

Course Outcomes

CO1: Interpret and apply the fundamental concepts of information theory, including information measure, entropy, mutual information, and channel capacity

CO2: Analyze and construct different coding techniques, such as Huffman coding and convolutional coding, and solve numerical problems related to source and channel coding

CO3: Explain the structure and properties of cyclic codes and perform encoding and decoding using generator polynomials and matrix representations

CO4: Apply error detection and correction methods to identify, detect, and correct errors in digital communication systems through appropriate numerical techniques

CO5: Design and evaluate linear block codes using systematic encoding, syndrome decoding, and solve related analytical and numerical problems.

CONTENTS

Unit-I:Information Theory

- Definition of Information,
- Measure of Information,
- Rate of Information,
- Shannon-Hartley Theorem,
- Entropy,
- Relationship between Entropy and Mutual Information.

QUESTION BANK:

1.Explain information and measure of information in detail. Derive the formula for information content.
2.Define entropy. Derive the entropy equation for a discrete memoryless source and explain its properties.
3.Explain the Shannon–Hartley theorem in detail. Derive the channel capacity expression and discuss its applications.
4.Explain rate of information and entropy. Show how they are related.
5.Define mutual information. Derive the relationship between entropy, conditional entropy, and mutual information.
6.With suitable examples, explain the relationship between entropy and mutual information.
7.A source emits symbols with probabilities {0.5,0.25,0.25}\{0.5, 0.25, 0.25\}{0.5,0.25,0.25}.
•Calculate the entropy of the source.
8.Calculate the information content of an event whose probability is 1/8.
9.A communication channel has bandwidth 3 kHz and signal-to-noise ratio 30 dB.
•Calculate the channel capacity using Shannon–Hartley theorem.
10.Given a binary source with equal probabilities, find:
a) Entropy
b) Rate of information

1.Explain information and measure of information in detail. Derive the formula for information content.

Information and Measure of Information

1. Information

- In Information Theory, information means reduction of uncertainty.
- When we receive a message that reduces our doubt, we gain information.

Example

- If a coin is tossed and we already know it will be “Head” → very little information
- If the result is unknown and we suddenly know it → more information

So,

More uncertainty → more information

Less uncertainty → less information

2. Measure of Information

- The measure of information tells us how much information is contained in a message or event.
- It depends on the probability of occurrence of the event:
- Rare events give more information
- Common events give less information

Example:

Getting “sunrise tomorrow” → very common → low information

Winning a lottery → very rare → high information

Derivation of Information Content Formula

- Let an event E occur with probability p(E).
- The information content I(E) should satisfy the following conditions:

Condition 1:

- If probability decreases, information should increase Information is inversely proportional to probability.

$$I(E) \propto \frac{1}{p(E)}$$

Condition 2:

- For independent events, total information should be additive.
- If two independent events $E_1$and E_2occur:

$$I(E_1, E_2) = I(E_1) + I(E_2)$$

- Probability of both events occurring together:

$$p(E_1, E_2) = p(E_1) \times p(E_2)$$

- To satisfy both conditions, the logarithmic function is used.

Final Formula (Information Content)

$$\boxed{I(E) = -\log_2 p(E)}$$

Where:

$I(E)$= information content (in bits)

$p(E)$= probability of event E

Base 2 is used because information is measured in binary digits (bits)

Example Calculation

If probability of an event is:

$$p(E) = \frac{1}{8}$$

$$I(E) = -\log_2\left(\frac{1}{8}\right)$$

$$I(E) = \log_2(8) = 3\ \textit{bits}$$

- Information is measured in bits
- Smaller probability → higher information
- Information theory does not consider meaning, only uncertainty
- Information is the reduction of uncertainty
- Measure of information quantifies how much information is obtained
- The information content of an event is given by:

$$\boxed{I = -\log_2(p)}$$

2.Define entropy. Derive the entropy equation for a discrete memoryless source and explain its properties.

Entropy

- Entropy is the average amount of information produced by a source.
- It measures the uncertainty of a random source.
- Entropy tells us how unpredictable the source is.

More uncertainty → higher entropy

Less uncertainty → lower entropy

Discrete Memoryless Source (DMS)

- A discrete memoryless source is a source where:
- It produces discrete symbols
- Each symbol is independent of previous symbols
- Probability of each symbol remains constant
- Example: Tossing a fair coin repeatedly.

Derivation of Entropy Equation

Let a discrete memoryless source produce symbols:

$$S = \{s_1, s_2, s_3, \ldots, s_n\}$$

with probabilities:

$$P = \{p_1, p_2, p_3, \ldots, p_n\}$$

Step 1: Information Content of a Symbol

Information content of symbol s_i is:

$$I(s_i) = -\log_2(p_i)$$

Step 2: Average Information (Entropy)

Entropy is the expected value (average) of information content.

$$H(S) = \sum_{i=1}^{n} p_i \cdot I(s_i)$$

Substitute $I(s_i)$:

$$H(S) = \sum_{i=1}^{n} p_i \left(-\log_2 p_i\right)$$

Final Entropy Equation

$$\boxed{H(S) = -\sum_{i=1}^{n} p_i \log_2 p_i}$$

Where:

$H(S)$= entropy (in bits per symbol)

p_i= probability of symbol s_i

Properties of Entropy

1. Entropy is Always Non-Negative

$$H(S) \geq 0$$

•Entropy is never negative

•If only one symbol occurs with probability 1 → entropy = 0

2. Maximum Entropy for Uniform Distribution

Entropy is maximum when all symbols are **equally likely**.

If there are $n$symbols:

$$H_{\max} = \log_2 n$$

3. Minimum Entropy

Entropy is minimum (zero) when the source is fully predictable.

Example:

$p = 1$for one symbol

$p = 0$for others

4. Entropy Measures Uncertainty

High entropy → more randomness

Low entropy → more predictability

5. Additivity Property

For independent sources $X$and Y:

$$H(X,Y) = H(X) + H(Y)$$

6. Entropy Depends Only on Probabilities

Meaning of symbols does not matter

Only probability values matter

Example

For a fair coin:

$$p(H) = p(T) = 0.5$$
$$H = -\left[0.5 \log_2 0.5 + 0.5 \log_2 0.5\right]$$
$$H = 1 \ bit$$

Entropy is the average information of a source

It measures uncertainty

For a discrete memoryless source:

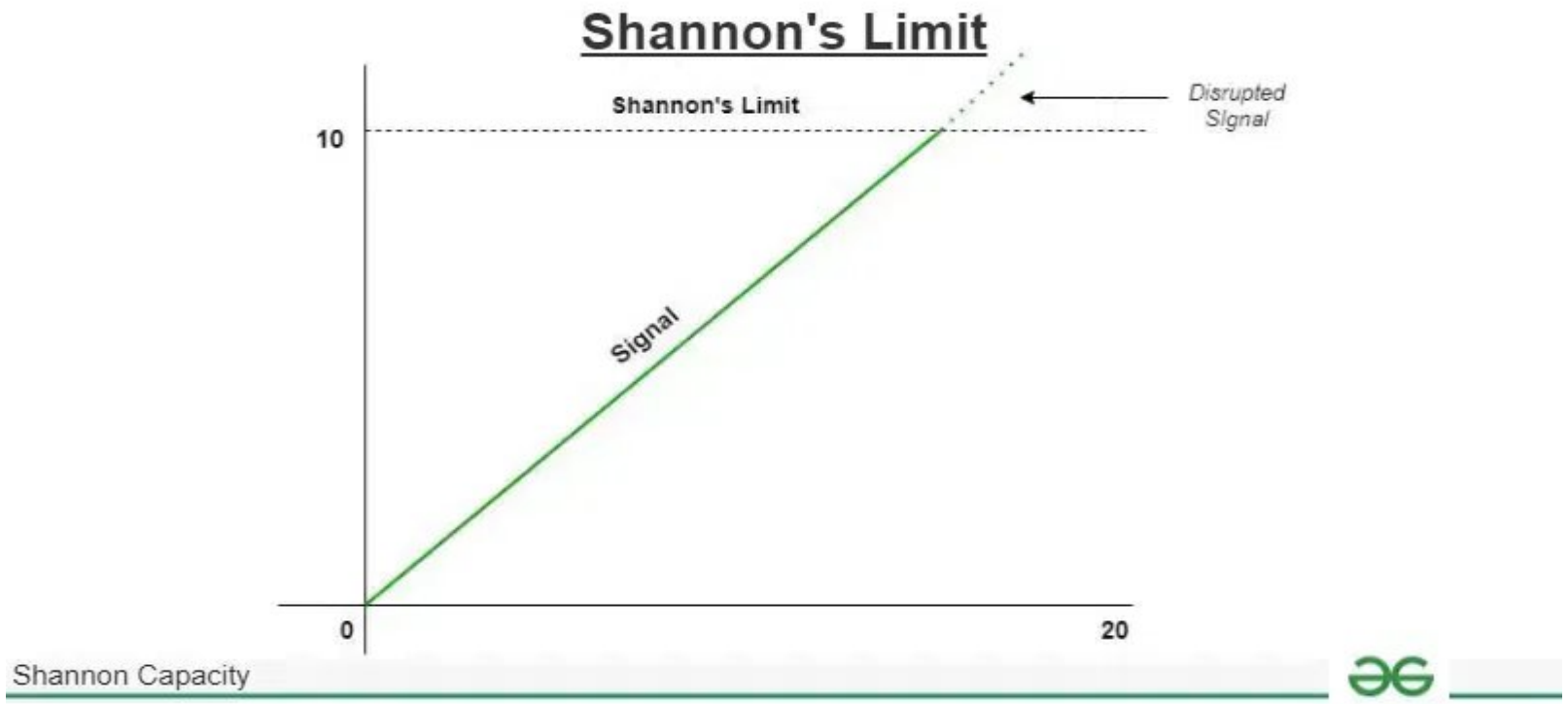
$$\boxed{H = -\sum p_i \log_2 p_i}$$

Entropy plays a very important role in information coding and data compression.

3.Explain the Shannon–Hartley theorem in detail. Derive the channel capacity expression and discuss its applications.

- In the 1940s, Claude Shannon Developed the Channel Capacity using the Concept of Nyquist and Hartley.
- He called this the channel capacity Theorem.
- The Shannon's Capacity theorem states that Shannon's Channel Capacity theorem States that the maximum rate at which information can be transmitted over a communication channel of a specified bandwidth in the presence of noise.
- Shannon's channel capacity tells us the theoretical highest limit in which any channel can transmit signals with the presence of noise.
- It tells us about any channel's capacity and its limitations.
- Shannon's channel capacity theorem is a fundamental concept in Digital Signals,
- It can tell us about the upper bound of any channel.

Let us understand this theorem with the help of a diagram.



- In this diagram, the signal reaching from point 0-10, 0-20 has a theoretical limit beyond which it may encounter several errors, data loss or packet loss.
- Shannon theorem has helped us derived this theoretical limit, Also this signal can be produced with noise, because noiseless communication is expensive.
- Such communication will not disrupt the signal in any way, this is also another advantages of shannon theorem.
- Shannon greatly increased the understanding of data communication after deriving this theorem for us.
- For years, sending data was stuck at a maximum rate of 9.6 kilobits per second and we tried increasing that rate a number of errors would occur in the data.

Some common terms related to Shannon Capacity

Channel: A channel is a medium used to transmit signals, data or any information.

Noise: A noise is any unwanted signal which can negatively impact the quality of any transmission.

Shannon's limit: Shannon's limit is a theoretical limit which tells us the maximum capacity for a signal to be transmitted through a channel.

Statement of the Theorem

Shannon's Channel Capacity theorem States that the maximum rate at which information can be transmitted over a communication channel of a specified bandwidth in the presence of noise.

$C=Blog_2(1+S/N)$

C is the channel capacity in bits per second.

B is the bandwidth of the channel.

S is the average received signal.

N is the average power or noise.

S/N is the signal to noise ratio.

Applications of Shannon's Channel Capacity

The theorem has numerous applications in a variety of fields

•**Telecommunications**: The Shannon theorem helps maintain the capacity of any telecommunication channel which can help in error-less communication and aid engineers, or system architects to build a better model.

•**In wireless communication**: The Shannon theorem helps us calculate the maximum data that can be transferred in a wireless channel without error or data loss.

•**Digital Signals**: Shannon theorem is also used in signals to calculate the amount of information that can be recovered from a signal with a particular noise.

•**Audio and Video compression:** Shannon theorem is also widely used in audio and video compression to derive an upper bound for the compression of data or data communication signals.

•**Storage and Maintenance of data:** Shannon theorem helps us understand a mathematical derivation for storing and maintaining data efficiently and preventing data loss by implementing concepts such as Quantization.

Advantages of Shannon Capacity

Given below is the some of the Advantages of Shannon Capacity

- **Structured framework:** Shannon's theorem breaks down communication processes into components such as sender, message channel, noise, receiver and feedback

- **Enables enhanced communication:** It helps boost the communication by outlining the major aspects that may cause an error in communication and makes the functioning easier.

- **Clarity and Simplicity:** It provides a straightforward way of communication and can be understood by any new scholar or practitioner.

- **Flexible:** Shannon capacity provides a wide range of applications ranging from high speed data transfer to simple voice communication

- **Cost effective:** Shannon capacity allows us to use noisy channels which are more cost effective as compared to noiseless channel.

Disadvantages of Shannon Capacity

Given below is the some of the Disadvantages of Shannon Capacity

- **Longer delays:** Shannon capacity may cause delays considering it requires theoretical proof to proceed with the communication inside a channel.

- **Error rate Increases with Information rate:** Shannon theorem may be error prone if the amount of information increases.

- **Errorless Channel:** Due to noise an errorless channel may not be possible considering this theorem only provides conceptual proof.

- **Increased Complexity:** Shannon capacity theorem may increase complexity since deriving a channel capacity requires complex and expensive methods which can largely impact the complexity of data communication.

- **Limited Scope:** Shannon theorem only considers noise while modern communication faces a lot of other challenges such as interference, fading, packet loss and security concerns.