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Heat Transfer

Course Code: 23BTMEL301

B. Tech - V Semester

Prepared by:

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Teaching and Evaluation Scheme

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Course Title: **Heat Transfer**										
Semester	V	Teaching Scheme				Evaluation Scheme				
						Theory			Practical	
Term	Odd	Th	Tu	Pr	Credits	TAE	CAE	ESE	INT	EXT
Course Category	PCC6	2	-	2	3	10	15	50	25	-
Course Code	23BTMEL301 23BTMEP301									
Teaching Mode	Offline	4 Hrs.			Total	75			25	
Duration of ESE	2.5 Hr					100				

Course Objectives and Outcomes

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Course Objectives	1. To understand the fundamental principles of heat transfer
	1. To analyze the heat transfer processes in practical applications.
	1. To learn the design principles of heat exchangers.
Course Outcomes	**Upon the successful completion of course, the student shall be able to:**
	CO1. Illustrate basic laws of heat transfer to solve real time heat conduction problem.
	CO2. Develop solutions for unsteady state heat conduction in simple geometries.
	CO3. Estimate the heat transfer coefficients and heat transfer rate for forced and natural convection.
	CO4. Analyze the performance of heat exchanger by using LMTD and NTU methods.
	CO5. Understand & evaluate the radiation heat transfer between various surfaces.

Mapping of Course Outcomes with Program Outcomes and Program Specific Outcomes:

Course Outcomes	Program Outcomes and Program Specific Outcomes														
	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO 10	PO 11	PO 12	PSO1	PSO2	PSO3
CO1	3	3	3	3	2	-	-	-	-	-	-	2	3	3	2
CO2	3	3	3	3	2	-	-	-	-	-	-	2	3	3	2
CO3	3	3	3	3	3	-	-	-	-	-	-	2	3	3	2
CO4	3	3	3	3	3	2	-	-	-	-	-	2	3	3	2
CO5	3	3	3	3	1	-	-	-	-	-	-	2	3	3	2

Syllabus

Unit	Contents
I	**One Dimensional Steady State Heat Conduction Without Heat Generation:** Introduction. Basic modes of heat transfer, conduction, convection and radiation. Laws of heat transfer. General heat conduction equation in Cartesian, Cylindrical and Spherical coordinates. Thermal conductivity and diffusivity. One dimensional steady state conduction equation for the plane wall, cylinder and sphere. Thermal resistance of composite structures, overall heat transfer coefficient, critical thickness of insulation**.**
II	**Conduction with Internal Heat Generation:** Internal heat generation for plane wall. **Extended Surfaces:** Types of fins. Fins of uniform cross section area, temperature distribution and heat transfer rate, fin efficiency and effectiveness. **Unsteady state heat transfer:** Unsteady state heat transfer, lumped heat capacity analysis.
III	**Forced Convection:** Physical signification of non-dimensional parameters. Flow of high, moderate & low prandtl number fluid over a flat surface. Concept of velocity & thermal boundary layer, local and average heat transfer coefficients, External, internal flow, laminar & turbulent flow. **Free or Natural Convection:** Grash off Number, Rayleigh Number for Horizontal and Vertical Plate. Empirical co-relations for cylinders and spheres. **Heat Transfer with Phase Change**: Pool boiling curve. Film & drop wise condensation,

Syllabus

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Unit	Contents
IV	**Heat Exchanger:** Classification, Overall Heat Transfer Coefficient, Fouling Factor, LMTD Method of Heat Exchanger Analysis for Parallel, Counter Flow & Cross Flow Arrangement. Effectiveness, NTU Method, Design Aspects of Heat Exchangers. Introduction to Compact Heat Exchanger.
V	**Radiation:** Nature of thermal radiation, black body radiation, radiation intensity, laws of radiation – Kirch offs, Planks, Weins displacement, Stefan Boltzmann & Lamberts co-sine law. Emissivity, absorptivity, transmissivity, reflectivity, radiosity, emissive power, irradiation. Concept of shape factor, Radiation shields, Introduction to mass transfer. Advanced Topic:

Study Material

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Text Books	1.	Heat and Mass Transfer – Fundamentals & Applications, Y. A. Cengel & A. J. Ghajar,5th edition, McGraw Hill Education, New York, 2015.
	2.	A Textbook of Heat and Mass Transfer, R. K. Rajput, 7th edition, S Chand Publishing, 2018.
E-Books	1.	Engineering Heat and Mass Transfer, Mahesh M. Rathore, 3rdedition, University Science Press, New Delhi, 2015.
	2.	Heat Transfer, J.P.Holman, 9th edition, Tata Mcgraw Hill, New Delhi, 2008.
Reference Books	1.	Heat and Mass Transfer, P. K. Nag, Tata McGraw-Hill, 2011
	2.	Heat and Mass Transfer, R. Yadav, 2nd edition Central Publishing House, Allahabad, 1992.
	3.	Fundamentals of Heat and Mass Transfer, Incropera& Dewitt, 7th edition, John Wiley and Sons, New York, 2011.
	4.	Heat Transfer, Y.V.C Rao, University Press, Hyderabad, 2001.
Data Book		Heat and Mass Transfer Data Book, C P Kothandaraman and S.Subramanyan, 9th Edition, New Age International Publishers, 2019.
Online TL Material (course)		Heat and Mass Transfer , Prof. S. P. Sukhatme, IIT Bombay NPTEL. https.//nptel.ac.in/courses/112/101/112101097/
		Conduction and Convection heat transfer, Prof. S. Chakraborty, NPTEL. https.//nptel.ac.in/courses/112/105/112105271/

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Unit 1

Scope of Heat Transfer

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Doors & Windows
Furniture & Curtains
Lights and Fans
ACs
Other Electronics gadgets
Paints
Books





Doors & Windows
Gas Stove
Ovens
Furniture
Utensils
Refrigerator
Chimney
Lights and Fans
Other Electronics gadgets



Doors & Windows
Beds
ACs
Coolers
Irons
Hair dryer
Furniture & Curtains
Lights and Fans
Other Electronics gadgets

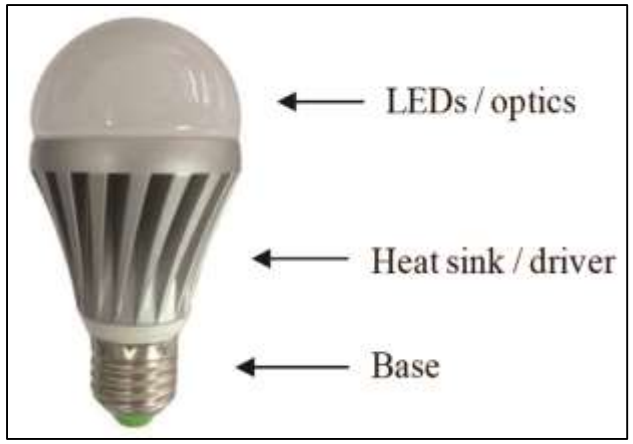




Doors & Windows
Showers
Taps and Fittings
Soap and Shampoo
Geysers
Washing Machine
Lights and Fans
Other Electronics gadgets

Scope of Heat Transfer



















Scope of Heat Transfer

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(a) Flame photographs of stable flame at different fuel injector size of brass burner



(b) Flame photographs of stable flame at different fuel injector size of Cast iron burner

Scope of Heat Transfer

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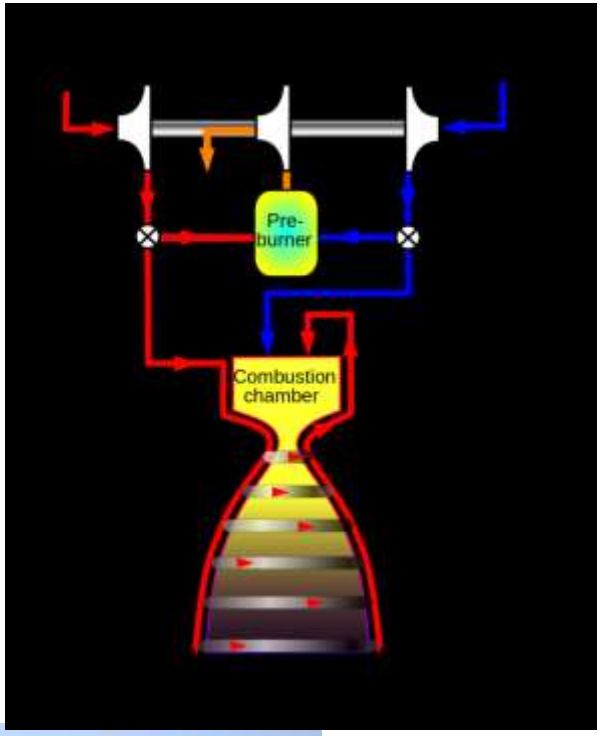


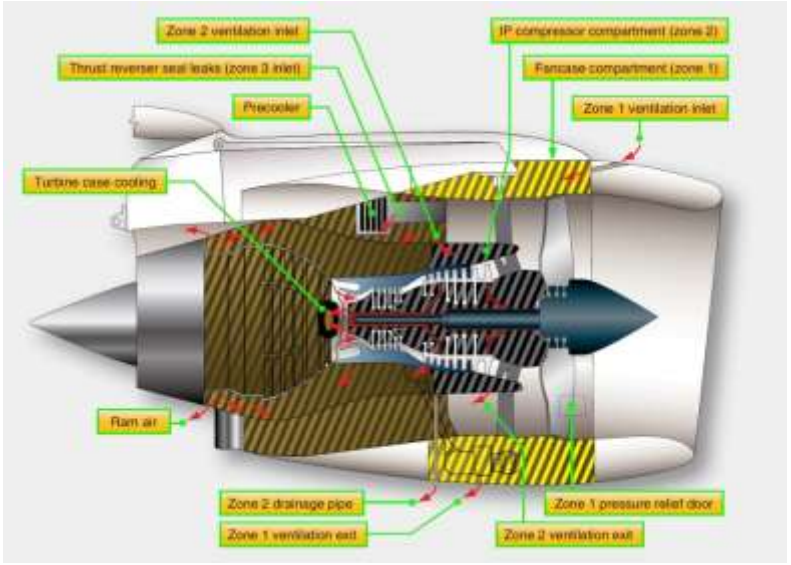
Scope of Heat Transfer

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Scope of Heat Transfer











THERMODYNAMICS AND HEAT TRANSFER

- **Heat:** The form of energy that can be transferred from one system to another as a result of temperature difference.
- **Thermodynamics** is concerned with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another.
- **Heat Transfer** deals with the determination of the *rates* of such energy transfers as well as variation of temperature.
- The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one.
- Heat transfer stops when the two mediums reach the same temperature.
- Heat can be transferred in three different modes:

conduction*, *convection*, *radiation

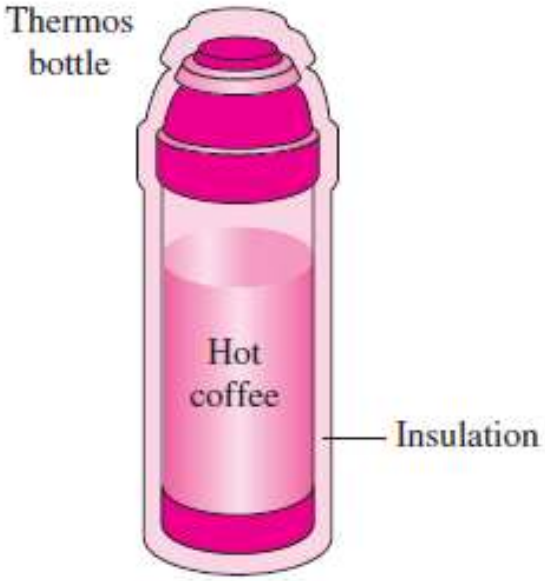


FIGURE 1–1
We are normally interested in how long it takes for the hot coffee in a thermos bottle to cool to a certain temperature, which cannot be determined from a thermodynamic analysis alone.

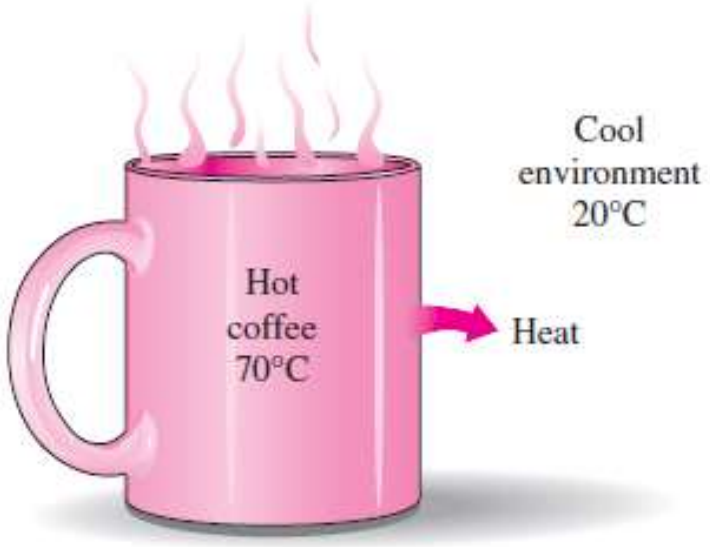


FIGURE 1–2
Heat flows in the direction of decreasing temperature.

Application Areas of Heat Transfer



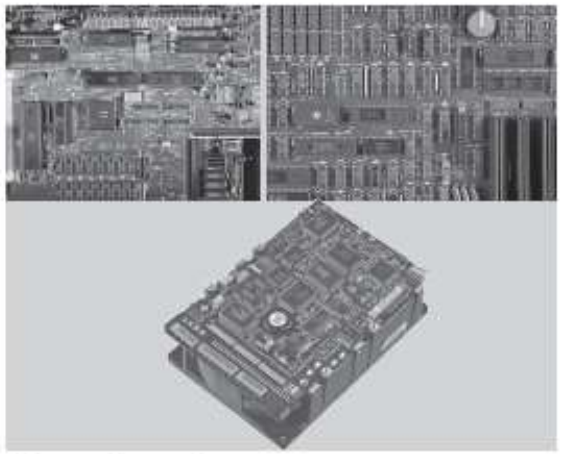
The human body
(© Vol. 121/Photo Disc.)



Air conditioning systems
(© The McGraw-Hill Companies, Inc./Jill Braaten, photographer.)



Heating systems
(© Comstock RF.)



Electronic equipment



Power plants



Refrigeration systems

ENGINEERING HEAT TRANSFER

Heat transfer equipment such as heat exchangers, boilers, condensers, radiators, heaters, furnaces, refrigerators, and solar collectors are designed primarily on the basis of heat transfer analysis.

The heat transfer problems encountered in practice can be considered in two groups: (1) *rating* and (2) *sizing* problems.

The rating problems deal with the determination of the heat transfer rate for an existing system at a specified temperature difference.

The sizing problems deal with the determination of the size of a system in order to transfer heat at a specified rate for a specified temperature difference.

An engineering device or process can be studied either *experimentally* (testing and taking measurements) or *analytically* (by analysis or calculations).

The experimental approach has the advantage that we deal with the actual physical system, and the desired quantity is determined by measurement, within the limits of experimental error. However, this approach is expensive, time consuming, and often impractical.

The analytical approach (including the numerical approach) has the advantage that it is fast and inexpensive, but the results obtained are subject to the accuracy of the assumptions, approximations, and idealizations made in the analysis.

HEAT TRANSFER MECHANISMS

- *Heat* as the form of energy that can be transferred from one system to another as a result of temperature difference.
- A thermodynamic analysis is concerned with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another.
- The science that deals with the determination of the *rates* of such energy transfers is the *heat transfer*.
- The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one, and heat transfer stops when the two mediums reach the same temperature.
- Heat can be transferred in three basic modes:
  - conduction
  - convection
  - radiation
- All modes of heat transfer require the existence of a temperature difference.

CONDUCTION

Conduction: The transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles.

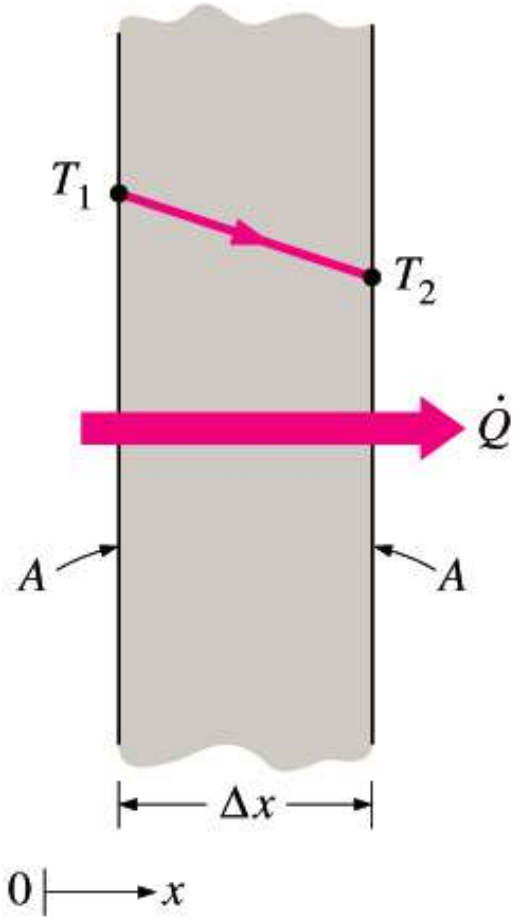
In gases and liquids, conduction is due to the *collisions* and *diffusion* of the molecules during their random motion.

In solids, it is due to the combination of *vibrations* of the molecules in a lattice and the energy transport by *free electrons*.

The rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, but is inversely proportional to the thickness of the layer.

$$\text{Rate of heat conduction} \propto \frac{(\text{Area})(\text{Temperature difference})}{\text{Thickness}}$$

$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x} \qquad \text{(W)}$$



Heat conduction through a large plane wall of thickness Δx and area A.

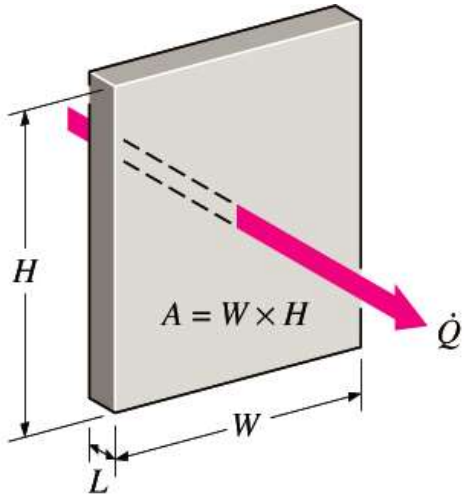
When $x \to 0$ $\dot{Q}_{\text{cond}} = -kA\dfrac{dT}{dx}$ **Fourier's law of heat conduction**

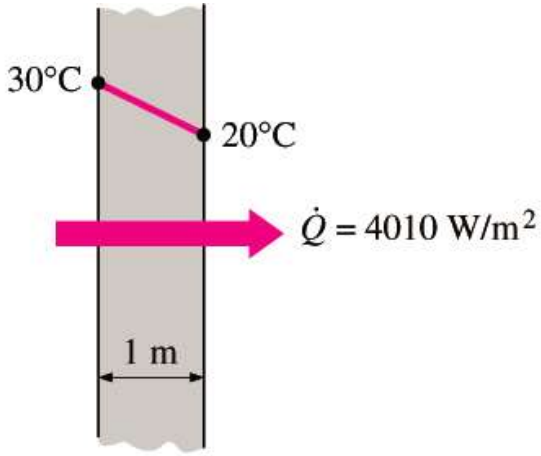
Thermal conductivity, *k*: A measure of the ability of a material to conduct heat.

Temperature gradient *dT/dx*: The slope of the temperature curve on a *T-x* diagram.

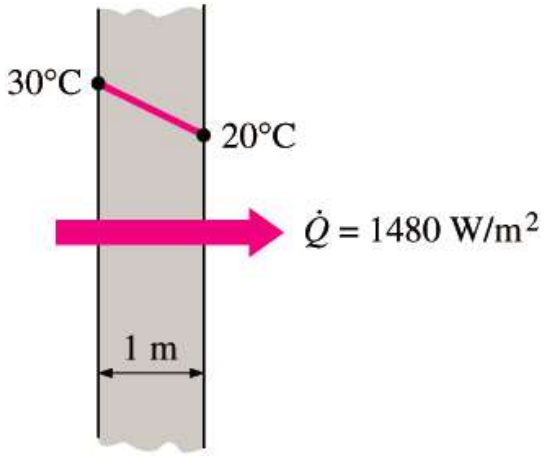
Heat is conducted in the direction of decreasing temperature, and the temperature gradient becomes negative when temperature decreases with increasing *x*. The *negative sign* in the equation ensures that heat transfer in the positive *x* direction is a positive quantity.

In heat conduction analysis, *A* represents the area *normal* to the direction of heat transfer.





(*a*) Copper ($k = 401$ W/m·°C)



(*b*) Silicon ($k = 148$ W/m·°C)

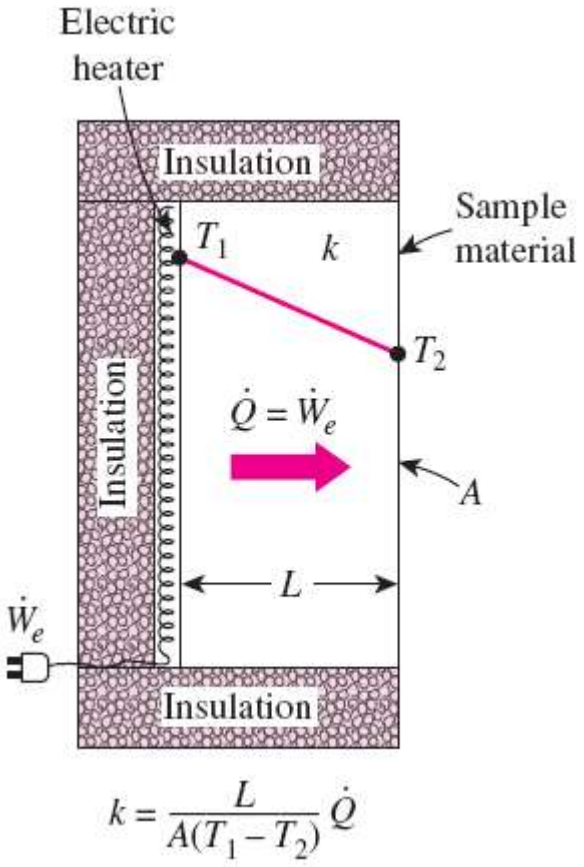
The rate of heat conduction through a solid is directly proportional to its thermal conductivity.

Thermal Conductivity

Thermal conductivity: The rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.

The thermal conductivity of a material is a measure of the ability of the material to conduct heat.

A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or *insulator*.

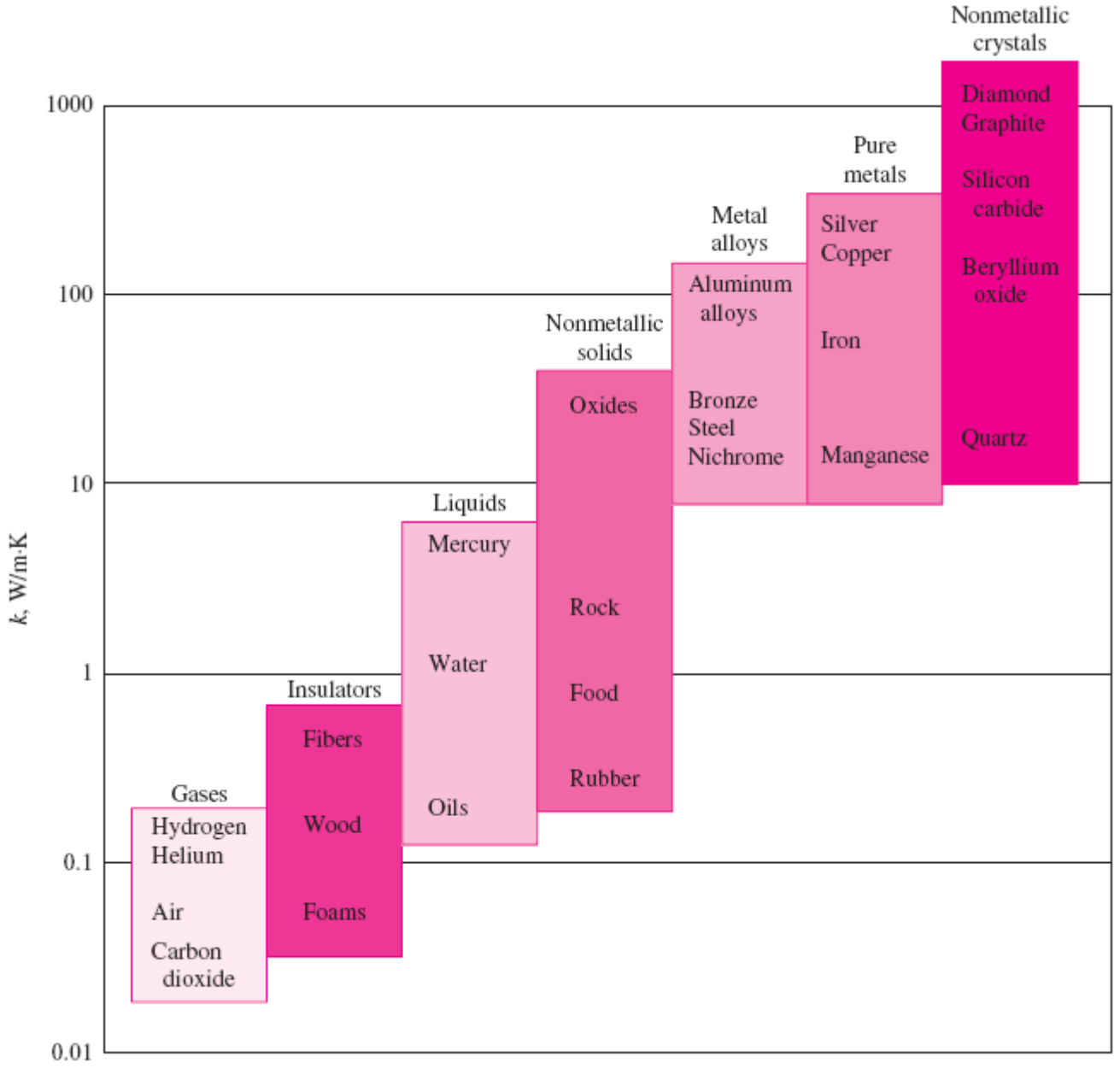


$$k = \frac{L}{A(T_1 - T_2)}\dot{Q}$$

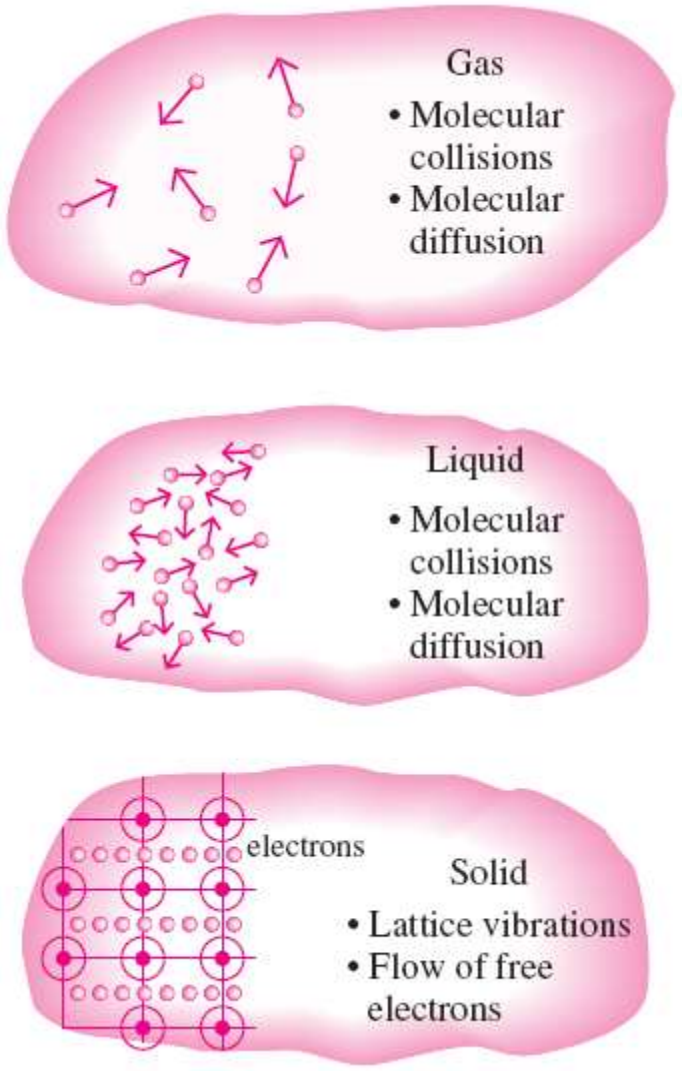
A simple experimental setup to determine the thermal conductivity of a material.

The thermal conductivities of some materials at room temperature

Material	k, W/m · °C*
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.607
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026



The range of thermal conductivity of various materials at room temperature.



The mechanisms of heat conduction in different phases of a substance.

The thermal conductivities of gases such as air vary by a factor of 10^4 from those of pure metals such as copper.

Pure crystals and metals have the highest thermal conductivities, and gases and insulating materials the lowest.

TABLE 1–2

The thermal conductivity of an alloy is usually much lower than the thermal conductivity of either metal of which it is composed

Pure metal or alloy	k, W/m·K, at 300 K
Copper	401
Nickel	91
Constantan (55% Cu, 45% Ni)	23
Copper	401
Aluminum	237
Commercial bronze (90% CU, 10% Al)	52

* Solids ⇒

$$k_{\text{non metallic crystal structure}} > k_{\text{pure metals}} > k_{\text{alloy}} > k_{\text{non metals}} > k_{\text{insulators}}$$

* Fluids ⇒

$$k_{\text{liquid metal}} > k_{\text{liquid}} > k_{\text{gas}}$$

In general ⇒ $k_{\text{solid}} > k_{\text{liquid}} > k_{\text{gas}}$

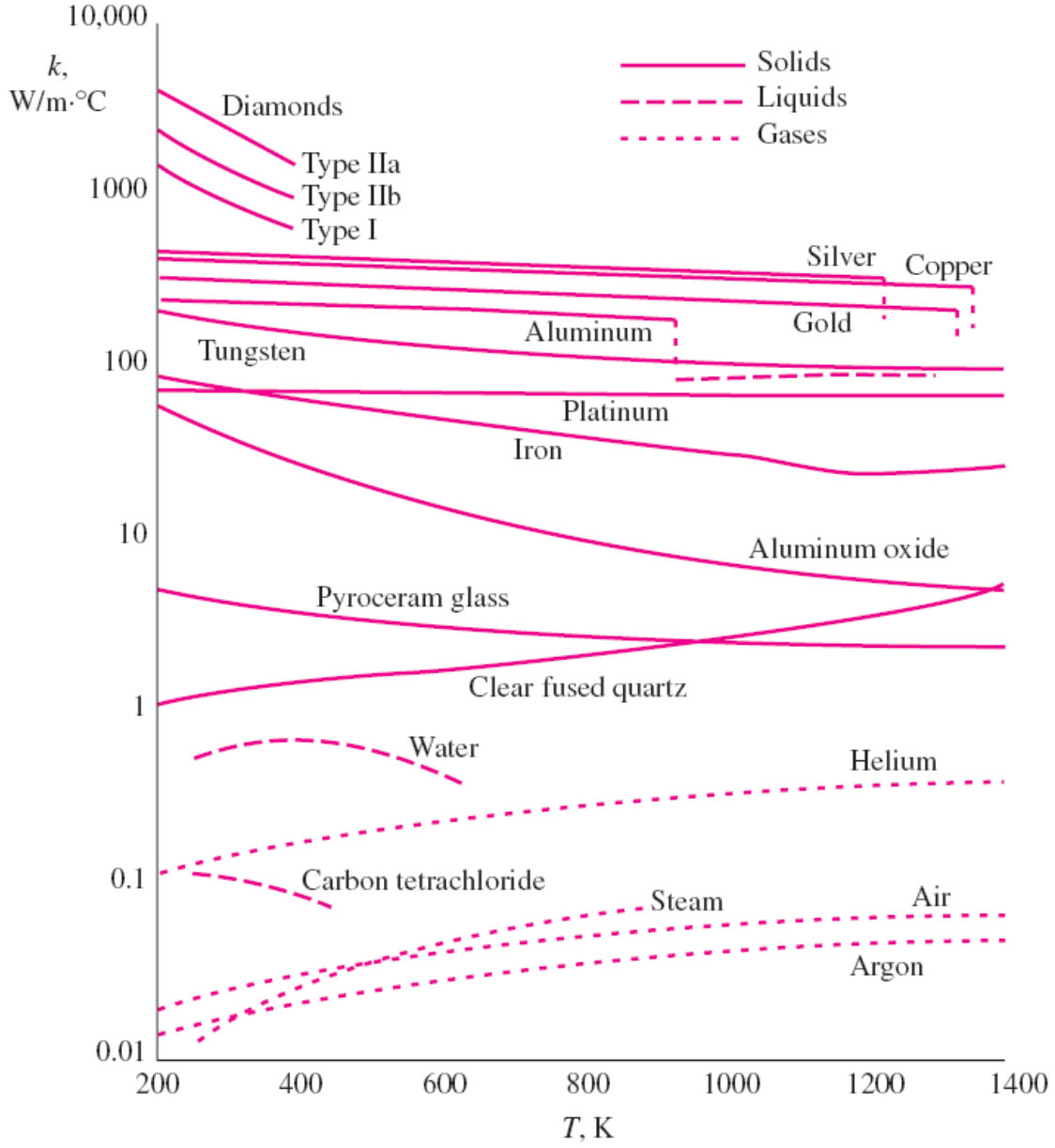


TABLE 1–3

Thermal conductivities of materials vary with temperature

	k, W/m·K	
T, K	Copper	Aluminum
100	482	302
200	413	237
300	401	237
400	393	240
600	379	231
800	366	218

Liquids ⇒ T↑ ⇒ k↓

Gases ⇒ T↑ ⇒ k↑

The variation of the thermal conductivity of various solids, liquids, and gases with temperature.

Thermal Diffusivity

c_p Specific heat, J/kg · °C: Heat capacity per unit mass

ρc_p Heat capacity, J/m³·°C: Heat capacity per unit volume

α Thermal diffusivity, m²/s: Represents how fast heat diffuses through a material

$$\alpha = \frac{\text{Heat conduction}}{\text{Heat storage}} = \frac{k}{\rho c_p} \qquad (\text{m}^2/\text{s})$$

A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity.

The larger the thermal diffusivity, the faster the propagation of heat into the medium.

A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat is conducted further.

The thermal diffusivities of some materials at room temperature

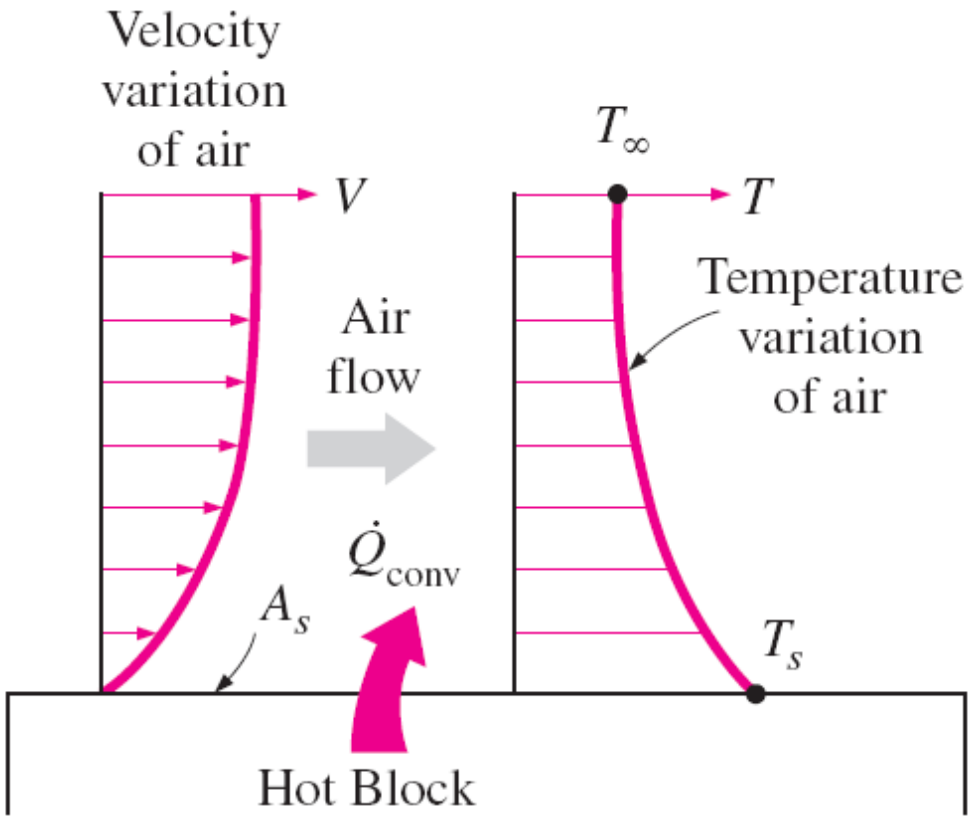
Material	α, m²/s*
Silver	149×10^{-6}
Gold	127×10^{-6}
Copper	113×10^{-6}
Aluminum	97.5×10^{-6}
Iron	22.8×10^{-6}
Mercury (l)	4.7×10^{-6}
Marble	1.2×10^{-6}
Ice	1.2×10^{-6}
Concrete	0.75×10^{-6}
Brick	0.52×10^{-6}
Heavy soil (dry)	0.52×10^{-6}
Glass	0.34×10^{-6}
Glass wool	0.23×10^{-6}
Water (l)	0.14×10^{-6}
Beef	0.14×10^{-6}
Wood (oak)	0.13×10^{-6}

CONVECTION

Convection: The convection heat transfer mode is comprised of two mechanisms. In addition to energy transfer due to random molecular motion (diffusion), energy is also transferred by the bulk, or macroscopic, motion of the fluid. Such motion, in the presence of a temperature gradient, contributes to heat transfer.

The faster the fluid motion, the greater the convection heat transfer.

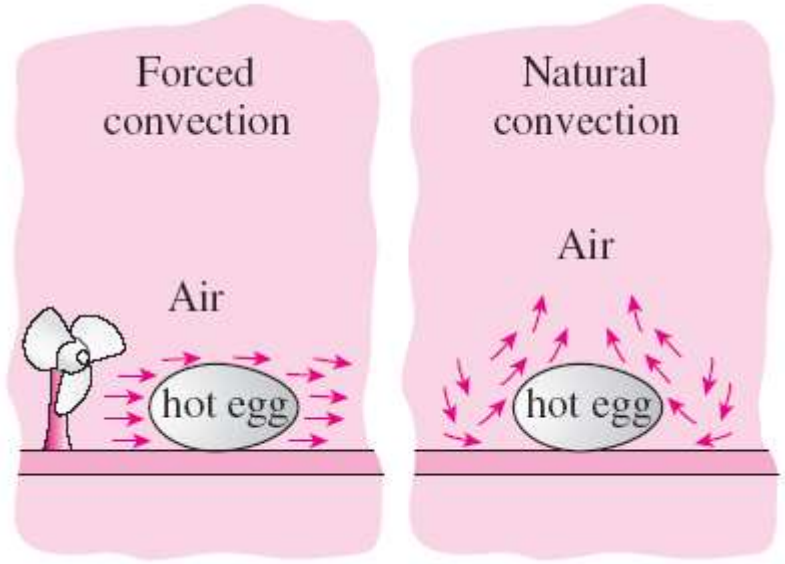
In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.



Heat transfer from a hot surface to air by convection.

Forced convection: If the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind.

Natural (or **free**) **convection:** If the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.



The cooling of a boiled egg by forced and natural convection.

Heat transfer processes that involve *change of phase* of a fluid are also considered to be convection because of the fluid motion induced during the process, such as the rise of the vapor bubbles during boiling or the fall of the liquid droplets during condensation.

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \quad \text{(W)}$$ **Newton's law of cooling**

h	convection heat transfer coefficient, W/m^2 · °C
A_s	the surface area through which convection heat transfer takes place
T_s	the surface temperature
T_∞	the temperature of the fluid sufficiently far from the surface.

The convection heat transfer coefficient *h* is not a property of the fluid.

It is an experimentally determined parameter whose value depends on all the variables influencing convection such as

- the surface geometry
- the nature of fluid motion
- the properties of the fluid
- the bulk fluid velocity

Typical values of convection heat transfer coefficient

Type of convection	h, W/m^2 · °C*
Free convection of gases	2–25
Free convection of liquids	10–1000
Forced convection of gases	25–250
Forced convection of liquids	50–20,000
Boiling and condensation	2500–100,000

RADIATION

- **Radiation:** The energy emitted by matter in the form of *electromagnetic waves* (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules.
- Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an *intervening medium.*
- In fact, heat transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum. This is how the energy of the sun reaches the earth.
- In heat transfer studies we are interested in *thermal radiation,* which is the form of radiation emitted by bodies because of their temperature.
- All bodies at a temperature above absolute zero emit thermal radiation.
- Radiation is a *volumetric phenomenon,* and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees.
- However, radiation is usually considered to be a *surface phenomenon* for solids.

$$\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4 \quad \text{(W)}$$ **Stefan–Boltzmann law**

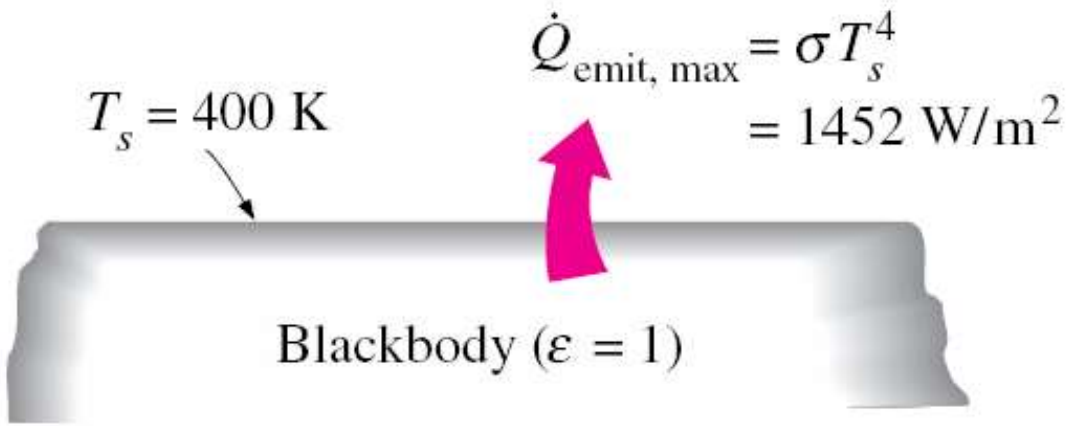
$\sigma = 5.670 \times 10^{-8}$ W/m$^2 \cdot$ K^4 *Stefan–Boltzmann constant*

Blackbody: The idealized surface that emits radiation at the maximum rate.

$$\dot{Q}_{\text{emit}} = \varepsilon \sigma A_s T_s^4 \quad \text{(W)}$$

Radiation emitted by real surfaces

Emissivity ε: A measure of how closely a surface approximates a blackbody for which $\varepsilon = 1$ of the surface. **$0 \le \varepsilon \le 1$**.



Emissivities of some materials at 300 K

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.03
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92–0.97
Asphalt pavement	0.85–0.93
Red brick	0.93–0.96
Human skin	0.95
Wood	0.82–0.92
Soil	0.93–0.96
Water	0.96
Vegetation	0.92–0.96

Blackbody radiation represents the *maximum amount of radiation that can be emitted from a surface at a specified temperature.*

When radiation and convection occur simultaneously between a surface and a gas:

$$\dot{Q}_{\text{total}} = h_{\text{combined}} A_s (T_s - T_\infty) \qquad \text{(W)}$$

Combined heat transfer coefficient h_{combined}

Includes the effects of both convection and radiation

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = h_{\text{conv}} A_s (T_s - T_{\text{surr}}) + \varepsilon\sigma A_s (T_s^4 - T_{\text{surr}}^4)$$

$$\dot{Q}_{\text{total}} = h_{\text{combined}} A_s (T_s - T_\infty) \qquad \text{(W)}$$

$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} = h_{\text{conv}} + \varepsilon\sigma (T_s + T_{\text{surr}})(T_s^2 + T_{\text{surr}}^2)$$

SIMULTANEOUS HEAT TRANSFER MECHANISMS

Heat transfer is only by conduction in *opaque solids,* but by conduction and radiation in *semitransparent solids.*

A solid may involve conduction and radiation but not convection. A solid may involve convection and/or radiation on its surfaces exposed to a fluid or other surfaces.

Heat transfer is by conduction and possibly by radiation in a *still fluid* (no bulk fluid motion) and by convection and radiation in a *flowing fluid.*

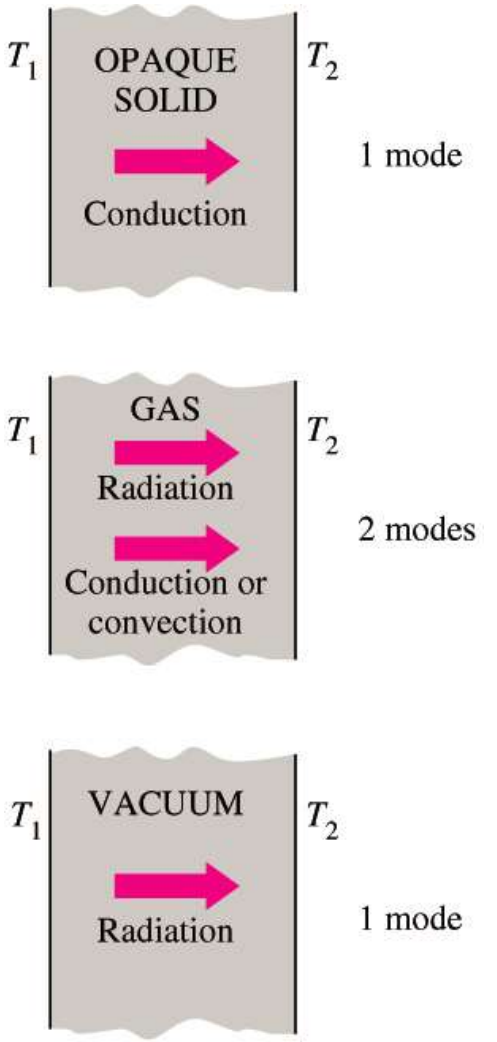
In the absence of radiation, heat transfer through a fluid is either by conduction or convection, depending on the presence of any bulk fluid motion.

Convection = Conduction + Fluid motion

Heat transfer through a *vacuum* is by radiation.

Most gases between two solid surfaces do not interfere with radiation.

Liquids are usually strong absorbers of radiation.



Although there are three mechanisms of heat transfer, a medium may involve only two of them simultaneously.

HEAT CONDUCTION EQUATION

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- Although heat transfer and temperature are closely related, they are of a different nature.
- Temperature has only magnitude. It is a *scalar* quantity.
- Heat transfer has direction as well as magnitude. It is a *vector* quantity.
- We work with a coordinate system and indicate direction with plus or minus signs.

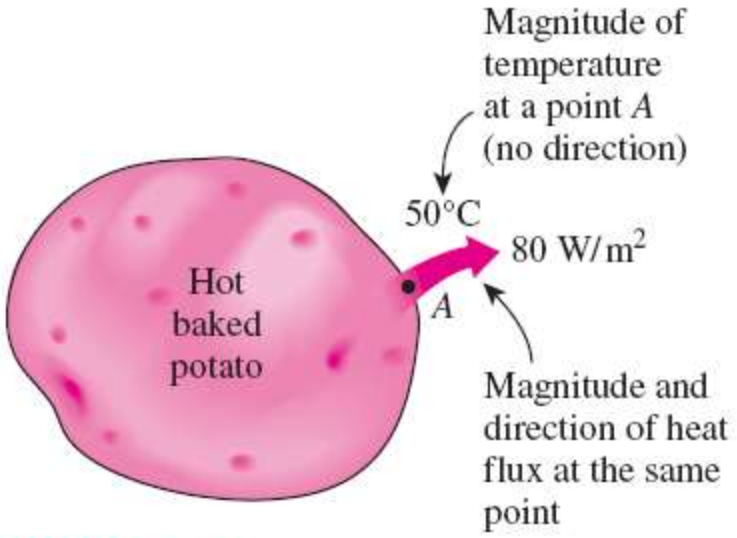


FIGURE 2–1
Heat transfer has direction as well as magnitude, and thus it is a *vector* quantity.

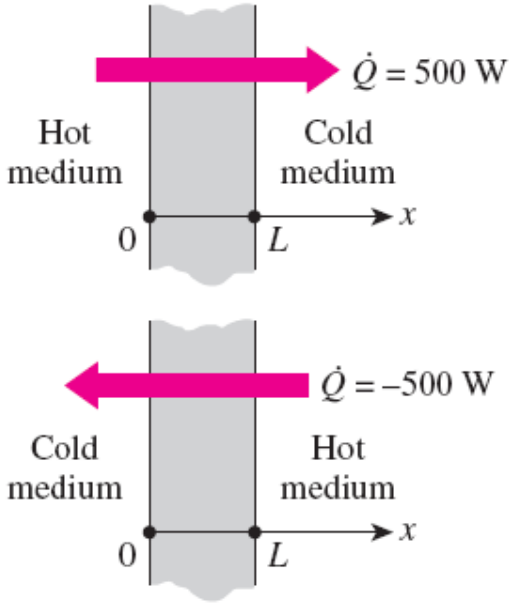
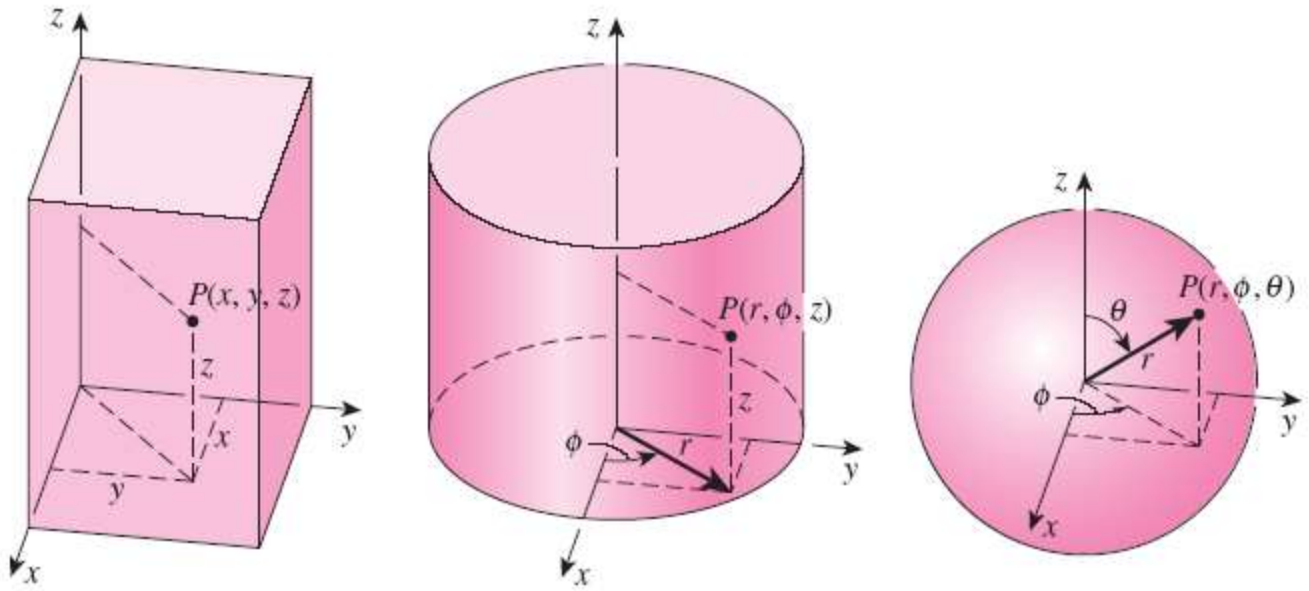


FIGURE 2–2
Indicating direction for heat transfer (positive in the positive direction; negative in the negative direction).

- The driving force for any form of heat transfer is the *temperature difference.*
- The larger the temperature difference, the larger the rate of heat transfer.
- Three prime coordinate systems:
  - rectangular $T(x, y, z, t)$
  - cylindrical $T(r, \phi, z, t)$
  - spherical $T(r, \phi, \theta, t)$.

FIGURE 2–3
The various distances and angles involved when describing the location of a point in different coordinate systems.



(*a*) Rectangular coordinates (*b*) Cylindrical coordinates (*c*) Spherical coordinates

Steady versus Transient Heat Transfer

- *Steady* implies *no change* with time at any point within the medium
- *Transient* implies *variation with time* or *time dependence*
- In the special case of variation with time but not with position, the temperature of the medium changes *uniformly* with time. Such heat transfer systems are called lumped systems.

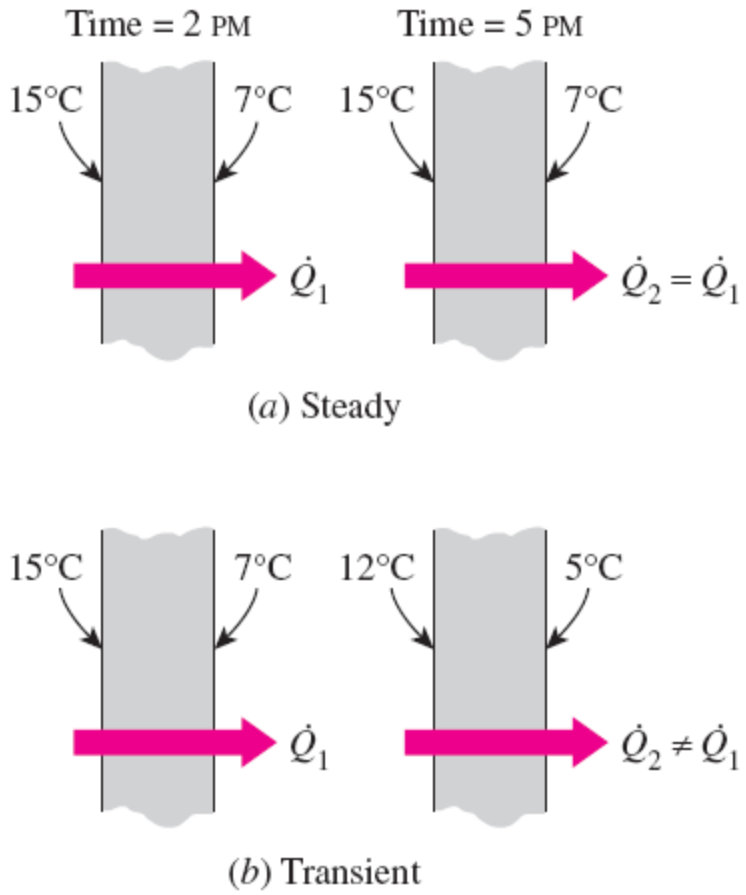


FIGURE 2–4

Transient and steady heat conduction in a plane wall.

Multidimensional Heat Transfer

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- Heat transfer problems are also classified as being:
  - *one-dimensional*
  - *two dimensional*
  - *three-dimensional*
- In the most general case, heat transfer through a medium is **three-dimensional**. However, some problems can be classified as two- or one-dimensional depending on the relative magnitudes of heat transfer rates in different directions and the level of accuracy desired.
- **One-dimensional** if the temperature in the medium varies in one direction only and thus heat is transferred in one direction, and the variation of temperature and thus heat transfer in other directions are negligible or zero.
- **Two-dimensional** if the temperature in a medium, in some cases, varies mainly in two primary directions, and the variation of temperature in the third direction (and thus heat transfer in that direction) is negligible.

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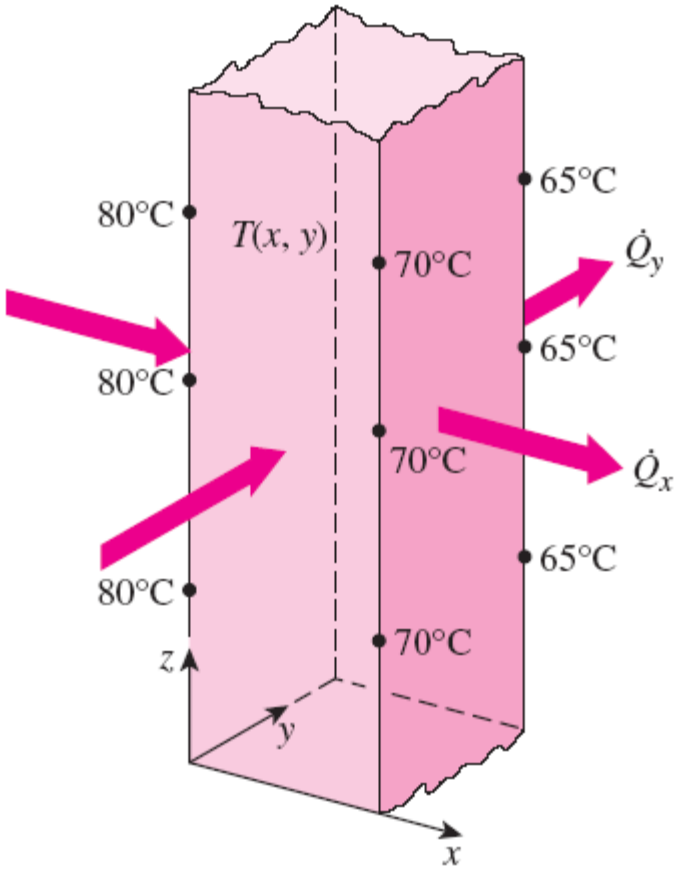


FIGURE 2–5
Two-dimensional heat transfer in a long rectangular bar.

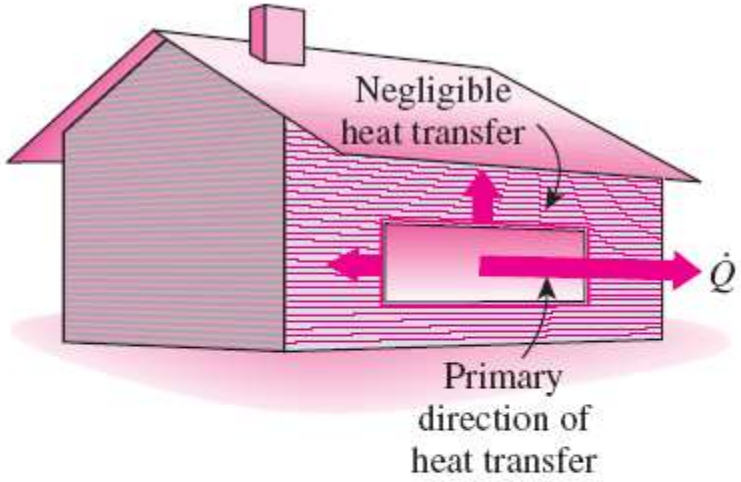


FIGURE 2–6
Heat transfer through the window of a house can be taken to be one-dimensional.

- The rate of heat conduction through a medium in a specified direction (say, in the *x*-direction) is expressed by **Fourier's law of heat conduction** for one-dimensional heat conduction as:

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

Heat is conducted in the direction of decreasing temperature, and thus the temperature gradient is negative when heat is conducted in the positive *x* -direction.

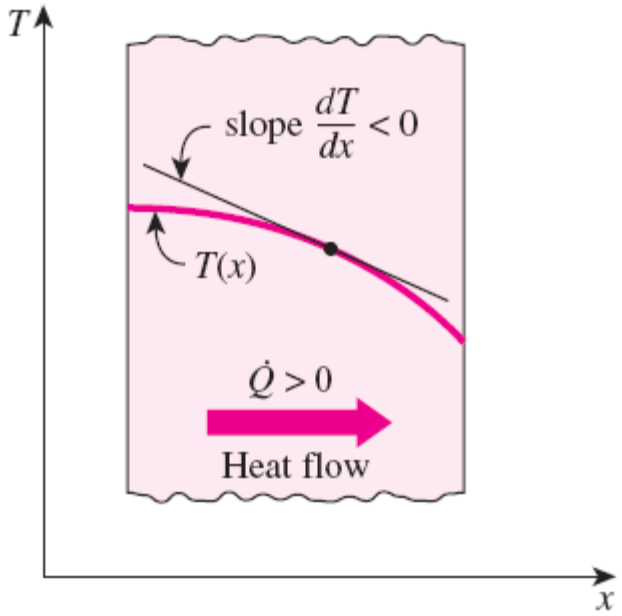


FIGURE 2–7

The temperature gradient *dT*/*dx* is simply the slope of the temperature curve on a *T*-*x* diagram.

- The heat flux vector at a point *P* on the surface of the figure must be perpendicular to the surface, and it must point in the direction of decreasing temperature
- If *n* is the normal of the isothermal surface at point *P*, the rate of heat conduction at that point can be expressed by Fourier's law as

$$\dot{Q}_n = -kA\frac{\partial T}{\partial n} \qquad \text{(W)}$$

$$\vec{\dot{Q}}_n = \dot{Q}_x\vec{i} + \dot{Q}_y\vec{j} + \dot{Q}_z\vec{k}$$

$$\dot{Q}_x = -kA_x\frac{\partial T}{\partial x}, \qquad \dot{Q}_y = -kA_y\frac{\partial T}{\partial y},$$

$$\dot{Q}_z = -kA_z\frac{\partial T}{\partial z}$$

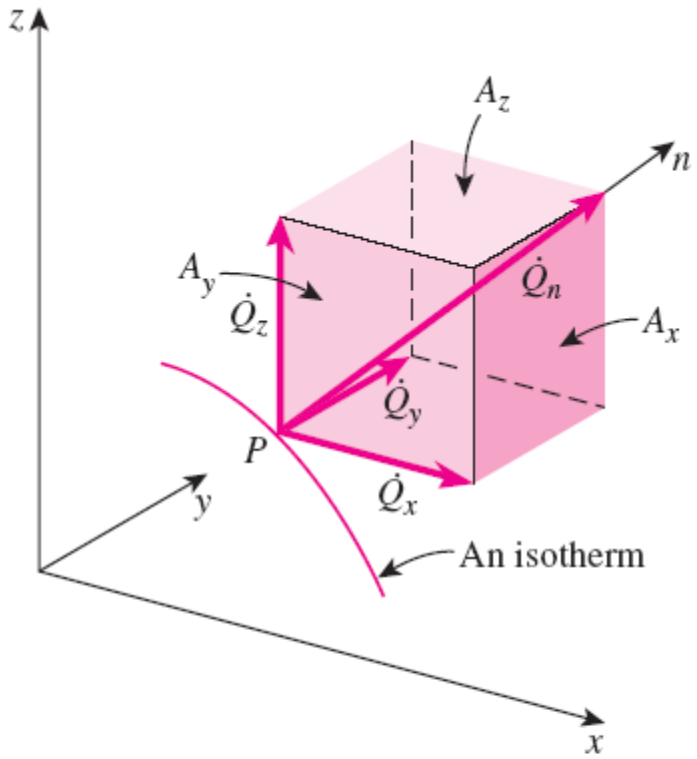


FIGURE 2–8
The heat transfer vector is always normal to an isothermal surface and can be resolved into its components like any other vector.

Heat Generation

- Examples:
  - electrical energy being converted to heat at a rate of I^2R,
  - fuel elements of nuclear reactors,
  - exothermic chemical reactions.
- Heat generation is a *volumetric phenomenon*.
- The rate of heat generation units : W/m^3 or $Btu/h \cdot ft^3$.
- The rate of heat generation in a medium may vary with time as well as position within the medium.



FIGURE 2–9

Heat is generated in the heating coils of an electric range as a result of the conversion of electrical energy to heat.

$$\dot{E}_{\text{gen}} = \int_V \dot{q}_g \, dV \qquad \text{(W)}$$

$$\dot{E}_{\text{gen}} = \dot{q}_g V,$$

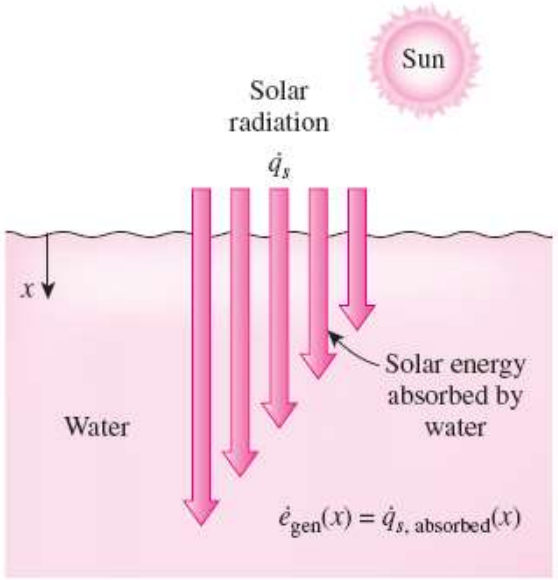


FIGURE 2–10

The absorption of solar radiation by water can be treated as heat generation.

ONE-DIMENSIONAL HEAT CONDUCTION EQUATION

Consider heat conduction through a large plane wall such as the wall of a house, the glass of a single pane window, the metal plate at the bottom of a pressing iron, a cast-iron steam pipe, a cylindrical nuclear fuel element, an electrical resistance wire, the wall of a spherical container, or a spherical metal ball that is being quenched or tempered.

Heat conduction in these and many other geometries can be approximated as being *one-dimensional* since heat conduction through these geometries is dominant in one direction and negligible in other directions.

Next we develop the onedimensional heat conduction equation in rectangular, cylindrical, and spherical coordinates.

Heat Conduction Equation in a Large Plane Wall

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$$\left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x\end{array}\right) - \left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x\end{array}\right) + \left(\begin{array}{c}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{array}\right) = \left(\begin{array}{c}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{array}\right)$$

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{E}_{\text{gen, element}} = \frac{\Delta E_{\text{element}}}{\Delta t} \qquad (2\text{-}6)$$

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho cA\Delta x(T_{t+\Delta t} - T_t)$$

$$\dot{E}_{\text{gen, element}} = \dot{q}_g V_{\text{element}} = \dot{q}_g A\Delta x$$

Substituting into Eq. 2–6, we get

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{q}_g A\Delta x = \rho cA\Delta x \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $A\Delta x$ gives

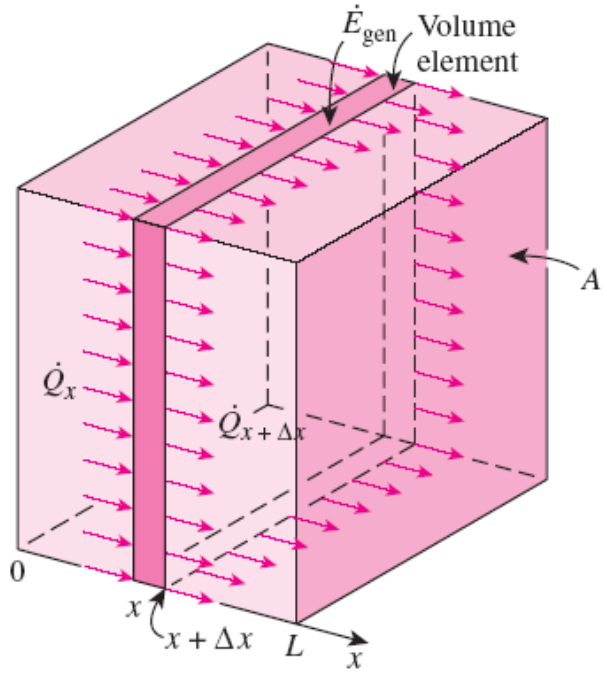
$$-\frac{1}{A}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} + \dot{q}_g = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta x \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right) + \dot{q}_g = \rho c \frac{\partial T}{\partial t}$$

$$\lim_{\Delta x \to 0} \frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \frac{\partial \dot{Q}}{\partial x} = \frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right)$$

from the definition of the derivative and Fourier's law of heat conduction



$A_x = A_{x+\Delta x} = A$

FIGURE 2–12

One-dimensional heat conduction through a volume element in a large plane wall.

Variable conductivity: $$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+q_g=\rho c\frac{\partial T}{\partial t}$$

Constant conductivity: $$\frac{\partial^2 T}{\partial x^2}+\frac{q_g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(1) *Steady-state:* $(\partial/\partial t = 0)$ $$\frac{d^2T}{dx^2}+\frac{q_g}{k}=0$$

(2) *Transient, no heat generation:* $(q_g = 0)$ $$\frac{\partial^2 T}{\partial x^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(3) *Steady-state, no heat generation:* $(\partial/\partial t = 0$ and $q_g = 0)$ $$\frac{d^2T}{dx^2}=0$$

General, one-dimensional:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\overbrace{q_g}^{\text{No generation}}}{k}=\frac{1}{\alpha}\overbrace{\frac{\partial T}{\partial t}}^{\text{Steady-state}}$$

Steady, one-dimensional:

$$\frac{d^2T}{dx^2}=0$$

The simplification of the one-dimensional heat conduction equation in a plane wall for the case of constant conductivity for steady conduction with no heat generation.

GENERAL HEAT CONDUCTION EQUATION

In the last section we considered one-dimensional heat conduction and assumed heat conduction in other directions to be negligible.

Most heat transfer problems encountered in practice can be approximated as being one-dimensional, and we mostly deal with such problems in this text.

However, this is not always the case, and sometimes we need to consider heat transfer in other directions as well.

In such cases heat conduction is said to be *multidimensional,* and in this section we develop the governing differential equation in such systems in rectangular, cylindrical, and spherical coordinate systems.

$$E_{in} - E_{out} + E_g = E_{st}$$

Rectangular Coordinates

The conduction heat rates at the opposite surfaces can then be expressed as a Taylor series expansion where, neglecting higher-order terms

$$Q_{x+dx} = Q_x + \frac{\partial Q_x}{\partial x} dx$$

$$Q_{y+dy} = Q_y + \frac{\partial Q_y}{\partial y} dy$$

$$Q_{z+dz} = Q_z + \frac{\partial q_z}{\partial z} dz$$

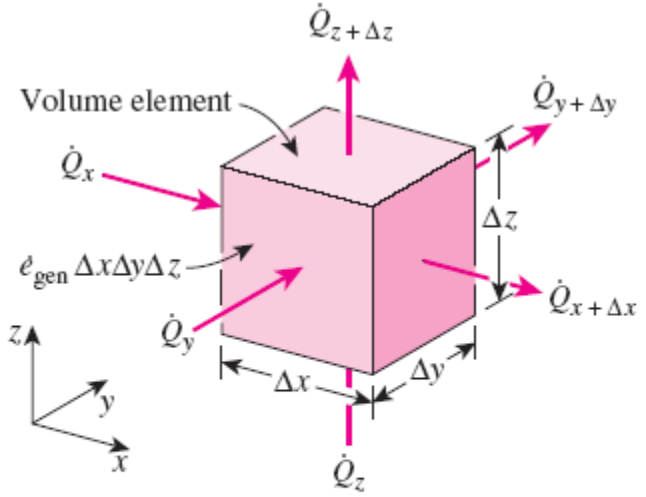


FIGURE 2–20

Three-dimensional heat conduction through a rectangular volume element.

Within the medium there may also be an *energy source* term associated with the rate of thermal energy generation. This term is represented as

$$\dot{E}_g = q_g dx\, dy\, dz$$

where is the rate at which energy is generated per unit volume of the medium (W/m^3).

The *energy storage* term may be expressed as

$$\dot{E}_{st} = \rho c_p \frac{\partial T}{\partial t} dx\, dy\, dz$$

where $\rho c_p \frac{\partial T}{\partial t}$ is the time rate of change of the sensible (thermal) energy of the medium per unit volume.

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

$$Q_x + Q_y + Q_z + q_g dx\,dy\,dz - Q_{x+dx} - Q_{y+dy} - Q_{z+dz} = \rho c_p \frac{\partial T}{\partial t} dx\,dy\,dz$$

$$-\frac{\partial Q_x}{\partial x}dx - \frac{\partial Q_y}{\partial y}dy - \frac{\partial Q_z}{\partial z}dz + q_g dx\,dy\,dz = \rho c_p \frac{\partial T}{\partial t} dx\,dy\,dz \quad \text{----- (1)}$$

The conduction heat rates in an isotropic material may be evaluated from Fourier's law

$$Q_x = -k\,dy\,dz\frac{\partial T}{\partial x}$$

$$Q_y = -k\,dx\,dz\frac{\partial T}{\partial y}$$

$$Q_z = -k\,dx\,dy\frac{\partial T}{\partial z}$$

Substituting above equations in eq.1 and dividing out the dimensions of the control volume (*dx dy dz*), we obtain

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + q_g = \rho c_p \frac{\partial T}{\partial t}$$

Above equation is the general form, in Cartesian coordinates, of the *heat diffusion equation.* This equation, often referred to as the *heat equation*, provides the basic tool for heat conduction analysis.

Noting that the heat transfer areas of the element for heat conduction in the x, y, and z directions are $A_x = \Delta y \Delta z$, $A_y = \Delta x \Delta z$, and $A_z = \Delta x \Delta y$, respectively, and taking the limit as Δx, Δy, Δz and $\Delta t \to 0$ yields

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + q_g = \rho c \frac{\partial T}{\partial t} \qquad \text{(2–38)}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta x \to 0} \frac{1}{\Delta y \Delta z}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \frac{1}{\Delta y \Delta z}\frac{\partial Q_x}{\partial x} = \frac{1}{\Delta y \Delta z}\frac{\partial}{\partial x}\left(-k\Delta y \Delta z \frac{\partial T}{\partial x}\right) = -\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)$$

$$\lim_{\Delta y \to 0} \frac{1}{\Delta x \Delta z}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} = \frac{1}{\Delta x \Delta z}\frac{\partial Q_y}{\partial y} = \frac{1}{\Delta x \Delta z}\frac{\partial}{\partial y}\left(-k\Delta x \Delta z \frac{\partial T}{\partial y}\right) = -\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)$$

$$\lim_{\Delta z \to 0} \frac{1}{\Delta x \Delta y}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} = \frac{1}{\Delta x \Delta y}\frac{\partial Q_z}{\partial z} = \frac{1}{\Delta x \Delta y}\frac{\partial}{\partial z}\left(-k\Delta x \Delta y \frac{\partial T}{\partial z}\right) = -\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)$$

Eq. 2–38 is the general heat conduction equation in rectangular coordinates. In the case of constant thermal conductivity, it reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{(2–39)}$$

where the property $\alpha = k/\rho c$ is again the *thermal diffusivity* of the material. Eq. 2–39 is known as the **Fourier-Biot equation**, and it reduces to these forms under specified conditions:

(1) *Steady-state:*
(called the **Poisson equation**)

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_g}{k}=0$$

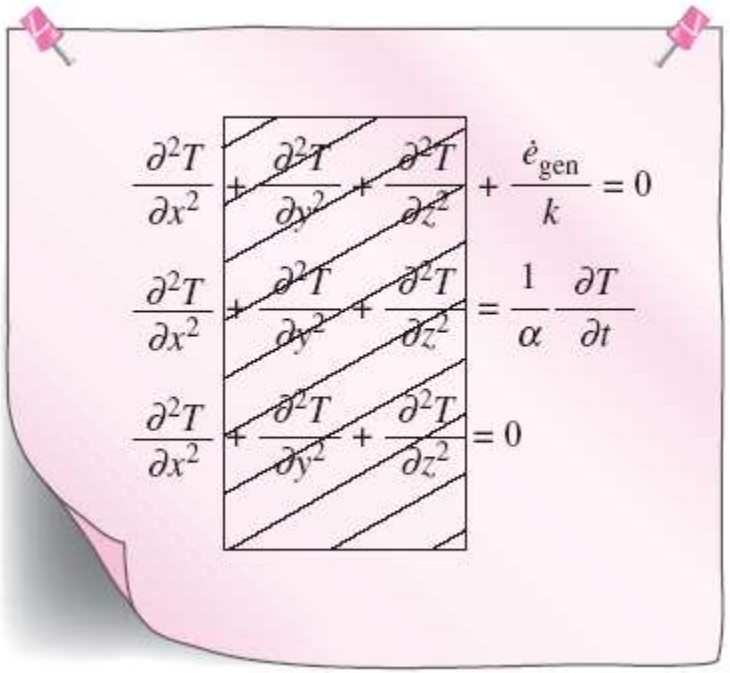
(2) *Transient, no heat generation:*
(called the **diffusion equation**)

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(3) *Steady-state, no heat generation:*
(called the **Laplace equation**)

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$



The three-dimensional heat conduction equations reduce to the one-dimensional ones when the temperature varies in one dimension only.

Cylindrical Coordinates

Relations between the coordinates of a point in rectangular and cylindrical coordinate systems:

$$x = r\cos\phi, \qquad y = r\sin\phi, \qquad \text{and} \qquad z = z$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial T}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{e}_{\text{gen}} = \rho c\frac{\partial T}{\partial t}$$

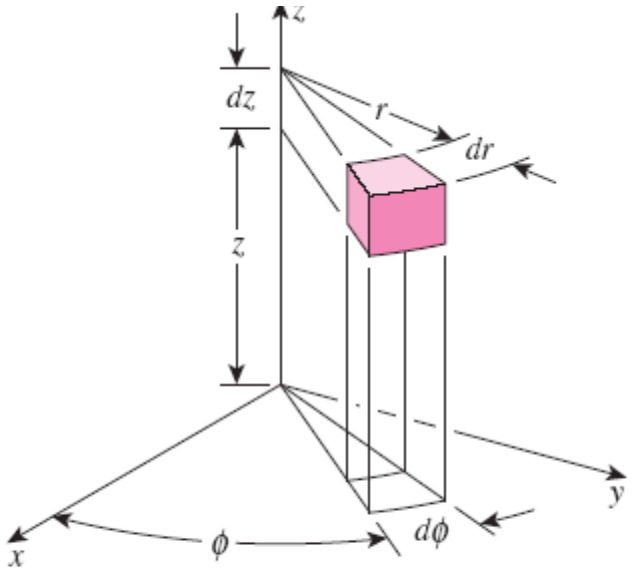


FIGURE 2–22

A differential volume element in cylindrical coordinates.

Spherical Coordinates

Relations between the coordinates of a point in rectangular and spherical coordinate systems:

$$x = r\cos\phi\sin\theta, \qquad y = r\sin\phi\sin\theta, \qquad \text{and} \qquad z = \cos\theta$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(k\frac{\partial T}{\partial\phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(k\sin\theta\frac{\partial T}{\partial\theta}\right) + \dot{e}_{\text{gen}} = \rho c\frac{\partial T}{\partial t}$$

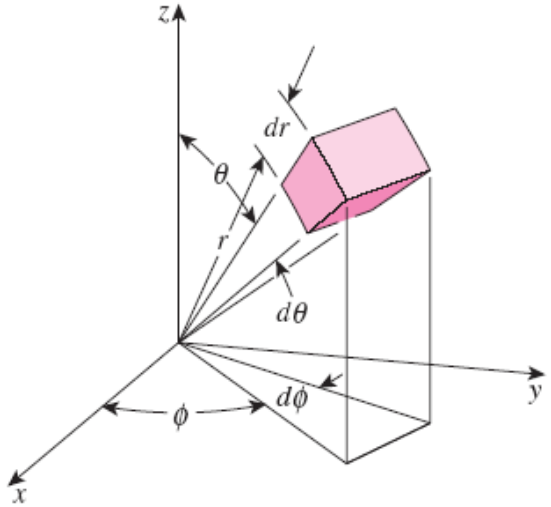


FIGURE 2–23

A differential volume element in spherical coordinates.

L-04

EXAMPLE 2–10 Heat Conduction in a Plane Wall

Consider a large plane wall of thickness $L = 0.2$ m, thermal conductivity $k = 1.2$ W/m·K, and surface area $A = 15$ m^2. The two sides of the wall are maintained at constant temperatures of $T_1 = 120°C$ and $T_2 = 50°C$, respectively, as shown in Fig. 2–40. Determine (*a*) the variation of temperature within the wall and the value of temperature at $x = 0.1$ m and (*b*) the rate of heat conduction through the wall under steady conditions.

SOLUTION A plane wall with specified surface temperatures is given. The variation of temperature and the rate of heat transfer are to be determined.

Assumptions **1** Heat conduction is steady. **2** Heat conduction is one-dimensional since the wall is large relative to its thickness and the thermal conditions on both sides are uniform. **3** Thermal conductivity is constant. **4** There is no heat generation.

Properties The thermal conductivity is given to be $k = 1.2$ W/m·K.

Analysis (*a*) Taking the direction normal to the surface of the wall to be the *x*-direction, the differential equation for this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

with boundary conditions

$$T(0) = T_1 = 120°C$$
$$T(L) = T_2 = 50°C$$

The differential equation is linear and second order, and a quick inspection of it reveals that it has a single term involving derivatives and no terms involving the unknown function T as a factor. Thus, it can be solved by direct integration. Noting that an integration reduces the order of a derivative by one, the general solution of the differential equation above can be obtained by two simple successive integrations, each of which introduces an integration constant.

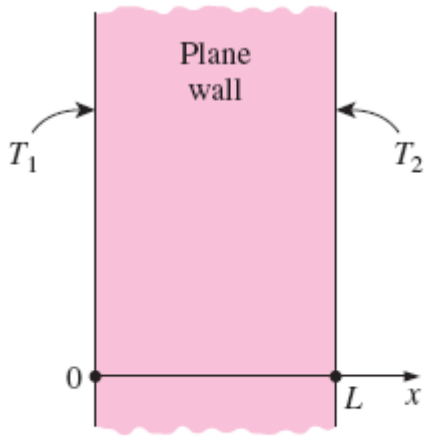


FIGURE 2–40
Schematic for Example 2–10.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Integrating the differential equation once with respect to x yields

$$\frac{dT}{dx} = C_1$$

where C_1 is an arbitrary constant. Notice that the order of the derivative went down by one as a result of integration. As a check, if we take the derivative of this equation, we will obtain the original differential equation. This equation is not the solution yet since it involves a derivative.

Integrating one more time, we obtain

$$T(x) = C_1 x + C_2$$

which is the general solution of the differential equation (Fig. 2–41). The general solution in this case resembles the general formula of a straight line whose slope is C_1 and whose value at $x = 0$ is C_2. This is not surprising since the second derivative represents the change in the slope of a function, and a zero second derivative indicates that the slope of the function remains constant. Therefore, *any straight line* is a solution of this differential equation.

The general solution contains two unknown constants C_1 and C_2, and thus we need two equations to determine them uniquely and obtain the specific solution. These equations are obtained by forcing the general solution to satisfy the specified boundary conditions. The application of each condition yields one equation, and thus we need to specify two conditions to determine the constants C_1 and C_2.

When applying a boundary condition to an equation, *all occurrences of the dependent and independent variables and any derivatives are replaced by the specified values.* Thus the only unknowns in the resulting equations are the arbitrary constants.

The first boundary condition can be interpreted as *in the general solution, replace all the x's by zero and T(x) by* T_1. That is (Fig. 2–42),

$$T(0) = C_1 \times 0 + C_2 \quad \rightarrow \quad C_2 = T_1$$

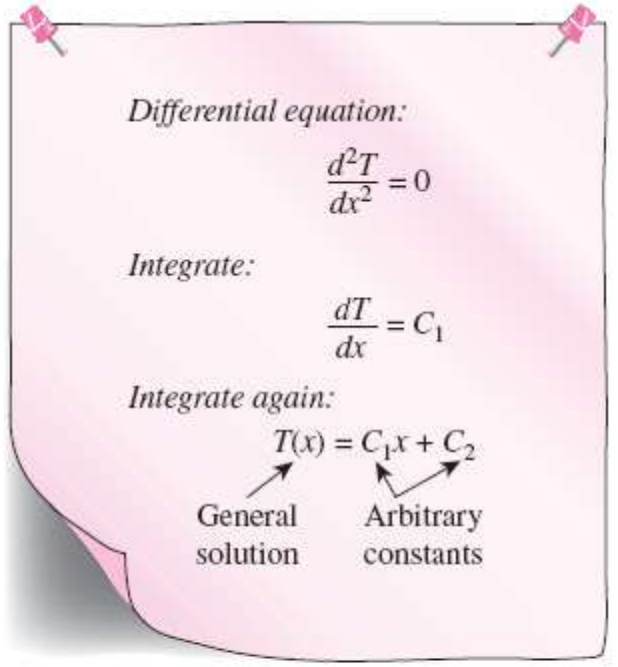


FIGURE 2–41

Obtaining the general solution of a simple second order differential equation by integration.

The second boundary condition can be interpreted as *in the general solution, replace all the x's by L and T(x) by T_2*. That is,

$$T(L) = C_1L + C_2 \quad \rightarrow \quad T_2 = C_1L + T_1 \quad \rightarrow \quad C_1 = \frac{T_2 - T_1}{L}$$

Substituting the C_1 and C_2 expressions into the general solution, we obtain

$$T(x) = \frac{T_2 - T_1}{L}x + T_1 \qquad \textbf{(2–56)}$$

which is the desired solution since it satisfies not only the differential equation but also the two specified boundary conditions. That is, differentiating Eq. 2–56 with respect to x twice will give d^2T/dx^2, which is the given differential equation, and substituting $x = 0$ and $x = L$ into Eq. 2–56 gives $T(0) = T_1$ and $T(L) = T_2$, respectively, which are the specified conditions at the boundaries.

Substituting the given information, the value of the temperature at $x = 0.1$ m is determined to be

$$T(0.1\text{ m}) = \frac{(50 - 120)^\circ\text{C}}{0.2\text{ m}}(0.1\text{ m}) + 120^\circ\text{C} = \mathbf{85^\circ C}$$

(*b*) The rate of heat conduction anywhere in the wall is determined from Fourier's law to be

$$\dot{Q}_{\text{wall}} = -kA\frac{dT}{dx} = -kAC_1 = -kA\frac{T_2 - T_1}{L} = kA\frac{T_1 - T_2}{L} \qquad \textbf{(2–57)}$$

The numerical value of the rate of heat conduction through the wall is determined by substituting the given values to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (1.2\text{ W/m·K})(15\text{ m}^2)\frac{(120 - 50)^\circ\text{C}}{0.2\text{ m}} = \mathbf{6300\text{ W}}$$

Discussion Note that under steady conditions, the rate of heat conduction through a plane wall is constant.

Boundary condition:

$$T(0) = T_1$$

General solution:

$$T(x) = C_1x + C_2$$

Applying the boundary condition:

$$T(x) = C_1x + C_2$$
$$\uparrow \quad \uparrow$$
$$0 \quad 0$$
$$\overline{T_1}$$

Substituting:

$$T_1 = C_1 \times 0 + C_2 \rightarrow C_2 = T_1$$

It cannot involve x or $T(x)$ after the boundary condition is applied.

FIGURE 2–42

When applying a boundary condition to the general solution at a specified point, all occurrences of the dependent and independent variables should be replaced by their specified values at that point.

EXAMPLE 2–14 Heat Loss through a Steam Pipe

Consider a steam pipe of length $L = 20$ m, inner radius $r_1 = 6$ cm, outer radius $r_2 = 8$ cm, and thermal conductivity $k = 20$ W/m·K, as shown in Fig. 2–49. The inner and outer surfaces of the pipe are maintained at average temperatures of $T_1 = 150°C$ and $T_2 = 60°C$, respectively. Obtain a general relation for the temperature distribution inside the pipe under steady conditions, and determine the rate of heat loss from the steam through the pipe.

SOLUTION A steam pipe is subjected to specified temperatures on its surfaces. The variation of temperature and the rate of heat transfer are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction, and thus $T = T(r)$. **3** Thermal conductivity is constant. **4** There is no heat generation.

Properties The thermal conductivity is given to be $k = 20$ W/m·K.

Analysis The mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

with boundary conditions

$$T(r_1) = T_1 = 150°C$$
$$T(r_2) = T_2 = 60°C$$

Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr} = C_1$$

where C_1 is an arbitrary constant. We now divide both sides of this equation by r to bring it to a readily integrable form,

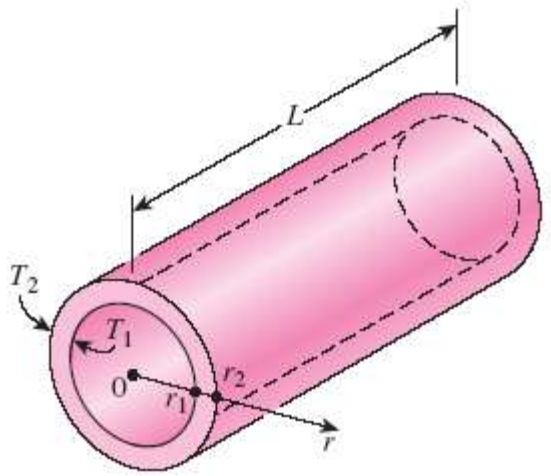


FIGURE 2–49
Schematic for Example 2–14.

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Again integrating with respect to r gives (Fig. 2–50)

$$T(r) = C_1 \ln r + C_2 \qquad (a)$$

We now apply both boundary conditions by replacing all occurrences of r and $T(r)$ in Eq. (a) with the specified values at the boundaries. We get

$$T(r_1) = T_1 \quad \rightarrow \quad C_1 \ln r_1 + C_2 = T_1$$
$$T(r_2) = T_2 \quad \rightarrow \quad C_1 \ln r_2 + C_2 = T_2$$

which are two equations in two unknowns, C_1 and C_2. Solving them simultaneously gives

$$C_1 = \frac{T_2 - T_1}{\ln(r_2/r_1)} \qquad \text{and} \qquad C_2 = T_1 - \frac{T_2 - T_1}{\ln(r_2/r_1)} \ln r_1$$

Substituting them into Eq. (a) and rearranging, the variation of temperature within the pipe is determined to be

$$T(r) = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}(T_2 - T_1) + T_1 \qquad (2\text{–}58)$$

The rate of heat loss from the steam is simply the total rate of heat conduction through the pipe, and is determined from Fourier's law to be

$$\dot{Q}_{\text{cylinder}} = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{C_1}{r} = -2\pi kLC_1 = 2\pi kL\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad (2\text{–}59)$$

The numerical value of the rate of heat conduction through the pipe is determined by substituting the given values

$$\dot{Q} = 2\pi(20\ \text{W/m·K})(20\ \text{m})\frac{(150 - 60)^\circ\text{C}}{\ln(0.08/0.06)} = \mathbf{786\ kW}$$

Discussion Note that the total rate of heat transfer through a pipe is constant, but the heat flux $\dot{q} = \dot{Q}/(2\pi rL)$ is not since it decreases in the direction of heat transfer with increasing radius.

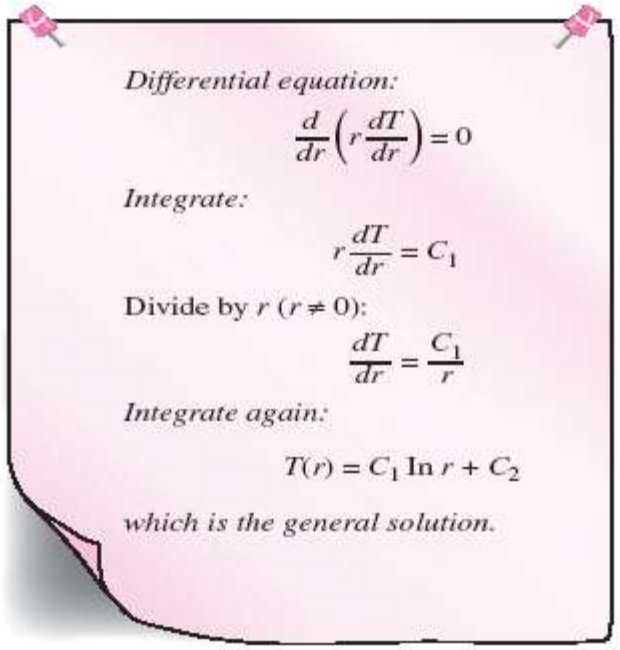


FIGURE 2–50

Basic steps involved in the solution of the steady one-dimensional heat conduction equation in cylindrical coordinates.

EXAMPLE 2–15 **Heat Conduction through a Spherical Shell**

Consider a spherical container of inner radius $r_1 = 8$ cm, outer radius $r_2 = 10$ cm, and thermal conductivity $k = 45$ W/m·K, as shown in Fig. 2–51. The inner and outer surfaces of the container are maintained at constant temperatures of $T_1 = 200°C$ and $T_2 = 80°C$, respectively, as a result of some chemical reactions occurring inside. Obtain a general relation for the temperature distribution inside the shell under steady conditions, and determine the rate of heat loss from the container.

SOLUTION A spherical container is subjected to specified temperatures on its surfaces. The variation of temperature and the rate of heat transfer are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint, and thus $T = T(r)$. **3** Thermal conductivity is constant. **4** There is no heat generation.

Properties The thermal conductivity is given to be $k = 45$ W/m·K.

Analysis The mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

with boundary conditions

$$T(r_1) = T_1 = 200°C$$
$$T(r_2) = T_2 = 80°C$$

Integrating the differential equation once with respect to r yields

$$r^2 \frac{dT}{dr} = C_1$$

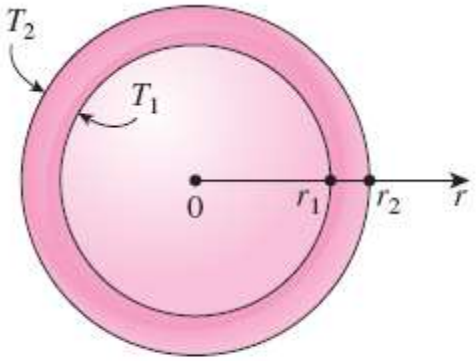


FIGURE 2–51

Schematic for Example 2–15.

where C_1 is an arbitrary constant. We now divide both sides of this equation by r^2 to bring it to a readily integrable form,

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

Again integrating with respect to r gives

$$T(r) = -\frac{C_1}{r} + C_2 \qquad (a)$$

We now apply both boundary conditions by replacing all occurrences of r and $T(r)$ in the relation above by the specified values at the boundaries. We get

$$T(r_1) = T_1 \quad \rightarrow \quad -\frac{C_1}{r_1} + C_2 = T_1$$

$$T(r_2) = T_2 \quad \rightarrow \quad -\frac{C_1}{r_2} + C_2 = T_2$$

which are two equations in two unknowns, C_1 and C_2. Solving them simultaneously gives

$$C_1 = -\frac{r_1 r_2}{r_2 - r_1}(T_1 - T_2) \qquad \text{and} \qquad C_2 = \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1}$$

Substituting into Eq. (a), the variation of temperature within the spherical shell is determined to be

$$T(r) = \frac{r_1 r_2}{r(r_2 - r_1)}(T_1 - T_2) + \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1} \qquad (2\text{–}60)$$

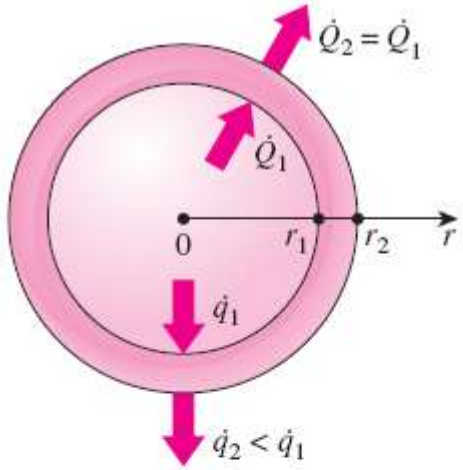
The rate of heat loss from the container is simply the total rate of heat conduction through the container wall and is determined from Fourier's law

$$\dot{Q}_{\text{sphere}} = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{C_1}{r^2} = -4\pi k C_1 = 4\pi k r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} \qquad (2\text{–}61)$$

The numerical value of the rate of heat conduction through the wall is determined by substituting the given values to be

$$\dot{Q} = 4\pi(45\ \text{W/m}\cdot\text{K})(0.08\ \text{m})(0.10\ \text{m})\frac{(200-80)^\circ\text{C}}{(0.10-0.08)\ \text{m}} = \mathbf{27.1\ kW}$$

Discussion Note that the total rate of heat transfer through a spherical shell is constant, but the heat flux $\dot{q} = \dot{Q}/4\pi r^2$ is not since it decreases in the direction of heat transfer with increasing radius as shown in Fig. 2–52.



$$\dot{q}_1 = \frac{\dot{Q}_1}{A_1} = \frac{27.1\ \text{kW}}{4\pi(0.08\ \text{m})^2} = 337\ \text{kW/m}^2$$

$$\dot{q}_2 = \frac{\dot{Q}_2}{A_2} = \frac{27.1\ \text{kW}}{4\pi(0.10\ \text{m})^2} = 216\ \text{kW/m}^2$$

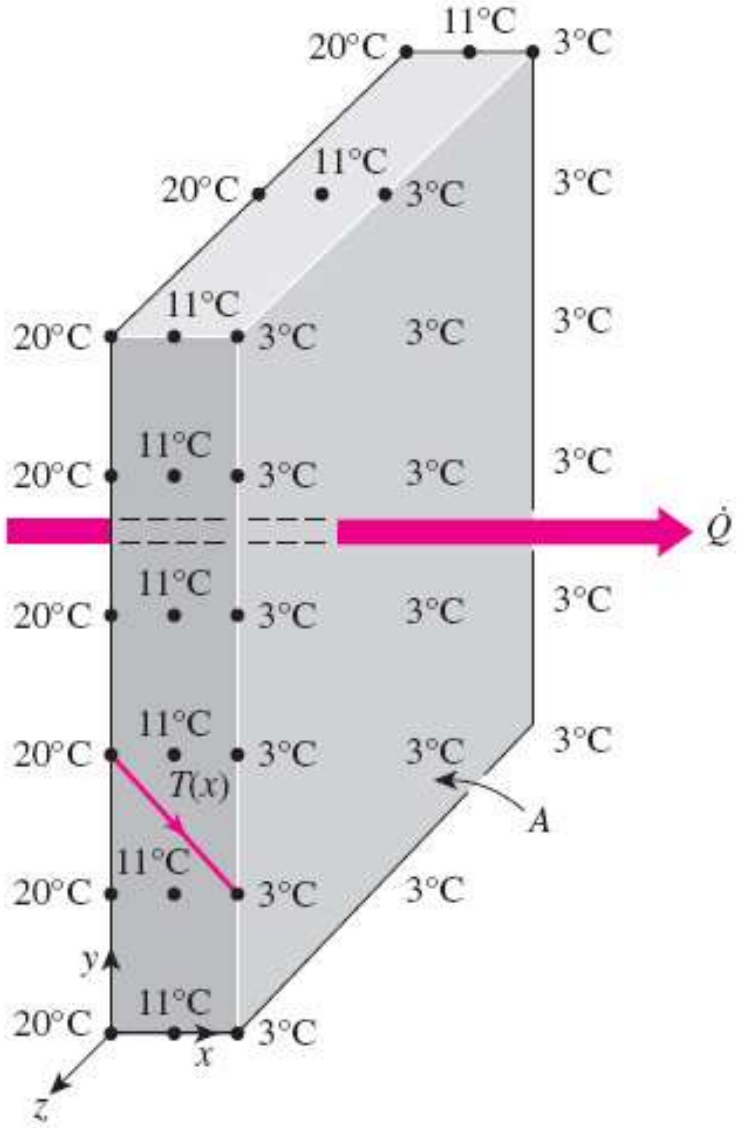
FIGURE 2–52

During steady one-dimensional heat conduction in a spherical (or cylindrical) container, the total rate of heat transfer remains constant, but the heat flux decreases with increasing radius.

	Plane Wall	Cylindrical Wall	Spherical Wall
Heat Equation	$\frac{d^2T}{dx^2}=0$	$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)=0$	$\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)=0$
Temperature Distribution	$T_{s,1}-\Delta T\frac{x}{L}$	$\frac{\Delta T}{ln\left(r_1/r_2\right)}ln\left(\frac{r}{r_2}\right)+T_{s,2}$	$T_{s,1}+\frac{\Delta T}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}\left[\frac{1}{r_1}-\frac{1}{r}\right]$
Heat Flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\,ln\left(r_2/r_1\right)}$	$\frac{k\,\Delta T}{r^2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$
Heat Rate (q)	$k\,A\frac{\Delta T}{L}$	$\frac{2\pi L k\,\Delta T}{ln\left(r_2/r_1\right)}$	$\frac{4\pi k\,\Delta T}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$
Thermal Resistance ($R_{t,\,cond}$)	$\frac{L}{kA}$	$\frac{ln\left(r_2/r_1\right)}{2\pi L k}$	$\frac{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{4\pi k}$

One-Dimensional Steady State Solutions to the Heat Equation with no generation

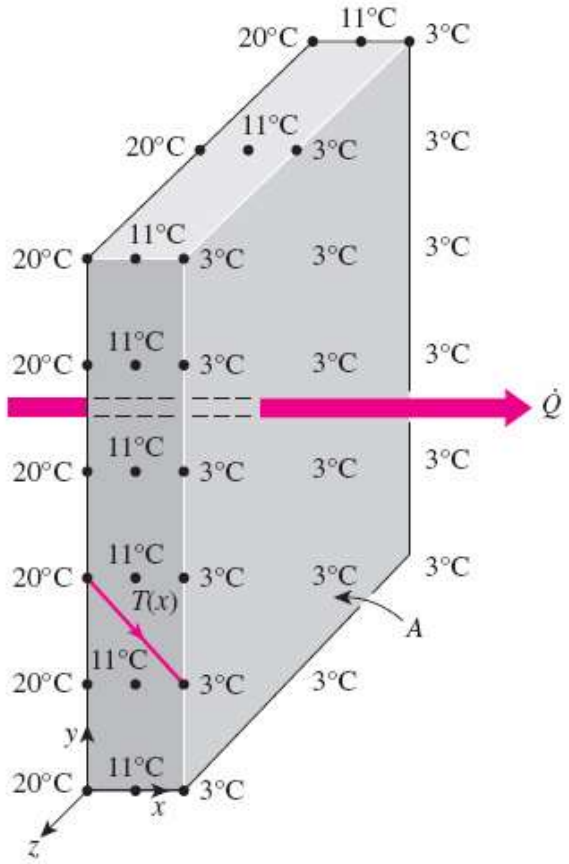
STEADY HEAT CONDUCTION IN PLANE WALLS



Heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional.*

The temperature of the wall in this case depends on one direction only (say the *x*-direction) and can be expressed as $T(x)$.

$$\begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

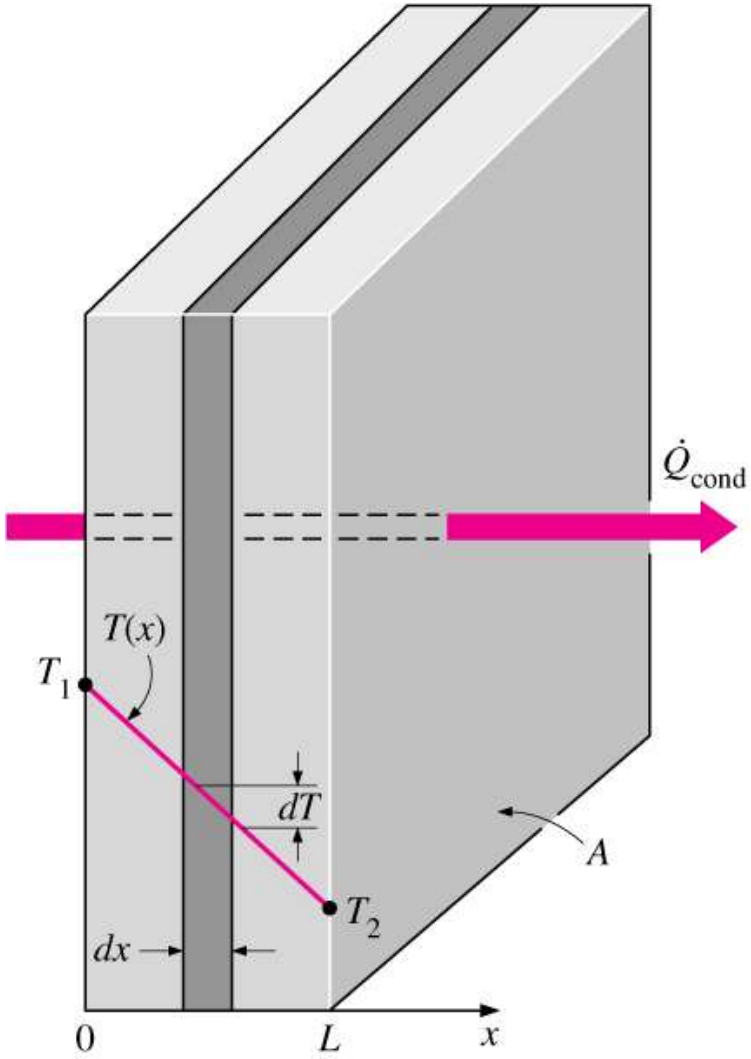


$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \frac{dE_{\text{wall}}}{dt}$$

$dE_{\text{wall}}/dt = 0$ for steady operation

In steady operation, the rate of heat transfer through the wall is constant.

$$\dot{Q}_{\text{cond, wall}} = \text{constant}$$



$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx}$$

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \quad \text{(W)}$$

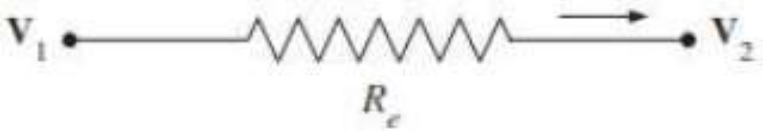
The rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness.

Once the rate of heat conduction is available,
the temperature $T(x)$ at any location x can be determined by replacing T_2 by T, and L by x.

Thermal Resistance Concept

- Just as an electrical resistance is associated with the conduction of electricity, a thermal resistance may be associated with the conduction of heat.
- Defining resistance as the ratio of a driving potential to the corresponding transfer rate,
- For electrical conduction in the same system, Ohm's law provides an electrical resistance of the form

$$R_e = \frac{V_1 - V_2}{I}$$



a. *thermal resistance for conduction in a plane wall is*

$$R_{t,\text{cond}} \equiv \frac{T_{s,1} - T_{s,2}}{q_x} = \frac{L}{kA}$$

b. thermal resistance for convection heat transfer at a surface. (Newton's cooling system)

$$q = hA(T_s - T_\infty)$$

$$R_{t,\text{conv}} \equiv \frac{T_s - T_\infty}{q} = \frac{1}{hA}$$

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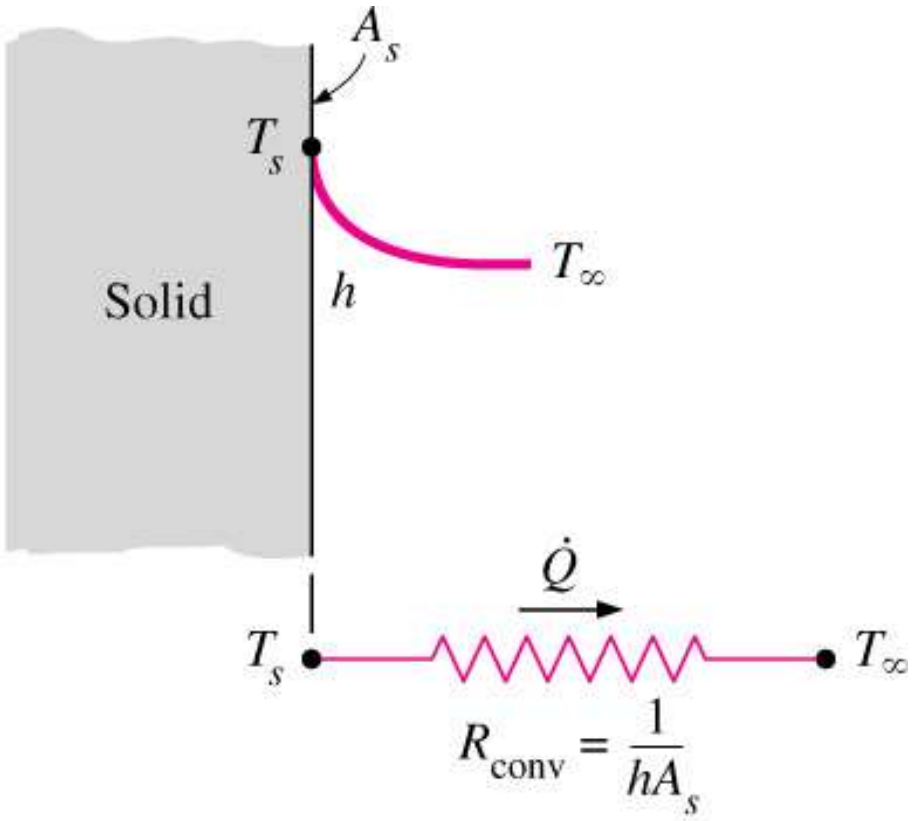
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Consider the convection heat transfer from a surface As at a temperature T_s to a convecting medium with a temperature T_∞ and convection heat transfer coefficient h.

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

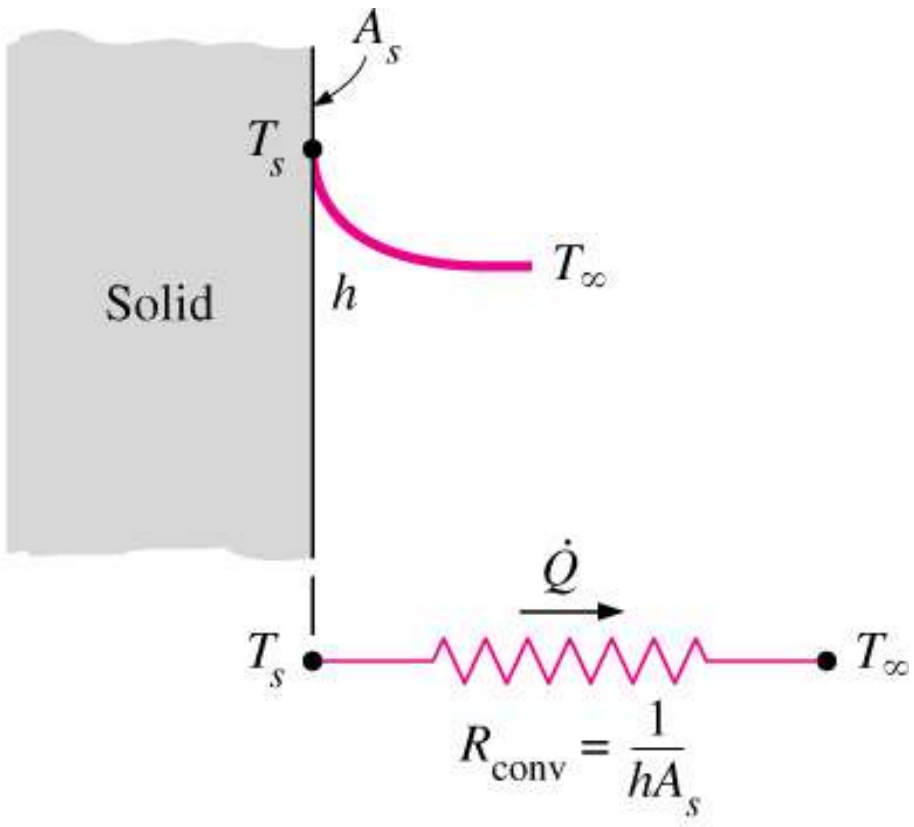
$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}} \qquad \text{(W)}$$

$$R_{\text{conv}} = \frac{1}{hA_s} \qquad (^\circ\text{C/W})$$



Convection Resistance

Convection resistance of the surface, R_{conv}, is the thermal resistance of the surface against heat convection.



When the convection heat transfer coefficient is very large ($h \to \infty$),

the convection resistance becomes *zero* and $T_s \approx T$.

That is, the surface offers no resistance to convection,

and thus, it does not slow down the heat transfer process.

This situation is approached in practice at surfaces where boiling and condensation occur.

In some cases radiation heat transfer from the surface could be important. The radiation heat transfer from a surface with a surface area As and a surface temperature T_{surr} will be

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) = h_{\text{rad}} A_s(T_s - T_{\text{surr}}) = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}}$$

Radiation resistance $$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_s} \quad (\text{K/W})$$

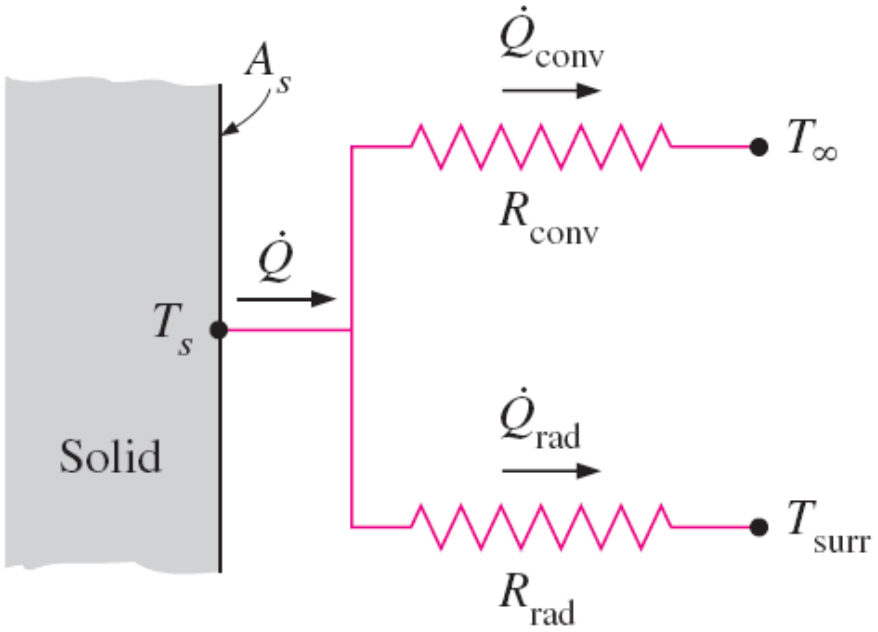
$$h_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{A_s(T_s - T_{\text{surr}})} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \quad (\text{W/m}^2 \cdot \text{K})$$

Radiation heat transfer coefficient

The radiative heat transfer coefficient allows to express radiative heat transfer in terms of a temperature difference, in analogy with convection heat transfer coefficient

Radiation resistance of the surface
is the thermal resistance of the surface against radiation.

When a surface is exposed to combined convection and radiation, we get



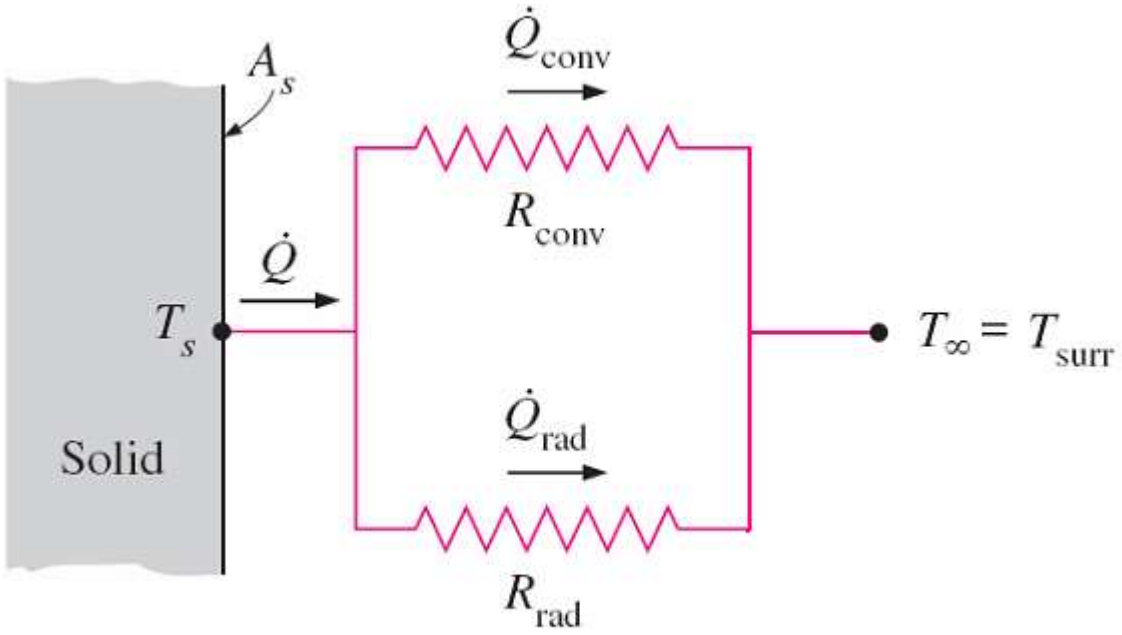
$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad}$$

When

$$T_{surr} \approx T_{\infty}$$

$$h_{combined} = h_{conv} + h_{rad}$$

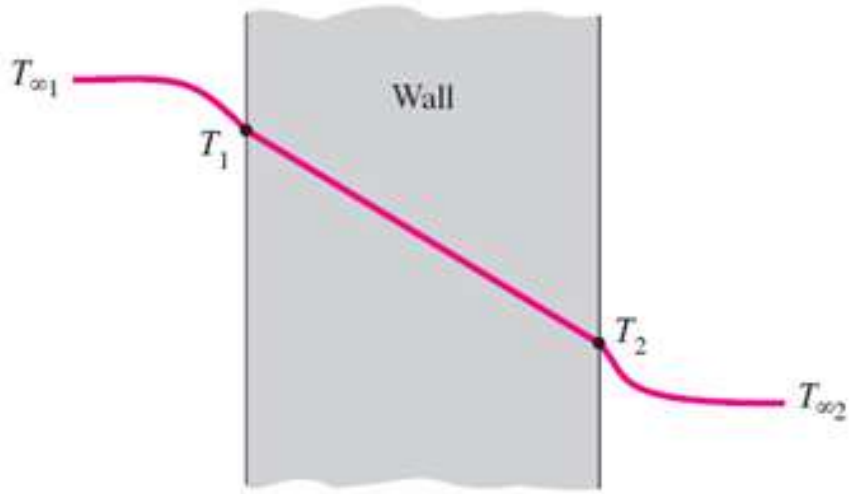
where $h_{combined}$ is the combined heat transfer coefficient.



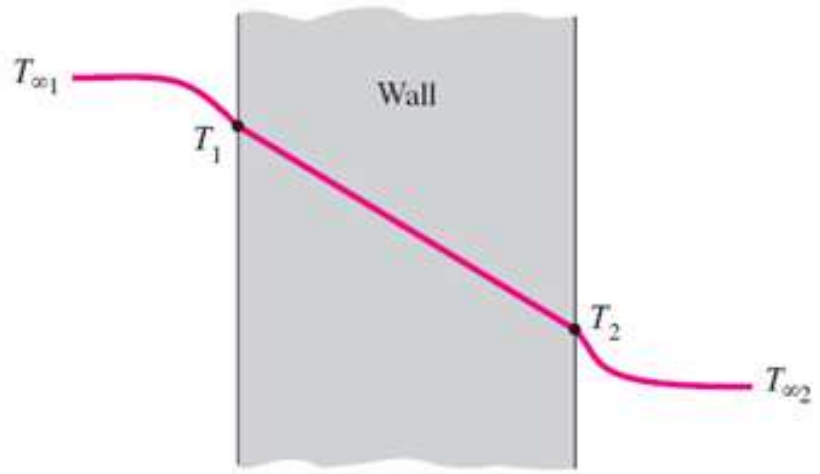
$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad}$$

Thermal Resistance Network

Consider steady one dimensional heat transfer through a plane wall of thickness L, area A, and thermal k and is exposed to convection on both sides to fluids of temperature $T_{\infty 1}$ and $T_{\infty 2}$ with heat transfer coefficients h_1 and h_2, respectively.



Assuming $T_{\infty 2} < T_{\infty 1}$, temperature will vary linearly within the wall and asymptotically approach $T_{\infty 2}$ and $T_{\infty 1}$ in the fluids as we move away from the wall.



Under steady conditions, we have

$$\begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{pmatrix}$$

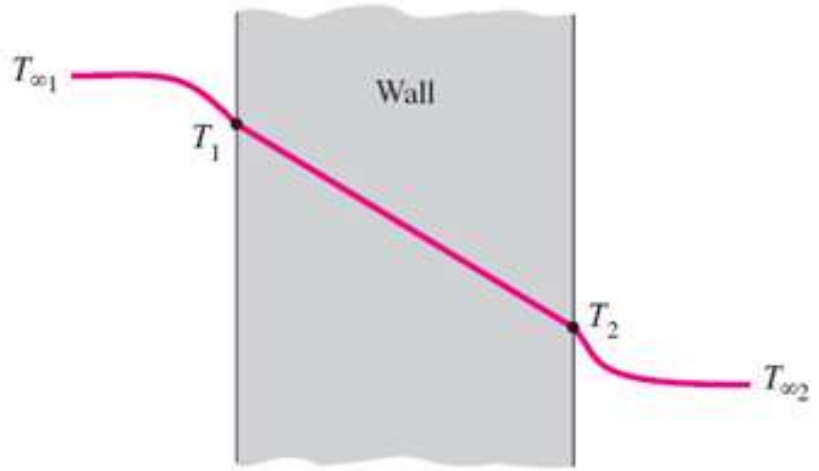
or

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA\frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2})$$

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA\frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2})$$

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

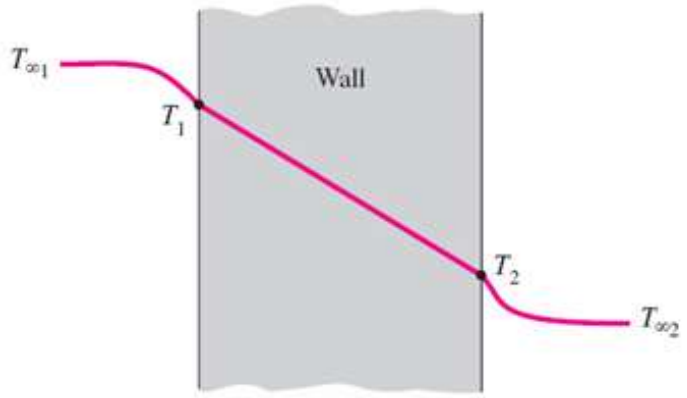
$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}}$$



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (°\text{C/W})$$

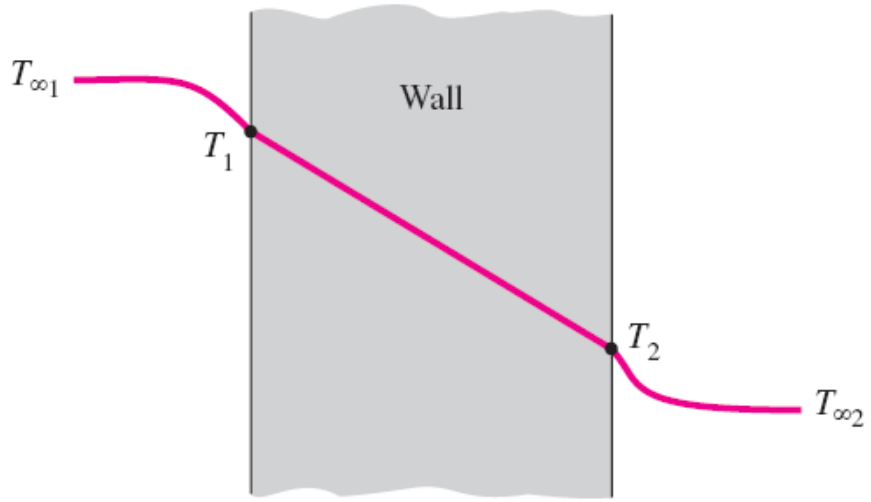
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$



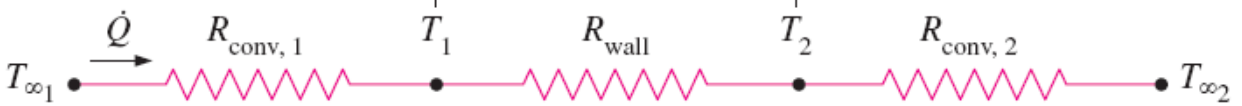
$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (°\text{C/W})$$

Heat transfer surface area is constant for plane wall, and the rate of heat transfer through the wall separating two media is equal to the overall temperature difference $T_{\infty 1} - T_{\infty 2}$ divided by the total thermal resistance between the media.

Note also that the thermal resistances are in series and the equivalent thermal resistance is determined by simply adding the individual resistances, just like the electrical resistances connected in series

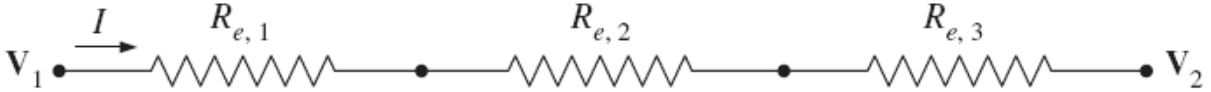


$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}}}$$



Thermal network

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_{e,1} + R_{e,2} + R_{e,3}}$$



Electrical analogy

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

The rate of heat transfer through the wall separating two media is equal to the overall temperature difference $T_{\infty 1} - T_{\infty 2}$ divided by the total thermal resistance between the media

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (^\circ\text{C/W})$$

It is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA\,\Delta T \qquad \text{(W)}$$

U is the overall heat transfer coefficient

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \qquad UA = \frac{1}{R_{\text{total}}}$$

For a unit area, the overall heat transfer coefficient is equal to the reciprocal of the total resistance

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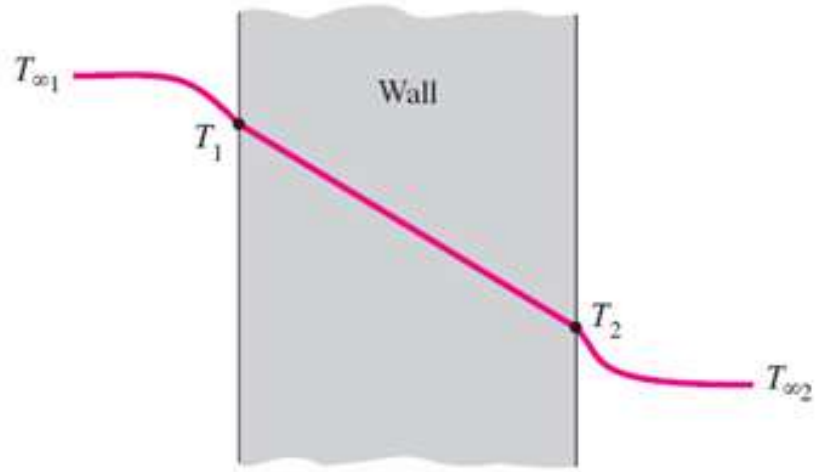
Note that we do not need to know the surface temperatures of the wall in order to evaluate the heat transfer rate through it.

All we need to know is the convection heat transfer coefficients and the fluid temperatures on both sides of the wall.

Once the heat transfer rate is evaluated, the surface temperature of the wall can be determined as

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_{\infty 1} - T_1}{1/h_1 A}$$

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 1}}} = \frac{T_2 - T_{\infty 2}}{1/h_1 A}$$

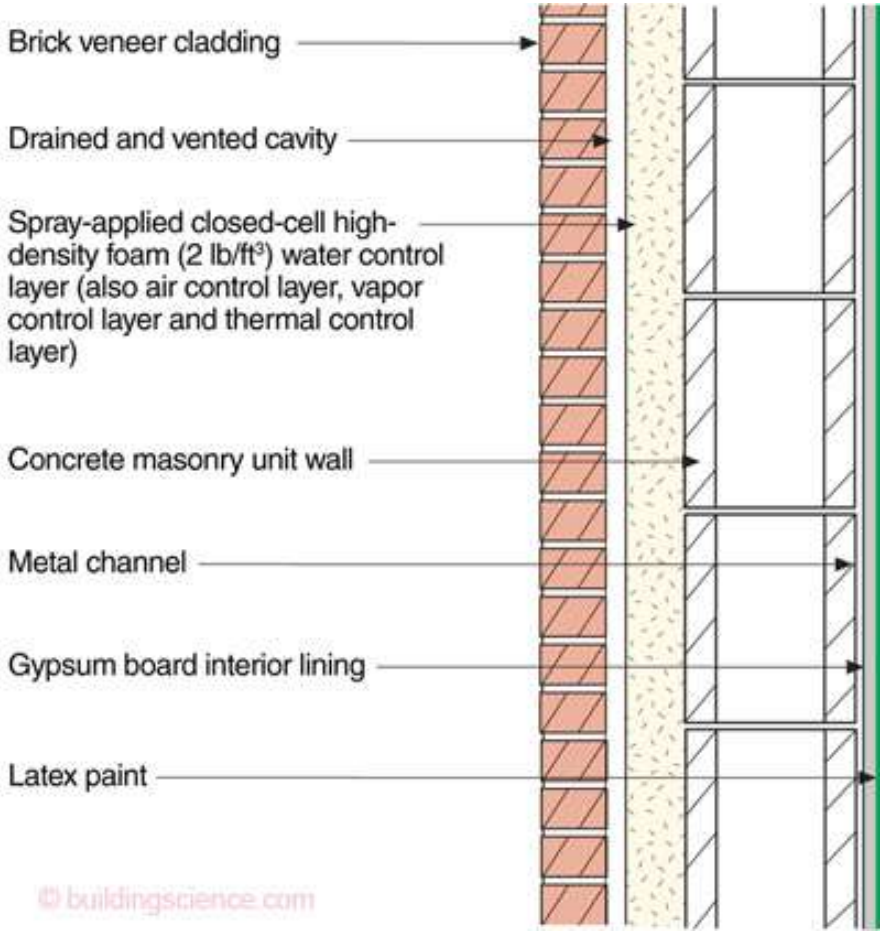


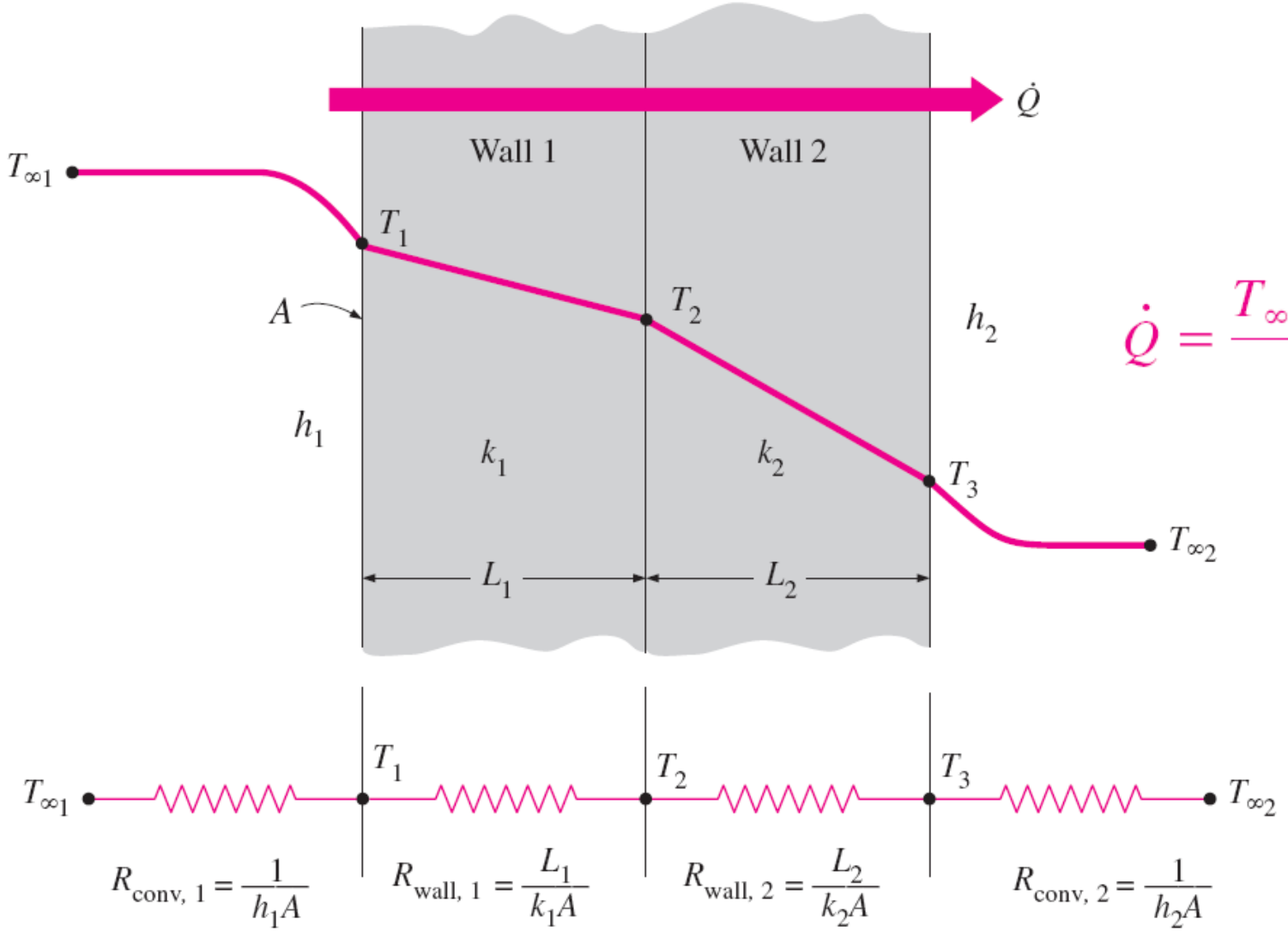
Multilayer Plane Walls

In practice we often encounter plane walls that consist of several layers of different materials.

The thermal resistance concept can still be used to determine the rate of steady heat transfer through such composite walls.

Noting that the conduction resistance L/kA of each wall is connected in series, we can use the electrical analogy to find the heat transfer rate through the wall.





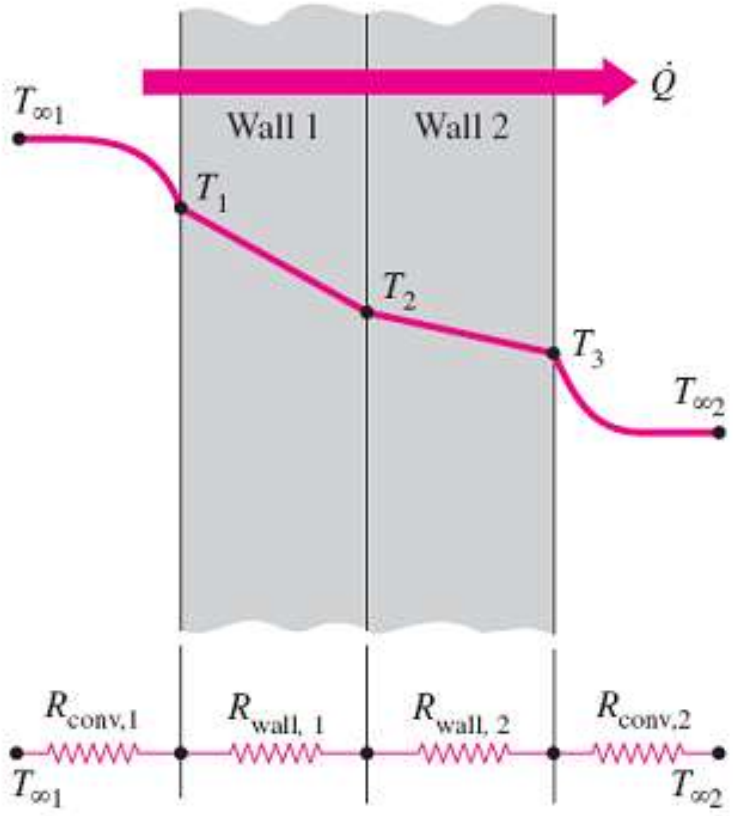
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall, 1}} + R_{\text{wall, 2}} + R_{\text{conv, 2}}$$

$$= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

Once $\dot{Q}$ is known, an unknown surface temperature T_j at any surface or interface j can be determined from

$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total},\, i-j}}$$



T_i is a known surface location i and $R_{\text{total,i-j}}$ is the total resistance between locations i and j.

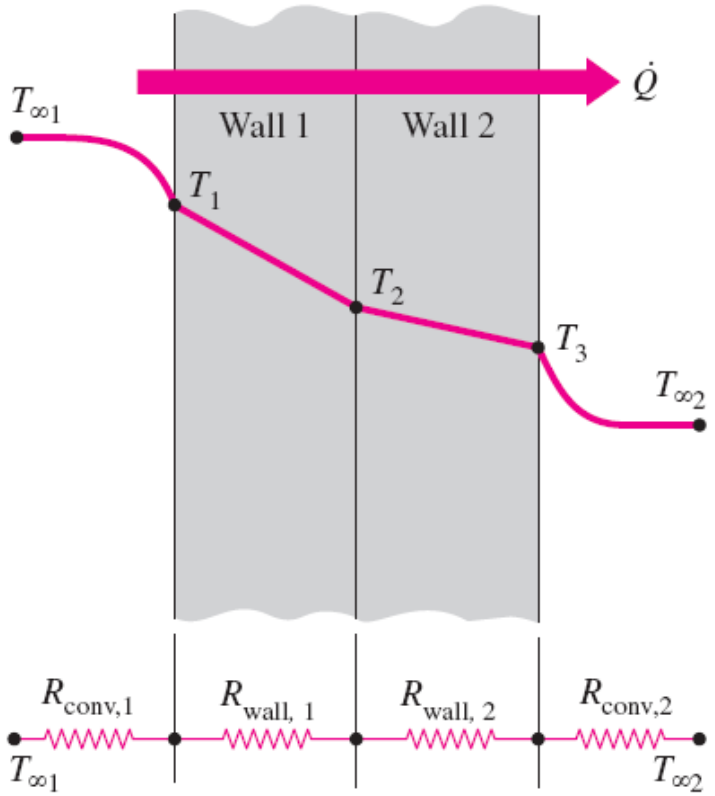
For example, the expression

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},\,1} + R_{\text{wall},\,1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}}$$

will give the interface temperature T_2.

L-08

The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.



To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{conv,1}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{conv,1} + R_{wall,\ 1}}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{conv,2}}$

EXAMPLE 3–1 Heat Loss through a Wall

Consider a 3-m-high, 5-m-wide, and 0.3-m-thick wall whose thermal conductivity is $k = 0.9$ W/m·K (Fig. 3–11). On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 16°C and 2°C, respectively. Determine the rate of heat loss through the wall on that day.

L-08

Assumptions **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.9$ W/m · °C.

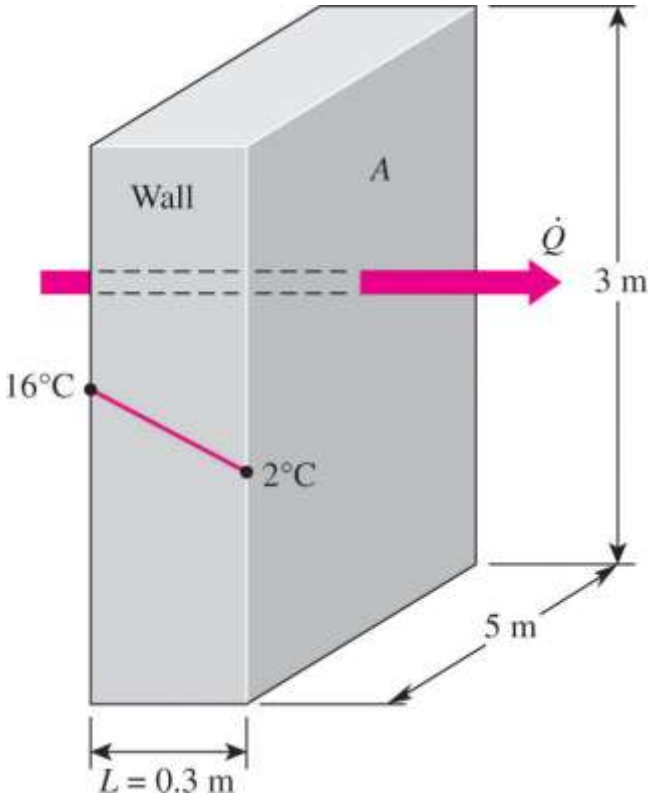
Analysis Noting that the heat transfer through the wall is by conduction and the area of the wall is $A = 3\text{ m} \times 5\text{ m} = 15\text{ m}^2$, the steady rate of heat transfer through the wall can be determined from Eq. 3–3 to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.9\text{ W/m} \cdot \text{°C})(15\text{ m}^2)\frac{(16 - 2)\text{°C}}{0.3\text{ m}} = \mathbf{630\text{ W}}$$

$$\dot{Q} = \frac{\Delta T_{\text{wall}}}{R_{\text{wall}}}$$

$$R_{\text{wall}} = \frac{L}{kA} = \frac{0.3\text{ m}}{(0.9\text{ W/m} \cdot \text{°C})(15\text{ m}^2)} = 0.02222\text{°C/W}$$

$$\dot{Q} = \frac{(16 - 2)\text{°C}}{0.02222\text{°C/W}} = 630\text{ W}$$



EXAMPLE 3–2 Heat Loss through a Single-Pane Window

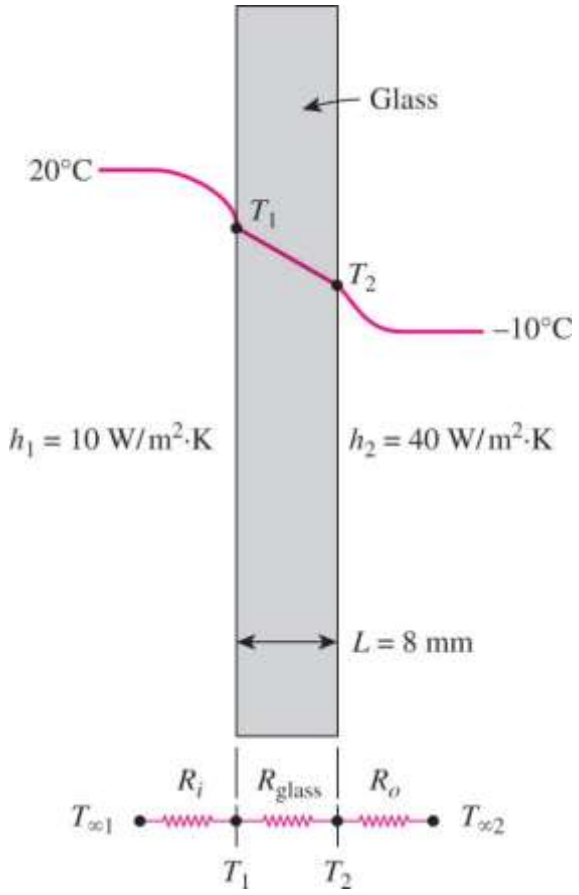
Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m·K. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m²·K and $h_2 = 40$ W/m²·K, which includes the effects of radiation.

SOLUTION Heat loss through a window glass is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

Assumptions 1 Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. 2 Heat transfer through the wall is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. 3 Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.78$ W/m · °C.

Analysis This problem involves conduction through the glass window and convection at its surfaces, and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 3–12. Noting that the area of the window is $A = 0.8\ \text{m} \times 1.5\ \text{m} = 1.2\ \text{m}^2$, the individual resistances are evaluated from their definitions to be



$$R_i = R_{\text{conv},1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.08333{}^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008\ \text{m}}{(0.78\ \text{W/m} \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.00855{}^\circ\text{C/W}$$

$$R_o = R_{\text{conv},2} = \frac{1}{h_2 A} = \frac{1}{(40\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.02083{}^\circ\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{glass}} + R_{\text{conv},2} = 0.08333 + 0.00855 + 0.02083$$
$$= 0.1127{}^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]{}^\circ\text{C}}{0.1127{}^\circ\text{C/W}} = \mathbf{266\ W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q} R_{\text{conv},1}$$
$$= 20{}^\circ\text{C} - (266\ \text{W})(0.08333{}^\circ\text{C/W})$$
$$= \mathbf{-2.2{}^\circ\text{C}}$$

EXAMPLE 3–3 Heat Loss through Double-Pane Windows

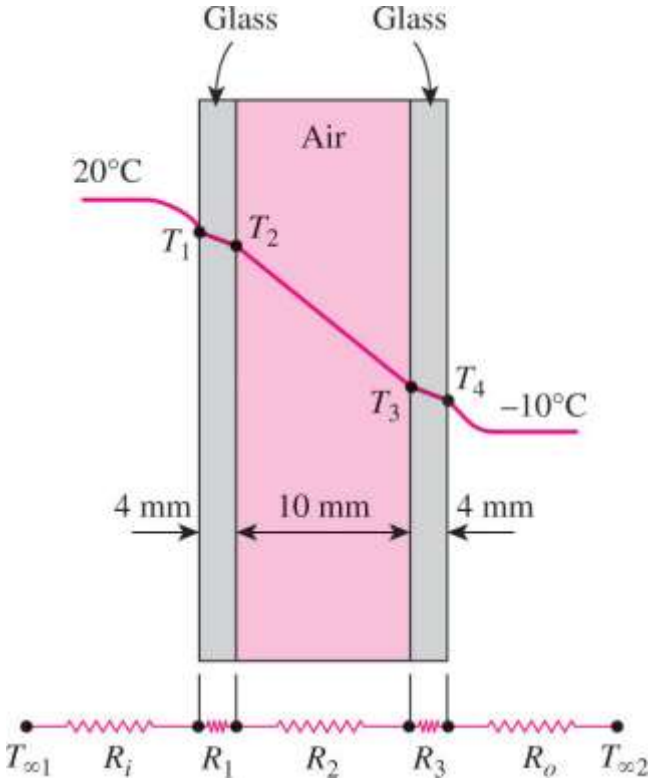
Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass ($k = 0.78$ W/m·K) separated by a 10-mm-wide stagnant air space ($k = 0.026$ W/m·K). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m²·K and $h_2 = 40$ W/m²·K, which includes the effects of radiation.

$$R_i = R_{\text{conv},1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.004\ \text{m}}{(0.78\ \text{W/m} \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.00427^\circ\text{C/W}$$

$$R_2 = R_{\text{air}} = \frac{L_2}{k_2 A} = \frac{0.01\ \text{m}}{(0.026\ \text{W/m} \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.3205^\circ\text{C/W}$$

$$R_o = R_{\text{conv},2} = \frac{1}{h_2 A} = \frac{1}{(40\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.02083^\circ\text{C/W}$$



$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{glass},1} + R_{\text{air}} + R_{\text{glass},2} + R_{\text{conv},2}$$
$$= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083$$
$$= 0.4332^\circ\text{C/W}$$

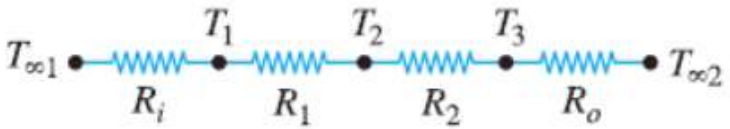
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.4332^\circ\text{C/W}} = 69.2\ \text{W}$$

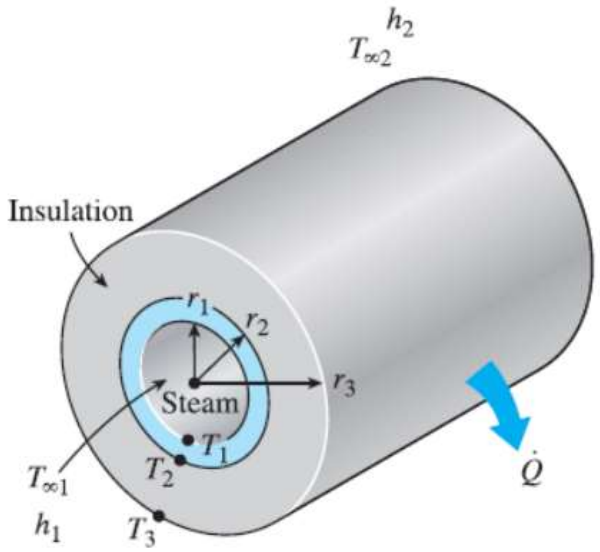


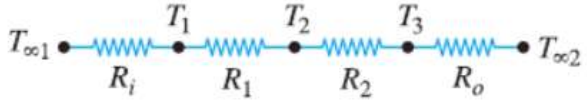


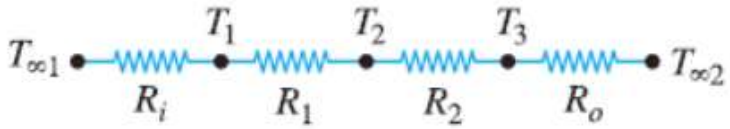
wiseGEEK

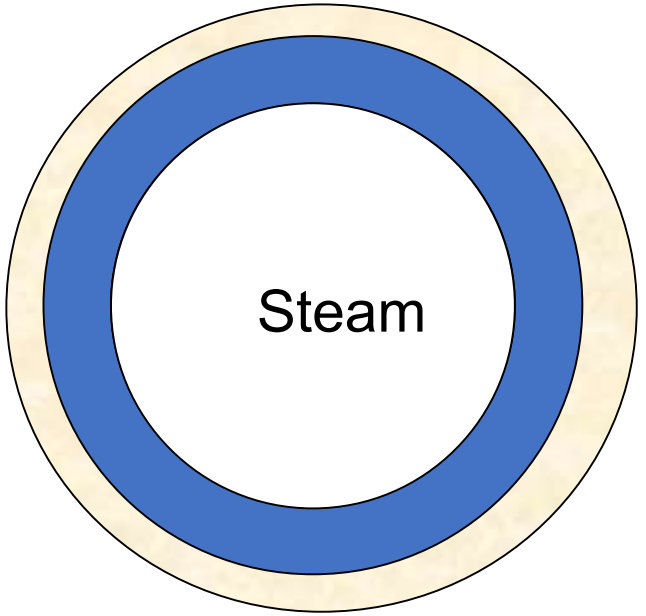
Steam at 320°C flows in a stainless steel pipe (k =15 W/m°C) whose inner and outer diameters are 5 cm and 5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation (k =*0.038* W/m°C*). Heat is lost to the surroundings* at 5°C by convection, with a natural convection heat transfer coefficient of 15 W/m^2°C. Taking the heat transfer coefficient inside the pipe to be 80 W/m^2°C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.











$$A_i = \pi D_i L = \pi(0.05\ \text{m})(1\ \text{m}) = 0.157\ \text{m}^2$$
$$A_o = \pi D_o L = \pi(0.055 + 0.06\ \text{m})(1\ \text{m}) = 0.361\ \text{m}^2$$

The individual thermal resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(80\ \text{W/m}^2.^\circ\text{C})(0.157\ \text{m}^2)} = 0.08\ ^\circ\text{C/W}$$
$$R_1 = R_{pipe} = \frac{\ln(r_2 / r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(15\ \text{W/m.}^\circ\text{C})(1\ \text{m})} = 0.00101\ ^\circ\text{C/W}$$
$$R_2 = R_{insulation} = \frac{\ln(r_3 / r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.038\ \text{W/m.}^\circ\text{C})(1\ \text{m})} = 3.089\ ^\circ\text{C/W}$$
$$R_o = \frac{1}{h_o A_o} = \frac{1}{(15\ \text{W/m}^2.^\circ\text{C})(0.361\ \text{m}^2)} = 0.1847\ ^\circ\text{C/W}$$
$$R_{total} = R_i + R_1 + R_2 + R_o = 0.08 + 0.00101 + 3.089 + 0.1847 = 3.355\ ^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(320-5)^\circ\text{C}}{3.355\ ^\circ\text{C/W}} = \mathbf{93.9\ W}$$

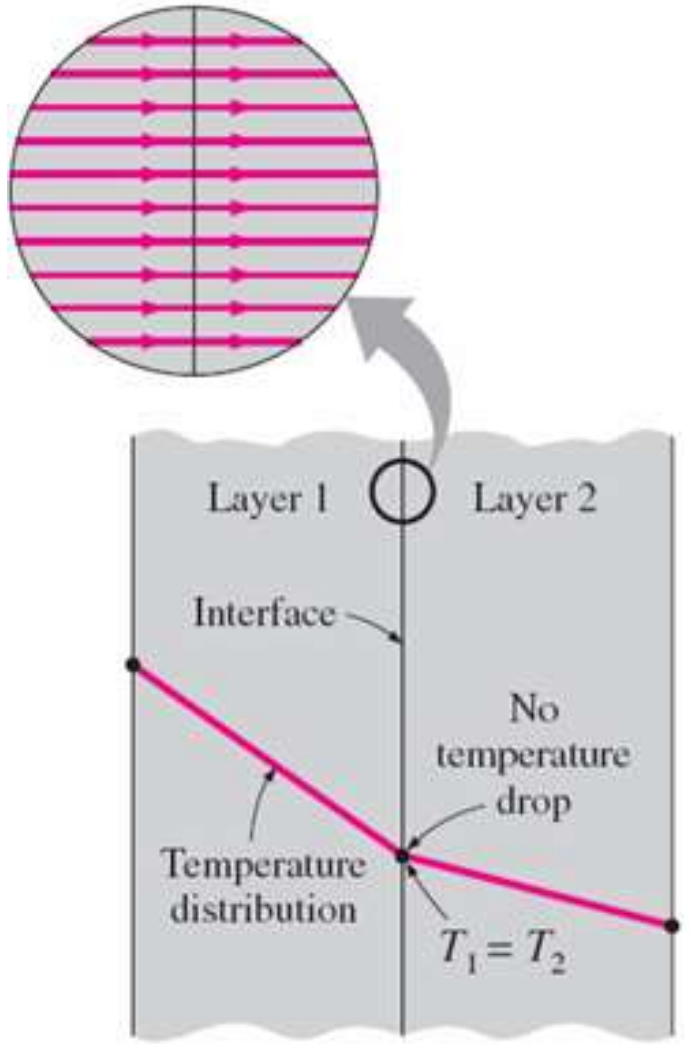
$$\Delta T_{pipe} = \dot{Q} R_{pipe} = (93.9\ \text{W})(0.00101\ ^\circ\text{C/W}) = \mathbf{0.095^\circ C}$$
$$\Delta T_{insulation} = \dot{Q} R_{insulation} = (93.9\ \text{W})(3.089\ ^\circ\text{C/W}) = \mathbf{290^\circ C}$$

THERMAL CONTACT RESISTANCE

In the analysis of heat conduction through multilayer solids,

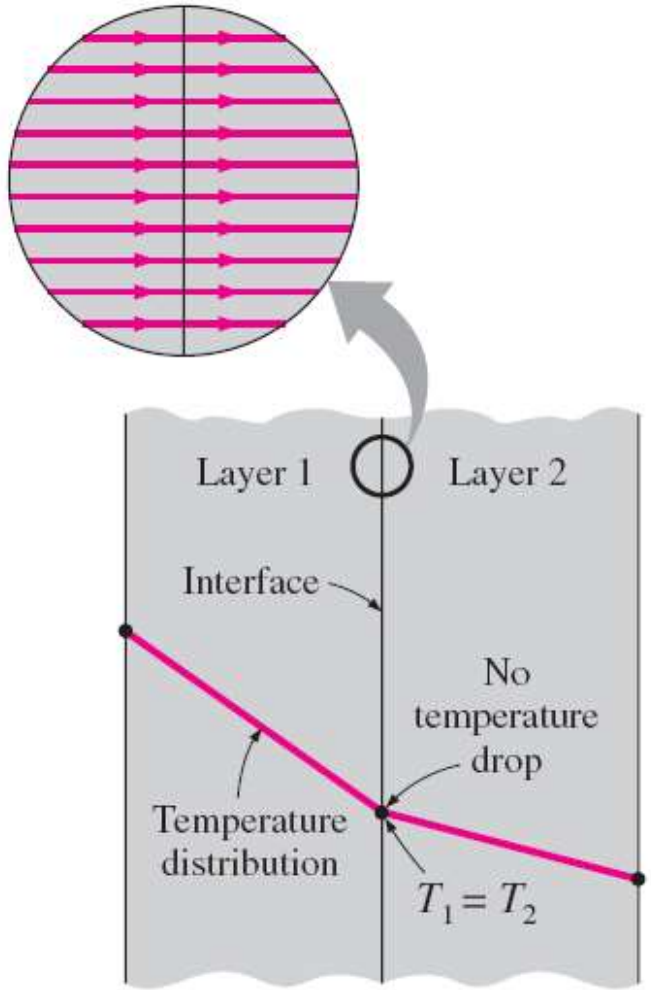
we assumed perfect contact at the interface of the two layers, and thus no temperature drop at the interface.

This would be the case when the surfaces are perfectly smooth and produce a perfect contact at each point.

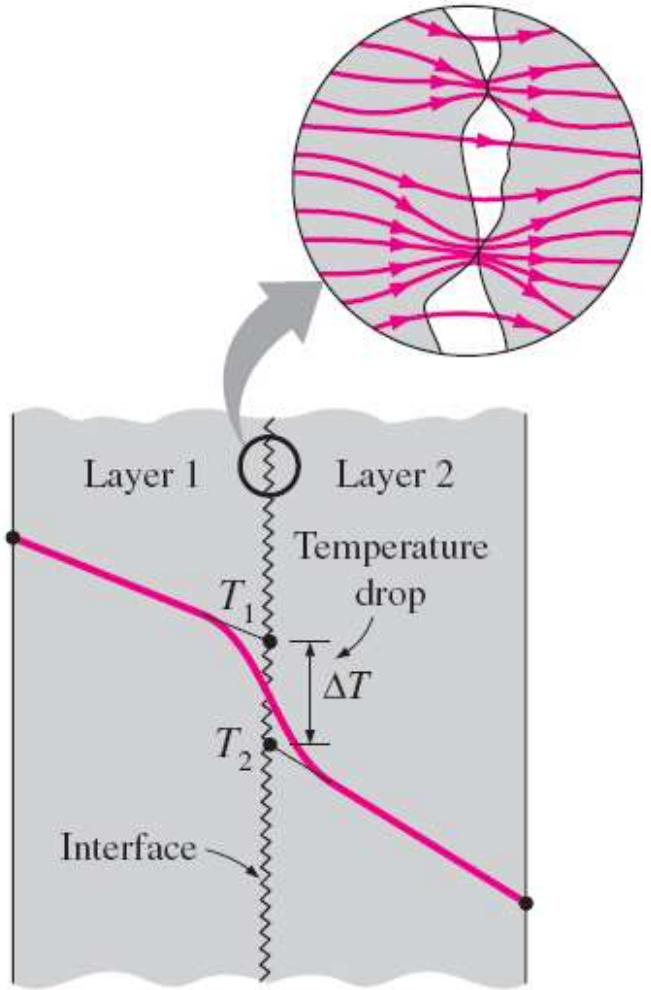


(*a*) Ideal (perfect) thermal contact

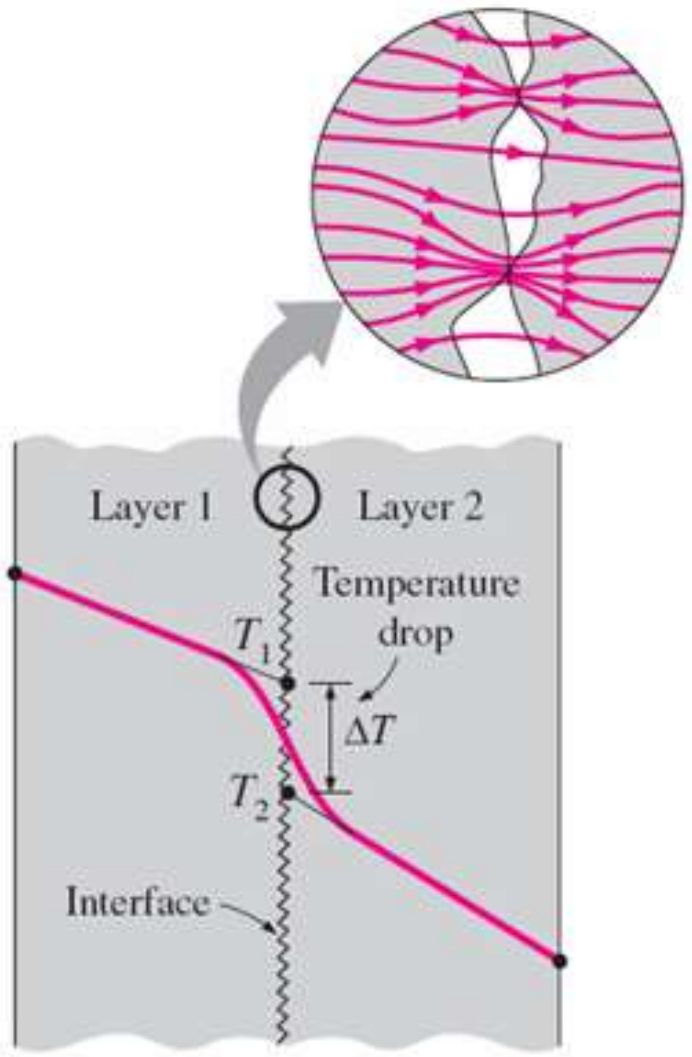
In reality, however, even flat surfaces that appear smooth to the eye turn out to be rather rough when examined under the microscope with numerous peaks and walleys.



(*a*) Ideal (perfect) thermal contact



(*b*) Actual (imperfect) thermal contact



Actual (imperfect) thermal contact

When two surfaces are pressed against each other, the peaks form good material contact but the walleys form voids filled with air in most cases.

As a result, an interface contains numerous air gaps of varying sizes that act as insulation because of the low thermal conductivity of air.

Thus, an interface offers some resistance to heat transfer, and this resistance for a unit interface area is called the thermal resistance, R_c

Consider heat transfer through two metal rods of cross sectional area A that are pressed against each other. Heat transfer through the interface of these two rods is the sum of the heat transfer through the solid contact spots (solid to solid conduction) and the gaps (conduction and or radiation across the gaps) in the noncontact areas (which is a major contributor to heat transfer) and can be expressed as

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}}$$

It can also be expressed in an analogous manner to Newton's law of cooling as

$$\dot{Q} = h_c A \, \Delta T_{\text{interface}}$$

where A is the apparent surface area (which is the same as the cross-sectional areas of the rod) and $\Delta T_{\text{interface}}$ is the effective temperature difference at the interface. h_c, which corresponds to the convection heat transfe coefficient is called the thermal conductance.

Thermal contact conductance $h_c = \dfrac{\dot{Q}/A}{\Delta T_{\text{interface}}}$ $(\text{W/m}^2 \cdot {}^\circ\text{C})$

$$\dot{Q} = h_c A \, \Delta T_{\text{interface}}$$

Thermal contact resistance $R_c = \dfrac{1}{h_c} = \dfrac{\Delta T_{\text{interface}}}{\dot{Q}/A}$ $(\text{m}^2 \cdot {}^\circ\text{C/W})$

Thermal contact resistance is the reciprocal of the thermal contact conductance

Usually, the thermal contact conductance is reported in the literature, but the concept of thermal contact resistance serves as a better vehicle for explaining the effect of interface on heat transfer.

Note that R_c represents thermal contact resistance of unit area. The thermal resistance of the entire interface is obtained by dividing R_c by the apparent surface area.

The value of thermal contact resistance depends on:

surface roughness,
material properties,
temperature and pressure at the interface,
type of fluid trapped at the interface,

The situation become more complex when plates are fastened by bolts or screws, or rivets, since the interface pressure in this case is not uniform.

The thermal conductance in this case will also depend on

the plate thickness,
the bolt radius
and the size of the contact zone.

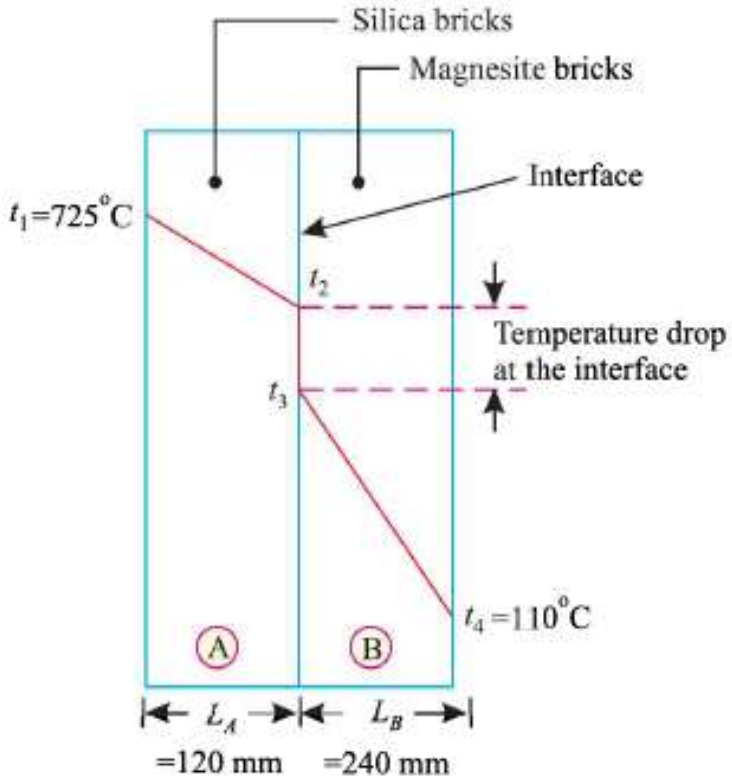
Example 2.5. *A wall of a furnace is made up of inside layer of silica brick 120 mm thick covered with a layer of magnesite brick 240 mm thick. The temperatures at the inside surface of silica brick wall and outside surface of magnesite brick wall are 725°C and 110°C respectively. The contact thermal resistance between the two walls at the interface is 0.0035°C/W per unit wall area. If thermal conductivities of silica and magnesite bricks are 1.7 W/m°C and 5.8 W/m°C, calculate.*

(*i*) *The rate of heat loss per unit area of walls, and*

(*ii*) *The temperature drop at the interface.*

L_A = 120 mm = 0.12 m;

L_B = 240 mm = 0.24 m;



The rate of heat loss per unit area of wall,

$$q = \frac{\Delta t}{\Sigma R_{th}} = \frac{\Delta t}{R_{th-A} + (R_{th})_{cont.} + R_{th-B}}$$

$$= \frac{(t_1 - t_4)}{L_A / k_A + 0.0035 + L_B / k_B}$$

$$= \frac{(725 - 110)}{0.12/1.7 + 0.0035 + 0.24/5.8}$$

$$= \frac{615}{0.0706 + 0.0035 + 0.0414}$$

$$= 5324.67 \text{ W/m}^2$$

The temperature drop at the interface, $(t_2 - t_3)$:

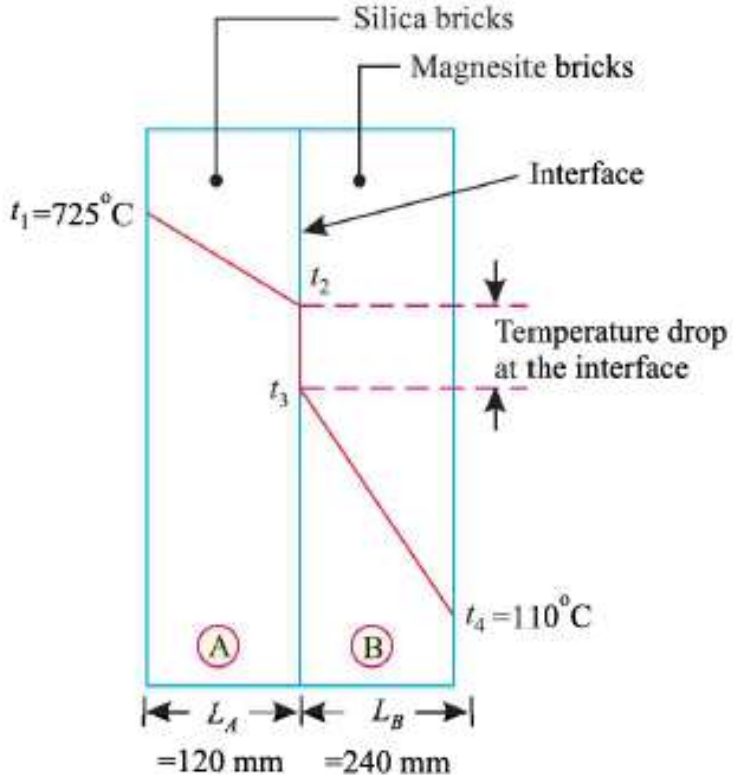
$$q = \frac{t_1 - t_2}{L_A / k_A} = \frac{t_3 - t_4}{L_B / k_B}$$

$$5324.67 = \frac{(725 - t_2)}{0.12/1.7}$$

$$5324.67 = \frac{(t_3 - 110)}{0.24/5.8}$$

Hence, the temperature drop at the interface = $t_2 - t_3$

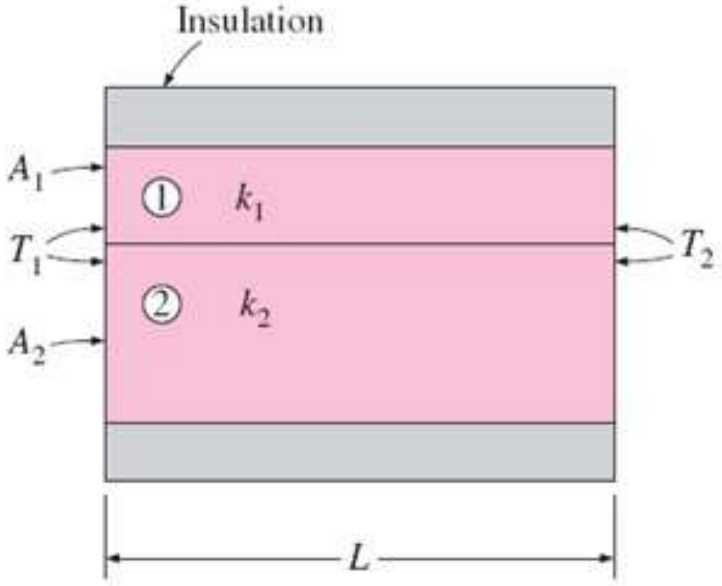
= 349.14 – 330.33 = **18.81°C** **(Ans.)**

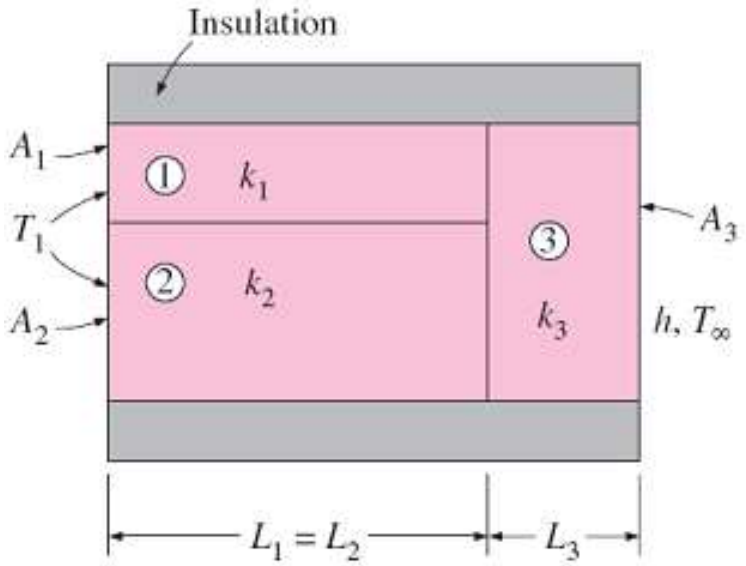


GENERALIZED THERMAL RESISTANCE NETWORKS

The thermal resistance concept or the electrical analogy can also be used to solve heat transfer problems that involve parallel layers or combined serial parallel arrangements.

Although such problems are often two- or even three-dimensional approximate solutions can be obtained by assuming one dimensional heat transfer and using the thermal resistance network.



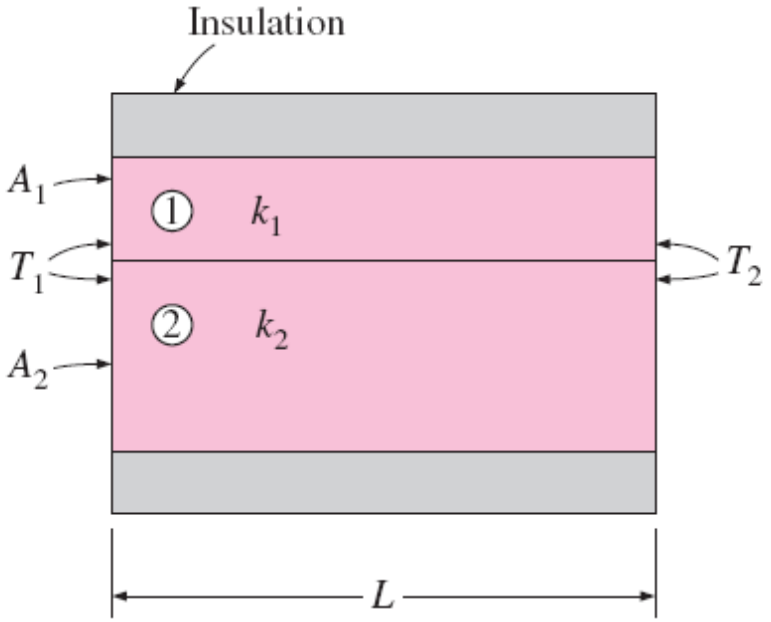


Consider a composite wall which consists of two parallel layers.

The total heat transfer is the sum of the heat transfer through each layer

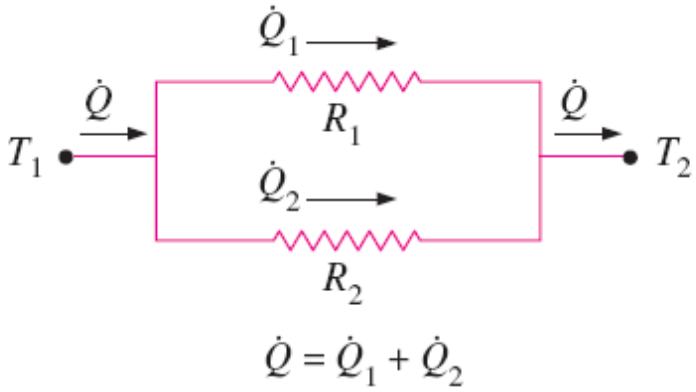
$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2}$$

$$= (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$



$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}}$$

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$



$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2$$

Now consider a combined series-parallel arrangement. The total rate of heat transfer through this composite system can be expressed as

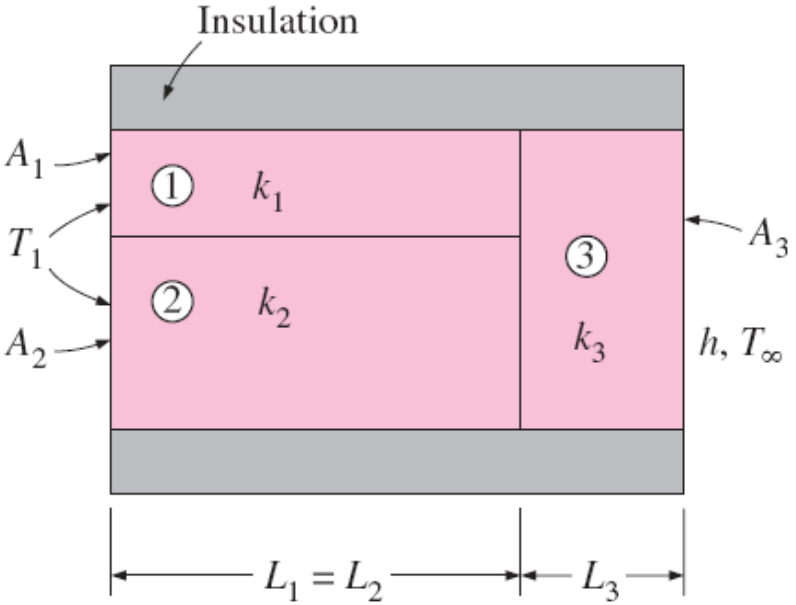
$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}}$$

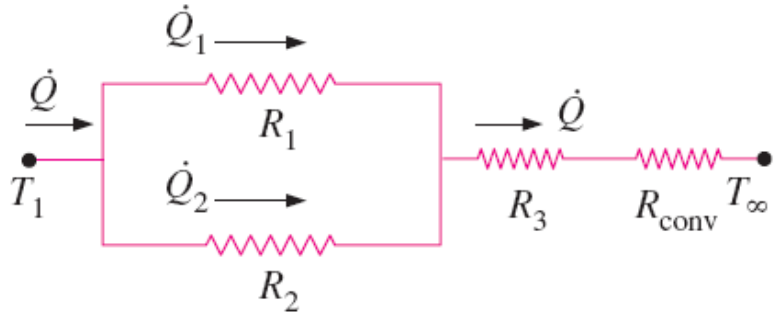
$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}}$$

$$= \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}}$$

$$R_1 = \frac{L_1}{k_1 A_1} \qquad R_2 = \frac{L_2}{k_2 A_2}$$

$$R_3 = \frac{L_3}{k_3 A_3} \qquad R_{\text{conv}} = \frac{1}{hA_3}$$



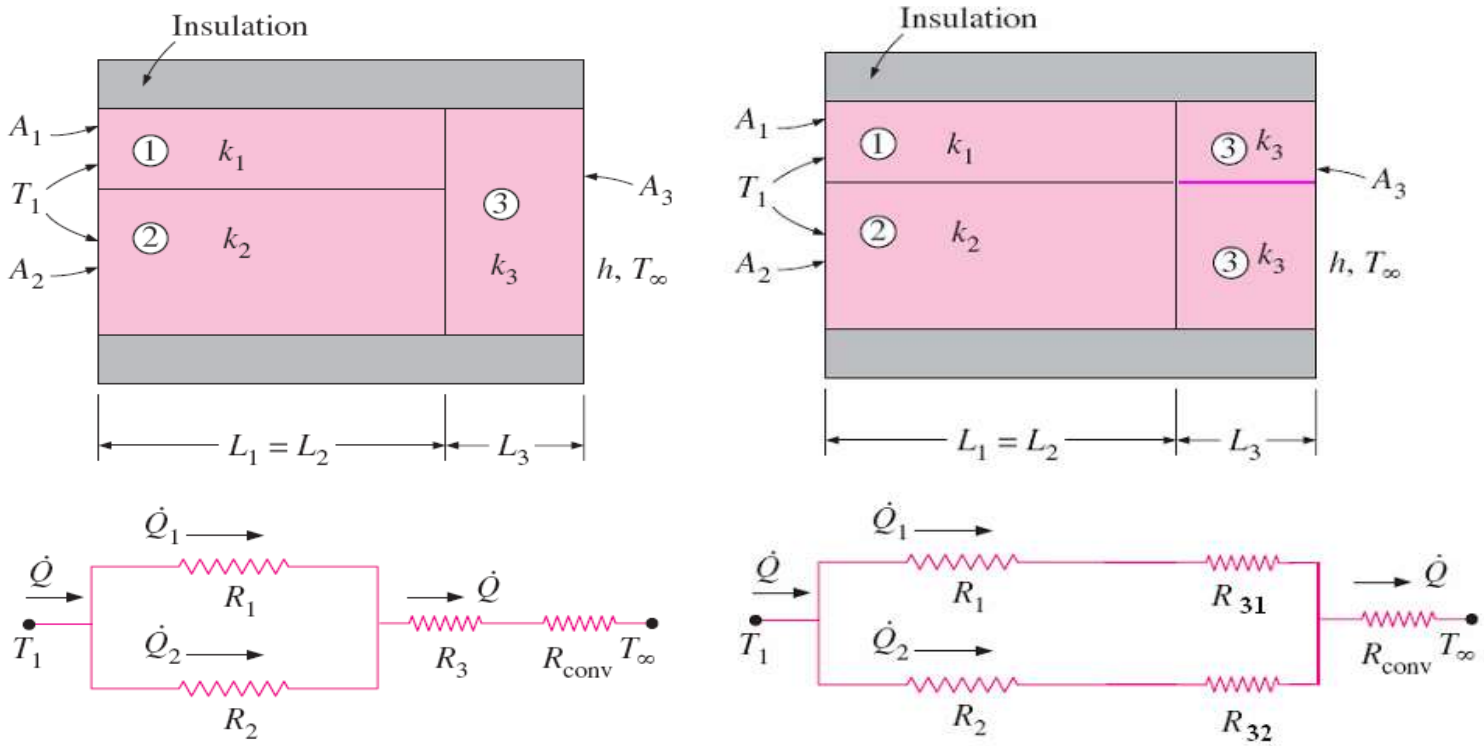


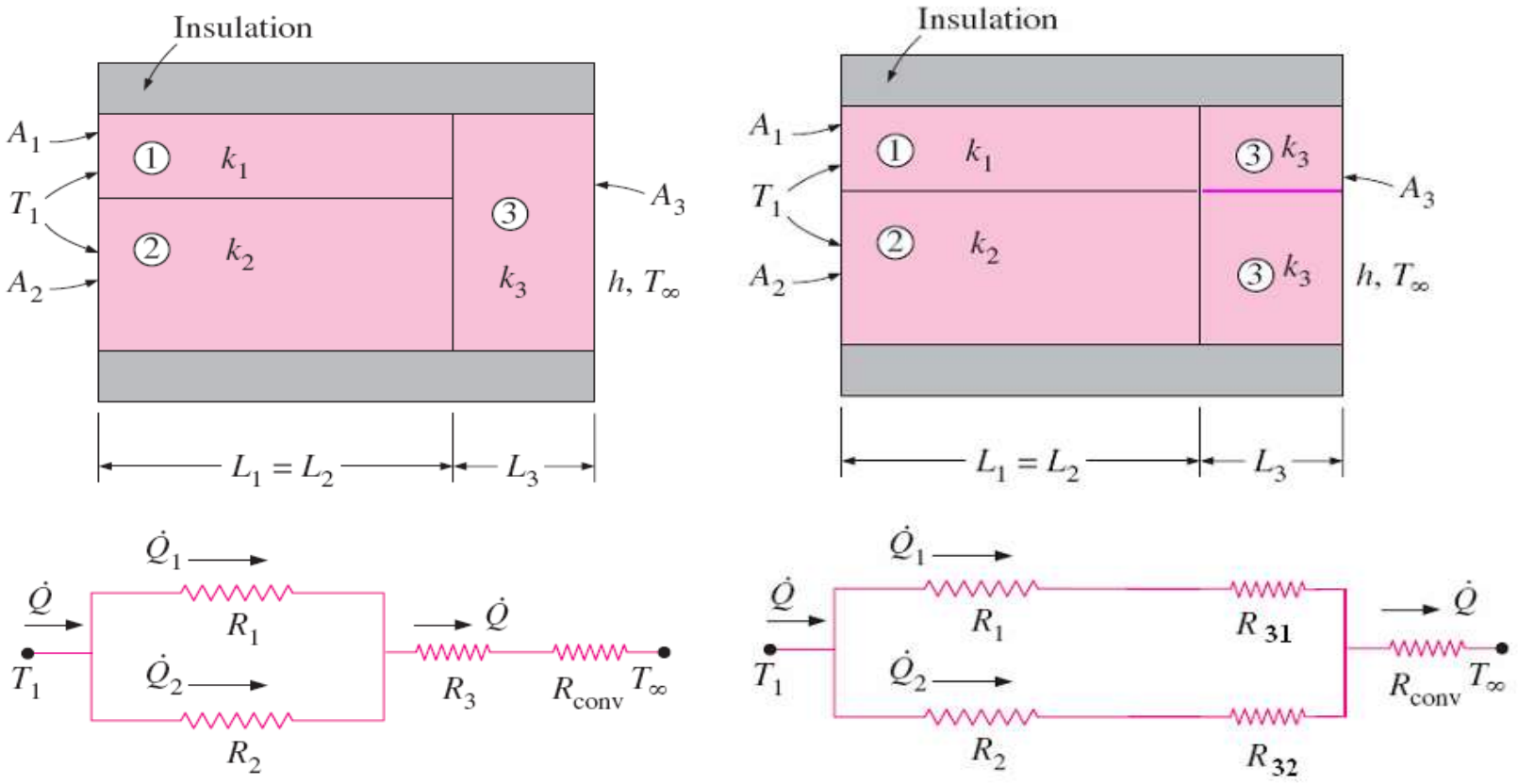
Two assumptions in solving complex multidimensional heat transfer problems by treating them as one-dimensional using the thermal resistance network are

(1)any plane wall normal to the *x*-axis is *isothermal* (i.e., to assume the temperature to vary in the *x*-direction only)

(2)any plane parallel to the *x*-axis is *adiabatic* (i.e., to assume heat transfer to occur in the *x*-direction only)

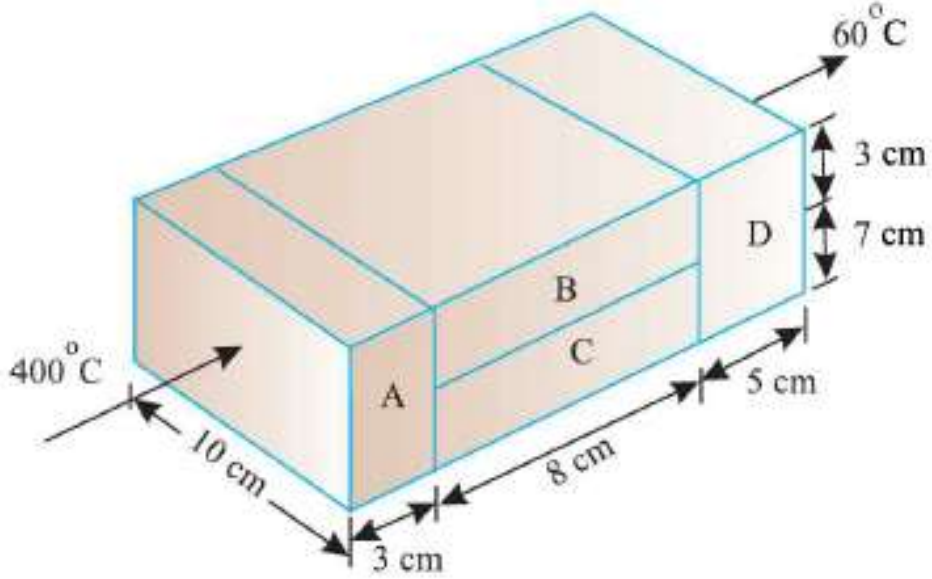
Do they give the same result?



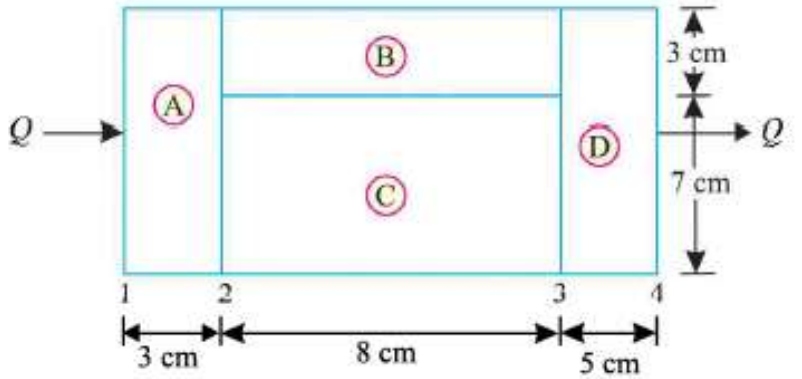


These two assumptions result in different networks and thus different (but usually close) values for the total thermal resistance and thus heat transfer. The actual result lies between these values. In geometries in which heat transfer occurs predominantly in one direction, each approach gives satisfactory results.

L-10



$k_A = 150\ W/m°C,$
$k_B = 30\ W/m°C,$
$k_C = 65\ W/m°C,\ and$
$k_D = 50\ W/m°C$



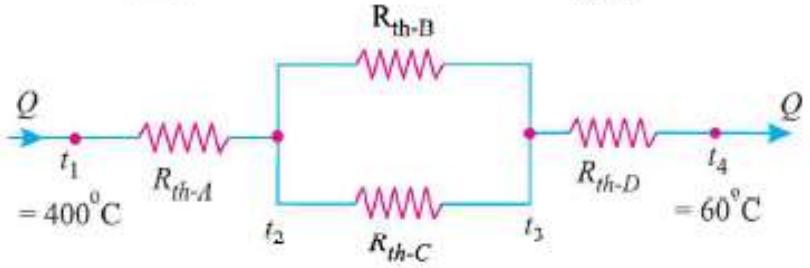
Thickness :

$L_A = 3\ cm = 0.03\ m;\ L_B = L_C = 8\ cm = 0.08\ m;\ L_D = 5\ cm = 0.05\ m$

Areas :

$A_A = 0.1 \times 0.1 = 0.01 m^2;\quad A_B = 0.1 \times 0.03 = 0.003 m^2$

$A_C = 0.1 \times 0.07 = 0.007 m^2;\quad A_D = 0.1 \times 0.1 = 0.01 m^2$



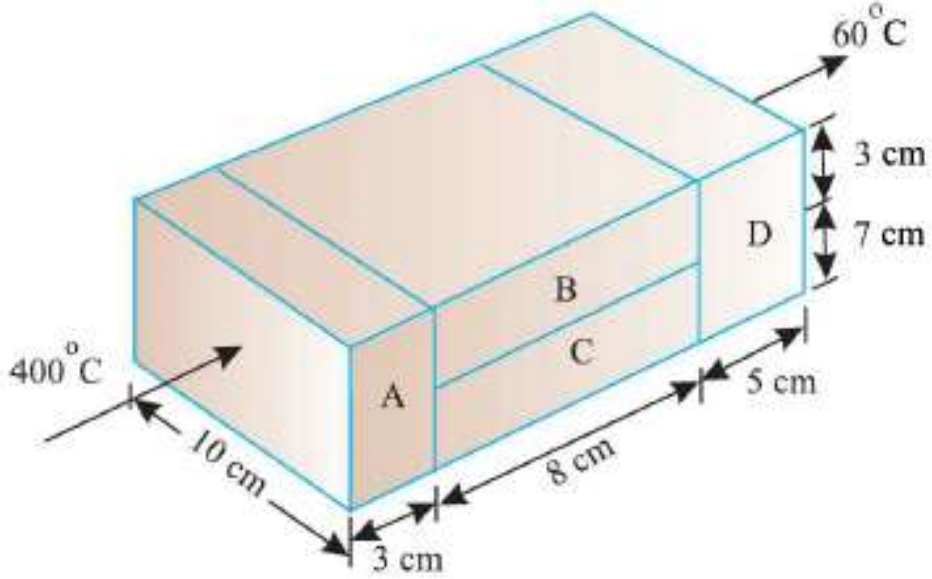
$$R_{th-A} = \frac{L_A}{k_A A_A} = \frac{0.03}{150 \times 0.01} = 0.02$$

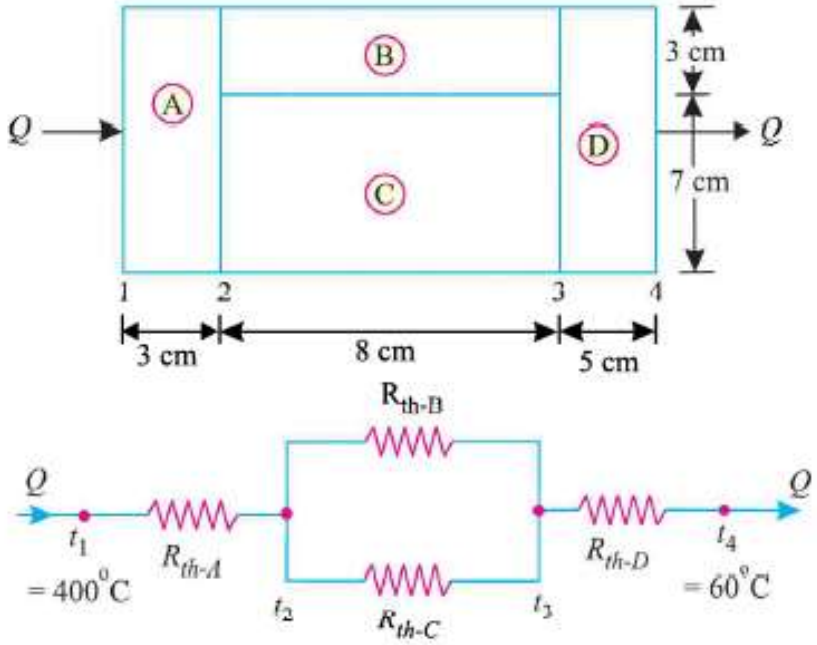
$$R_{th-B} = \frac{L_B}{k_B A_B} = \frac{0.08}{30 \times 0.003} = 0.89$$

$$R_{th-C} = \frac{L_C}{k_C A_C} = \frac{0.08}{65 \times 0.007} = 0.176$$

$$R_{th-D} = \frac{L_D}{k_D A_D} = \frac{0.05}{50 \times 0.01} = 0.1$$

L-10





$$\frac{1}{(R_{th})_{eq}} = \frac{1}{R_{th-B}} + \frac{1}{R_{th-C}} = \frac{1}{0.89} + \frac{1}{0.176} = 6.805$$

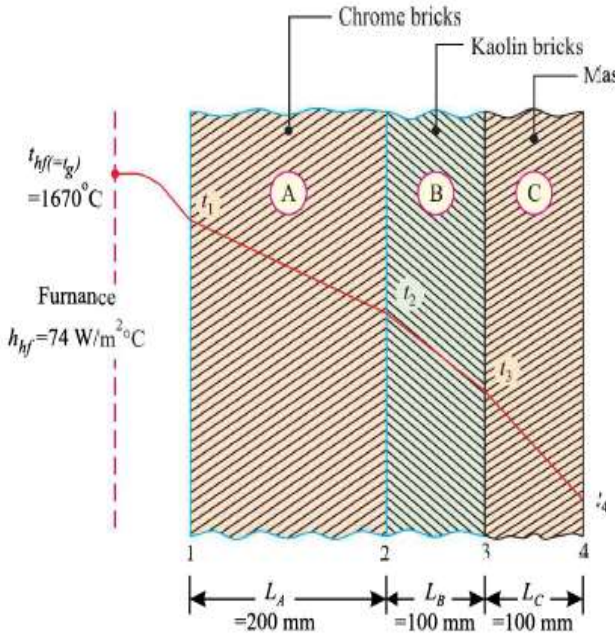
$$(R_{th})_{eq.} = \frac{1}{6.805} = 0.147$$

$$(R_{th})_{\text{total}} = R_{th-A} + (R_{th})_{\text{eq.}} + R_{th-D} = 0.02 + 0.147 + 0.1 = 0.267$$

$$Q = \frac{(\Delta t)_{overall}}{(R_{th})_{total}} = \frac{(400 - 60)}{0.267} = \mathbf{1273.4\,W\ (Ans.)}$$

Calculate the rate of heat flow per m^2 through a furnace wall consisting of 200 mm thick inner layer of chrome brick, a centre layer of kaolin brick 100 mm thick and an outer layer of masonry brick 100 mm thick. The unit surface conductance at the inner surface is 74 $W/m^2°C$ and the outer surface temperature is 70°C. The temperature of the gases inside the furnace is 1670°C. What temperatures prevail at the inner and outer surfaces of the centre layer ?

Take : $k_{chrome\ brick} = 1.25\ W/m°C$; $k_{kaolin\ brick} = 0.074\ W/m°C$; $k_{masonry\ brick} = 0.555\ W/m°C$



Solution.

Thickness of chrome bricks, $L_A = 200\ mm = 0.2\ m$

Thickness of kaolin bricks, $L_B = 100\ mm = 0.1\ m$

Thickness of masonry bricks, $L_C = 100\ mm = 0.1m$

Thermal conductivities : $k_A = 1.25\ W/m°C$;

$k_B = 0.074\ W/m°C$; $k_C = 0.555\ W/m°C$;

The unit surface conductance, $h_{hf} = 74\ W/m^2°C$

Temperature of hot fluid, $t_{hf}\ (= t_g) = 1670°C$

Temperature of the outer surface, $t_4 = 70°C$

(*i*) Rate of heat flow per m^2, q :

$$q = \frac{(t_{hf} - t_4)}{\frac{1}{h_{hf}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}}$$

(*i*) Rate of heat flow per m², q :

$$q = \frac{(t_{hf} - t_4)}{\frac{1}{h_{hf}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}}$$

$$= \frac{(1670 - 70)}{\frac{1}{74} + \frac{0.2}{1.25} + \frac{0.1}{0.074} + \frac{0.1}{0.555}}$$

$$= \frac{1600}{0.0135 + 0.16 + 1.351 + 0.1802}$$

$$= \mathbf{938.58\ W/m^2} \qquad \textbf{(Ans.)}$$

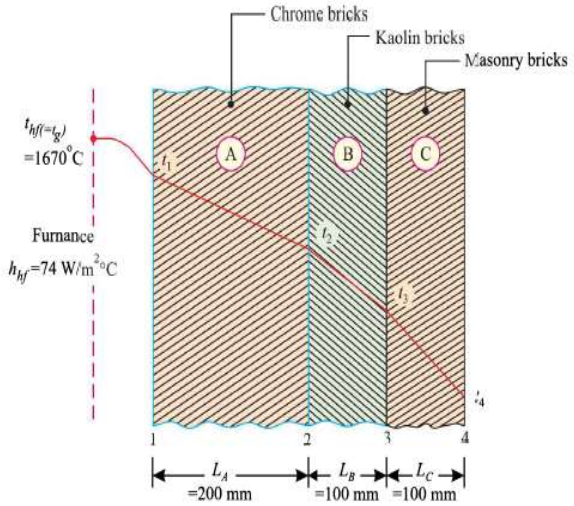
(*ii*) Temperatures; t_2, t_3 :

The heat flow is given by

$$q = \frac{(t_{hf} - t_1)}{1/h_{hf}} = \frac{(t_1 - t_2)}{L_A/k_A} = \frac{(t_2 - t_3)}{(L_B/k_B)}$$

$$\therefore \qquad 938.58 = \frac{1670 - t_1}{1/74}$$

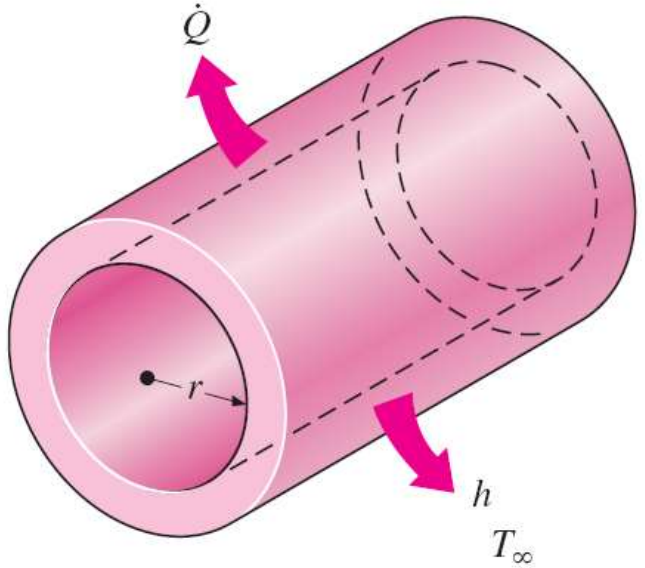
$$t_1 = 1670 - 938.58 \times \frac{1}{74} = 1657.3°\text{C}$$



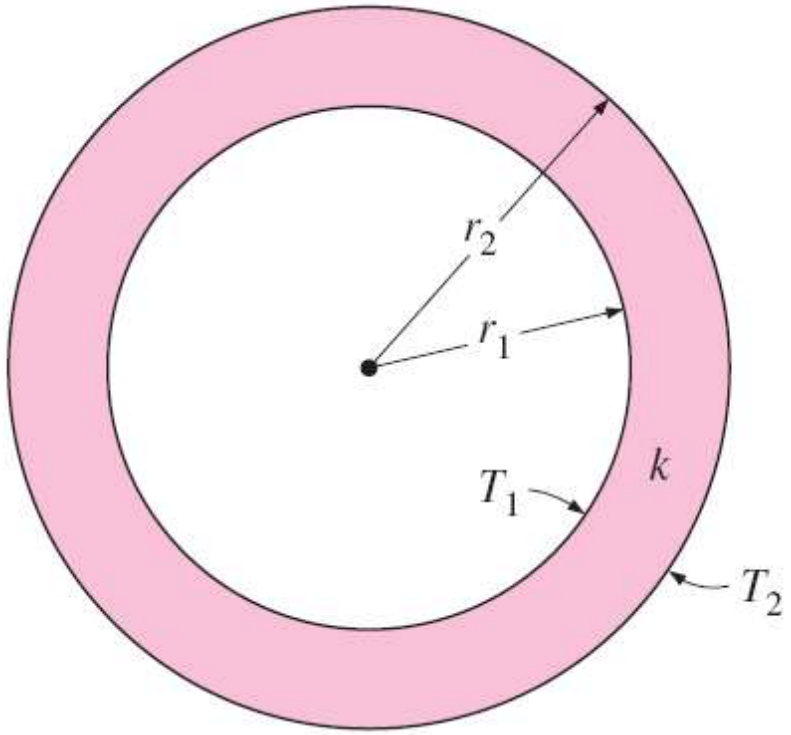
Similarly, $938.58 = \frac{(1657.3 - t_2)}{0.2/1.25}$ or $t_2 = 1657.3 - 938.58 \times \frac{0.2}{1.25} = \mathbf{1507.1°\,C}$ **(Ans)**

$938.58 = \frac{(1507.1 - t_3)}{(0.1/0.074)}$ or $t_3 = 1507.1 - 938.58 \times \frac{0.1}{0.074} = \mathbf{238.7°\,C}$ **(Ans.)**

HEAT CONDUCTION IN CYLINDERS AND SPHERES



Heat transfer through the pipe can be modeled as steady and one-dimensional.
The temperature of the pipe depends on one direction only (the radial *r*-direction) and can be expressed as T = T(r).
The temperature is independent of the azimuthal angle or the axial distance. This situation is approximated in practice in long cylindrical pipes and spherical containers.



$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \qquad \text{(W)}$$

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\,dr = -\int_{T=T_1}^{T_2} k\,dT$$

$$A = 2\pi rL$$

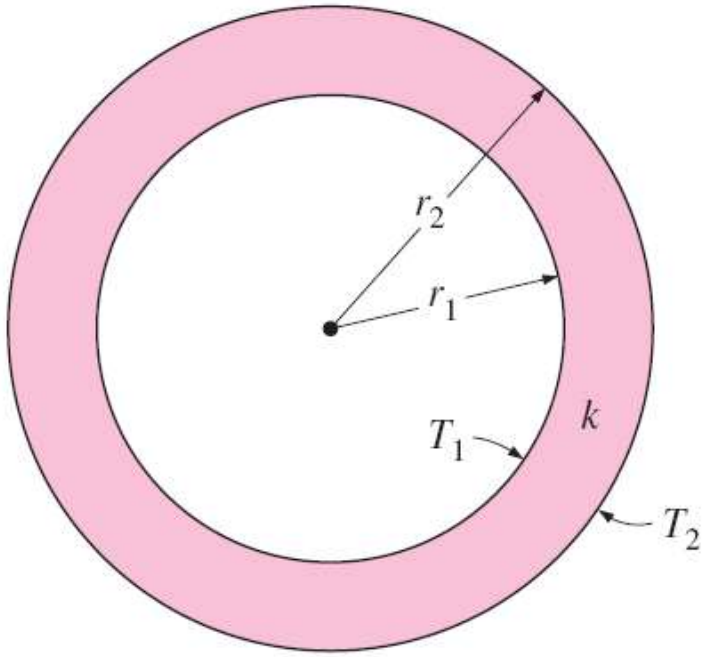
$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \text{(W)}$$

A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \qquad \text{(W)}$$

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times \text{Length} \times \text{Thermal conductivity}}$$

is the conduction resistance of the cylinder layer.



A spherical shell with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{\text{cond, sph}} = \frac{T_1 - T_2}{R_{\text{sph}}}$$

$$R_{\text{sph}} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$$

is the conduction resistance of the spherical layer.

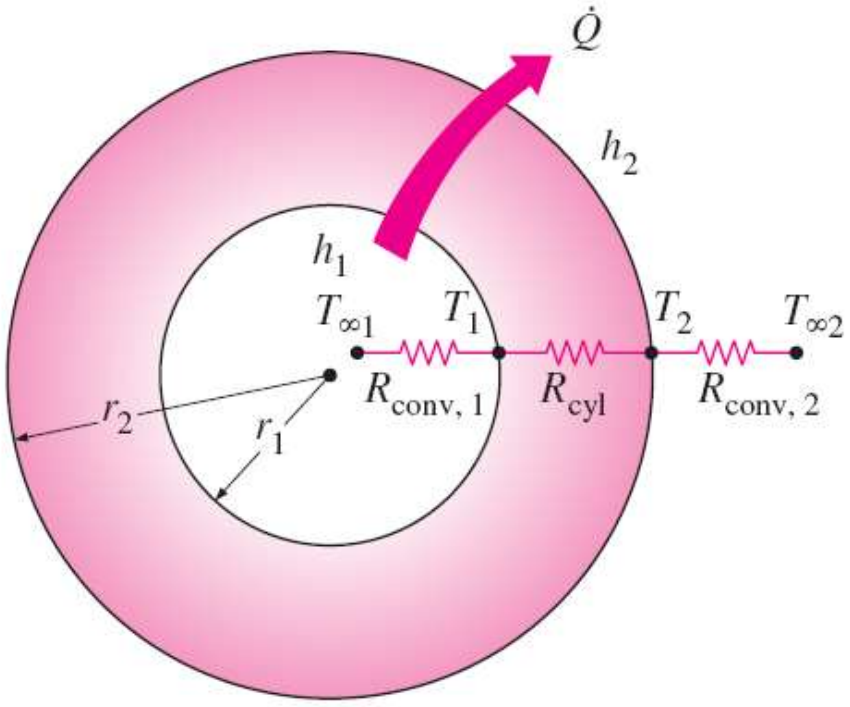
The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

where

$$\boxed{\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{cyl}} + R_{\text{conv, 2}} \\ &= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2} \end{aligned}}$$

for a *cylindrical* layer, and



$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{cyl}} + R_{\text{conv, 2}}$$

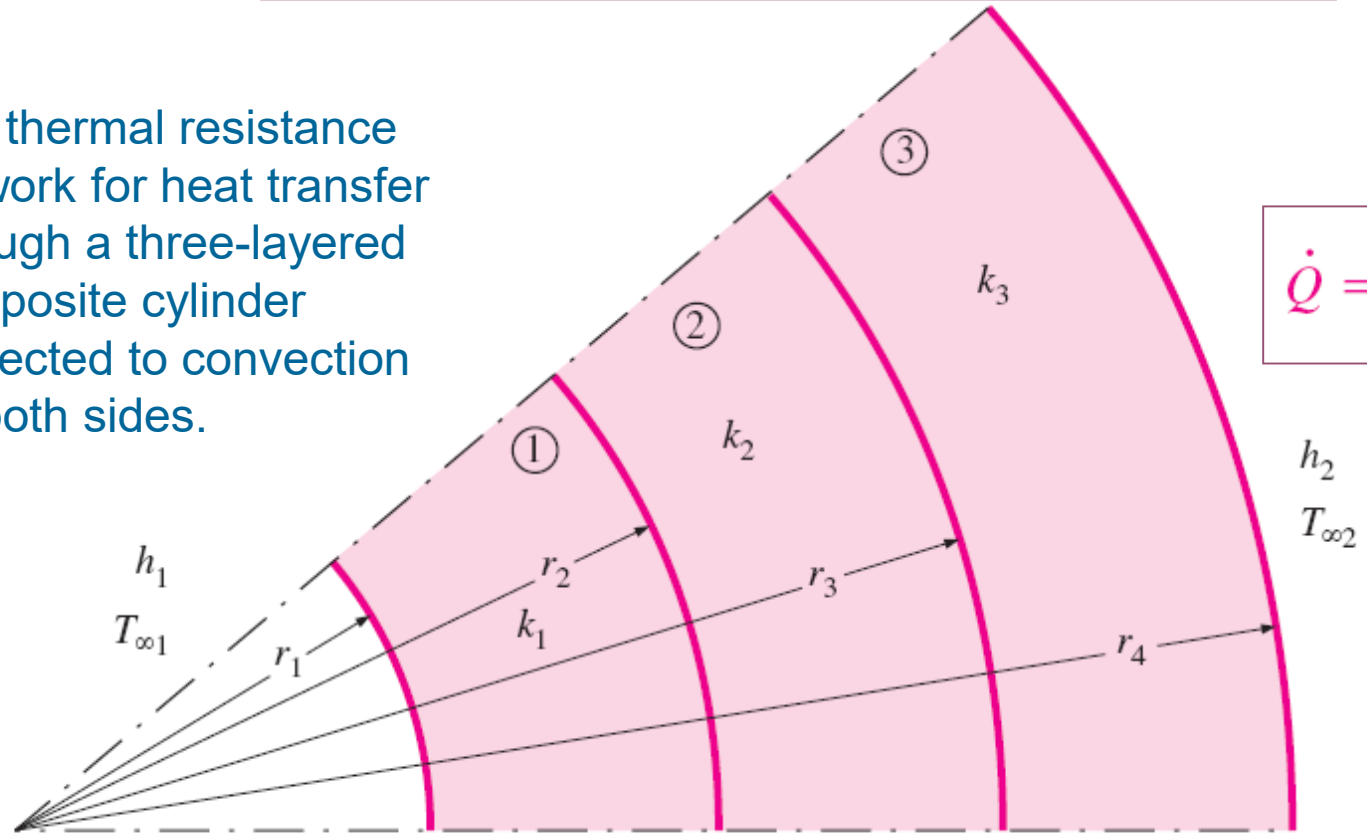
$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{sph}} + R_{\text{conv, 2}} \\ &= \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2} \end{aligned}$$

for a *spherical* layer

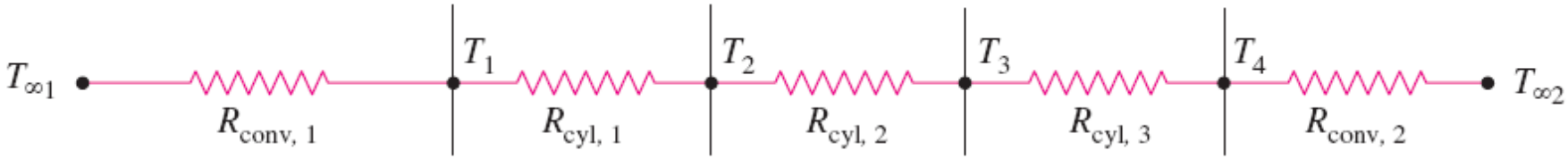
Multilayered Cylinders and Spheres

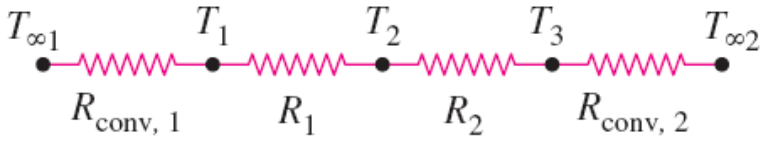
$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{cyl, 1}} + R_{\text{cyl, 2}} + R_{\text{cyl, 3}} + R_{\text{conv, 2}}$$
$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$





$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$$

$$= \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1}$$

$$= \frac{T_1 - T_3}{R_1 + R_2}$$

$$= \frac{T_2 - T_3}{R_2}$$

$$= \frac{T_2 - T_{\infty 2}}{R_2 + R_{\text{conv},2}}$$

$$= \cdots$$

The ratio ΔT/R across any layer is is equal to $\dot{Q}$, which remains constant in one dimensional coduction

Once heat transfer rate Q has been calculated, the interface temperature T_2 can be determined from any of the following two relations:

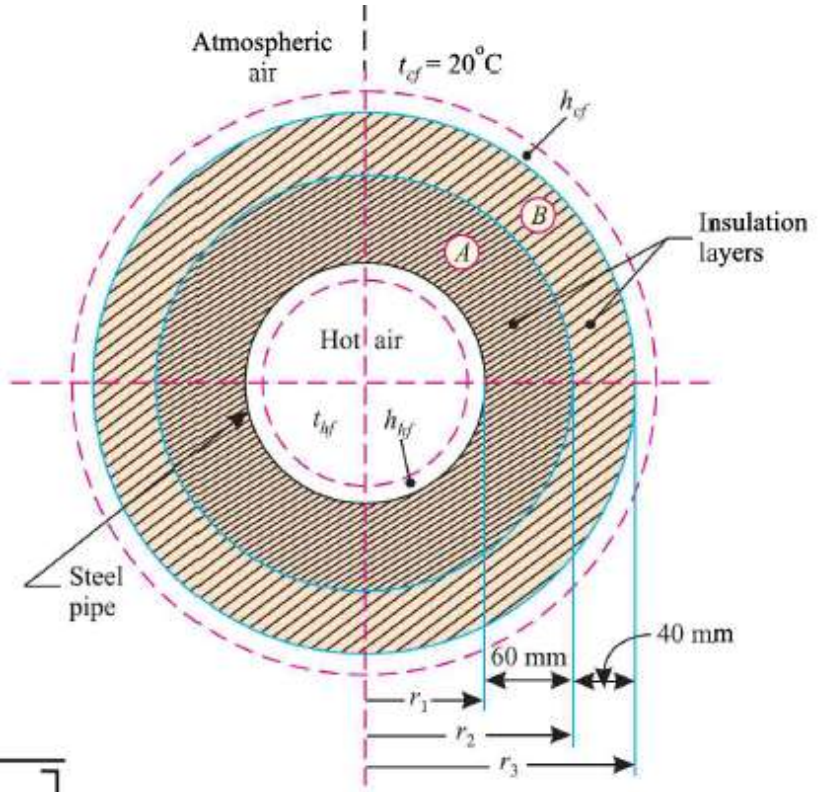
$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv, 1}} + R_{\text{cyl, 1}}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}}$$

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{\text{conv, 2}}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}}$$

L-12

	Plane Wall	Cylindrical Wall	Spherical Wall
Heat Equation	$\frac{d^2T}{dx^2}=0$	$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)=0$	$\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)=0$
Temperature Distribution	$T_{s,1}-\Delta T\frac{x}{L}$	$\frac{\Delta T}{ln\left(r_1/r_2\right)}ln\left(\frac{r}{r_2}\right)+T_{s,2}$	$T_{s,1}+\frac{\Delta T}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}\left[\frac{1}{r_1}-\frac{1}{r}\right]$
Heat Flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\Delta T}{r\,ln\left(r_2/r_1\right)}$	$\frac{k\Delta T}{r^2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$
Heat Rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\Delta T}{ln\left(r_2/r_1\right)}$	$\frac{4\pi k\Delta T}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$
Thermal Resistance ($R_{t,\,cond}$)	$\frac{L}{kA}$	$\frac{ln\left(r_2/r_1\right)}{2\pi Lk}$	$\frac{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{4\pi k}$

Example 2.11. Hot air at a temperature of 65°C is flowing through a steel pipe of 120 mm diameter. The pipe is covered with two layers of different insulating materials of thickness 60 mm and 40 mm, and their corresponding thermal conductivities are 0.24 and 0.4 W/m°C. The inside and outside heat transfer coefficients are 60 W/m°C and 12 W/m°C respectively. The atmosphere is at 20°C. Find the rate of heat loss from 60 m length of pipe.



Given :

$$r_1 = \frac{120}{2} = 60\,\text{mm} = 0.06\,\text{m}$$

$$r_2 = 60 + 60 = 120\,\text{mm} = 0.12\,\text{m}$$

$$r_3 = 60 + 60 + 40 = 160\,\text{mm} = 0.16\,\text{m}$$

$$k_A = 0.24\,\text{W/m°C}; \qquad k_B = 0.4\,\text{W/m°C}$$

$$h_{hf} = 60\,\text{W/m}^2\text{°C}; \qquad h_{cf} = 12\,\text{W/m}^2\text{°C}$$

$$t_{hf} = 65°C; \; t_{cf} = 20°C$$

Length of pipe, $L = 60$ m

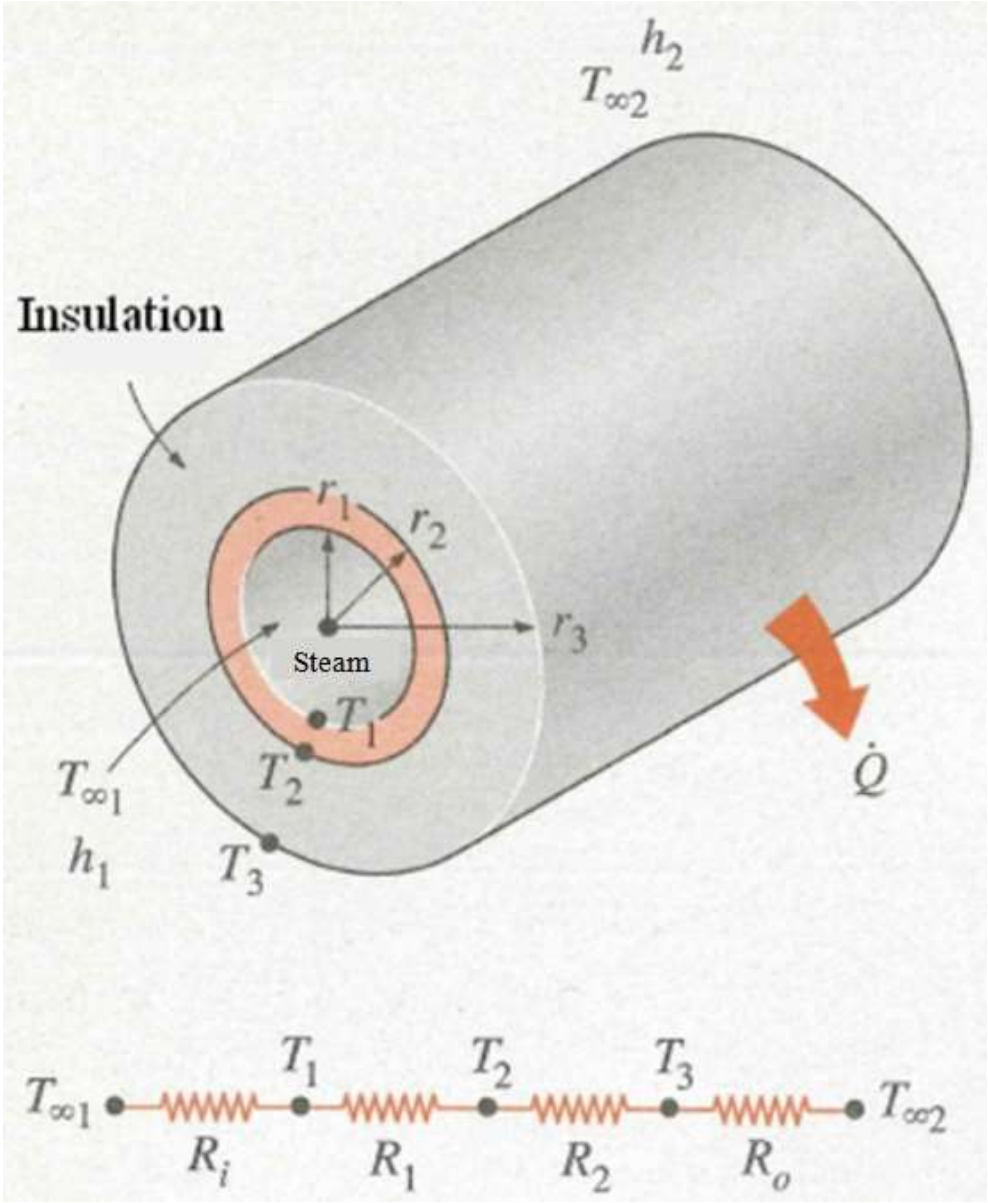
Rate of heat loss, Q :

Rate of heat loss is given by

$$Q = \frac{2\pi L (t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} \cdot r_1} + \frac{\ln (r_2 / r_1)}{k_A} + \frac{\ln (r_3 / r_2)}{k_B} + \frac{1}{h_{cf} \cdot r_3}\right]}$$

Rate of heat loss = **3850.5 W**

Heat Loss from an Insulated pipe



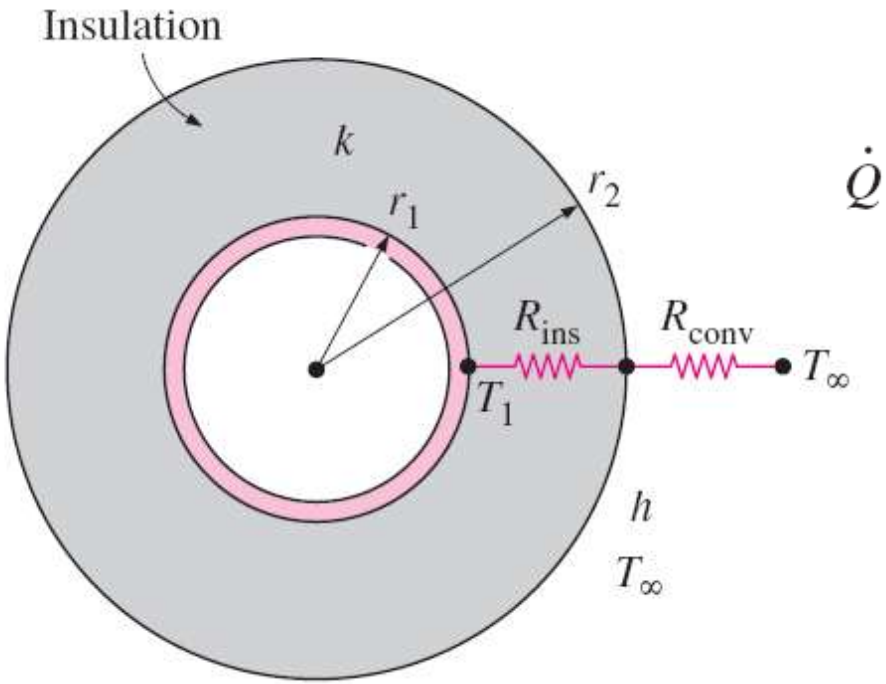
CRITICAL RADIUS OF INSULATION

Adding more insulation to a wall or to the attic always decreases heat transfer since the heat transfer area is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

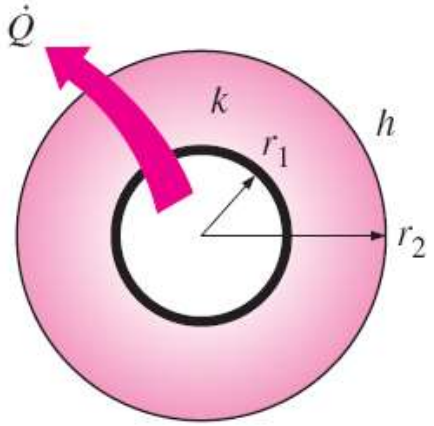
In a cylindrical pipe or a spherical shell, the additional insulation **increases** the conduction resistance of the insulation layer but **decreases** the convection resistance of the surface because of the increase in the outer surface area for convection. The heat transfer from the pipe may increase or decrease, depending on which effect dominates.

L-13

L-13



$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{h(2\pi r_2 L)}}$$



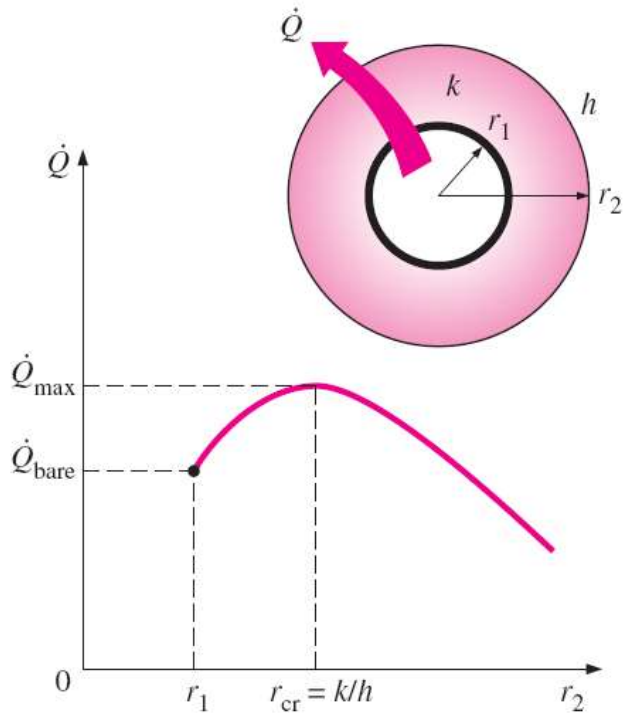
$$\frac{d\dot{Q}}{dr_2} = 0$$

$$r_{cr,\,cylinder} = \frac{k}{h} \qquad \text{(m)}$$

Note that the critical radius of insulation depends on the thermal conductivity of the insulation *k and the external convection heat transfer coefficient h.* The rate of heat transfer from the cylinder increases with the addition of insulation for $r_2 < r_{cr}$, *reaches a maximum when* $r_2 = r_{cr}$, *and starts to decrease for* $r_2 > r_{cr}$. *Thus, insulating the pipe may actually increase the rate of heat transfer* from the pipe instead of decreasing it when $r_2 < r_{cr}$.

$$r_{\text{cr, cylinder}} = \frac{k}{h} \qquad \text{(m)}$$

$$r_{\text{cr, sphere}} = \frac{2k}{h}$$



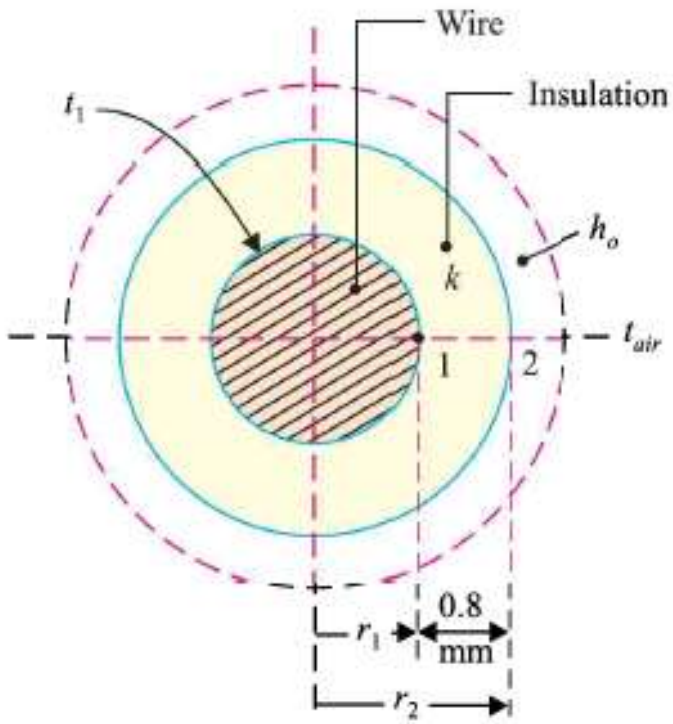
L-13

A wire of 6.5 mm diameter at a temperature of 60°C is to be insulated by a material having k = 0.174 W/m°C. Convection heat transfer coefficient (h_o) = 8.722 W/m²°C. The ambient temperature is 20°C. For maximum heat loss, what is the minimum thickness of insulation and heat loss per metre length? Also find percentage increase in the heat dissipation too.

$$r_1 = \frac{6.5}{2} = 3.25\,\text{mm} = 0.00325\,\text{m}$$

$k = 0.174$ W/m°C,

$h_o = 8.722$ W/m²°C.



$$r_c = \frac{k}{h_0} = \frac{0.174}{8.722} = 0.01995\,\text{m} \text{ or } 19.95\,\text{mm}$$

∴ *Minimum insulation thickness*

$= r_c - r_1 = 19.95 - 3.25 =$ **16.7 mm (Ans.)**

Heat loss per metre length :

Case I. *Without insulation* :

$$Q_1 = \frac{2\pi L(t_1 - t_{air})}{\frac{1}{h_o \cdot r_1}} = \frac{2\pi \times 1(60 - 20)}{\frac{1}{8.722 \times 0.00325}}$$

$= 7.124$ W/m

Heat loss per metre length :

Case I. *Without insulation* :

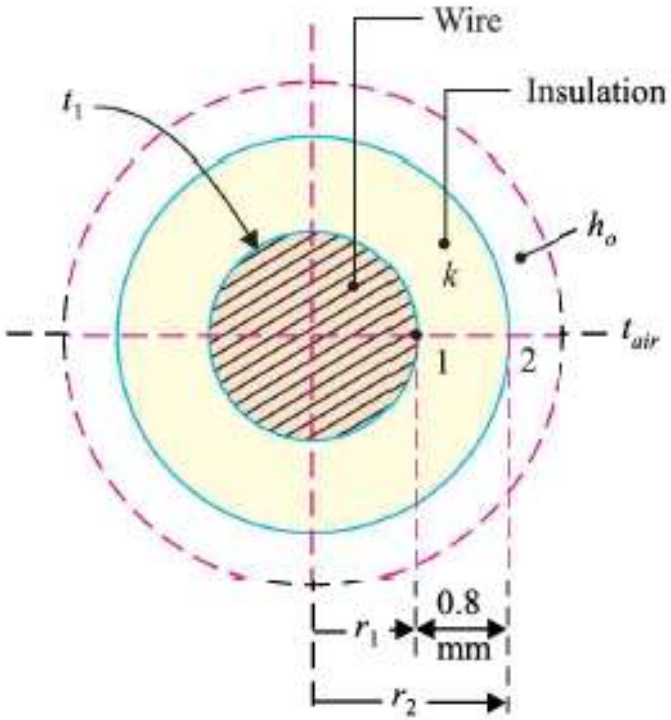
$$Q_1 = \frac{2\pi L(t_1 - t_{air})}{\frac{1}{h_o \cdot r_1}} = \frac{2\pi \times 1(60 - 20)}{\frac{1}{8.722 \times 0.00325}}$$

$$= 7.124 \text{ W/m}$$

Case II. *With insulation when critical thickness is used* :

$$Q_2 = \frac{2\pi L(t_1 - t_{air})}{\frac{\ln(r_c / r_1)}{k} + \frac{1}{h_o \cdot r_c}} = \frac{2\pi \times 1 \times (60 - 20)}{\frac{\ln(0.01995/0.00325)}{0.174} + \frac{1}{8.722 \times 0.01995}}$$

$$= 15.537 \text{ W/m}$$



$\therefore$ Percentage increase in heat dissipation $= \frac{Q_2 - Q_1}{Q_1} \times 100$

$$= \frac{(15.537 - 7.124)}{7.124} \times 100 = \mathbf{118.09\ \%} \qquad \textbf{(Ans.)}$$

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Unit 2

Conduction with Internal Heat Generation

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Objectives

- One dimensional steady state conduction in plane wall and radial systems with thermal energy generation is appreciated.
- Here again, the approach is to reduce the heat diffusion equation for the case chosen.
- Using the appropriate boundary conditions, the heat diffusion equation is solved for temperature distribution and heat transfer rates are computed.

In the preceding lecture we considered conduction problems for which the temperature distribution in a medium was determined solely by conditions at the boundaries of the medium. We shall now consider the additional effect on the temperature distribution of processes that may be occurring within the medium. Specifically we will consider situations for which thermal energy is being generated due to conversion from some other energy form.

A very common thermal energy generation process involves the conversion from electrical to thermal energy in a current carrying medium (resistance heating). The rate at which energy is generated by passing a current I through a medium of electrical resistance R_e is

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$$\dot{E}_g = \dot{E}_e \qquad (2.59)$$

If this power generation occurs uniformly throughout the medium of volume V, the volumetric generation rate (W/m^3) is then

$$\dot{q} = \frac{\dot{E}_g}{V} = \frac{\dot{E}_e}{V} \qquad (2.60)$$

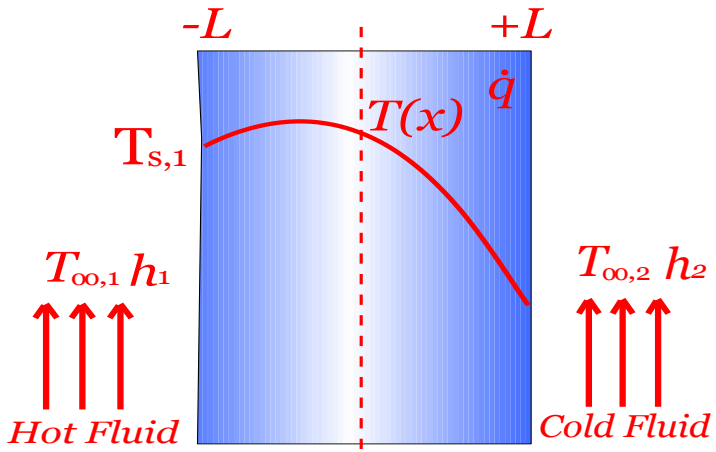
THE PLANE WALL

Consider the plane wall of Figure 2.17, in which there is uniform energy generation per unit volume ($\dot{q}$ is constant) and the surfaces are maintained at $T_{s,1}$ and $T_{s,2}$. For constant thermal conductivity k , the appropriate form of the heat equation,

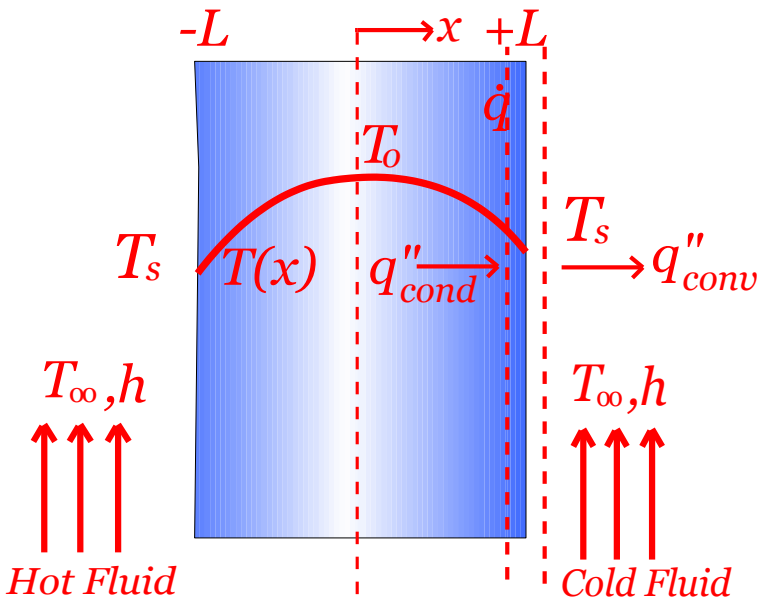
$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \qquad (2.61)$$

The general solution is

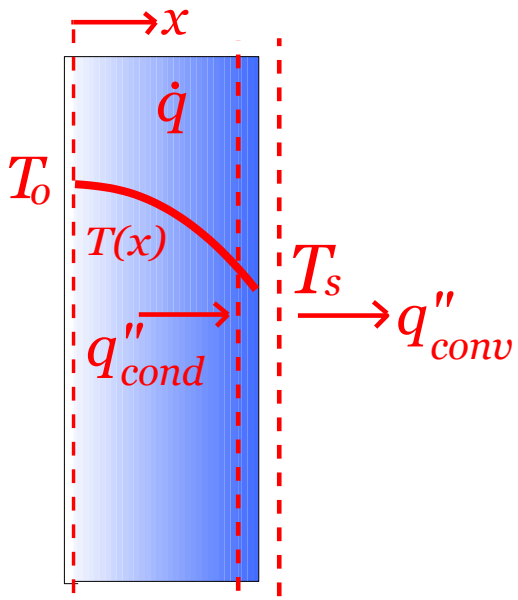
$$T = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 \qquad (2.62)$$



Asymmetrical Boundary Condition



Symmetrical Boundary Condition



Adiabatic Surface At Midplane

Figure 2.17 Conduction In A Plane Wall With Uniform Generation

where C_1 and C_2 are the constants of integration. For the prescribed boundary conditions,

$$T(-L) = T_{s,1} \quad \text{and} \quad T(L) = T_{s,2}$$

The constants may be evaluated and are of the form

$$C_1 = \frac{T_{s,2} - T_{s,1}}{2L} \quad \text{and} \quad C_2 = \frac{\dot{q}}{2k} L + \frac{T_{s,2}}{2}$$

in which case the temperature distribution is

$$T(x) = \frac{\dot{q}}{2k}\left(L^2 - x^2\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \qquad (2.63)$$

The heat flux at any point in the wall may, be determined by using Equation 2.63 with Fourier's Law.

NOTE: That with generation, the heat flux is no longer independent of x.

The preceding result simplifies when both surfaces are maintained at a common temperature, $T_{s,1}=T_{s,2}=T_s$. The temperature distribution is then symmetrical about the midplane, Figure 2.17 b, and is given by

$$T(x) = \frac{\dot{q}}{2k}\left(L^2 - x^2\right) + T_s \qquad (2.64)$$

The maximum temperature exists at the midplane

$$T(0) = T_0 = \frac{\dot{q}L^2}{2k} + T_s \qquad (2.65)$$

In which case the temperature distribution, Equation 2.64, may be expressed as

$$\frac{T(x)-T_0}{T_s-T_0}=\left(\frac{x}{L}\right)^2 \qquad (2.66)$$

- It is important to note that the plane of symmetry in Figure 2.17b, the temperature gradient is zero, $(dT/dx)_{x=0}=0$.
- Accordingly, there is no heat transfer across this plane, and it may be represented by the adiabatic surface shown in Figure. 21.7 c.
- One implication of this result is that Equation 2.64 also applies to plane walls that are perfectly insulated on one side ($x=0$) and maintained at a fixed temperature T_s on the other side ($x=L$).

To use the above results the surface temperature T_s must be known. However, a common situation is one for which it is the temperature of an adjoining fluid,, and not T_s, which is known. It then becomes necessary to relate T_s to. This relation may be developed by applying a surface energy balance. Consider the surface at $x=L$ for the symmetrical plane wall (Figure 2.17 b) or the insulated plane wall (Figure 2.17 c). Neglecting the radiation and substituting the appropriate rate equations, the energy balance is given by

$$-k\left.\frac{dT}{dx}\right|_{x=L} = h\left(T_s - T_\infty\right) \qquad (2.67)$$

Substituting from Equation 2.64 to obtain the temperature gradient at $x=L$, it follows that

$$T_s = T_\infty + \frac{\dot{q}}{h} \qquad (2.68)$$

Hence, T_s may be computed from knowledge of T_∞, $\dot{q}$,L and h .

RADIAL SYSTEMS

Consider the long, solid cylinder of Figure 2.18, which could represent a current carrying wire. For steady state conditions the rate at which heat is generated within the cylinder must equal the rate at which heat is convected from the surface of the cylinder to a moving fluid. This condition allows the surface temperature to be maintained at a fixed value of T_s.

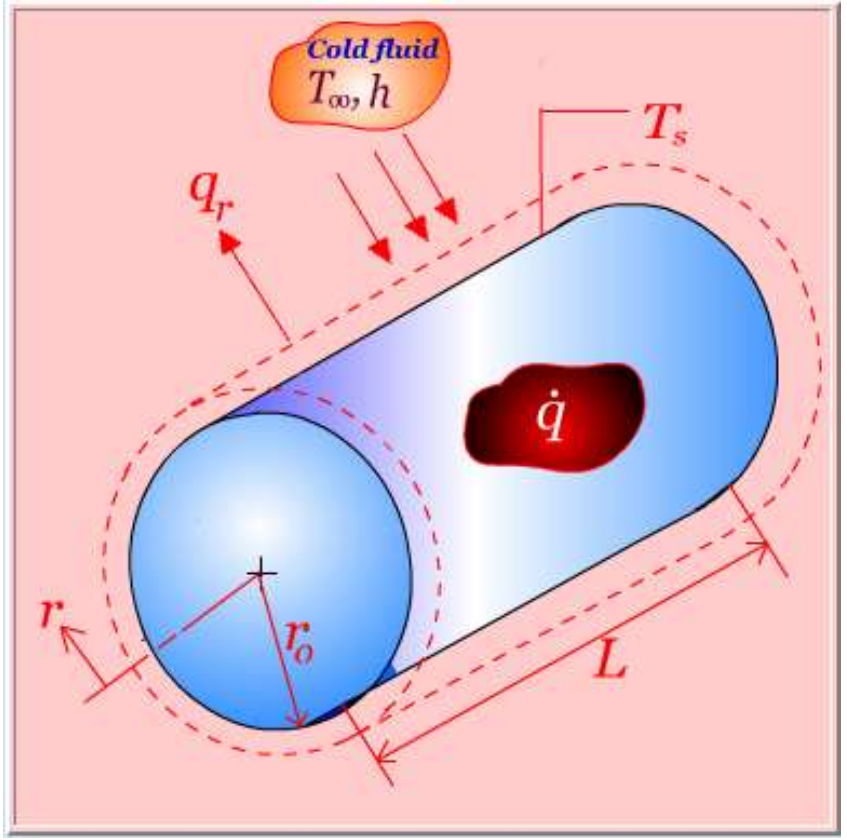


Figure 2.18 Conduction In A Solid Cylinder With Uniform Heat Generation

To determine the temperature distribution in the cylinder, we begin with the appropriate form of the heat equation. For constant thermal conductivity k , heat equation reduces to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0 \qquad (2.69)$$

Separating variables and assuming uniform generation, this expression may be integrated to obtain

$$r\frac{dT}{dr} = -\frac{\dot{q}}{2k}r^2 + C_1 \qquad (2.70)$$

Repeating the procedure, the general solution for the temperature distribution becomes

$$T(r) = -\frac{q}{4k}r^2 + C_1 \ln(r) + C_2 \qquad (2.71)$$

To obtain the constants of integration C_1 and C_2, we apply the boundary conditions

$$\left.\frac{dT}{dr}\right|_{r=0} = 0 \quad \text{and} \quad T(r_o) = T_s$$

The first condition results from the symmetry of the situation. That is, for the solid cylinder the centerline is a line of symmetry for the temperature distribution and the temperature gradient must be zero. Recall the similar conditions existed at the midplane of a wall having symmetrical boundary conditions (Figure 2.17b). From the symmetry condition at $r=0$ and Equation 2.70, it is evident that $C_1=0$. Using the surface boundary condition at $r = r_o$ with Equation 2.70, we then obtain

$$C_2 = T_s + \frac{\dot{q}}{4k} r_0^2 \qquad (2.72)$$

The temperature distribution is therefore

$$T(r) = T_s + \frac{\dot{q}}{4k}\left(1 - \frac{r^2}{r_0^2}\right) \qquad (2.73)$$

Evaluating Equation 2.73 at the centerline and dividing the result into Equation 2.73, we obtain the temperature distribution in non-dimensional form,

$$\frac{T(r) - T_s}{T_0 - T_s} = 1 - \left(\frac{r}{r_0}\right)^2 \qquad (2.73)$$

Where T_0 is the center line temperature. The heat rate at any radius in the cylinder may, be evaluated by using Equation 2.73 with Fourier's Law.

The surface temperature, T_s, may be related to the temperature of the cold fluid, T_∞ , by surface energy balance. We obtain

$$\dot{q}(\pi r_0^2 L) = h(2\pi r_0 L)(T_s - T_\infty) \qquad (2.74)$$

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or,

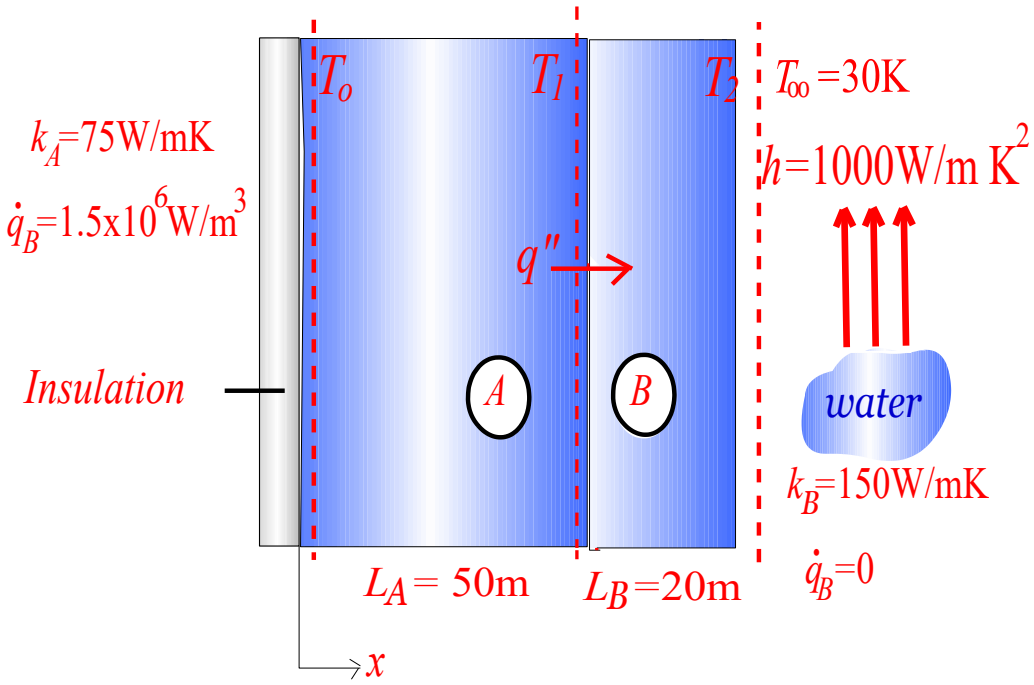
$$T_s = T_\infty + \frac{\dot{q}}{2h} \qquad (2.75)$$

Problem 2.5:

A plane wall is a composite of two materials, A and B. The wall of material A has uniform heat generation $\dot{q}$ = 1.5 x 10^6 W/m^3 k_A=75 $W/m.K$, and thickness L_A=50 mm. The wall material B has no generation with k_B=150 $W/m.K$ and thickness L_A= 20 mm. The inner surface of material A is well insulated, while the outer surface of material B is cooled by water stream T_∞ = 30^0C and h=1000 $W/m^2.K$.

- Sketch the temperature distribution that exists in the composite under steady state conditions.
- Determine the temperature T_o of the insulated surface and the temperature T_2 of the cooled surface.

Figure



Known : Plane wall of material A with internal heat generation is insulated on one side and bounded by a second wall of material B, which is without heat generation and is subjected to convection cooling

Find : Convection heat transfer coefficient between the outer surface of the wire and the air in the room.

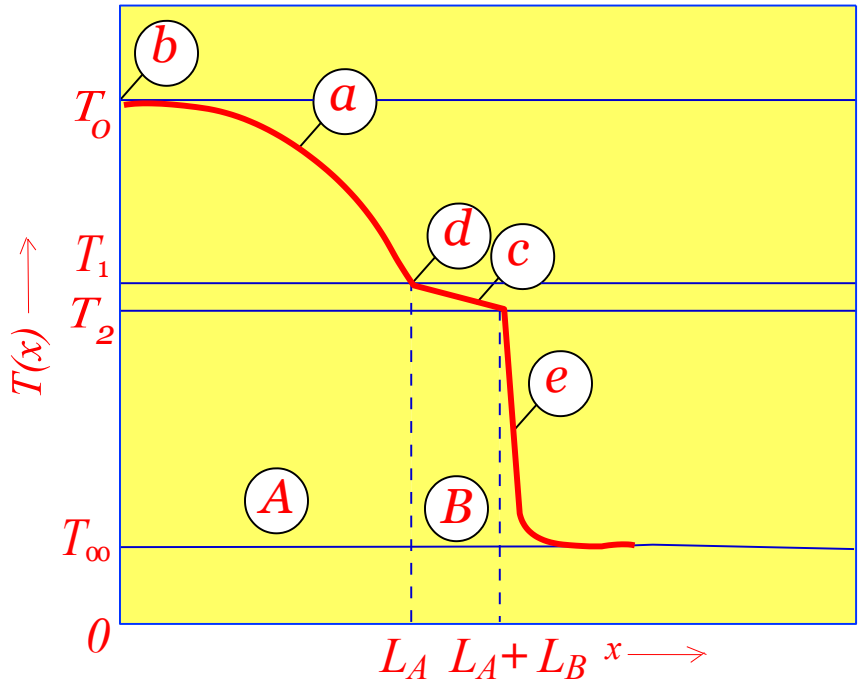
Assumptions:

1. Heat transfer is steady since there is no indication of any change with time.
2. Heat transfer is one dimensional since there is thermal symmetry about the center line and no variation in the axial direction.
3. Thermal conductivities are constant.
4. The thermal contact resistance at the interface is negligible.
5. Heat transfer coefficient incorporates the radiation effects, if any.

Analysis:

1. From the prescribed physical conditions, the temperature distribution in the composite is known to have the following features, as shown:
    1. Parabolic in material A
    2. Zero slope at insulated boundary
    3. Linear in material A

4. The temperature distribution in the water is characterized by large gradients near the surface
5. Slope change = $k_A / k_B = 2$ at interface



The outer surface temperature T_2 may be obtained by performing an energy balance on the control volume about material B. since there is no generation in this material, it follows that, for steady state conditions and a unit surface area , the heat flux into the material at $x = L_A$ must equal the heat flux from the material due to convection at $x = L_A + L_B$. Hence

$$q'' = h(T_2 - T_\infty)$$

3. The heat flux may be determined by performing a second energy balance on a control volume about material A. In particular, since the surface at *x = 0* is adiabatic, there is no inflow and the rate at which the energy is generated must equal the outflow. Accordingly, for a unit surface area,

Combining above two Equations , the outer surface temperature is,

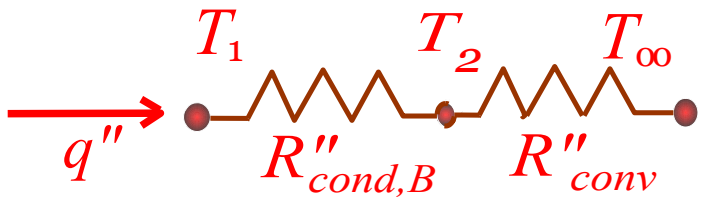
$$T_2 = T_\infty + \frac{\dot{q}''}{h}$$

$$T_2 = 30 + \frac{1.5 \times 10^6 \,(0.05)}{1000} = 105°C$$

From Equation 2.65, the temperature at the insulated surface is,

$$T_o = T_1 + \frac{\dot{q}''}{2k_A}$$

where T_1 may be obtained from the thermal circuit:



That is,

$$T_1 = T_\infty + \left(R''_{cond,B} + R''_{conv}\right) q''$$

where the resistances for a unit surface area

$$R''_{cond,B} = \frac{L_B}{k_B} \quad and \quad R''_{conv} = \frac{1}{h}$$

Hence

$$T_1 = 30 + \left(\frac{0.02}{150} + \frac{1}{1000}\right) \times 1.5 \times 10^6 \times 0.05$$

$$T_1 = 30 + 85 = 115°C$$

Comments:

1. Material A, having heat generation, cannot be represented by a thermal circuit element
2. Since the resistance to heat transfer by convection is significantly larger than due to conduction in material B, $R''_{cond,B} / R''_{conv} = 7.5$ the surface-to-fluid temperature difference is much larger than the temperature drop across material B,

$$\left(T_2 - T_\infty\right) / \left(T_1 - T_2\right) = 7.5$$

This result is consistent with the temperature distribution plotted in part (1).

3. The surface and interface temperatures (T_o,T_1,*and* T_2) depend on the generation rate , the thermal conductivities k_A and k_B, and the convection coefficient h. Each material will have a maximum allowable operating temperature, which must not be exceeded if thermal failure of the system is to be avoided. We explore the effect of one of these parameters by computing and plotting the temperature distribution for values of h=200 and 1000 $W/m^2.K$, which would be representative of air and liquid cooling respectively.

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HEAT TRANSFER FROM FINNED SURFACES

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

Newton's law of cooling The rate of heat transfer from a surface to the surrounding medium

When T_s and T_∞ are fixed, there are *two ways* to increase the rate of heat transfer:

Increase the convection heat transfer coefficient h.

This may require the installation of a pump or fan, or replacing the existing one with a larger one, but this approach may or may not be practical. Besides, it may not be adequate.

Increase the surface area A_s

by attaching to the surface **extended surfaces** called **fins** made of highly conductive materials such as aluminum.

L-16





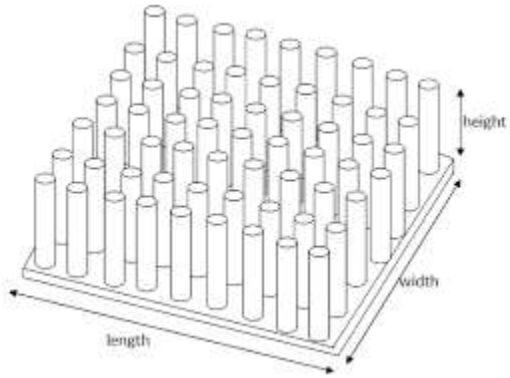




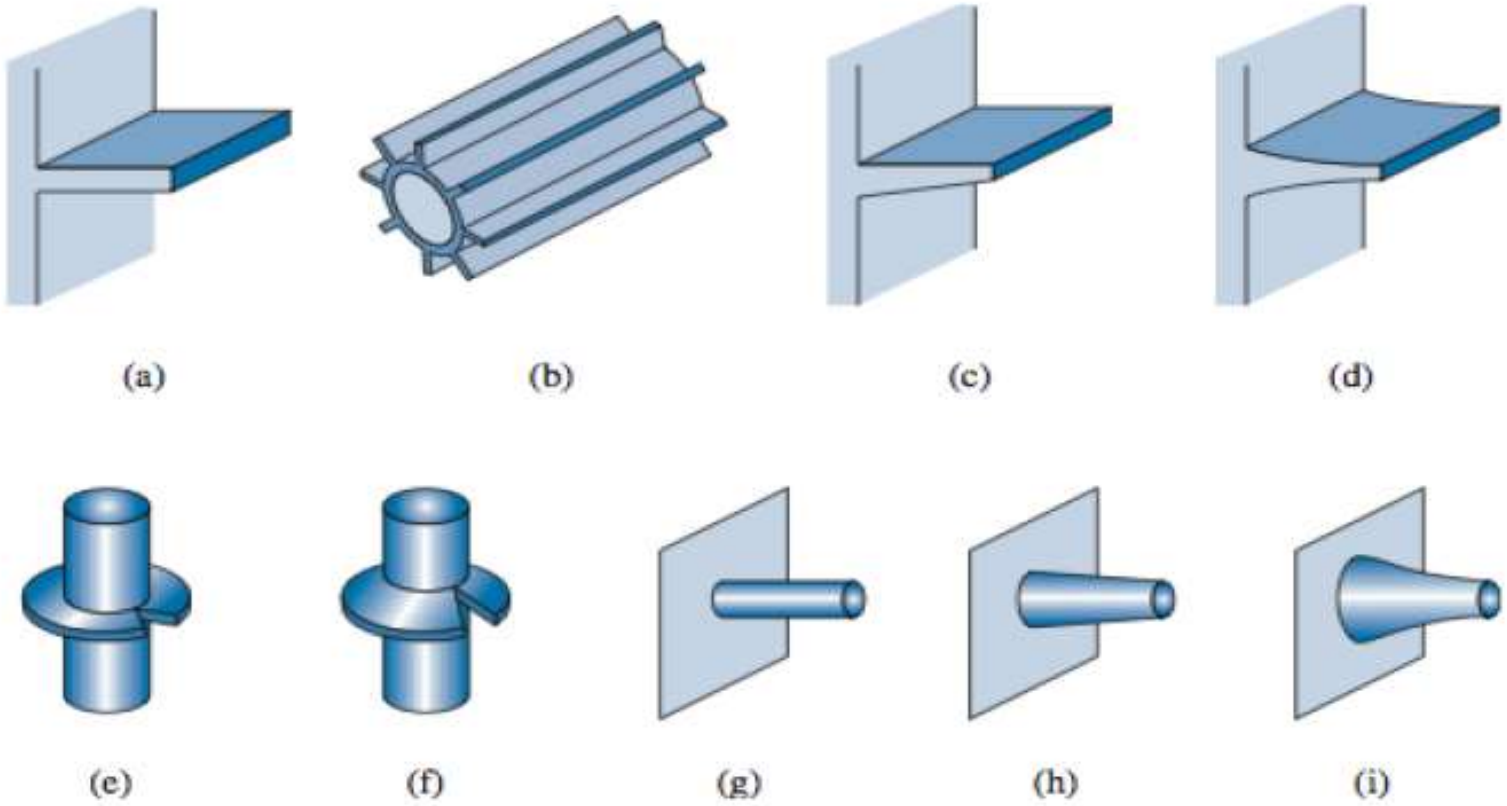





wikiHow to Clean Refrigerator Coils


height
width
length





(a) Longitudinal fin – Rectangular profile

(b) Longitudinal fin – Rectangular profile

(c) Longitudinal fin - Trapezoidal profile

(d) Longitudinal fin - Concave parabolic

(e) Radial fin – Rectangular profile

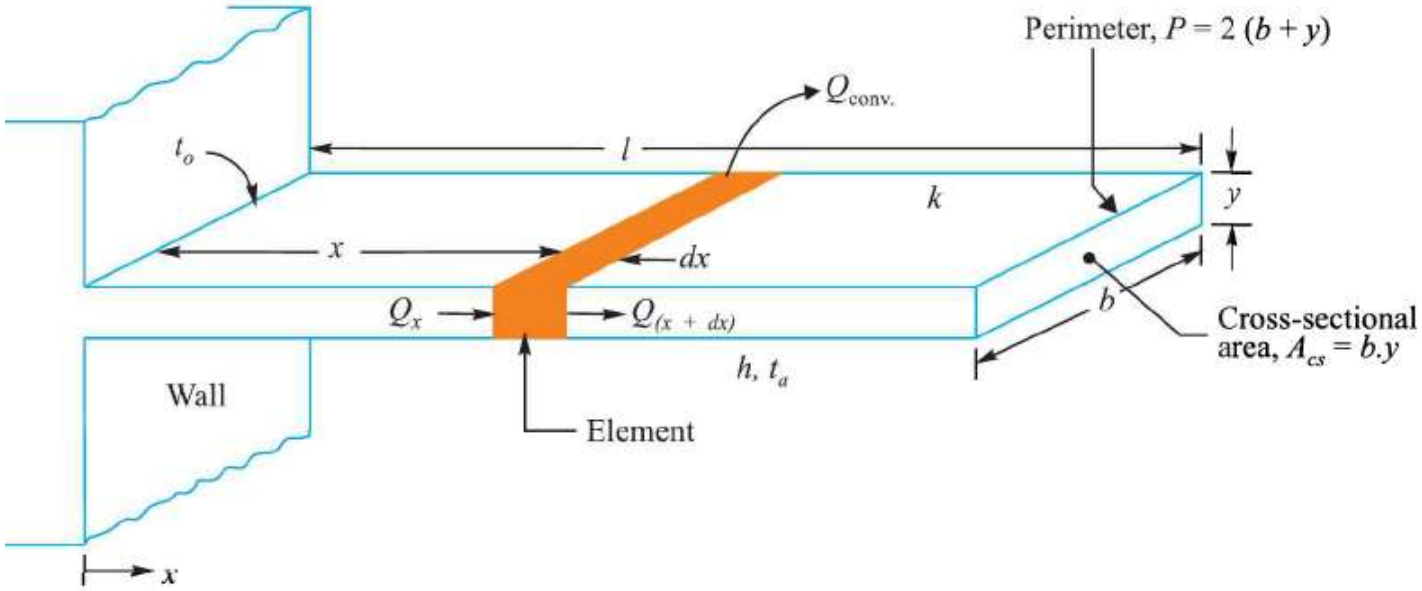
(f) Radial fin – Triangular profile

(g) Pin fin – Cylindrical

(h) Pin fin – Tapered profile

(i) Pin fin – Concave parabolic

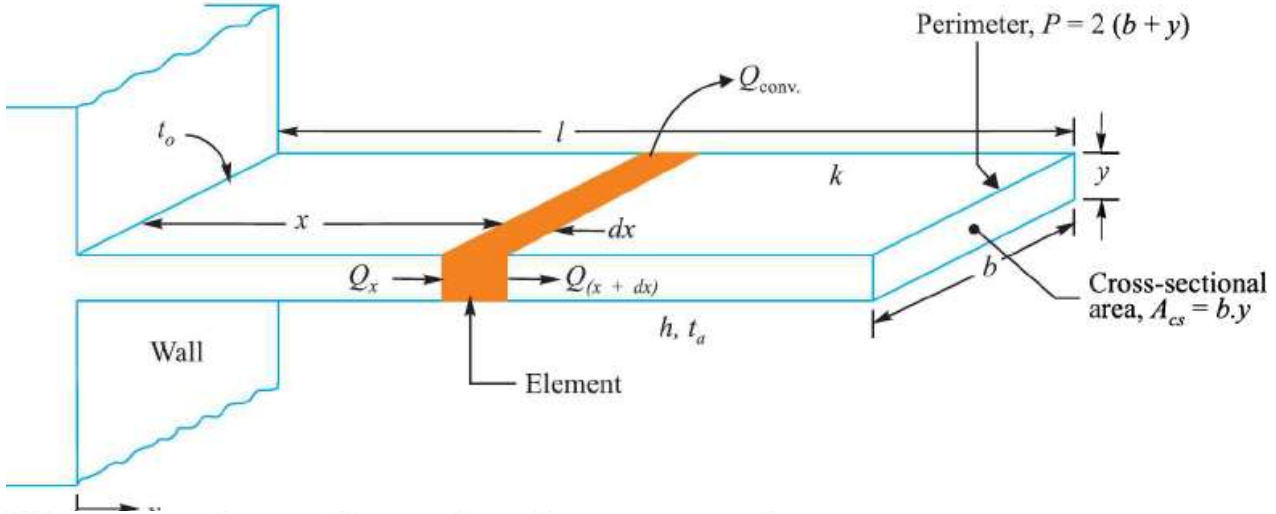
Fin Equation



The following **assumptions** are made for the analysis of heat flow through the fin :

1. Steady state heat conduction.
2. No heat generation within the fin.
3. Uniform heat transfer coefficient (h) over the entire surface of the fin.
4. Homogeneous and isotropic fin material (*i.e.* thermal conductivity of material constant).
5. Negligible contact thermal resistance.
6. Heat conduction one-dimensional.
7. Negligible radiation.

Fin Equation



Heat conducted into the element at plane x,

$$Q_x = -k\,A_{cs}\left[\frac{dt}{dx}\right]_x$$

Heat conducted out of the element at plane $(x + dx)$

$$Q_{(x+dx)} = -k\,A_{cs}\left[\frac{dt}{dx}\right]_{x+dx}$$

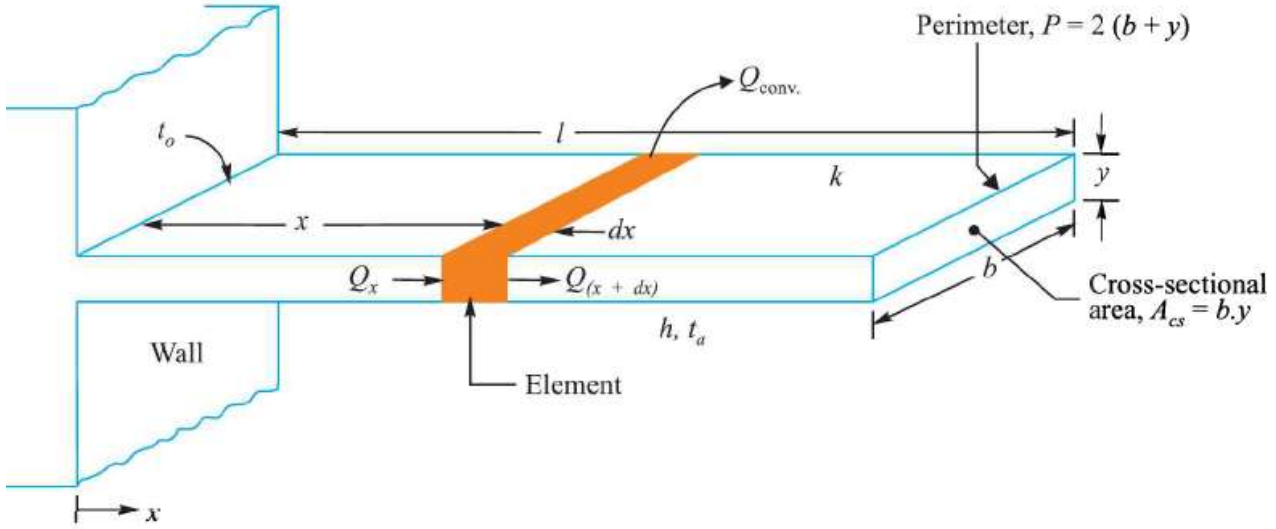
Heat convected out of the element between the planes x and $(x + dx)$,

$$Q_{conv} = h\,(P\,.\,dx)\,(t - t_a)$$

Applying an energy balance on the element, we can write

$$Q_x = Q_{(x+dx)} + Q_{conv.}$$

Fin Equation



$$Q_x = Q_{(x+dx)} + Q_{conv.}$$

$$-k\,A_{cs}\left[\frac{dt}{dx}\right]_x = -k\,A_{cs}\left[\frac{dt}{dx}\right]_{x+dx} + h\,(P.dx)\,(t - t_a)$$

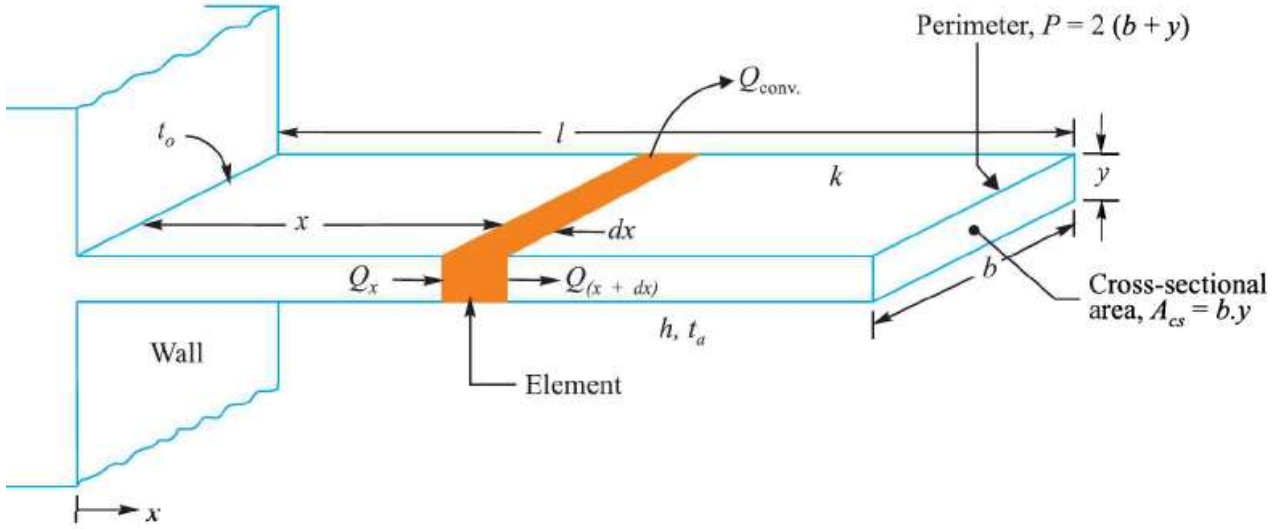
Making a Taylor's expansion of the temperature gradient at $(x + dx)$ in terms of that at x, we get

$$\left(\frac{dt}{dx}\right)_{x\,+\,dx} = \left(\frac{dt}{dx}\right)_x + \frac{d}{dx}\left(\frac{dt}{dx}\right)_x dx + \frac{d^2}{dx^2}\left(\frac{dt}{dx}\right)\frac{(dx)^2}{2!} + \ldots$$

Neglecting higher terms as $dx \to 0$, we have

$$-k\,A_{cs}\left[\frac{dt}{dx}\right] = -k\,A_{cs}\left[\frac{dt}{dx}\right] - k\,A_{cs}\left[\frac{d^2t}{dx^2}\right]dx + h\,(P.dx)\,(t - t_a)$$

Fin Equation



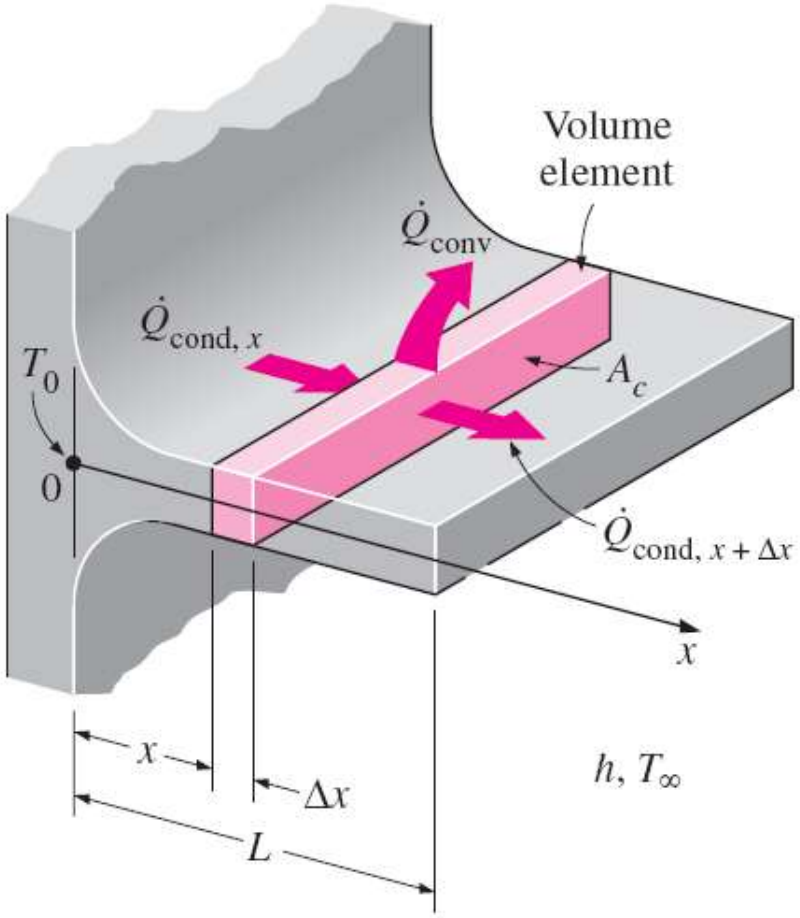
$$k\,A_{cs}\left[\frac{d^2t}{dx^2}\right]dx - h(P.dx)(t - t_a) = 0$$

Dividing both sides by $A_{cs}\,dx$, we get,

$$k\frac{d^2t}{dx^2} - \frac{hP}{A_{cs}}(t - t_a) = 0$$

or,

$$\frac{d^2t}{dx^2} - \frac{h\,P}{k\,A_{cs}}(t - t_a) = 0$$



$$\frac{d}{dx}\left(kA_c\frac{dT}{dx}\right) - hp(T - T_\infty) = 0$$

Excess Temperature

$$\theta = T - T_\infty$$

Assuming that A_c does not vary with x

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$m^2 = \frac{hp}{kA_c}$$

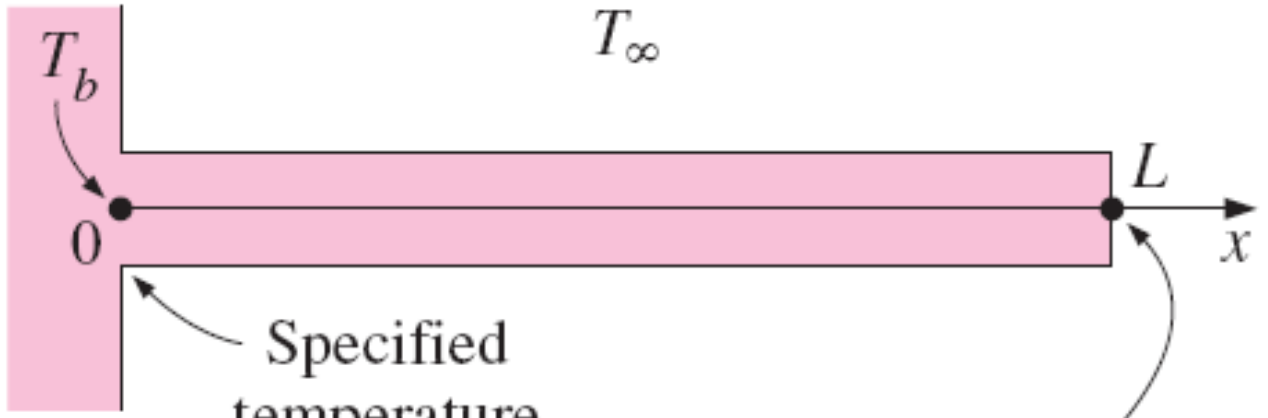
$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad m^2 = \frac{hp}{kA_c}$$

$$\theta = T - T_\infty$$

The general solution of the differential equation

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

To determine the constants we need to specify the boundary conditions.



(*a*) Specified temperature
(*b*) Negligible heat loss
(*c*) Convection
(*d*) Convection and radiation

Boundary conditions at the fin base and the fin tip.

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary condition at fin base

$$\theta(0) = \theta_b = T_b - T_\infty$$

Boundary condition at fin tip

1 Infinitely Long Fin ($T_{\text{fin tip}} = T_\infty$)

$$\theta(L) = T(L) - T_\infty = 0 \quad L \rightarrow \infty$$

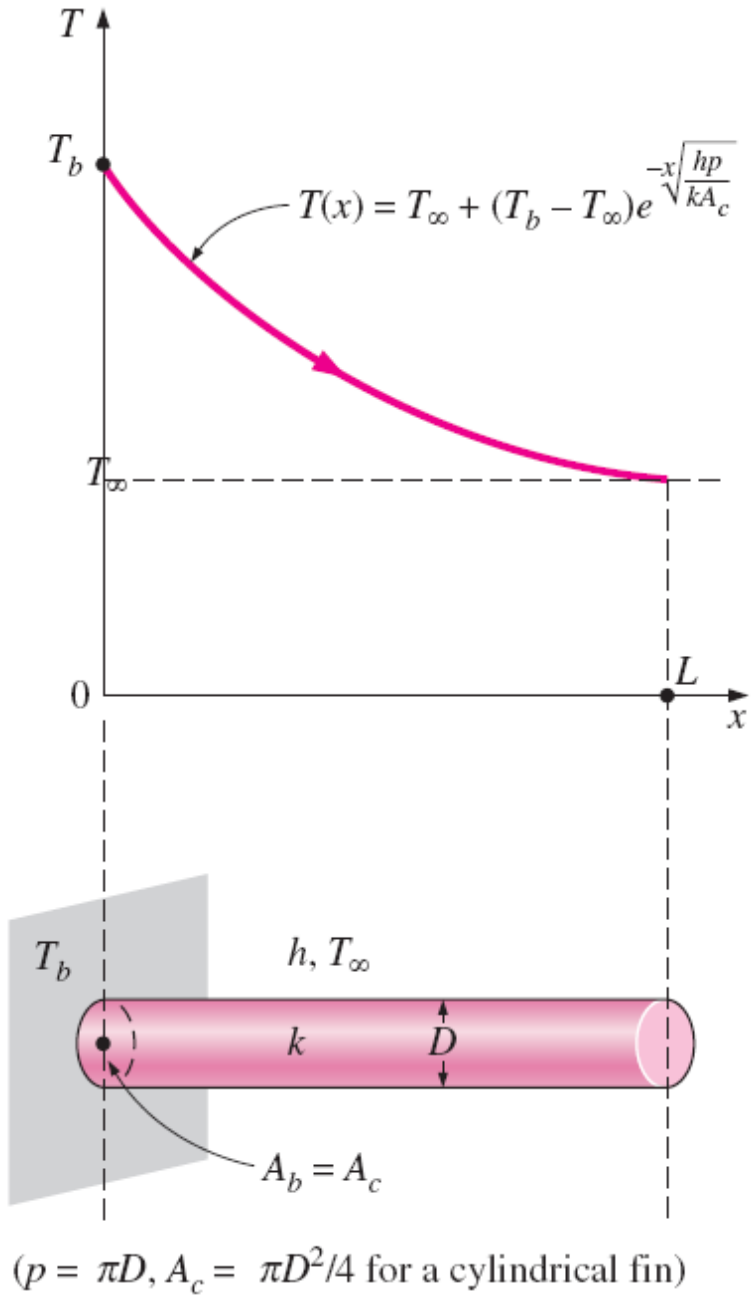
$$C_1 = 0$$

$$C_2 = \theta_0$$

The variation of temperature along the fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}} \qquad m = \sqrt{hp/kA_c}$$

$e^0 = 1$ and $e^{-\infty} = 0$

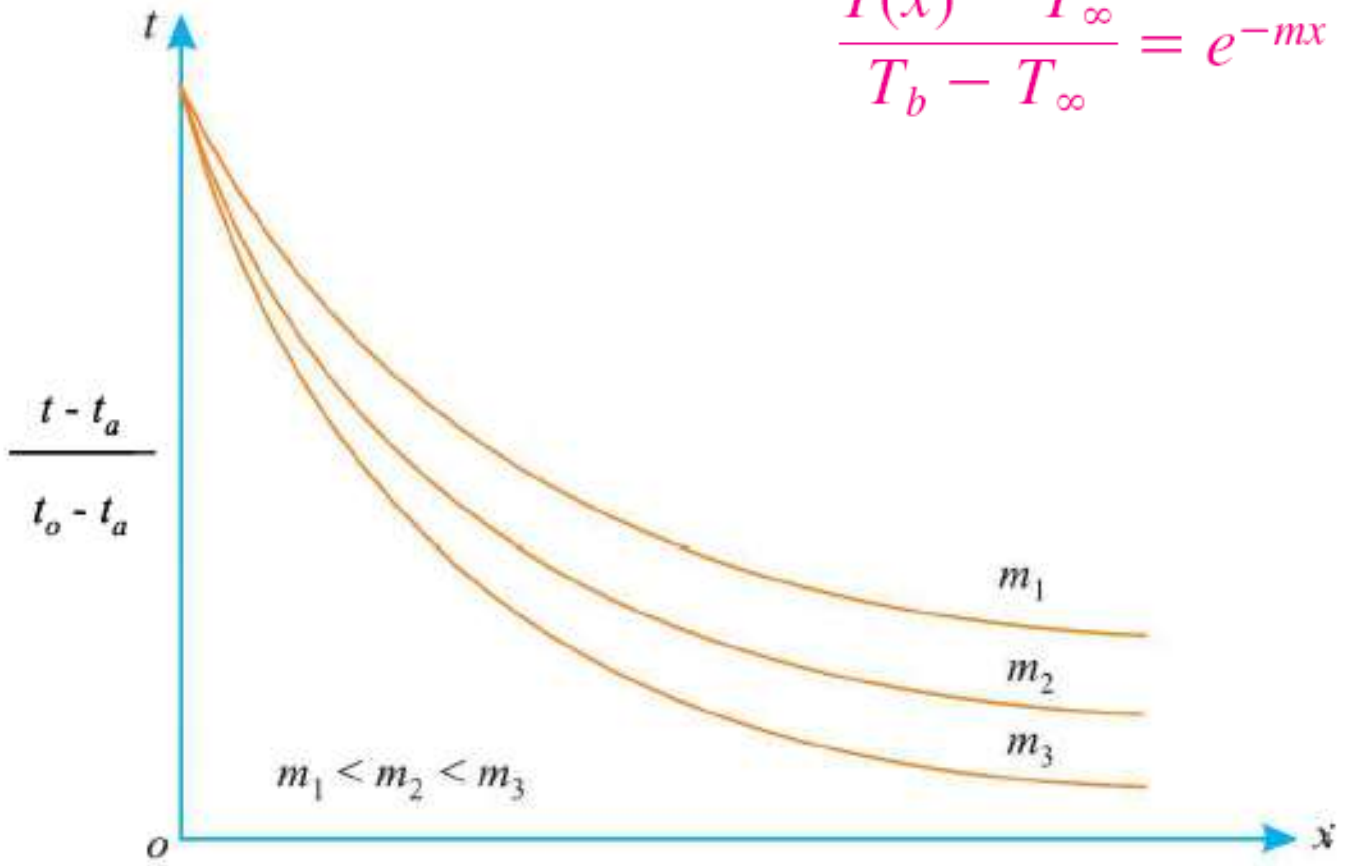


($p = \pi D$, $A_c = \pi D^2/4$ for a cylindrical fin)

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}}$$

Temperature decreases exponentially
from the fin base to the fin tip

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}}$$



(*i*) As the value of m increases, the dimensionless temperature falls;

(*ii*) As the length of fin increases to infinity all the curves approach $\frac{t - t_a}{t - t} = 0$ *asymptotically*.

The steady rate of heat transfer from the entire fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}}$$

$$Q_{fin} = -k\,A_{cs}\left[\frac{dt}{dx}\right]_{x=0}$$

$$\left[\frac{dt}{dx}\right]_{x=0} = \left[-m\,(t_0 - t_a)\,e^{-mx}\right]_{x=0} = -m\,(t_0 - t_a)$$

$$Q_{fin} = -k\,A_{cs} \times [-\,m\,(t_o - t_a)] = k\,A_{cs}\,m\,(t_o - t_a)$$

$$Q_{fin} = k\,A_{cs}\,m\,(t_o - t_a)$$

$$Q_{fin} = k\,A_{cs}\sqrt{\frac{Ph}{kA_{cs}}}\,(t_0 - t_a)$$

$$Q_{fin} = \sqrt{Ph\,k\,A_{cs}}\,(t_0 - t_a)$$

$$\dot{Q}_{\text{long fin}} = -kA_c\left.\frac{dT}{dx}\right|_{x=0}$$

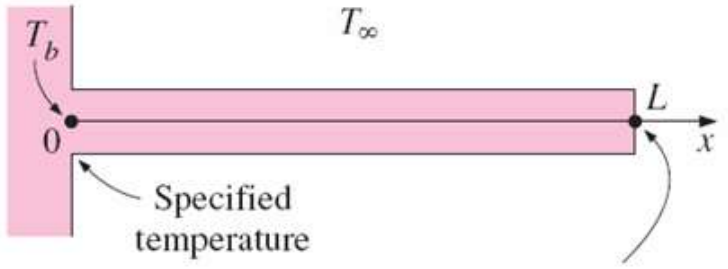
$$= \sqrt{hpkA_c}\,(T_b - T_\infty)$$

2 Negligible Heat Loss from the Fin Tip (Adiabatic fin tip, $Q_{\text{fin tip}} = 0$)

Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic assumption is for heat transfer from the fin tip to be negligible since the surface area of the fin tip is usually a negligible fraction of the total fin area.

Boundary condition at fin tip

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$



The variation of temperature along the fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$$

Heat transfer from the entire fin

$$\dot{Q}_{\text{adiabatic tip}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$= \sqrt{hpkA_c}\,(T_b - T_\infty) \tanh mL$$

Infinitely long fin

$$\dot{Q}_{\text{long fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

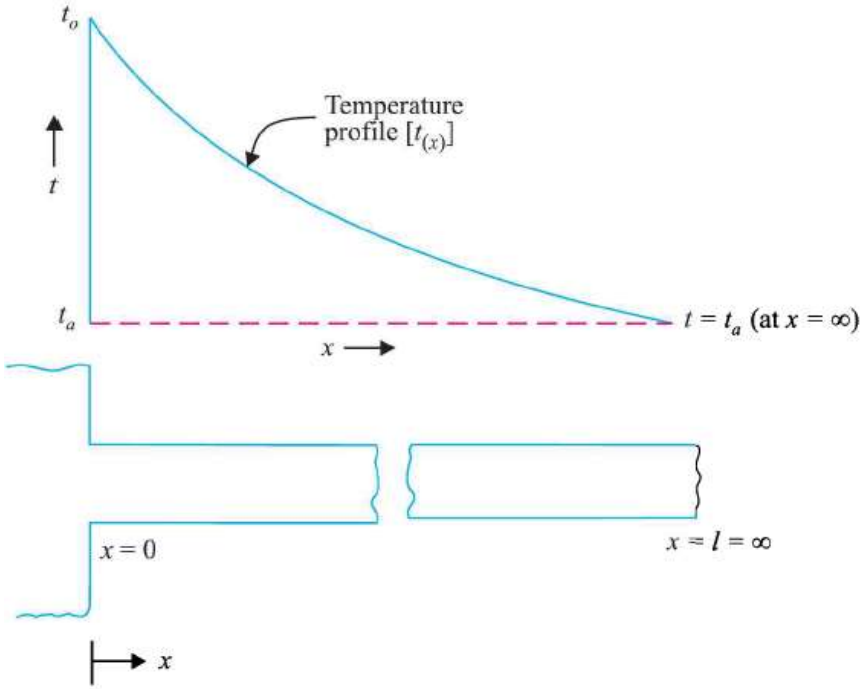
$$= \sqrt{hpkA_c}\,(T_b - T_\infty)$$

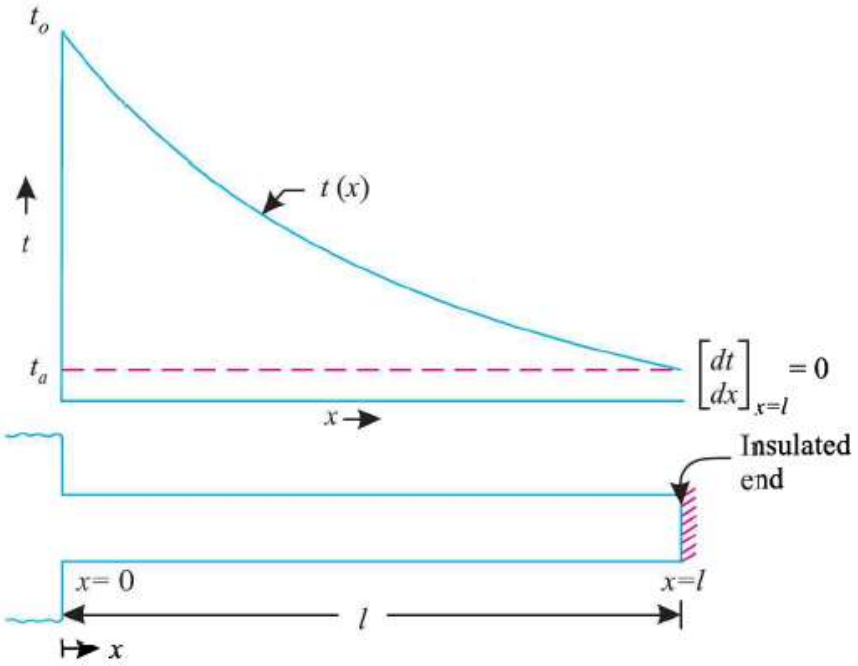
Finite fin with adiabatic tip

$$\dot{Q}_{\text{adiabatic tip}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$= \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL$$

Note that heat transfer rate from an infinitely long fin differs from the heat transfer rate from a fin with adiabatic tip by a factor of tanh(mL) and as L becomes larger and larger this factor approaches one.


t_o
Temperature profile $[t_{(x)}]$
t
t_a
$t = t_a$ (at $x = \infty$)
x
$x = 0$
$x = l = \infty$
x


t_o
$t(x)$
t
t_a
$\left[\frac{dt}{dx}\right]_{x=l} = 0$
x
Insulated end
$x = 0$
$x = l$
l
x

3 Specified Temperature ($T_{fin,tip} = T_L$)

In this case the temperature at the end of the fin (the fin tip) is fixed at a specified temperature T_L.

This case could be considered as a generalization of the case of Infinitely Long Fin where the fin tip temperature was fixed at T_∞.

Boundary condition at fin tip: $\theta(L) = \theta_L = T_L - T_\infty$

Specified fin tip temperature:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{[(T_L - T_\infty)/(T_b - T_\infty)]\sinh mx + \sinh m(L-x)}{\sinh mL}$$

Specified fin tip temperature:

$$\dot{Q}_{\text{specified temp.}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$= \sqrt{hpkA_c}(T_b - T_\infty)\frac{\cosh mL - [(T_L - T_\infty)/(T_b - T_\infty)]}{\sinh mL}$$

4 Convection from Fin Tip

The fin tips, in practice, are exposed to the surroundings, and thus the proper boundary condition for the fin tip is convection that may also include the effects of radiation.

Consider the case of convection only at the tip. The condition at the fin tip can be obtained from an energy balance at the fin tip.

$$(\dot{Q}_{\text{cond}} = \dot{Q}_{\text{conv}})$$

Boundary condition at fin tip:

$$-kA_c \left.\frac{dT}{dx}\right|_{x=L} = hA_c[T(L) - T_\infty]$$

Convection from fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x) + (h/mk)\sinh m(L - x)}{\cosh mL + (h/mk)\sinh mL}$$

Convection from fin tip:

$$\dot{Q}_{\text{convection}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$= \sqrt{hpkA_c}(T_b - T_\infty)\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

Temperature distribution and heat loss for fins of uniform cross section

Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
Convection heat transfer: $h\theta(L) = -kd\theta/dx\|_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
Adiabatic $d\theta/dx\|_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M\tanh mL$
Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$
Infinite fin ($L \to \infty$): $\theta(L) = 0$	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

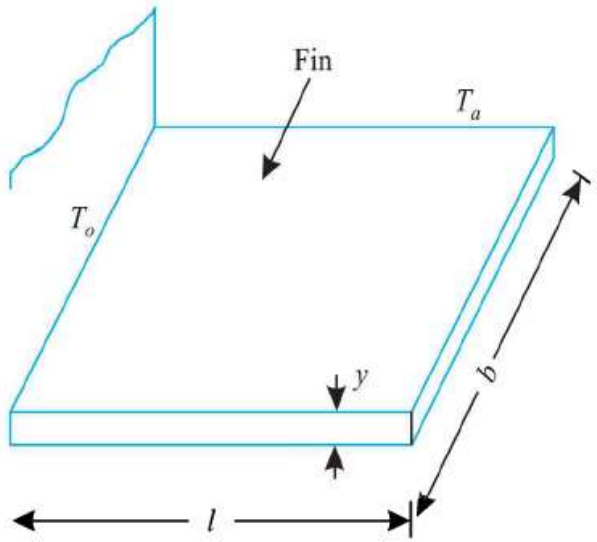
Find out the amount of heat transferred through an iron fin of length 50 mm, width 100 mm and thickness 5 mm. Assume k = 210 kJ/mh°C and h = 42 kJ/m²h°C for the material of the fin and the temperature at the base of the fin as 80°C. Also determine the temperature at tip of the fin, if the atmosphere temperature is 20°C.

Amount of heat transferred through the fin, Q :

Perimeter, $P = 2\,(b + y) = 2\,(0.1 + 0.005) = 0.21$ m

Area, $A_{cs} = b \times y = 0.1 \times 0.005 = 0.0005$ m²

$$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{42 \times 0.21}{210 \times 0.0005}} = 9.165$$



$$Q_{\text{fin}} = \sqrt{PhkA_{cs}}\,(t_0 - t_a)\tanh(ml)$$

$$= \sqrt{0.21 \times 42 \times 210 \times 0.0005}$$

$$\times (80 - 20)\tanh(9.165 \times 0.05)$$

$$= 0.9263 \times 60 \times 0.4286$$

$$= \mathbf{24.75\ kJ/h\ (Ans.)}$$

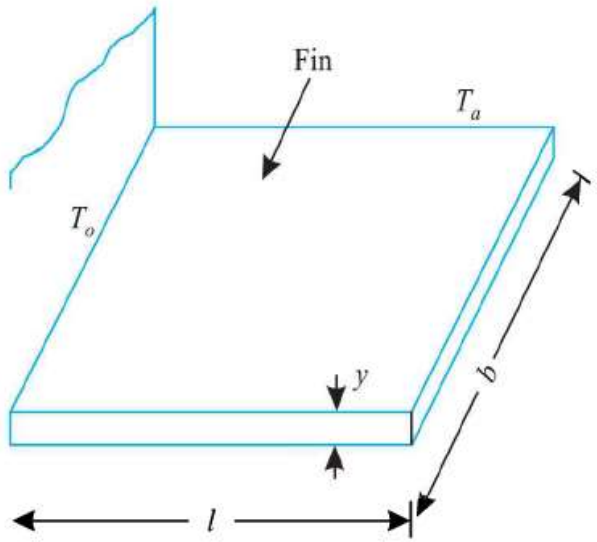
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Temperature at the tip of the fin, Q_L :

We know that,

$$\frac{\theta}{\theta_0} = \frac{t - t_a}{t_0 - t_a} = \frac{\cosh\{m(l - x)\}}{\cosh(ml)}$$

...[Eqn. (2.134)]

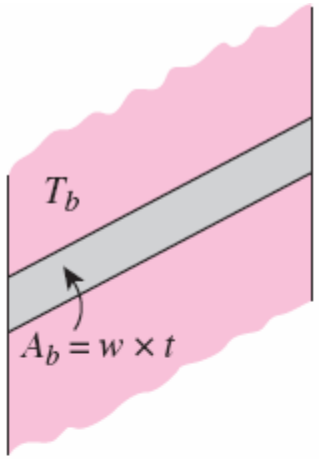


At $x = l$, we have

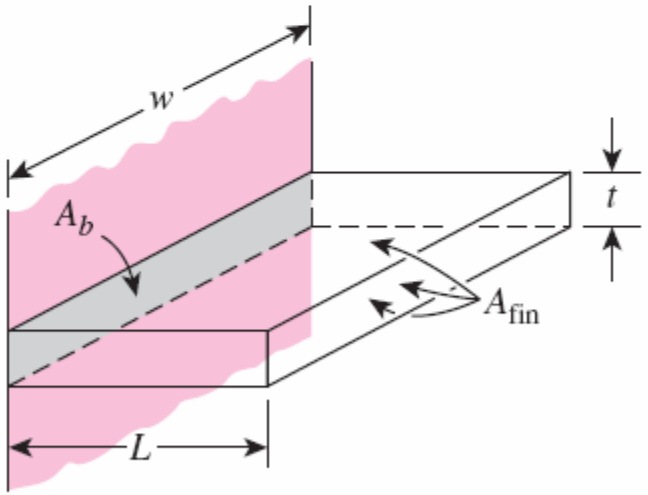
$$\frac{t - 20}{80 - 20} = \frac{1}{\cosh\ ml}$$

$$t = \frac{60}{\cosh(9.165 \times 0.05)} + 20 = \mathbf{74.21°C}\ \textbf{(Ans.)}$$

Fin Efficiency

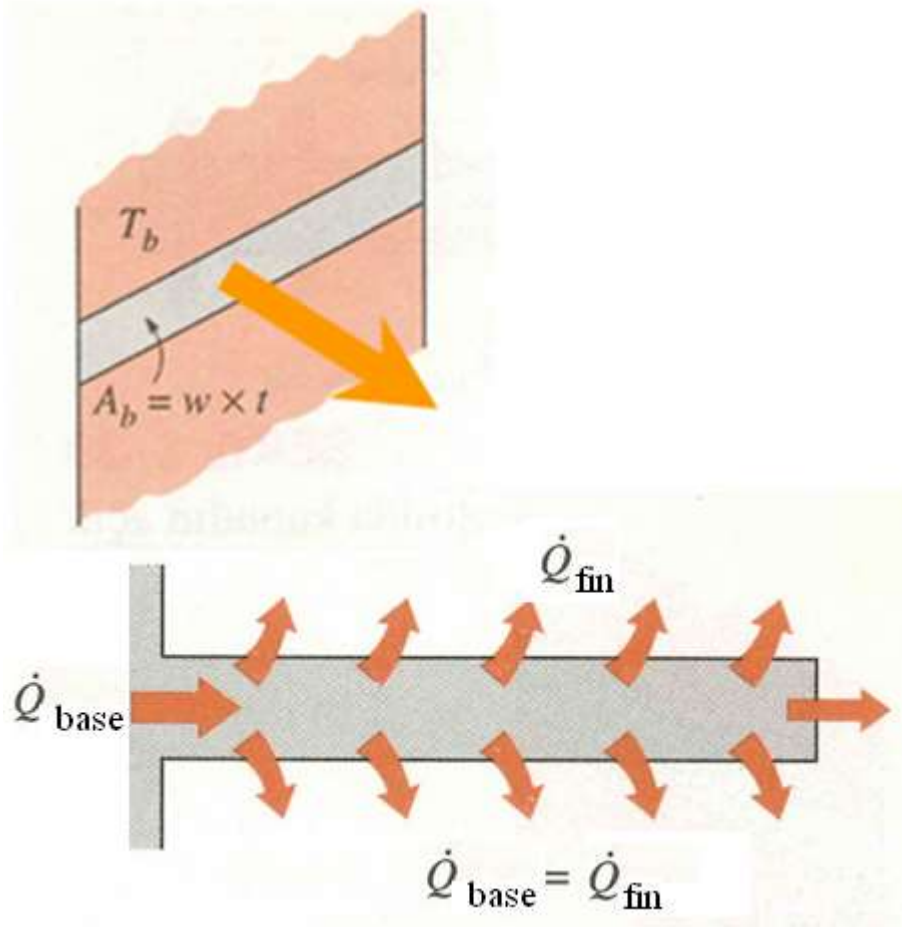


(*a*) Surface without fins



(*b*) Surface with a fin

$$A_{fin} = 2 \times w \times L + w \times t$$
$$\cong 2 \times w \times L$$



Fins enhance heat transfer from a surface by enhancing surface area.

Fin Efficiency

Efficiency of fin (η_{fin}) :

The efficiency of a fin is defined as the ratio of the actual heat transferred by the fin to the maximum heat transferable by fin, if entire fin area were at base temperature.

i.e.,

$$\eta_{fin} = \frac{\text{Actual heat transferred by the fin } (Q_{fin})}{\text{Maximum heat that would be transferred if whole surface of the fin is maintained at the base temperature } (Q_{max})}$$

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{\text{fin}}\,(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{mL}$$

$$\eta_{\text{adiabatic tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL}{hA_{\text{fin}}\,(T_b - T_\infty)} = \frac{\tanh mL}{mL}$$

L-17

It is better to use 10 fins of 5 cm length than 5 fins of 10 cm length. State and prove correctness of the statement. Take properties as follows : Diameter of fin = 10 mm, Thermal conductivity = 45 W/m.K Heat transfer coefficient = 95 W/m2.K.

The length of fin in either case is much larger than its diameter, therefore, assuming negligible heat loss from fin tip. For a fin

$$A_c = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.01)^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$P = \pi d = \pi \times 0.01 = 0.01\,\pi \text{ m}$$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{95 \times 0.01\,\pi}{45 \times 78.54 \times 10^{-6}}} = 29.06$$

For arrangement 1 :

$$mL_1 = 29.06 \times 0.05 = 1.453$$

$$\tanh (mL_1) = 0.896$$

The fin efficiency

$$\eta_{fin} = \frac{\tanh mL_1}{mL_1} = \frac{0.896}{1.453} = 61.68\%$$

The heat transfer rate from 10 fins

$$Q_{fin} = N_{fin} \sqrt{hPkA_c}\ (T_0 - T_\infty) \tanh (mL_1)$$

$$= 10 \times \sqrt{95 \times 0.01 \times \pi \times 45 \times 78.54 \times 10^{-6}} \times (T_0 - T_\infty) \times 0.896$$

$$= 9.2\ (T_0 - T_\infty) \text{ Watts.}$$

For arrangement 2 : 5 fins 10 cm long

$$mL_2 = 29.06 \times 0.1 = 2.906$$

$$\tanh (mL_2) = 0.994$$

$$\eta_{fin} = \frac{\tanh mL}{mL} = \frac{0.994}{2.906} = 34.2\%$$

and heat transfer rate with this arrangement

$$Q_{fin} = 5 \times \sqrt{95 \times 0.01\,\pi \times 45 \times 78.54 \times 10^{-6}} \times (T_0 - T_\infty) \times 0.994$$

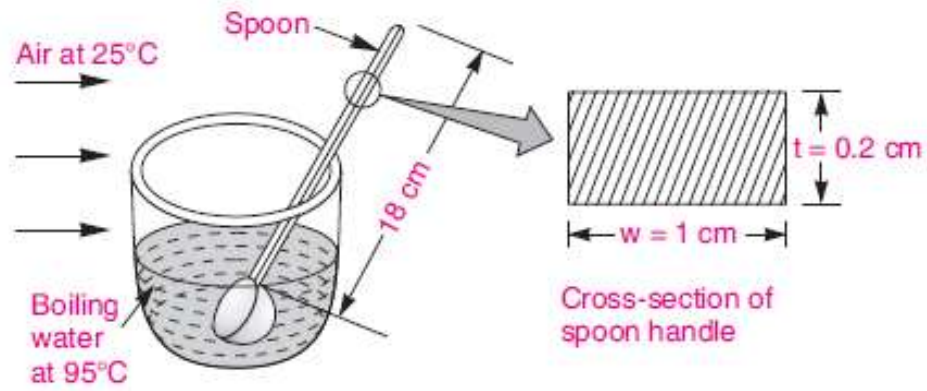
$$= 5.10\ (T_0 - T_\infty) \text{ Watts.}$$

The short fins are always effective.

Therefore, *large number of short fins should be installed on a surface to increase the heat transfer rate.*

L-18

Consider a stainless steel spoon (k = 15.1 W/m.K), partially immersed in the boiling water at 95°C in a kitchen at 25°C. The handle of the spoon has a cross-section 0.2 cm × 1 cm and it extends 18 cm in the air from the free surface of the water. If the heat transfer coefficient on the exposed surface of the spoon is 15 W/m2.K, calculate the temperature difference across the exposed surface of the spoon handle. State your assumptions, if any.



$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$$

$$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{1}{\cosh mL}$$

$$P = 2(w + t) = 2(1\text{ cm} + 0.2\text{ cm})$$
$$= 2.4\text{ cm} = 0.024\text{ m}$$
$$A_c = wt = 1\text{ cm} \times 0.2\text{ cm}$$
$$= 0.2\text{ cm}^2 = 0.2 \times 10^{-4}\text{ m}^2$$

$$m = \sqrt{\frac{15 \times 0.024}{15.1 \times 0.2 \times 10^{-4}}} = 34.52\text{ m}^{-1}$$

$$mL = 34.52 \times 0.18 = 6.21$$

$$\cosh mL = 250.0$$

$$T_L - T_\infty = \frac{95 - 25}{250} = 0.2799 \approx 0.28°C$$

$$T_L = 25 + 0.28 = 25.28°C,$$

$$T_0 - T_L = 95 - 25.28 = \mathbf{69.72°C.}$$

Fin Effectiveness

Fins are used to enhance heat transfer, and the use of fins on a surface cannot be recommended unless the enhancement in heat transfer justifies the added cost and complexity associated with fins.

in fact, there is no assurance that adding fins on a surface will enhance heat transfer.

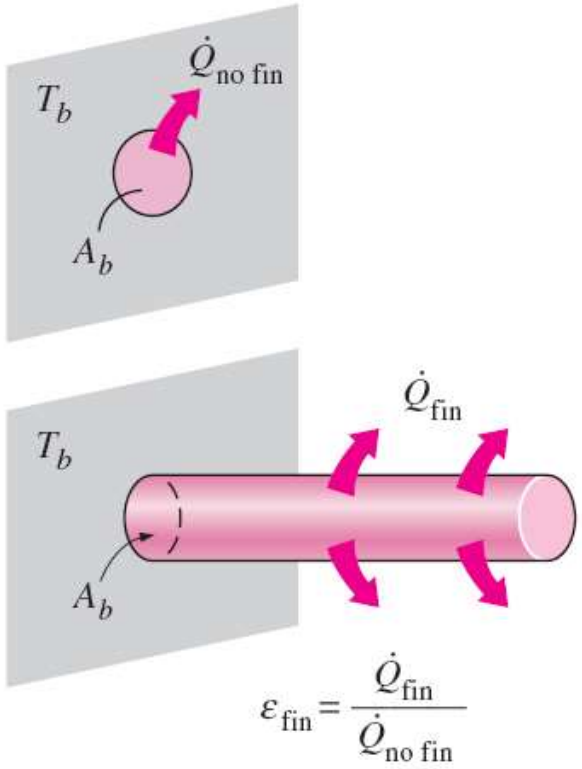
The performance of the fins is judged on the basis on the basis of the enhancement in heat transfer relative to the no-fin case.

Fin effectiveness is defined as

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b}$$

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)}$$

$$= \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b}$$



An effectiveness of $\varepsilon_{fin} = 1$ indicates that the addition of fins to the surface does not affect heat transfer at all. That is heat conducted to the fin through the base area A_b is equal to the heat transferred from the same area A_b to the surrounding medium.

An effectiveness of $\varepsilon_{fin} < 1$ indicates that the fin actually acts as an insulation, slowing down the heat transfer from the surface. This situation can occur when fins made of low thermal conductivity materials are used.

An effectiveness of $\varepsilon_{fin} > 1$ indicates that fins are enhancing heat transfer from the surface, as they should.

The Relationship Between the Fin Efficiency and the Fin Effectiveness

Both the fin efficiency and the fin effectiveness are related to the performance of the fin, but they are different quantities.

They are related to each other as

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\eta_{\text{fin}}$$

Therefore, the fin effectiveness can be determined easily when the fin efficiency is known, or vice versa.

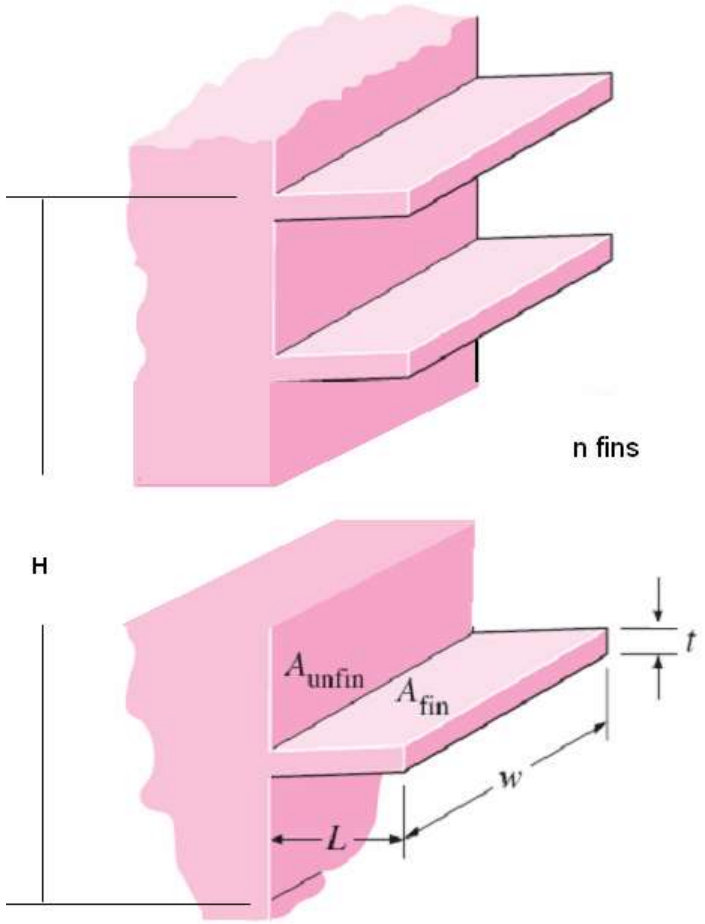
The effectiveness of a long fin is given by

$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}$$

- The thermal conductivity **k** of the fin material should be as high as possible.
  Use aluminum, copper, iron.

- The ratio of the perimeter to the cross-sectional area of the fin $\mathbf{p/A_c}$ should be as high as possible.
  Use slender pin fins.

- Low convection heat transfer coefficient **h**.
  Place fins on the gas (air) side.



Overall Effectiveness of Finned Surfaces

The total rate of heat transfer from a finned surface with n fins will be

$A_{no\ fin} = wxH$

$A_{unfin} = wxH - nx(txw)$

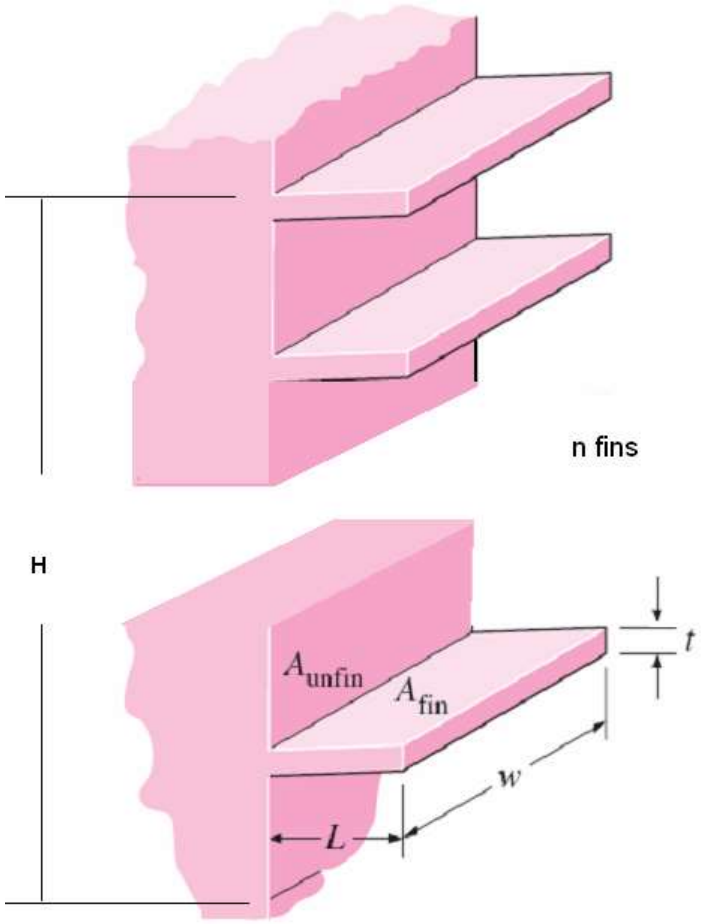
$A_{single\ fin} = 2xLxw + txw \approx 2xLxw$

$A_{fin} = n\,A_{single\ fin}$

$$\dot{Q}_{\text{total, fin}} = \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}}$$
$$= hA_{\text{unfin}}(T_b - T_\infty) + \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$
$$= h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)$$

Overall effectiveness for a finned surface with n fins will then be

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}\,(T_b - T_\infty)}$$

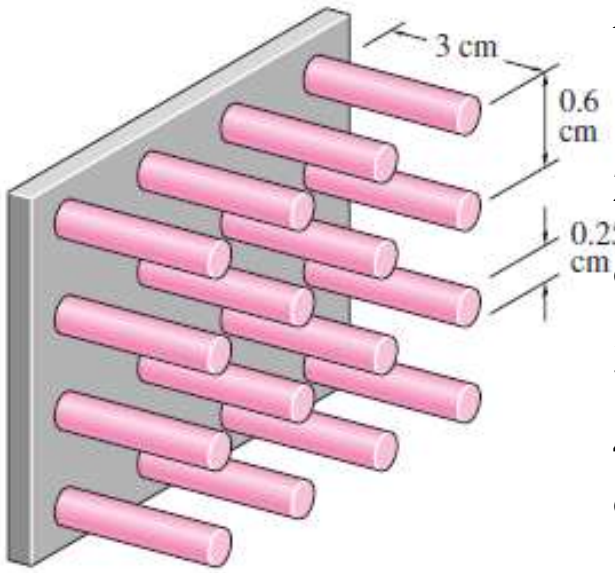


The overall fin effectiveness depends on the fin density (number of fins per unit length) as well as the effectiveness of the individual fins.

The overall effectiveness is a better measure of the performance of a finned surface than the effectiveness of the individual fins.

L-18

A hot surface at 100°C is to be cooled by attaching 3-cm-long, 0.25-cm-diameter aluminum pin fins (k 237 W/m · °C) to it, with a center-to-center distance of 0.6 cm. The temperature of the surrounding medium is 30°C, and the heat transfer coefficient on the surfaces is 35 W/m2 · °C. Determine the rate of heat transfer from the surface for a 1-m X 1-m section of the plate. Also determine the overall effectiveness of the fins.



Assumptions

1 Steady operating conditions exist.

2 The temperature along the fins varies in one direction only (normal to the plate).

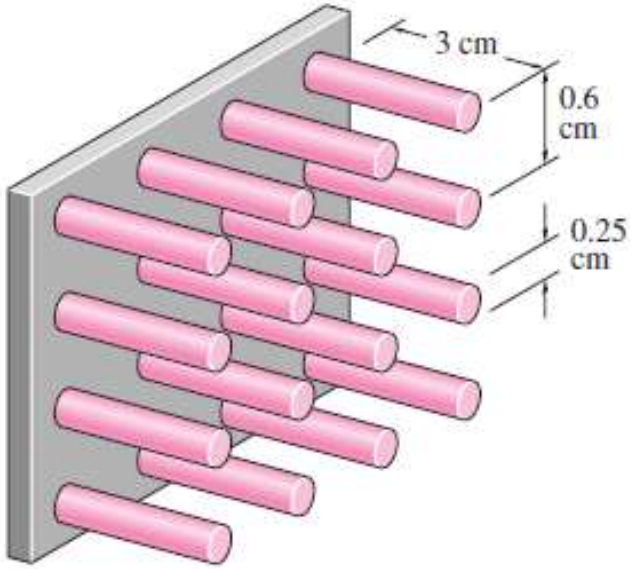
3 Heat transfer from the fin tips is negligible.

4 The heat transfer coefficient is constant and uniform over the entire fin surface.

5 The thermal properties of the fins are constant.

6 The heat transfer coefficient accounts for the effect of radiation from the fins.

L-18



$$a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(35\ \text{W/m}^2.^\circ\text{C})}{(237\ \text{W/m.}^\circ\text{C})(0.0025\ \text{m})}} = 15.37\ \text{m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(15.37\ \text{m}^{-1} \times 0.03\ \text{m})}{15.37\ \text{m}^{-1} \times 0.03\ \text{m}} = 0.935$$

Note : a = m

The number of fins, finned and unfinned surface areas, and heat transfer rates from those areas are

$$n = \frac{1\ \text{m}^2}{(0.006\ \text{m})(0.006\ \text{m})} = 27{,}777$$

$$A_{\text{fin}} = 27777\left[\pi DL + \frac{\pi D^2}{4}\right] = 27777\left[\pi(0.0025)(0.03) + \frac{\pi(0.0025)^2}{4}\right] = 6.68\ \text{m}^2$$

$$A_{\text{unfinned}} = 1 - 27777\left(\frac{\pi D^2}{4}\right) = 1 - 27777\left[\frac{\pi(0.0025)^2}{4}\right] = 0.86\ \text{m}^2$$

$$\dot{Q}_{\text{finned}} = \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}}hA_{\text{fin}}(T_b - T_\infty)$$
$$= 0.935(35\ \text{W/m}^2.^\circ\text{C})(6.68\ \text{m}^2)(100-30)^\circ\text{C}$$
$$= 15{,}300\ \text{W}$$

$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_b - T_\infty) = (35\ \text{W/m}^2.^\circ\text{C})(0.86\ \text{m}^2)(100-30)^\circ\text{C}$$
$$= 2107\ \text{W}$$

Then the total heat transfer from the finned plate becomes

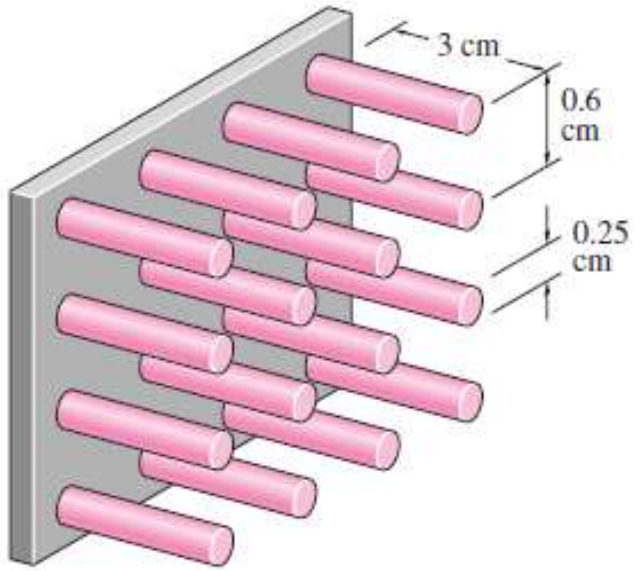
$$\dot{Q}_{\text{total,fin}} = \dot{Q}_{\text{finned}} + \dot{Q}_{\text{unfinned}} = 15{,}300 + 2107 = 1.74\times10^4\ \text{W} = \mathbf{17.4\ kW}$$

The rate of heat transfer if there were no fin attached to the plate would be

$$A_{\text{no fin}} = (1\ \text{m})(1\ \text{m}) = 1\ \text{m}^2$$
$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (35\ \text{W/m}^2.^\circ\text{C})(1\ \text{m}^2)(100-30)^\circ\text{C} = 2450\ \text{W}$$

Then the fin effectiveness becomes

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{17{,}400}{2450} = \mathbf{7.10}$$



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TRANSIENT HEAT CONDUCTION

TIME DEPENDENT CONDUCTION - Temperature history inside a conducting body that is immersed suddenly in a bath of fluid at a different temperature.

Ex: Quenching of special alloys

The temperature of such a body varies with time as well as position.

$$T(x,y,z,t)$$

***(x,y,z)* - Variation in the *x,y* and *z* directions**

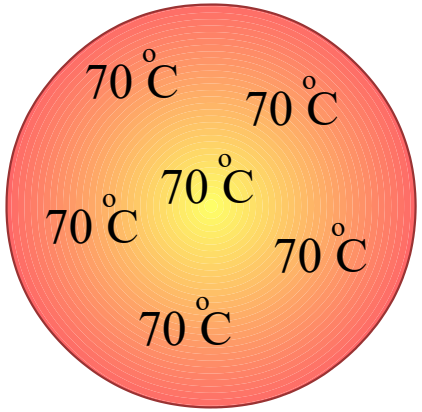
t **- Variation with time**

In this chapter, we consider the variation of temperature with *time* as well as *position* in one and multi-dimensional systems.

LUMPED SYSTEM ANALYSIS

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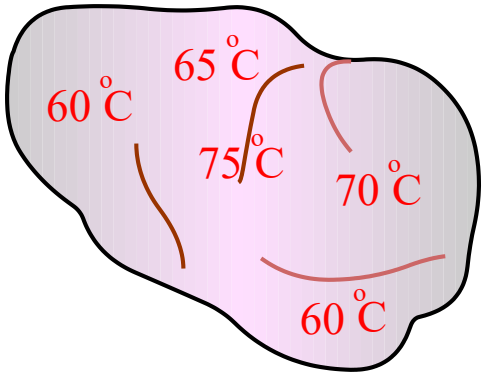
COPPER BALL WITH UNIFORM TEMPERATURE



Temperature of the copper ball changes with time, but it does not change with position at any given time.

Temperature of the ball remains uniform at all times

POTATA TAKEN FROM BOILING WATER



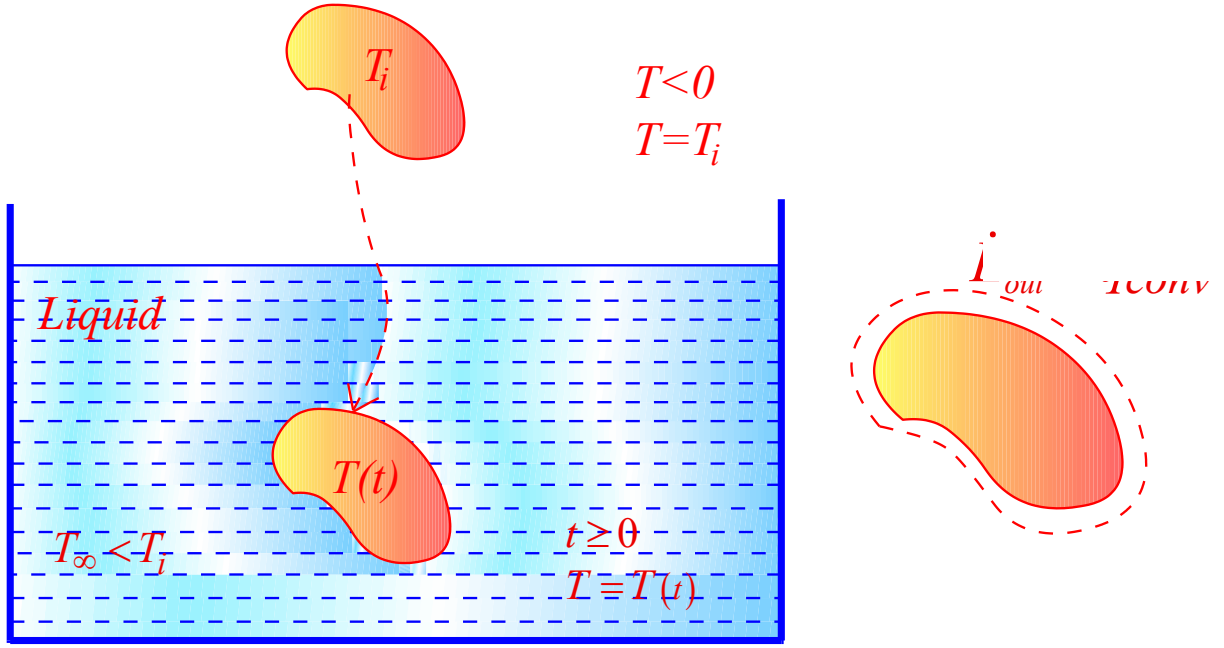
Large potato put in a vessel with boiling water.

After few minutes, if you take out the potato, temperature distribution within the potato is not even close to being uniform.

Thus, lumped system analysis is not applicable in this case.

Hot metal forging that is initially at a uniform temperature T_i and is quenched by immersing it in a liquid of lower temperature $T_\infty < T_i$

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During a differential time interval *dt*, the temperature of the body rises by a differential amount *dT*. An energy balance of the solid for the time interval *dt* can be expressed as

$$\begin{pmatrix} \text{Heat transfer int o the} \\ \text{body during } dt \end{pmatrix} = \begin{pmatrix} \text{The increase in the energy} \\ \text{of the body during } dt \end{pmatrix}$$

$$-hA(T-T_\infty)dt = \rho V C_p dT$$

$$-hA(T-T_\infty)dt=\rho V C_p dT$$

V - Body volume

A_s - surface area

ρ - density of the body material

C_p - specific heat of the body material

$$\theta=T-T_\infty$$

$$\frac{\rho V C_P}{hA_s}\frac{d\theta}{dt}=-\theta$$

$t = 0$ and $T(0) = T_i$

$$\frac{\rho V C_P}{hA_s}\int_{\theta_i}^{\theta}\frac{d\theta}{\theta}=-\int_0^t dt \qquad \theta_i=T_i-T_\infty$$

$$\frac{\rho V C_P}{hA_s}\ln\frac{\theta}{\theta_i}=-t$$

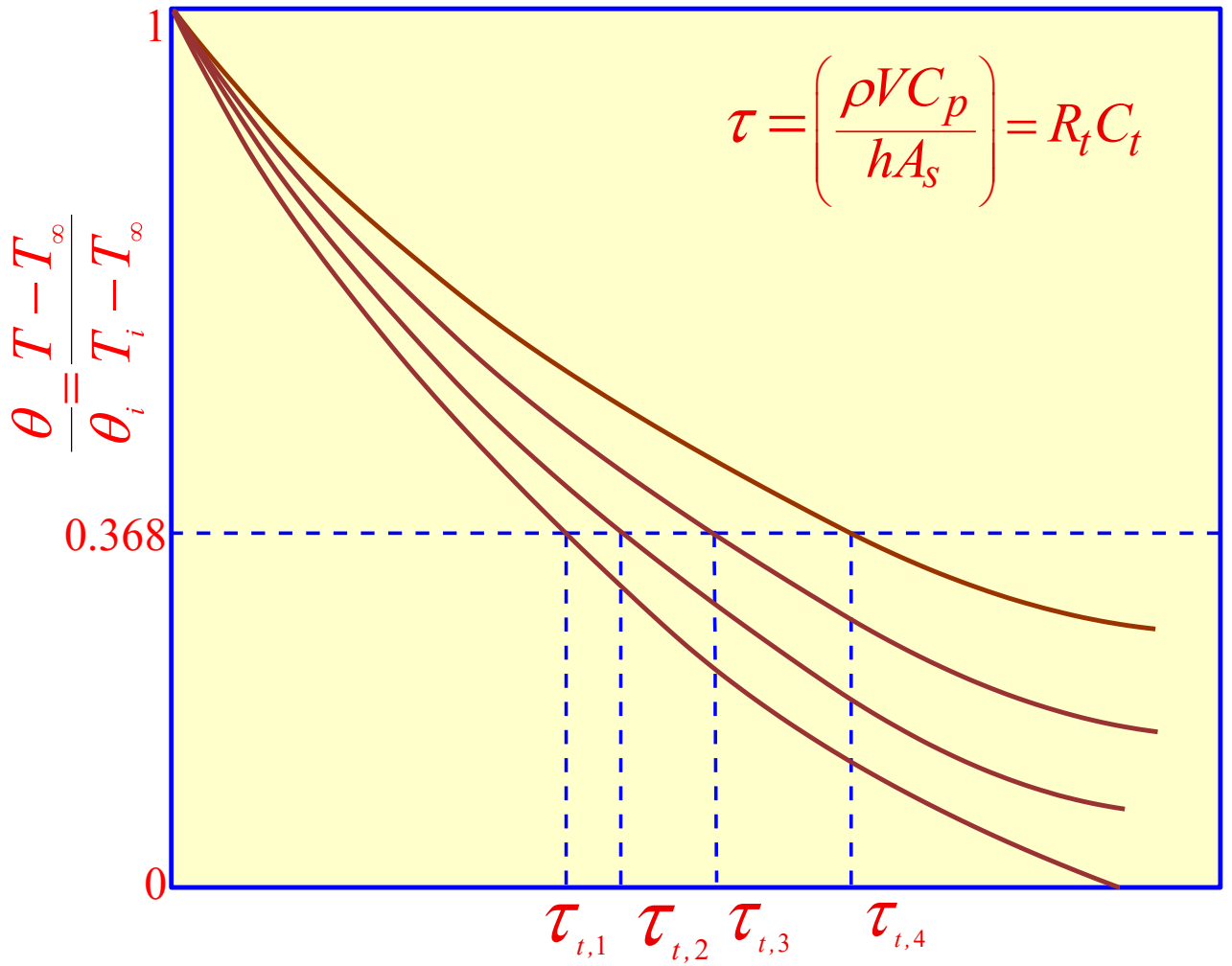
$$\frac{\theta}{\theta_i}=\frac{T-T_\infty}{T_i-T_\infty}=exp\left[-\left(\frac{hA_s}{\rho V C_P}\right)t\right]$$

$$\tau=\left(\frac{1}{hA_s}\right)\left(\rho V C_p\right)=R_t C_t$$

R_t - Resistance to convection heat transfer

C_t - Lumped thermal capacitance of the solid

Transient Temperature Response of Lumped Capacitance Solids



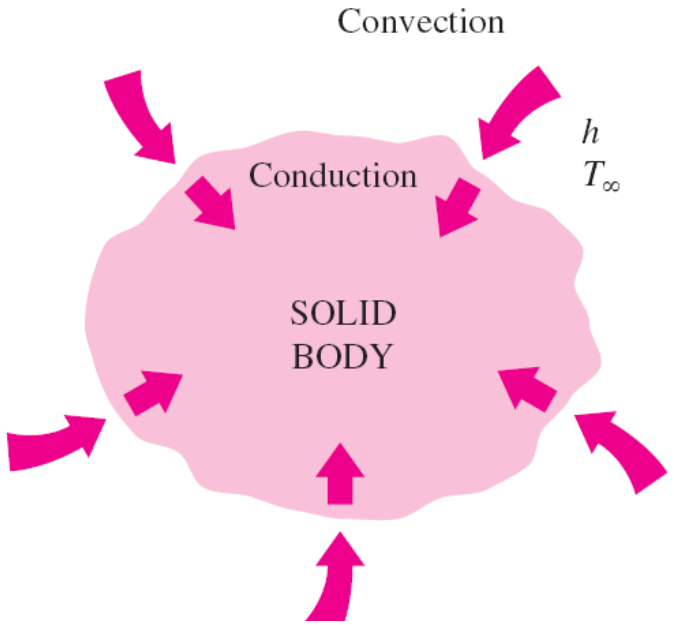
The rate of convection heat transfer between the body and its environment at that time can be determined from Newton's law of cooling

$$Q = h A_s \left[T(t) - T_\infty \right]$$

CRITERIA OF THE LUMPED SYSTEM ANALYSIS

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Biot number Bi



$$Bi = \frac{hL_c}{k}$$

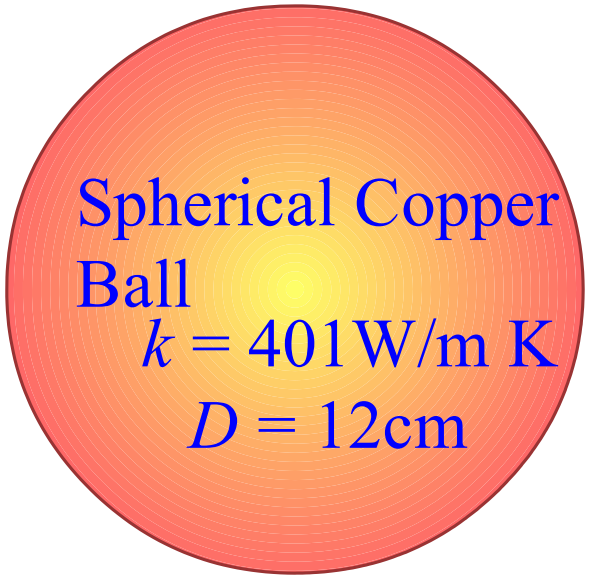
$$Bi = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{Convection\ at\ the\ surface\ of\ the\ body}{Conduction\ within\ the\ body}$$

$$Bi = \frac{L_c/k}{1/h} = \frac{Conduction\ resistance\ within\ the\ body}{Convective\ resistance\ at\ the\ surface\ of\ the\ body}$$

Lumped system analysis is *exact* when *Bi = 0*

Generally, accepted norm for assuming lumped system analysis

$$Bi \leq 0.1$$



$$L_c = \frac{V}{A} = \frac{\frac{\pi D^3}{6}}{\pi D^2} = \frac{D}{6} = 0.02\,m$$

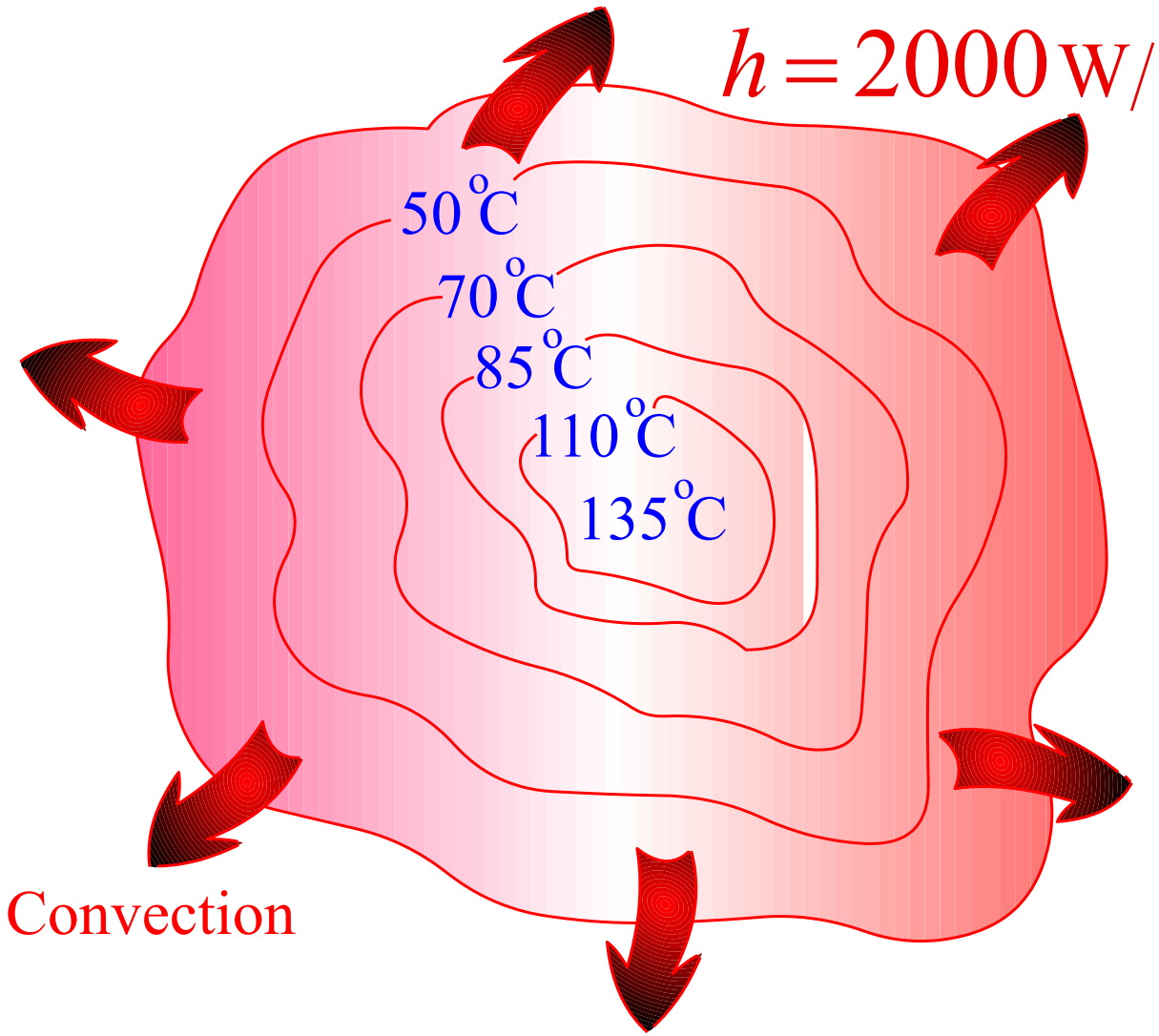
$$Bi = \frac{h L_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 < 0.1$$

Small bodies with higher thermal conductivities and low convection coefficients are most likely to satisfy the criterion for lumped system analysis

Heat conduction in a specified direction *n* per unit surface area is expressed as

$$\dot{q} = -k \frac{\partial T}{\partial n}$$

Larger the thermal conductivity ⇒ the smaller the temperature gradient



When the convection coefficient *h* is high and *k* is low, large temperature differences occur between the inner and outer regions of a large solid

Problem: A thermocouple junction, which may be approximated as a sphere, is to be used for temperature measurement in a gas stream. The convection coefficient between the junction surface and the gas is known to be h = 400 W/m^2.K and the junction thermophysical properties are k = 20 W/m.K, C_p = 400 U/kg.K, and ρ = 8500 kg/m^3. Determine the junction diameter needed for the thermocouple to have a time constant of one second. If the junction is at 25^0 C and is placed in a gas stream that is at 200^0 C, how long will it take for the junction to reach 199^0 C?

Known:

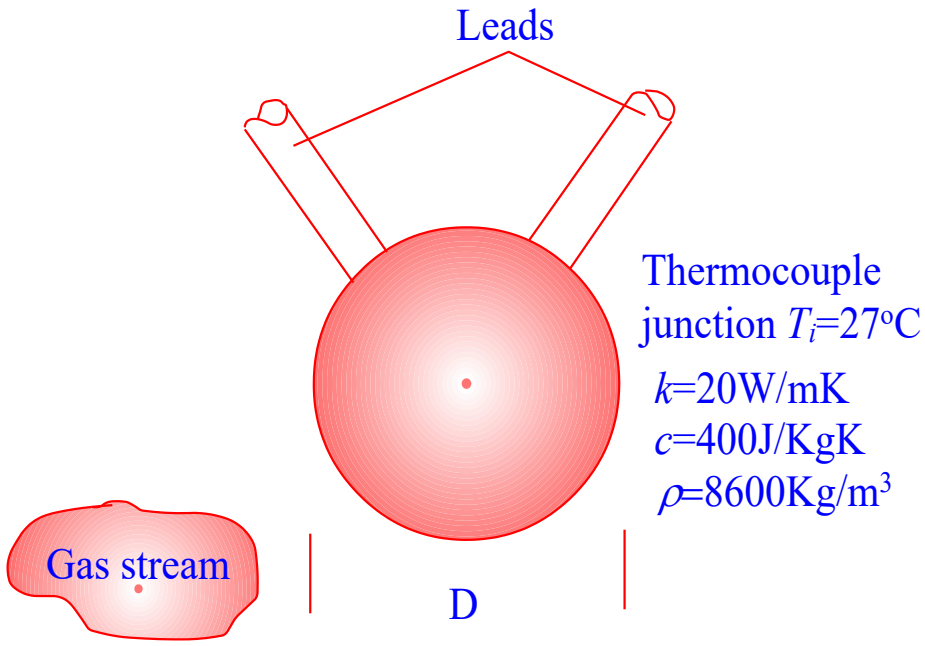
Thermophysical properties of thermocouple junction used to measure temperature of a gas stream

Find:

Junction diameter needed for a time constant of 1 second

Time required to reach 199^0 C in gas stream at 200^0 C

Schematic:



Assumptions:

Temperature of the junction is uniform at any instant

Radiation exchange with the surroundings is negligible

Losses by conduction through the leads are negligible

Constant properties

Analysis:

Because the junction diameter is unknown, it is not possible to begin the solution by determining whether the criterion for using the lumped capacitance method, $Bi \ll 0.1$. However, a reasonable approach is to use the method to find the diameter and to then determine whether the criterion is satisfied.

$A_s = \pi D^2$ and $V = \pi D^3/6$ for a sphere, $\Rightarrow L_c = r_0/3$

$$\tau_t = \frac{1}{h\pi D^2} \times \frac{\rho\,\pi D^3}{6} C_p \qquad D = \frac{6h\tau_t}{\rho C_p} = \frac{6\times 400\times 1}{8500\times 400} = 7.06\times 10^{-4}\ m$$

With $L_c = r_0/3$ it follows that

$$Bi = \frac{h\left(r_o/3\right)}{k} = \frac{400\times 3.53\times 10^{-4}}{3\times 20} = 2.35\times 10^{-3}$$

Criterion for using the lumped capacitance method, $Bi \ll 0.1$ is satisfied and the lumped capacitance method may be used to an excellent approximation.

The time required for the junction to reach $T = 199^{o}$ C

$$t = \frac{\rho\left(\pi D^3/6\right)C_p}{h\left(\pi D^2\right)} \ln \frac{T_i - T_\infty}{T - T_\infty} = \frac{\rho D C_p}{6h} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{8500 \times 7.06 \times 10^{-4} \times 400}{6 \times 400} \ln \frac{25 - 200}{199 - 200} = 5.2\,s$$

Comments:

Heat transfer due to radiation exchange between the junction and the surroundings and conduction through the leads would affect the time response of the junction and would, in fact, yield an equilibrium temperature that differs from T_∞.

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Unit 3

PHYSICAL MECHANISM OF CONVECTION

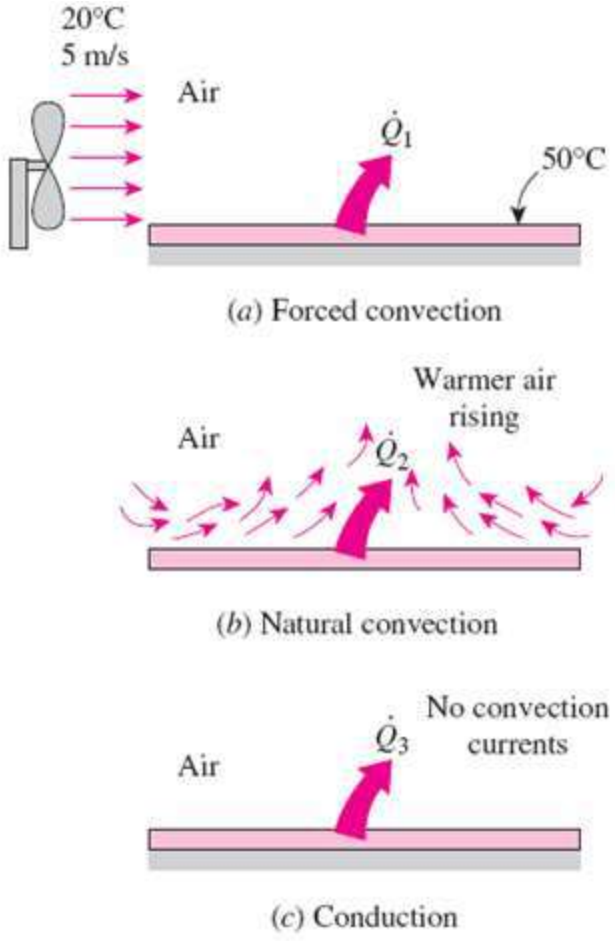


FIGURE 5.1
Heat transfer from a hot surface to the surrounding fluid by convection and conduction.

Conduction and convection both require the presence of a material medium but convection requires fluid motion.

Convection involves fluid motion as well as heat conduction.

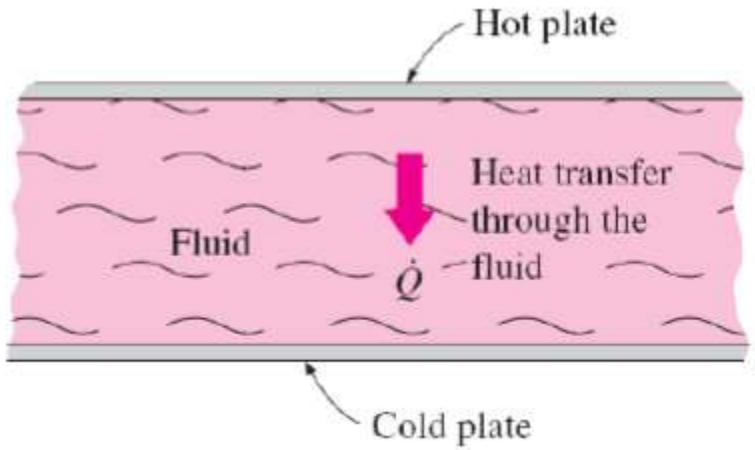
Heat transfer through a solid is always by conduction.

Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it.

Therefore, conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid.

PHYSICAL MECHANISM OF CONVECTION

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The fluid motion enhances heat transfer, since it brings warmer and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid.

•The rate of heat transfer through a fluid is much higher by convection than it is by conduction.

•In fact, the higher the fluid velocity, the higher the rate of heat transfer.

E.g Heat transfer through a fluid sandwiched between two parallel plates.

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- Convection heat transfer strongly depends on fluid properties *dynamic viscosity*, *thermal conductivity*, *density*, and *specific heat*, as well as *fluid velocity*.
- It also depends on the *geometry* and the *roughness* of the solid surface, in addition to the *type of fluid flow* (such as being streamlined or turbulent).

Newton's law of cooling

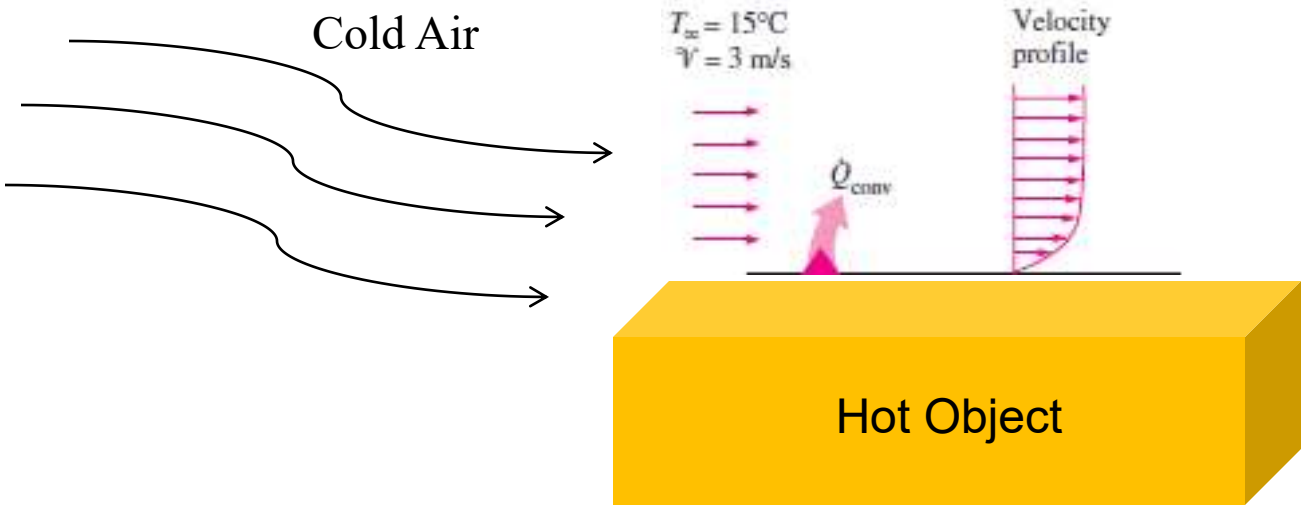
$$\dot{q}_{conv} = h(T_s - T_\infty) \qquad (W/m^2)$$

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad (W)$$

h = convection heat transfer coefficient, $W/m^2 \cdot °C$
A_s = heat transfer surface area, m^2
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C

Convection heat transfer coefficient, h: The rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference.

Concept of boundary layer



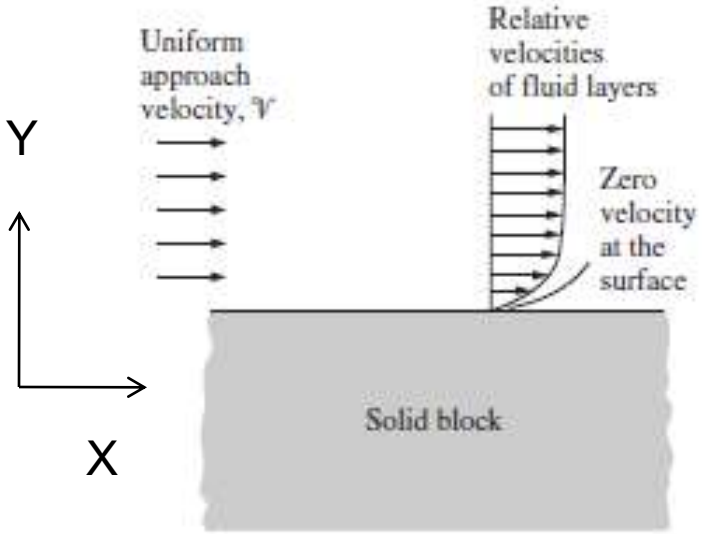
No-slip condition: A fluid in direct contact with a solid "sticks" to the surface due to viscous effects, and there is no slip.

Boundary layer: The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant.

The fluid property responsible for the no-slip condition and the development of the boundary layer is *viscosity.*

Concept of boundary layer

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$$\dot{q}_{conv} = \dot{q}_{cond} = -k_{fluid}\left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad (W/m^2)$$

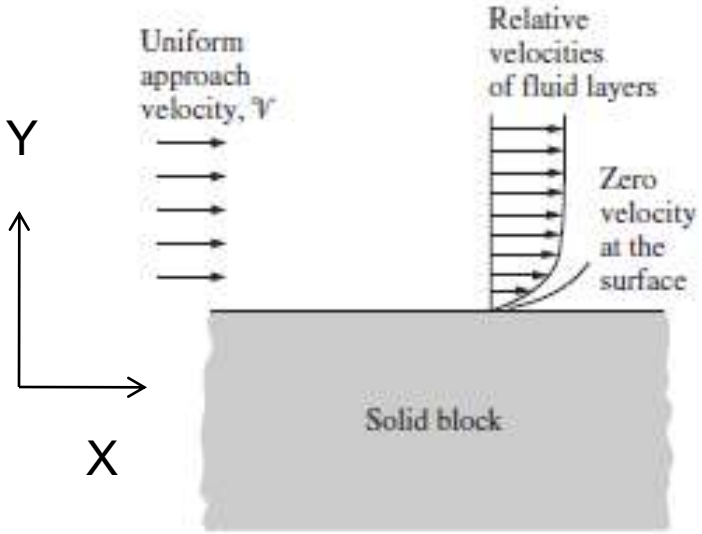
$$h = \frac{-k_{fluid}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} \qquad (W/m^2 \cdot {}^\circ C)$$

An implication of the no-slip condition is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by *pure conduction, since* the fluid layer is motionless, and can be expressed as

To determine the convection heat transfer coefficient when the temperature distribution within the fluid is known:

Concept average heat transfer coefficient

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The convection heat transfer coefficient, in general, varies along the flow (or x-) direction.

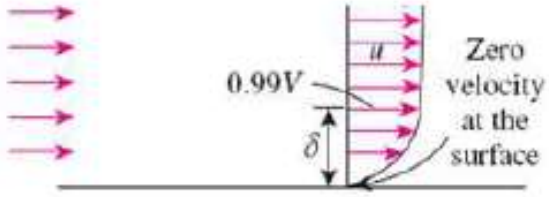
The average or mean convection heat transfer coefficient for a surface in such cases is determined by properly averaging the local convection heat transfer coefficients over the entire surface area As or length L as

$$h = \frac{1}{A_s}\int_{A_s} h_{\text{local}}\, dA_s \quad \text{and} \quad h = \frac{1}{L}\int_0^L h_x\, dx$$

Velocity Boundary Layer

Velocity boundary layer: The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt.

The *boundary layer thickness*, δ, is typically defined as the distance y from the surface at which $u = 0.99V$.

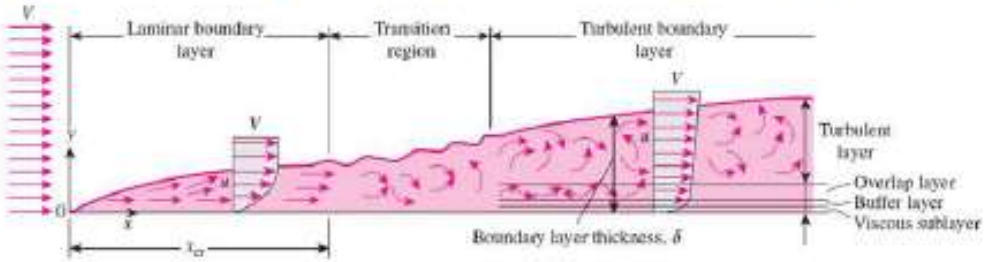


The development of a boundary layer on a surface is due to the no-slip condition and friction.

The hypothetical line of $u = 0.99V$ divides the flow over a plate into two regions:

Boundary layer region: The viscous effects and the velocity changes are significant.

Irrotational flow region: The frictional effects are negligible and the velocity remains essentially constant.

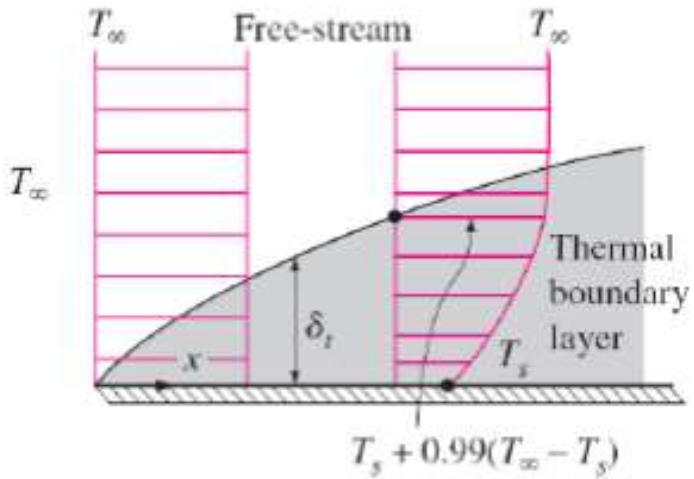


The very thin layer next to the wall where the viscous effects are dominant is the laminar sublayer. The velocity profile in this layer is nearly linear, and the flow is streamlined. Next to the laminar sublayer is the buffer layer, in which the turbulent effects are significant but not dominant of the diffusion effects, and next to it is the turbulent layer, in which the turbulent effects dominate.

Thermal Boundary Layer

A *thermal boundary layer* develops when a fluid at a specified temperature flows over a surface that is at a different temperature.

A thermal boundary layer on a flat plate (the fluid is hotter than the plate surface). Shown on the fig



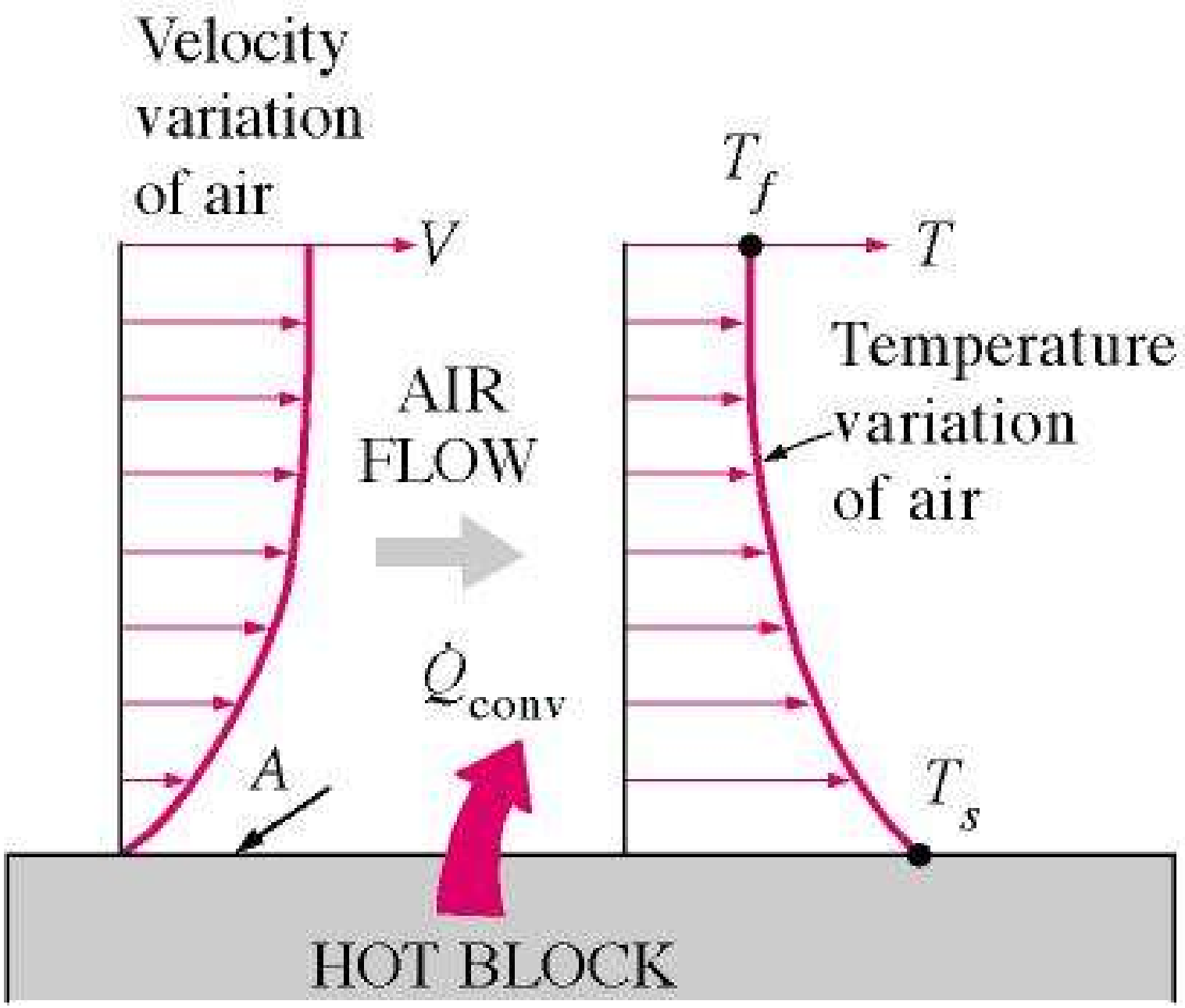
Thermal boundary layer: The flow region over the surface in which the temperature variation in the direction normal to the surface is significant.

The *thickness* of the thermal boundary layer δ_t at any location along the surface is defined as *the distance from the surface at which the temperature difference* $T - T_s$ *equals* $0.99(T_\infty - T_s)$.

The thickness of the thermal boundary layer increases in the flow direction, since the effects of heat transfer are felt at greater distances from the surface further down stream.

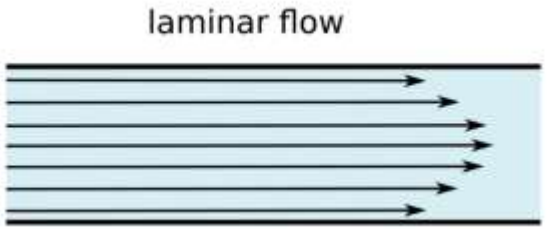
The shape of the temperature profile in the thermal boundary layer dictates the convection heat transfer between a solid surface and the fluid flowing over it.

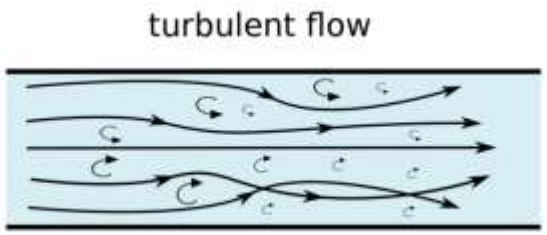
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LAMINAR AND TURBULENT FLOWS

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The flow regime in the first case is said to be laminar, characterized by smooth streamlines and highly-ordered motion, and turbulent in the second case, where it is characterized by velocity fluctuations and highly-disordered motion.

The transition from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.

Both the friction and heat transfer coefficients reach maximum values when the flow becomes *fully turbulent.*

Transport Phenomena

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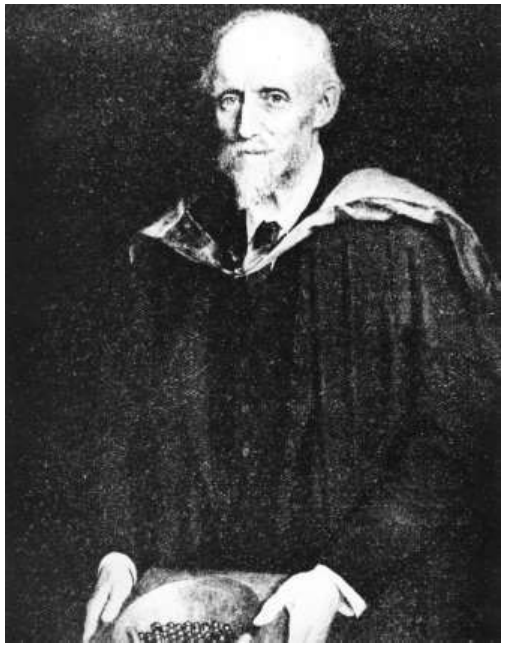
Viscosity → Momentum Transport

Thermal Conductivity → Heat Transport

Diffusivity → Mass Transport

Important dimensionless numbers in forced convection

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Osborne Reynolds
(1842—1912)

Re



Ludwig Prandtl
(1875—1953)

Pr



Wilhelm Nusselt
(1882-1957)

Nu

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Reynolds Number

The transition from laminar to turbulent flow depends on the *geometry*, *surface roughness*, *flow velocity*, *surface temperature*, and *type of fluid*.

The flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* (Reynolds number).

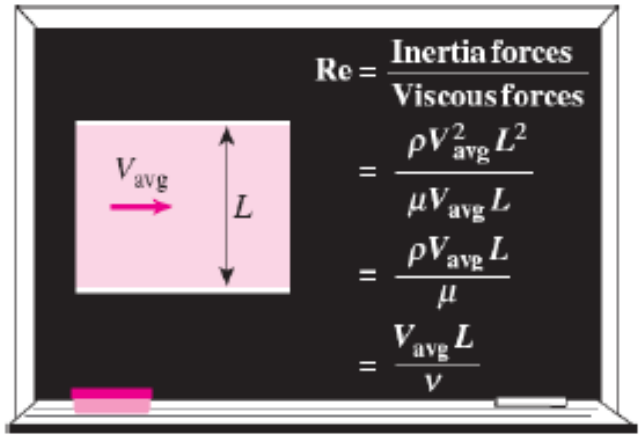
$$\text{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg} D}{\nu} = \frac{\rho V_{avg} D}{\mu}$$

At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (turbulent).

Critical Reynolds number, Re_{cr}: The Reynolds number at which the flow becomes turbulent.

The value of the critical Reynolds number is different for different geometries and flow conditions.

(5)



At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid "in line" (laminar).

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For flow over a flat plate, transition from laminar to turbulent is usually taken to occur at the *critical Reynolds number* of

$$\mathrm{Re}_{cr} = \frac{\rho V x_{cr}}{\mu} = 5 \times 10^5$$

The value of the critical Reynolds number for a flat plate may vary from 10^5 to 3×10^6, depending on the surface roughness and the turbulence level of the free stream.

Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless* parameter Prandtl number

$$\Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k} \qquad (4)$$

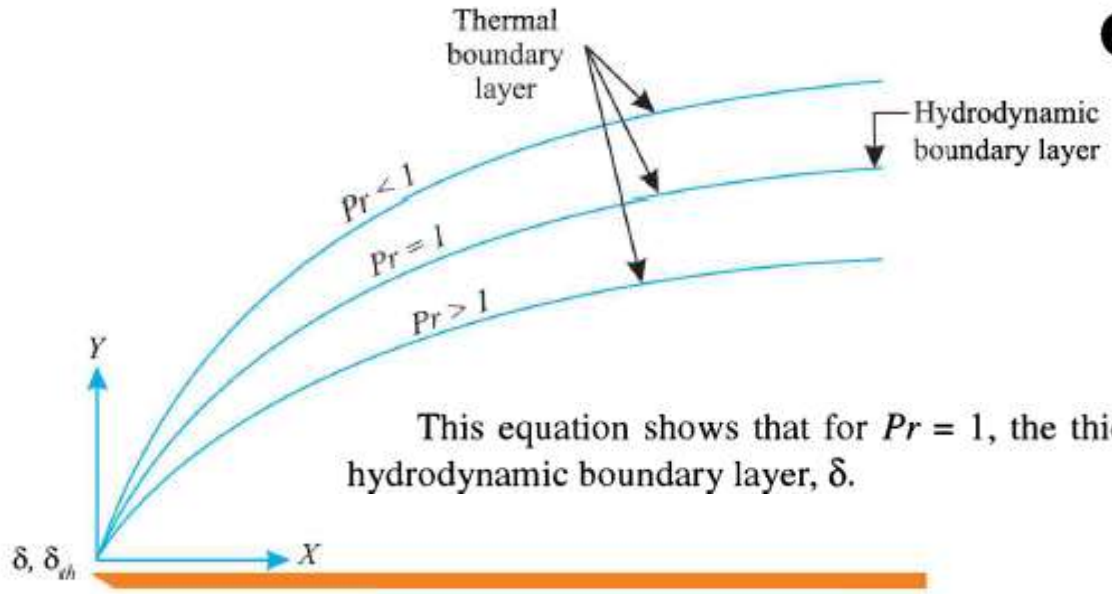
- The Prandtl numbers of **gases** are about **1**, which indicates that both momentum and heat dissipate through the fluid at about the same rate.
- Heat diffuses very quickly in liquid metals ($\Pr << 1$) and very slowly in oils ($\Pr >> 1$) relative to momentum.

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

- Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

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Case I. *When Pr = 1.*

$$\frac{\delta_{th}}{x} = \frac{5.0}{\sqrt{(xU)/\nu}} = \frac{5.0}{\sqrt{Re_x}} = \frac{\delta}{x}$$

This equation shows that for $Pr = 1$, the thickness of thermal boundary layer, δ_{th} is *equal to* hydrodynamic boundary layer, δ.

Case II. *When Pr < 1.*

$$\frac{\delta_{th}}{x} > \frac{5.0}{\sqrt{Re_x}} > \frac{\delta}{x}$$

This equation shows that for $Pr < 1$, $\delta_{th} > \delta$.

Case III. *When Pr > 1.*

$$\frac{\delta_{th}}{x} < \frac{5.0}{\sqrt{Re_x}} < \frac{\delta}{x}$$

This equation shows that for $Pr > 1$, $\delta_{th} < \delta$

$$\delta_{th} = \frac{\delta}{(Pr)^{1/3}}$$

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In convection studies, it is common practice to non-dimensionalize the governing equations and combine the variables, which group together into *dimensionless numbers* in order to reduce the number of total variables.

Nusselt Number

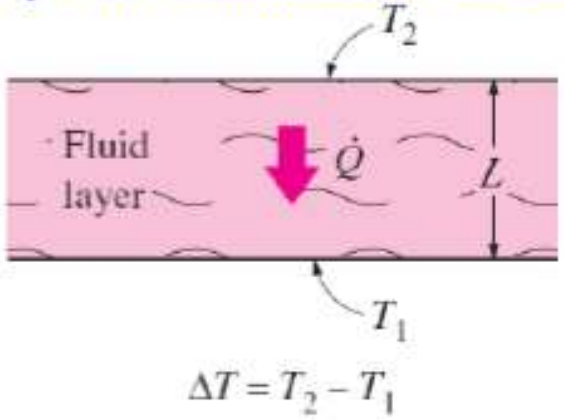
Nusselt number: Dimensionless convection heat transfer coefficient.

$$\mathrm{Nu} = \frac{hL_c}{k} \qquad (3)$$

$$\dot{q}_{\text{conv}} = h\Delta T \qquad \dot{q}_{\text{cond}} = k\frac{\Delta T}{L}$$

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \mathrm{Nu}$$

L_c is the characteristic length.



$$\Delta T = T_2 - T_1$$

The Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer.

The larger the Nusselt number, the more effective the convection.

A Nusselt number of Nu = 1 for a fluid layer represents heat transfer across the layer by pure conduction.

HEAT AND MOMENTUM TRANSFER IN TURBULENT FLOW

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Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress and heat transfer.

However, turbulent flow is a complex mechanism dominated by fluctuations, and the theory of turbulent flow is still not fully understood.

Therefore, we must rely on experiments and the empirical or semi-empirical correlations developed for various situations.

Turbulent flow is characterized by disorderly and rapid fluctuations of swirling regions of fluid, called eddies, throughout the flow.

These fluctuations provide an additional mechanism for momentum and energy transfer.

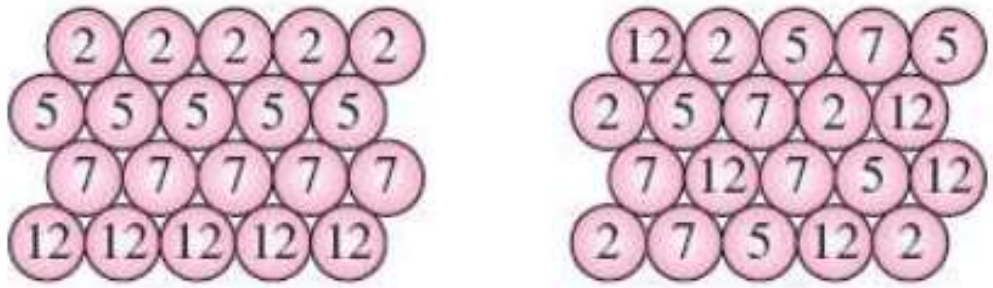
HEAT AND MOMENTUM TRANSFER IN TURBULENT FLOW

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The swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer.

Turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients.

The intense mixing in turbulent flow brings fluid particles at different temperatures into close contact, and thus enhances heat transfer.



(*a*) Before turbulence (*b*) After turbulence

Film temperature

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The fluid temperature in the thermal boundary layer varies from Ts at the surface to about T ∞ at the outer edge of the boundary.

The fluid properties also vary with temperature, and thus with position across the boundary layer.

In order to account for the variation of the properties with temperature, the fluid properties are usually evaluated at the so-called film temperature

$$T_f = \frac{T_s + T_\infty}{2}$$

Which is the *arithmetic average of the surface and the free-stream temperatures*. The fluid properties are then assumed to remain constant at those values during the entire flow.

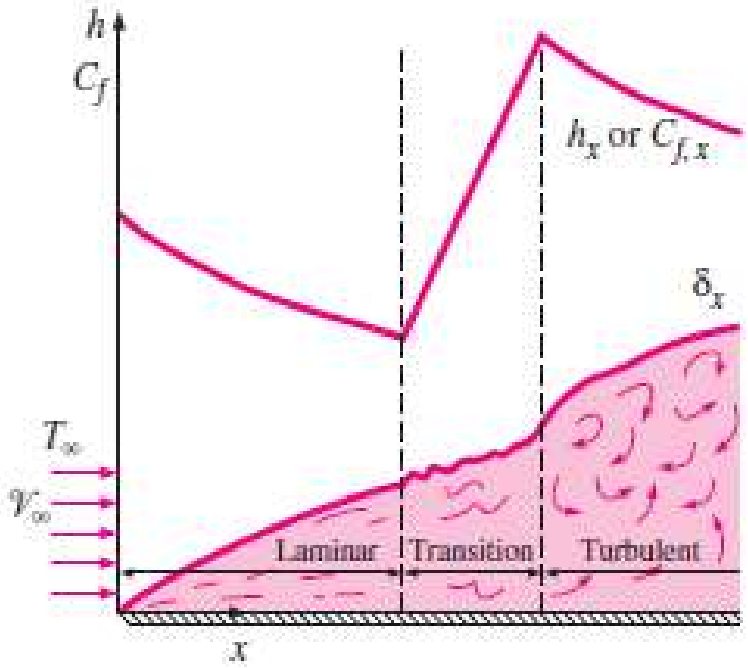
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Part –A

PARALLEL FLOW OVER FLAT PLATES

Local Heat Transfer Coefficient

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The local Nusselt number at a location *x for laminar flow over a flat plate was d*etermined by solving the differential energy equation to be

Laminar: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.60$$

The corresponding relation for turbulent flow is

Turbulent: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \qquad 0.6 \le \mathrm{Pr} \le 60,\quad 5 \times 10^5 \le \mathrm{Re}_x \le 10^7$$

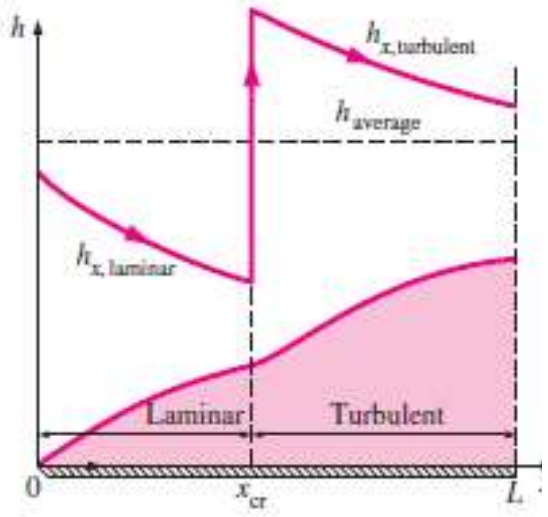
The average Nusselt number over the entire plate

Laminar: $$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Re}_L < 5 \times 10^5$$

Turbulent: $$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} \qquad 0.6 \le \mathrm{Pr} \le 60,\quad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$$

The first relation gives the average heat transfer coefficient for the entire plate when the flow is laminar over the entire plate. The second relation gives the average heat transfer coefficient for the entire plate only when the flow is turbulent over the entire plate, or when the laminar flow region of the plate is too small relative to the turbulent flow region.

In some cases, a flat plate is sufficiently long for the flow to become turbulent, but not long enough to disregard the laminar flow region. In such cases, the average heat transfer coefficient over the entire plate



Graphical representation of the average heat transfer coefficient for a flat plate with combined laminar and turbulent flow.

$$\text{Nu} = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\text{Pr}^{1/3} \qquad \begin{matrix} 0.6 \le \text{Pr} \le 60 \\ 5 \times 10^5 \le \text{Re}_L \le 10^7 \end{matrix}$$

It is desirable to have a single correlation that applies to *all fluids*, including liquid metals. By curve-fitting existing data, Churchill and Ozoe (Ref. 3) proposed the following relation which is applicable for *all Prandtl numbers* and is claimed to be accurate to ±1%,

$$\text{Nu}_x = \frac{h_x x}{k} = \frac{0.3387\,\text{Pr}^{1/3}\,\text{Re}_x^{1/2}}{[1 + (0.0468/\text{Pr})^{2/3}]^{1/4}}$$

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Uniform Heat Flux

When a flat plate is subjected to *uniform heat flux* instead of uniform temperature, the local Nusselt number is given by

Laminar: $$\mathrm{Nu}_x = 0.453\ \mathrm{Re}_x^{0.5}\ \mathrm{Pr}^{1/3}$$

Turbulent: $$\mathrm{Nu}_x = 0.0308\ \mathrm{Re}_x^{0.8}\ \mathrm{Pr}^{1/3}$$

Air at atmospheric pressure and 40° C flows with a velocity of U = 5 m/s over a 2 m long flat plate whose surface is kept at a uniform temperature of 120°C. Determine the average heat transfer coefficient over the 2 m length of the plate. Also find out the rate of heat transfer between the plate and the air per 1 m width of the plate. [Air at 1 atm. and 80°C, $\nu = 2.107 \times 10^{-5}$ m²/s, k = 0.03025 W/mK ; Pr = 0.6965]

The properties of air at mean bulk temperature of $\left(\frac{120+40}{2}\right) = 80°C$ are :

$\nu = 2.107 \times 10^{-5}$ m²/s; $k = 0.03025$ W/m K; $Pr = 0.6965$.

Average heat transfer coefficient, $\bar{h}$:

$$Re = \frac{UL}{\nu} = \frac{5 \times 2}{2.107 \times 10^{-5}} = 4.746 \times 10^5$$

Assuming $Re_{cr} = 5 \times 10^5$, the flow is *laminar*.

$$\overline{Nu} = 0.664\,(Re_L)^{1/2}\,(Pr)^{1/3}$$

$$\frac{\bar{h}L}{k} = 0.664\,(4.746 \times 10^5)^{1/2}\,(0.6965)^{1/3} = 405.48$$

$$\bar{h} = \frac{k}{L} \times 405.48 = \frac{0.03025}{2} \times 4054.48 = \mathbf{6.133\ W/m^3K}$$

Rate of heat transfer, *Q* :

$$Q = \bar{h}\,A_s\,(t_s - t_\infty)$$

$$= 6.133 \times (2 \times 1)\,(120 - 40) = \mathbf{981.28\ W}$$

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In a certain glass making process, a square plate of glass 1 m^2 area and 3 mm thick heated uniformly to 90° C is cooled by air at 20°C flowing over both sides parallel to the plate at 2 m/s. Calculate the initial rate of cooling the plate.

Neglect temperature gradient in the glass plate and consider only forced connection.

Take for glass : $\rho = 2500\ kg/m^3$ and $c_p = 0.67\ kJ/kg\ K$

Take the following properties of air :

$\rho = 1.076\ kg/m^2$; $c_p = 1008\ J/kg\ K$, $k = 0.0286\ W/m°\ C$ and $\mu = 19.8 \times 10^{-6}\ N\text{-}s/m^2$.

Initial rate of cooling :

The average heat transfer co-efficient for the air flow parallel to the plate is given by

$$\overline{Nu} = \frac{\bar{h}\,L}{k} = 0.664\,(Re_L)^{1/2}\,(Pr)^{1/3}$$

(valid for $Pr > 0.5$)

Laminar: $\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}$ $\quad \mathrm{Re}_L < 5 \times 10^5$

Turbulent: $\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}$ $\quad 0.6 \le \mathrm{Pr} \le 60$, $5 \times 10^5 \le \mathrm{Re}_L \le 10^7$

$$Re_L = \frac{\rho UL}{\mu} = \frac{1.076 \times 2 \times 1}{19.8 \times 10^{-6}} = 1.087 \times 10^5$$

$$Pr = \frac{\mu c_p}{k} = \frac{19.8 \times 10^{-6} \times 1008}{0.0286} = .0698$$

$$\bar{h} = \frac{k}{L} \times 194.19 = \frac{0.0286}{1} \times 194.19 = 5.55\ \mathrm{W/m^2\,°C}$$

The heat flow (Q) from both sides of the plate is given by :

$$Q = 2\bar{h}\,A\,(t_s - t_\infty) = 2 \times 5.55 \times 1 \times (90 - 20) = 777\ \mathrm{W}$$

In a certain glass making process, a square plate of glass 1 m^2 area and 3 mm thick heated uniformly to 90° C is cooled by air at 20°C flowing over both sides parallel to the plate at 2 m/s. Calculate the initial rate of cooling the plate.

Neglect temperature gradient in the glass plate and consider only forced connection.

Take for glass : $\rho = 2500\ kg/m^3$ and $c_p = 0.67\ kJ/kg\ K$

Take the following properties of air :

$\rho = 1.076\ kg/m^2$; $c_p = 1008\ J/kg\ K$, $k = 0.0286\ W/m°\ C$ and $\mu = 19.8 \times 10^{-6}\ N\text{-}s/m^2$.

The heat lost by the plate instantaneously is given by :

$$Q = m\, c_p\, \Delta t = 777$$

where, $m = (\text{Area} \times \text{thickness}) \times \rho$

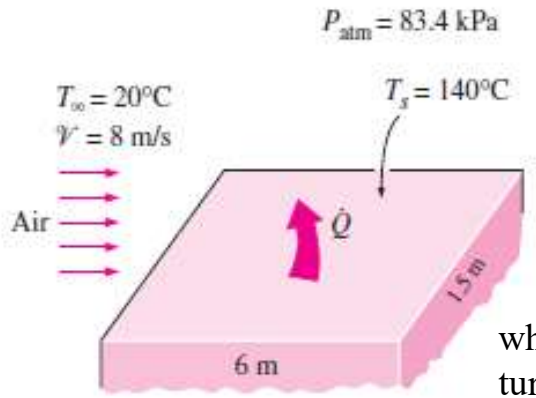
$$= 1 \times \frac{3}{1000} \times 2500 = 7.5 \text{ kg}$$

$\therefore$ $$777 = 7.5 \times (0.67 \times 10^3) \times \Delta t$$

$\therefore$ $$\Delta t = \frac{777}{7.5 \times (0.67 \times 10^3)} = \mathbf{0.155°C/s}$$

Air at 20°C flows with a velocity of 8 m/s over a 1.5 m X 6 m flat plate whose temperature is 140°C. Determine the rate of heat transfer from the plate if the air flows parallel to the (*a) 6-m-long* side and (*b) the 1.5-m side.*

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Film Temperature
Properties of medium

Analysis (*a*) When air flow is parallel to the long side, we have $L = 6$ m, and the Reynolds number at the end of the plate becomes

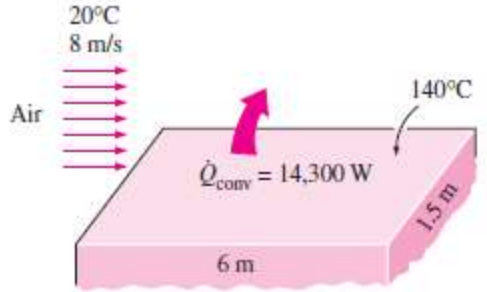
$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(8\ \mathrm{m/s})(6\ \mathrm{m})}{2.548 \times 10^{-5}\ \mathrm{m^2/s}} = 1.884 \times 10^6$$

which is greater than the critical Reynolds number. Thus, we have combined laminar and turbulent flow, and the average Nusselt number for the entire plate is determined to be

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\ \mathrm{Re}_L^{0.8} - 871)\mathrm{Pr}^{1/3}$$
$$= [0.037(1.884 \times 10^6)^{0.8} - 871]0.7154^{1/3}$$
$$= 2687$$

$$h = \frac{k}{L}\mathrm{Nu} = \frac{0.02953\ \mathrm{W/m \cdot °C}}{6\ \mathrm{m}}(2687) = 13.2\ \mathrm{W/m^2 \cdot °C}$$
$$A_s = wL = (1.5\ \mathrm{m})(6\ \mathrm{m}) = 9\ \mathrm{m^2}$$

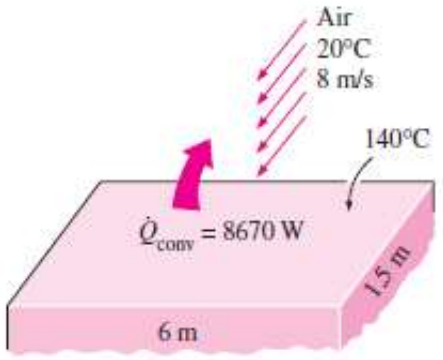
$$\dot{Q} = hA_s(T_s - T_\infty) = (13.2\ \mathrm{W/m^2 \cdot °C})(9\ \mathrm{m^2})(140 - 20)°\mathrm{C} = 1.43 \times 10^4\ \mathrm{W}$$



(*a*) Flow along the long side

Air at 20°C flows with a velocity of 8 m/s over a 1.5 m X 6 m flat plate whose temperature is 140°C. Determine the rate of heat transfer from the plate if the air flows parallel to the *(a) 6-m-long* side and (*b) the 1.5-m side.*

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(*b*) Flow along the short side

Case-b

$$Re_L = \frac{VL}{\nu} = \frac{(8\ \text{m/s})(1.5\ \text{m})}{2.548 \times 10^{-5}\ \text{m}^2/\text{s}} = 4.71 \times 10^5$$

hich is less than the critical Reynolds number. Thus, we have to consider laminar and the average Nusselt number for the entire plate is determined to be

$$\text{Nu} = \frac{hL}{k} = 0.664\ \text{Re}_L^{0.5}\ \text{Pr}^{1/3} = 0.664 \times (4.71 \times 10^5)^{0.5} \times 0.7154^{1/3} = 408$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.02953\ \text{W/m} \cdot {}^\circ\text{C}}{1.5\ \text{m}}(408) = 8.03\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

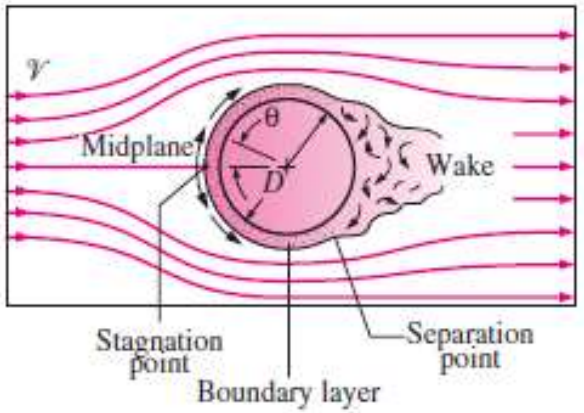
$$\dot{Q} = hA_s(T_s - T_\infty) = (8.03\ \text{W/m}^2 \cdot {}^\circ\text{C})(9\ \text{m}^2)(140 - 20)^\circ\text{C} = \mathbf{8670\ W}$$

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Part –B

FLOW ACROSS CYLINDERS AND SPHERES

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The characteristic length for a circular cylinder or sphere is taken to be the external diameter D.

The critical Reynolds number for flow across a circular cylinder or sphere is about Re cr 2 X10^5. That is, the boundary layer remains laminar for about Re 2 X10^5 and becomes turbulent for Re 2 X 10^5.

Flows across cylinders and spheres, in general, involve flow separation, which is difficult to handle analytically. Therefore, such flows must be studied experimentally or numerically. Indeed, flow across cylinders and spheres has been studied experimentally by numerous investigators, and several empirical correlations have been developed for the heat transfer coefficient.

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The average Nusselt number for flow across cylinders can be expressed compactly as

Churchill and Bernstein equation

Nusselt number for cross flow over a cylinder

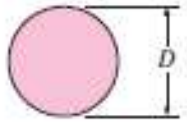
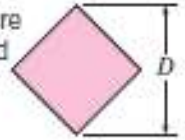
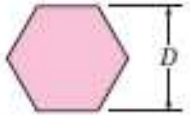
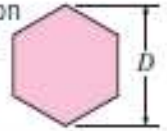
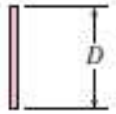
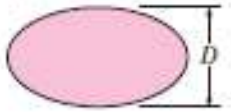
$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

This relation is quite comprehensive in that it correlates available data well for $\mathrm{Re}\,\mathrm{Pr} > 0.2$. The fluid properties are evaluated at the *film temperature* $T_f = \frac{1}{2}(T_\infty + T_s)$, which is the average of the free-stream and surface temperatures.

For flow over a *sphere*

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

which is valid for $3.5 \le \mathrm{Re} \le 80{,}000$ and $0.7 \le \mathrm{Pr} \le 380$. The fluid properties in this case are evaluated at the free-stream temperature T_∞, except for μ_s, which is evaluated at the surface temperature T_s. Although the two relations above are considered to be quite accurate, the results obtained from them can be off by as much as 30 percent.

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\ Pr^{1/3}$ $Nu = 0.911Re^{0.385}\ Pr^{1/3}$ $Nu = 0.683Re^{0.466}\ Pr^{1/3}$ $Nu = 0.193Re^{0.618}\ Pr^{1/3}$ $Nu = 0.027Re^{0.805}\ Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\ Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\ Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\ Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\ Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\ Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\ Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\ Pr^{1/3}$

$$Nu_{cyl} = \frac{hD}{k} = C\,Re^{m}\,Pr^{n}$$

Where, n= 1/3

C and M are given in table

A long 10-cm-diameter steam pipe whose external surface temperature is 110°C passes through some open area that is not protected against the winds . Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 10°C and the wind is blowing across the pipe at a velocity of 8 m/s.

Mean film Temp
Prop of medium

The Reynolds number is

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(8\ \mathrm{m/s})(0.1\ \mathrm{m})}{1.896 \times 10^{-5}\ \mathrm{m^2/s}} = 4.219 \times 10^4$$

The Nusselt number can be determined from

$$\mathrm{Nu} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(4.219 \times 10^4)^{1/2}\,(0.7202)^{1/3}}{[1 + (0.4/0.7202)^{2/3}]^{1/4}}\left[1 + \left(\frac{4.219 \times 10^4}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 124$$

$$h = \frac{k}{D}\mathrm{Nu} = \frac{0.02808\ \mathrm{W/m \cdot °C}}{0.1\ \mathrm{m}}(124) = 34.8\ \mathrm{W/m^2 \cdot °C}$$

Then the rate of heat transfer from the pipe per unit of its length becomes

$$A_s = pL = \pi DL = \pi(0.1\ \mathrm{m})(1\ \mathrm{m}) = 0.314\ \mathrm{m^2}$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (34.8\ \mathrm{W/m^2 \cdot C})(0.314\ \mathrm{m^2})(110 - 10)°\mathrm{C} = 1093\ \mathrm{W}$$

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- Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks.
- The fluid in such applications is usually **forced to flow** by a **fan** or **pump** through a flow section.
- Although the theory of fluid flow is reasonably well understood, theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe.
- Therefore, we must rely on experimental results and empirical relations for most fluid flow problems rather than closed-form analytical solutions.



Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any distortion, but the noncircular pipes cannot

For a fixed surface area, the circular tube gives the most heat transfer for the least pressure drop.

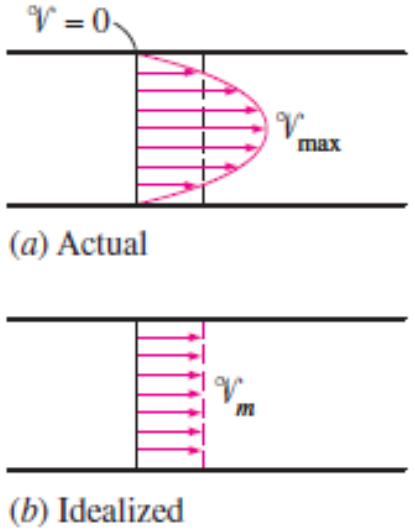
Average Velocity and Temperature in Tubes

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Average Velocity

The fluid velocity in a pipe changes from *zero at the* wall because of the no-slip condition to a maximum at the pipe center.

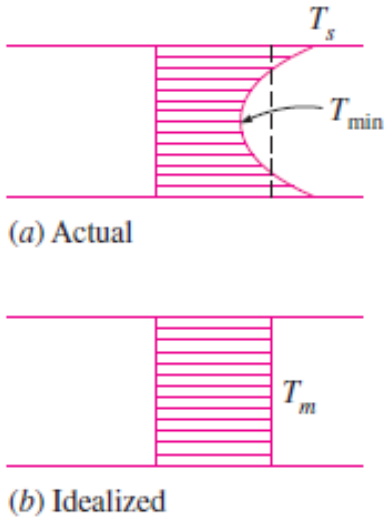
In fluid flow, it is convenient to work with an *average velocity Vavg, which remains constant in* incompressible flow when the cross-sectional area of the pipe is constant.



Average Temperature

In fluid flow, it is convenient to work with an average or mean temperature Tm, which remains constant at a cross section.

But, in practice, we evaluate the fluid properties at some average temperature and treat them as constants.



Laminar and Turbulent Flow in Tubes

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- Flow in a tube can be laminar or turbulent, depending on the **flow conditions**.
- Fluid flow is streamlined and thus laminar at **low velocities**, but turns turbulent as the **velocity is increased** beyond a critical value.
- **Transition** from laminar to turbulent flow does not occur suddenly; rather, it occurs over some range of velocity where the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.
- Most pipe flows encountered in practice are turbulent.
- ***Laminar flow*** is encountered when **highly viscous** fluids such as **oils** flow in **small diameter tubes** or narrow passages.
- Transition from laminar to turbulent flow depends on the **Reynolds number** as well as the degree of disturbance of the flow by ***surface roughness***, *pipe vibrations*, and the *fluctuations in the flow*.
- The flow in a pipe is laminar for $Re < 2300$, fully turbulent for $Re > 10{,}000$, and transitional in between.

$Re < 2300$	laminar flow
$2300 \le Re \le 10{,}000$	transitional flow
$Re > 10{,}000$	turbulent flow

Laminar and Turbulent Flow in Tubes

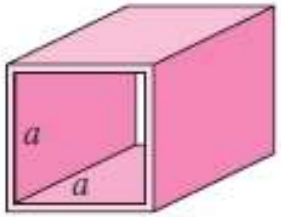
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Circular tube:



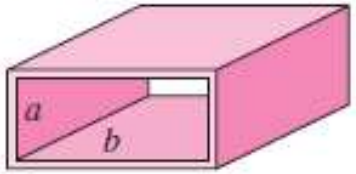
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:



$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.

$$\mathrm{Re} = \frac{\rho V_m D}{\mu} = \frac{V_m D}{\nu}$$

$$D_h = \frac{4A_c}{p}$$

Circular tubes: $D_h = \dfrac{4A_c}{p} = \dfrac{4\pi D^2/4}{\pi D} = D$

$\mathrm{Re} < 2300$	laminar flow
$2300 \le \mathrm{Re} \le 10{,}000$	transitional flow
$\mathrm{Re} > 10{,}000$	turbulent flow

The Entrance Region

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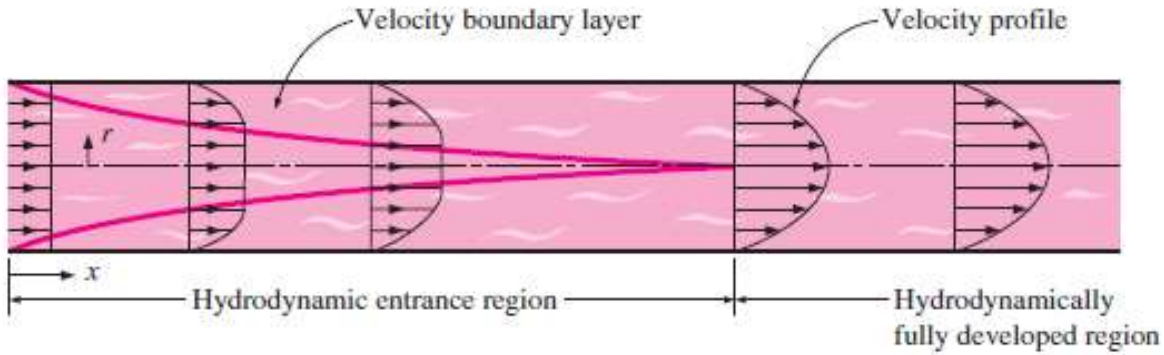
•The fluid particles in the layer in contact with the surface of the tube will come to a complete stop.

• This layer will also cause the fluid particles in the adjacent layers to slow down gradually as a result of friction.

• To make up for this velocity reduction, the velocity of the fluid at the midsection of the tube will have to increase to keep the mass flow rate through the tube constant.

•As a result, a *velocity boundary layer develops along the tube. The thickness of* this boundary layer increases in the flow direction until the boundary layer reaches the tube centre and thus fills the entire tube.

Uniform velocity



Hydrodynamically fully developed region

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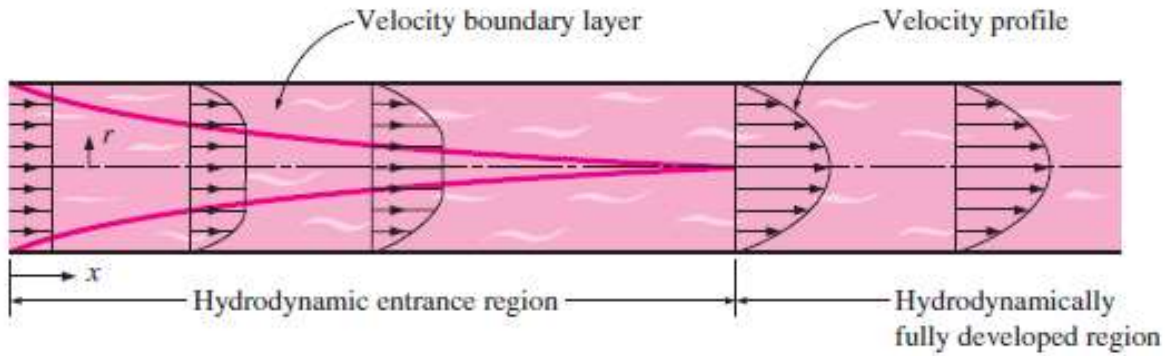
•The region from the tube inlet to the point at which the boundary layer merges at the centreline is called the hydrodynamic entrance region, and the length of this region is called the hydrodynamic entry length Lh.

• Flow in the entrance region is called hydrodynamic ally developing flow since this is the region where the velocity profile develops .

• The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged is called the hydro dynamically fully developed region.

•The velocity profile in the fully developed region is parabolic in laminar flow and somewhat flatter in turbulent flow due to eddy motion in radial direction.

Uniform velocity



Thermal entrance region

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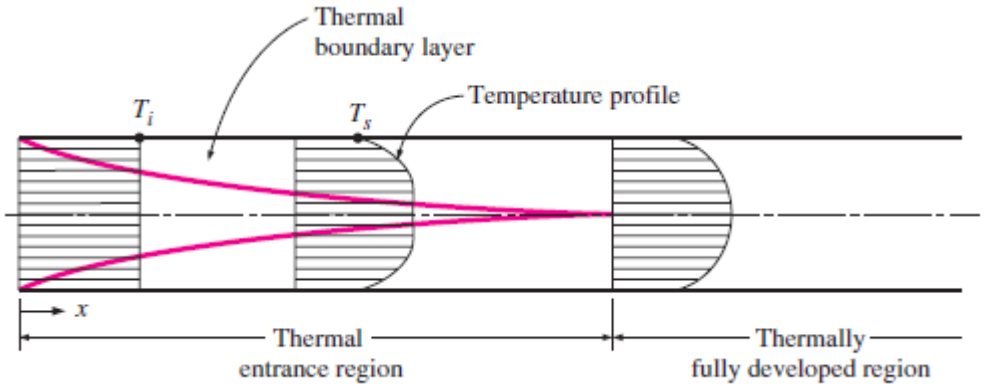
•This time, the fluid particles in the layer in contact with the surface of the tube will assume the surface temperature.

• This will initiate convection heat transfer in the tube and the development of a thermal boundary layer along the tube.

• The thickness of this boundary layer also increases in the flow direction until the boundary layer reaches the tube centre and thus fills the entire tube.

•The region of flow over which the thermal boundary layer develops and reaches the tube centre is called the thermal entrance region, and the length of this region is called the thermal entry length Lt.

Uniform Temperature



Fully developed flow

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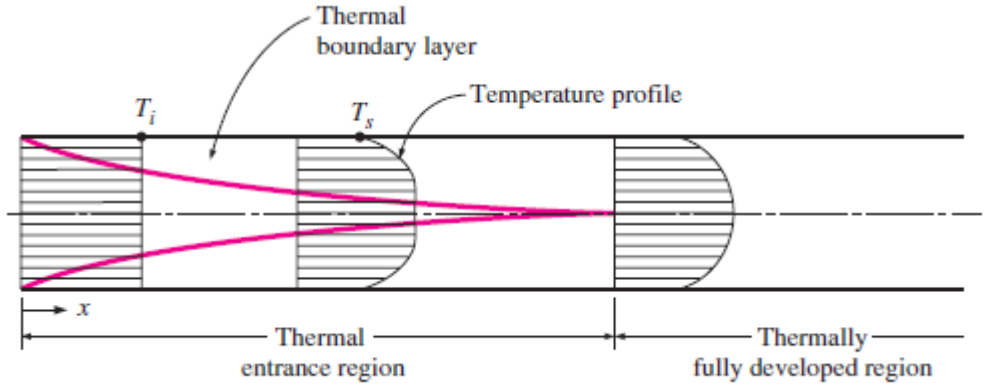
•Flow in the thermal entrance region is called thermally developing flow since this is the region where the temperature profile develops.

•The region beyond the thermal entrance region in which the dimensionless temperature profile expressed as (Ts -T)/ (Ts -Tm) remains unchanged is called the thermally fully developed region.

• The region in which the flow is both hydro dynamically and thermally developed and thus both the velocity and dimensionless temperature profiles remain unchanged is called fully developed flow.

•Thus we conclude that in the thermally fully developed region of a tube, the local convection coefficient is constant (does not vary with x). Therefore, both the friction and convection coefficients remain constant in the fully developed region of a tube.

Uniform Temperature



Fully developed flow

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•Temperature profile in the thermally fully developed region may vary with x in the flow direction. That is, unlike the velocity profile, the temperature profile can be different at different cross sections of the tube in the developed region.

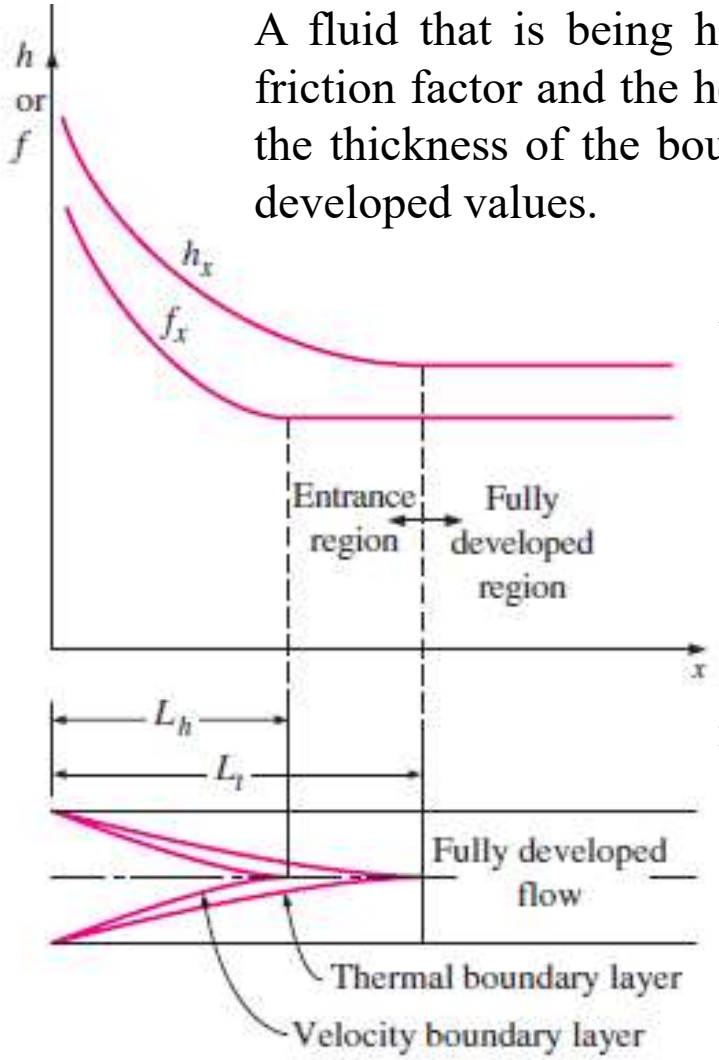
•However, the dimensionless temperature profile remains unchanged in the thermally developed region when the temperature or heat flux at the tube surface remains constant.

•During laminar flow in a tube, the magnitude of the dimensionless Prandtl number Pr is a measure of the relative growth of the velocity and thermal boundary layers.

•For fluids with Pr =1, such as gases, the two boundary layers essentially coincide with each other. For fluids with Pr >1, such as oils, the velocity boundary layer outgrows the thermal boundary layer. Thus, hydrodynamic entry length is smaller than the thermal entry length. The opposite is true for fluids with Pr < 1 such as liquid metals.

Fully developed flow

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(Pr > 1).

A fluid that is being heated (or cooled) in a tube as it flows through it. The friction factor and the heat transfer coefficient are *highest at the tube* inlet where the thickness of the boundary layers is zero, and decrease gradually to the fully developed values.

Therefore, the pressure drop and heat flux are higher in the entrance regions of a tube.

The effect of the entrance region is always to enhance the average friction and heat transfer coefficients for the entire tube.

This enhancement can be significant for short tubes but negligible for long ones.

Entry lengths

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The hydrodynamic entry length is usually taken to be the distance from the tube entrance where the friction coefficient reaches within about 2 percent of the fully developed value. In laminar flow, the hydrodynamic and thermal entry lengths are given approximately as

$$L_{h,\,\text{laminar}} \approx 0.05\,\text{Re}\,D$$

$$L_{t,\,\text{laminar}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = \text{Pr}\,L_{h,\,\text{laminar}}$$

In turbulent flow, the intense mixing during random fluctuations usually overshadows the effects of momentum and heat diffusion, and therefore the hydrodynamic and thermal entry lengths are of about the same size and independent of the Prandtl number.

Also, the friction factor and the heat transfer coefficient remain constant in fully developed laminar or turbulent flow since the velocity and normalized temperature profiles do not vary in the flow direction.

Entry lengths

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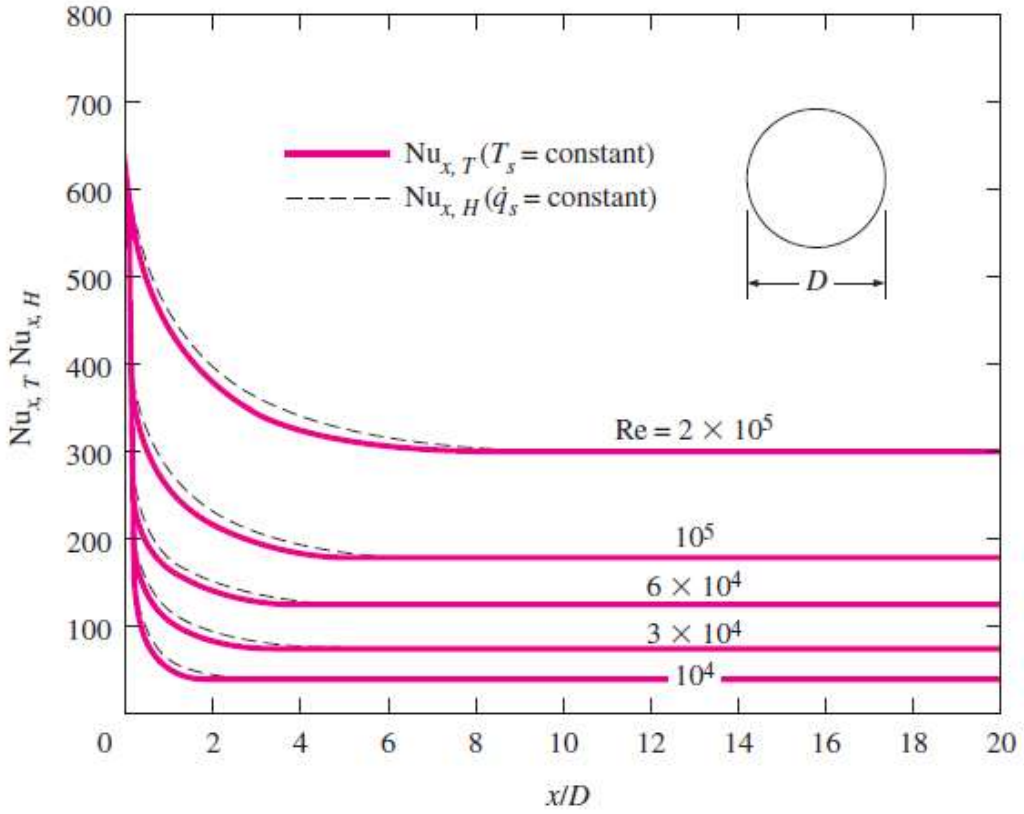
The hydrodynamic entry length for turbulent flow can be determined

$$L_{h,\ \text{turbulent}} = 1.359\ \text{Re}^{1/4}$$

In practice, it is generally agreed that the entrance effects are confined within a tube length of 10 diameters, and the hydrodynamic and thermal entry lengths are approximately taken to be

$$L_{h,\ \text{turbulent}} \approx L_{t,\ \text{turbulent}} \approx 10D$$

Turbulent flows in Tubes



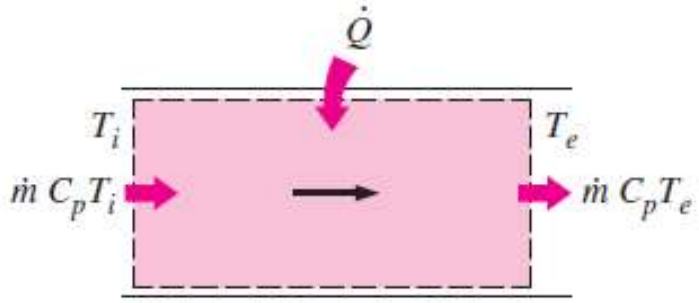
The Nusselt numbers and thus the convection heat transfer coefficients are much higher in the entrance region.

The Nusselt number reaches a constant value at a distance of less than 10 diameters, and thus the flow can be assumed to be fully developed for $x > 10D$.

The Nusselt numbers for the uniform surface temperature and uniform surface heat flux conditions are identical in the fully developed regions, and nearly identical in the entrance regions.

We will assume the fully developed flow in the analysis

Thermal Analysis of internal flows



Energy balance:

$$\dot{Q} = \dot{m}\,C_p(T_e - T_i)$$

The thermal conditions at the surface can usually be approximated with reasonable accuracy to be *constant surface temperature (Ts constant)* – *Phase change , boiling/condensation on the outer side of the tube*

or constant surface heat flux (qs constant) -*tube is subjected to radiation or electric resistance heating uniformly from all directions*

$$\dot{Q} = \dot{m}C_p(T_e - T_i) \qquad \text{(W)}$$

Surface heat flux is expressed as

$$\dot{q}_s = h_x(T_s - T_m) \qquad \text{(W/m}^2\text{)}$$

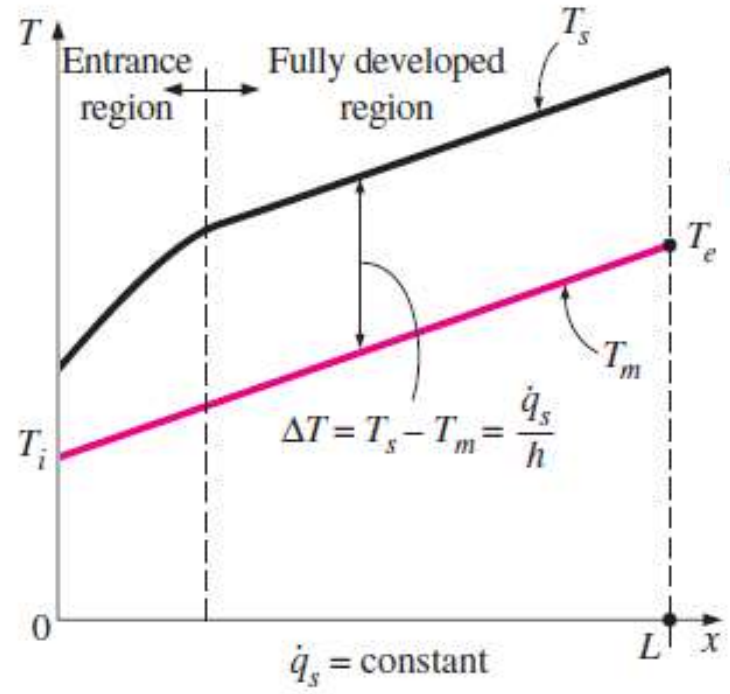
Local HT coeff — Surface T — Mean T

qs must change in constant Temp and Ts must change in constant qs

Hence, Either case would exist not both

Constant Surface Heat Flux

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$$\dot{Q} = \dot{q}_s A_s = \dot{m} C_p (T_e - T_i) \qquad \text{(W)}$$

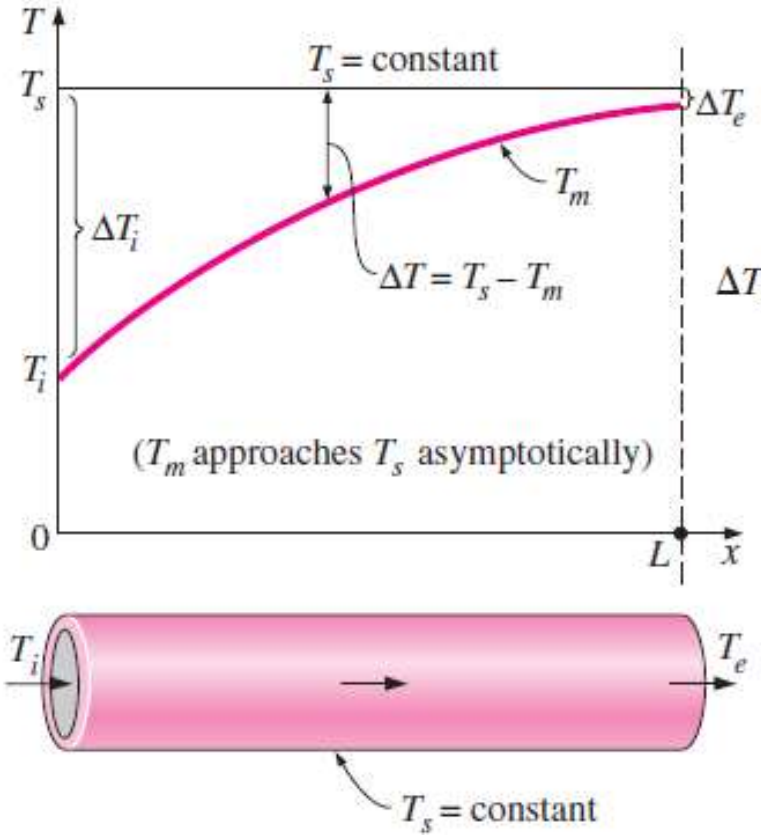
Then the mean fluid temperature at the tube exit becomes

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} C_p}$$

$$\dot{q}_s = h(T_s - T_m) \longrightarrow T_s = T_m + \frac{\dot{q}_s}{h}$$

Mean fluid temperature increases *linearly in the flow direction* in the case of constant surface heat flux

Constant Surface Temp



$$\dot{Q} = hA_s\Delta T_{ave} = hA_s(T_s - T_m)_{ave}$$

Arithmetic Mean Temperature Difference (AMTD)

$$\Delta T_{ave} \approx \Delta T_{am} = \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2} = T_s - \frac{T_i + T_e}{2}$$

$$= T_s - T_b$$

Bulk Mean Temperature Tb = (Ti + Te)/2

$$\dot{m}C_p\,dT_m = h(T_s - T_m)dA_s$$

$$\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}C_p}$$

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p)$$

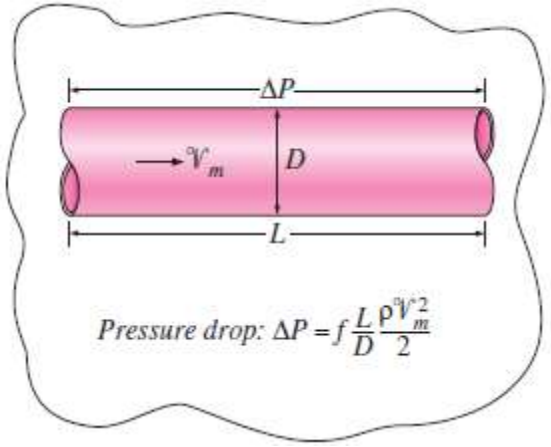
LAMINAR FLOW IN TUBES

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Re < 2300

we consider the steady laminar flow of an incompressible fluid with constant properties in the fully developed region of a straight circular tube

the dimensionless quantity *f is the friction factor (also called the Darcy friction factor*



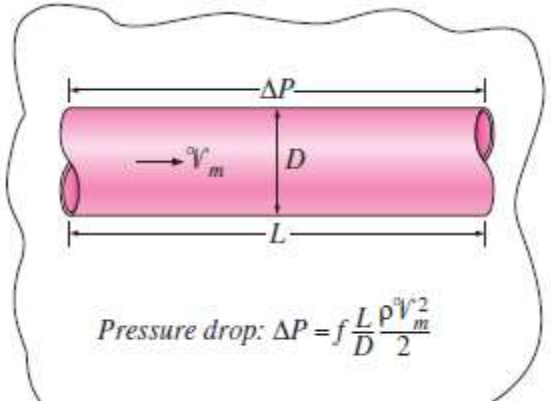
Circular tube, laminar: $$f = \frac{64\mu}{\rho D V_m} = \frac{64}{\mathrm{Re}}$$

This equation shows that *in laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the tube*

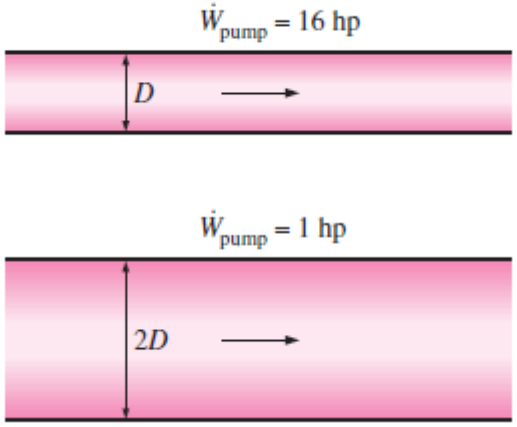
LAMINAR FLOW IN TUBES

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Circular tube, laminar: $f = \frac{64\mu}{\rho D V_m} = \frac{64}{\mathrm{Re}}$



$\dot{W}_{\text{pump}} = \dot{V}\Delta P$



where $\dot{V}$ is the volume flow rate of flow, which is expressed as

$$\dot{V} = V_{\text{ave}} A_c = \frac{\Delta P R^2}{8\mu L}\pi R^2 = \frac{\pi R^4 \Delta P}{8\mu L} = \frac{\pi D^4 \Delta P}{128\mu L}$$

This equation is known as the Poiseuille's Law, and this flow is called the *Hagen–Poiseuille flow*

for a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the tube and the viscosity of the fluid, but it is inversely proportional to the fourth power of the radius (or diameter) of the tube. Therefore, the pumping power requirement for a piping system can be reduced by a factor of 16 by doubling the tube diameter

Temperature Profile and the Nusselt Number

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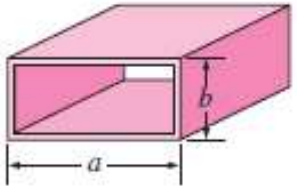
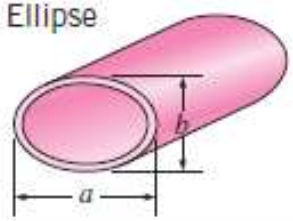
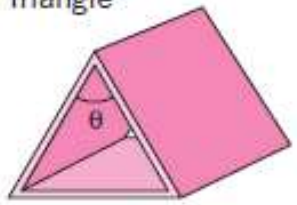
Circular tube, laminar (q_s = constant): $\mathrm{Nu} = \frac{hD}{k} = 4.36$

Circular tube, laminar (T_s = constant): $\mathrm{Nu} = \frac{hD}{k} = 3.66$

Therefore, for fully developed laminar flow in a circular tube subjected to constant surface heat flux, the Nusselt number is a constant. There is no dependence on the Reynolds or the Prandtl numbers.

The thermal conductivity *k for use in the Nu relations above should be evaluated* at the bulk mean fluid temperature, which is the arithmetic average of the mean fluid temperatures at the inlet and the exit of the tube. For laminar flow, the effect of *surface roughness on the friction factor and the heat transfer coefficient* is negligible.

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $\mathrm{Re} = \mathcal{V}_m D_h/\nu$, and $\mathrm{Nu} = hD_h/k$)

Tube Geometry	a/b or $\theta°$	Nusselt Number		Friction Factor f
		T_s = Const.	$\dot{q}_s$ = Const.	
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Developing Laminar Flow in the Entrance Region

For a circular tube of length L subjected to constant surface temperature, the average Nusselt number for the *thermal entrance region* can be determined from (Edwards et al., 1979)

Entry region, laminar:
$$\mathrm{Nu} = 3.66 + \frac{0.065\,(D/L)\,\mathrm{Re}\,\mathrm{Pr}}{1 + 0.04[(D/L)\,\mathrm{Re}\,\mathrm{Pr}]^{2/3}}$$

The average Nusselt number for the thermal entrance region of flow between *isothermal parallel plates* of length L is expressed as (Edwards et al., 1979)

Entry region, laminar:
$$\mathrm{Nu} = 7.54 + \frac{0.03\,(D_h/L)\,\mathrm{Re}\,\mathrm{Pr}}{1 + 0.016[(D_h/L)\,\mathrm{Re}\,\mathrm{Pr}]^{2/3}}$$

where D_h is the hydraulic diameter, which is twice the spacing of the plates. This relation can be used for $\mathrm{Re} \leq 2800$.

L-24

Water entering at 10°C is heated to 40°C in the tube of 0.02 m ID at a mass flow rate of 0.01 kg/s. The outside of the tube is covered with an insulated electric heating element that produces a uniform heat flux of 15000 W/m2 over the surface. Neglecting any entrance effect, determine

a) Reynolds number ;

(b) The heat transfer coefficient ;

(c) The length of pipe needed for a 30°C increase in average temperature ;

(d) The inner tube surface temperature at the outlet ;

(e) The friction factor ;

(f) The pressure drop in the pipe ;

(g) The pumping power required, if the pump is 50% efficient.

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Properties : The properties of water at its mean temperature

$$T_m = \frac{40 + 10}{2} = 25°C$$

The properties of water at 25°C (from Table A-7)

$$\rho = 997 \text{ kg/m}^3, \quad k_f = 0.608 \text{ W/m.K},$$

$$C_p = 4180 \text{ J/kgK}, \quad \mu = 910 \times 10^{-6} \text{ Ns/m}^2.$$

Analysis : (*a*) The Reynolds number:

$$Re = \frac{u_m D_i}{\nu} = \frac{4\dot{m}}{\pi D_i \mu}$$

$$= \frac{4 \times (0.01 \text{ kg/s})}{\pi \times (0.02 \text{ m}) \times (910 \times 10^{-6} \text{ Ns/m}^2)}$$

$$= \mathbf{700.} \quad \textbf{Ans.}$$

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(*b*) Since uniform flux is on the pipe surface, hence using eqn.

$$Nu_D = 4.36$$

And heat transfer coefficient

$$h = \frac{Nu_D\, k_f}{D_i} = \frac{4.36 \times (0.608\ \text{W/m.K})}{(0.02\ \text{m})}$$

$$= \mathbf{132.5\ W/m^2.K.} \quad \textbf{Ans.}$$

(*c*) Length of the pipe needed for 30°C temperature rise :

$$Q = q\ A_s = q(\pi D_i L) = \dot{m}\, C_p (T_o - T_i)$$

or

$$L = \frac{(0.01\ \text{kg/s}) \times (4180\ \text{J/kg.K}) \times (30\ \text{K})}{(15000\ \text{W/m}^2) \times (\pi \times 0.02\ \text{m})}$$

$$= \mathbf{1.33\ m.} \quad \textbf{Ans.}$$

(*d*) Inner tube surface temperature at the outlet :

$$q = h(T_{s,o} - T_o)$$

$$T_{s,o} = \frac{q}{h} + T_o = \frac{15000\ \text{W/m}^2}{132.5\ \text{W/m}^2.\text{K}} + 40$$

$$= \mathbf{153.6°C.} \quad \textbf{Ans.}$$

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(*e*) Friction factor :

$$f = \frac{64}{\mathrm{Re_D}} = \frac{64}{700} = \mathbf{0.0914.} \quad \textbf{Ans.}$$

(*f*) Pressure drop in pipe :

$$\Delta p = f \frac{\mathrm{L}}{\mathrm{D}_i} \frac{\rho u_m^{\,2}}{2}$$

where, $u_m = \dfrac{4\dot{m}}{\pi \mathrm{D}_i^{\,2} \rho}$ since $\dot{m} = \rho u_m \mathrm{A}_c$

$$= \frac{(4 \times 0.01\ \mathrm{kg/s})}{\pi \times (0.02\ \mathrm{m})^2 \times (997\ \mathrm{kg/m^3})}$$

$$= 0.032\ \mathrm{m/s}$$

$$\text{Hence } \Delta p = 0.0914 \times \frac{(1.33\ \mathrm{m})}{0.02} \times \frac{(997\ \mathrm{kg/m^3}) \times (0.032\ \mathrm{m/s^2})}{2}$$

$$= \mathbf{3.1\ N/m^2.} \quad \textbf{Ans.}$$

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(*g*) The pumping power :

$$= \frac{\dot{m}}{\rho} \Delta p = \frac{(0.01\,\text{kg/s})}{(997\,\text{kg/m}^3)} \times (3.1\,\text{N/m}^2)$$

$$= 3.11 \times 10^{-5}\,\text{W}$$

Actual power required

$$= \frac{3.11 \times 10^{-5}\,\text{W}}{0.5} = \mathbf{6.22 \times 10^{-5}\,W.}$$ **Ans.**

TURBULENT FLOW IN TUBES

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In smooth tubes is fully turbulent for Re > 10,000

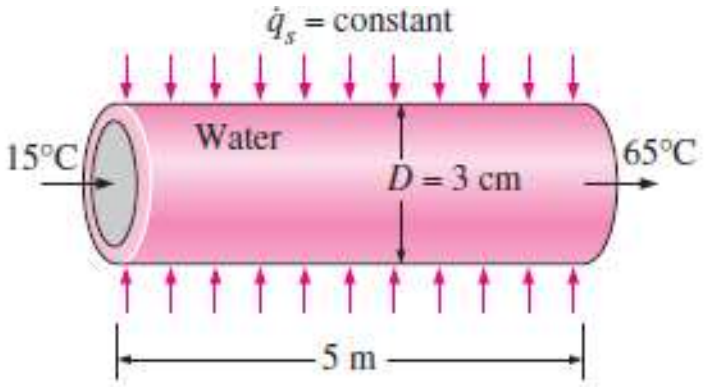
Turbulent flow is commonly utilized in practice because of the higher heat transfer coefficients associated with it. Most correlations for the friction and heat transfer coefficients in turbulent flow are based on experimental studies because of the difficulty in dealing with turbulent flow theoretically.

Dittus–Boelter equation

$$Nu = 0.023\, Re^{0.8}\, Pr^{n}$$

where *n = 0.4 for heating and*
n = 0.3 for cooling of the fluid flowing through the tube.

The fluid properties are evaluated at the *bulk mean fluid temperature Tb* (*Ti +Te)/2. When the temperature difference between the fluid and the wall is* very large, it may be necessary to use a correction factor to account for the different viscosities near the wall and at the tube center.

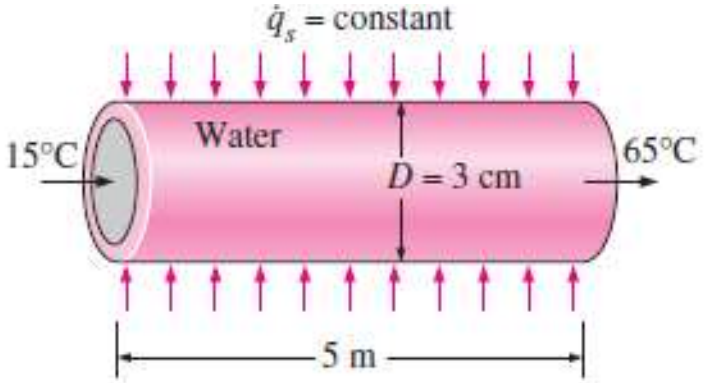


Water is to be heated from 15°C to 65°C as it flows through a 3-cm-internal diameter 5-m-long tube The tube is equipped with an electric resistance heater that provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 10 L/min, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at the exit.

Assumptions
1 Steady flow conditions exist.
2 The surface heat flux is uniform.
3 The inner surfaces of the tube are smooth.

Properties The properties of water at the bulk mean temperature of $T_b = (T_i + T_e)/2 = (15 + 65)/2 = 40°C$ are (Table A-9).

$\rho = 992.1\ kg/m^3$	$C_p = 4179\ J/kg \cdot °C$
$k = 0.631\ W/m \cdot °C$	$Pr = 4.32$
$\nu = \mu/\rho = 0.658 \times 10^{-6}\ m^2/s$	



The cross sectional and heat transfer surface areas are

$$A_c = \tfrac{1}{4}\pi D^2 = \tfrac{1}{4}\pi(0.03\ \text{m})^2 = 7.069 \times 10^{-4}\ \text{m}^2$$
$$A_s = pL = \pi DL = \pi(0.03\ \text{m})(5\ \text{m}) = 0.471\ \text{m}^2$$

$$\dot{m} = \rho\dot{V} = (992.1\ \text{kg/m}^3)(0.01\ \text{m}^3/\text{min}) = 9.921\ \text{kg/min} = 0.1654\ \text{kg/s}$$

$$\dot{Q} = \dot{m}C_p(T_e - T_i)$$
$$= (0.1654\ \text{kg/s})(4.179\ \text{kJ/kg}\cdot{}^\circ\text{C})(65 - 15)^\circ\text{C}$$
$$= 34.6\ \text{kJ/s} = 34.6\ \text{kW}$$

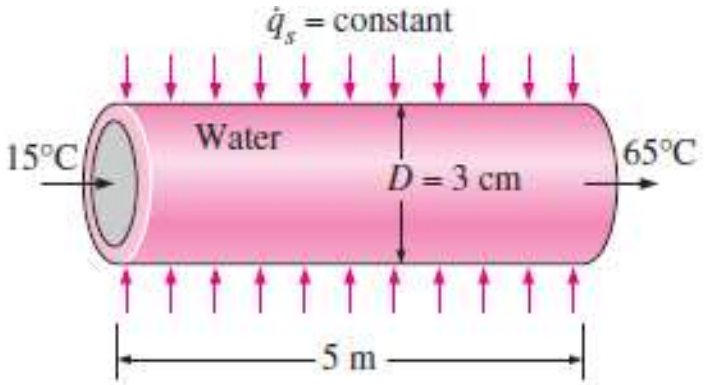
$$q_s = h(T_s - T_m) \quad \rightarrow \quad T_s = T_m + \frac{\dot{q}_s}{h}$$

$$q_s = \frac{\dot{Q}}{A_s} = \frac{34.6\ \text{kW}}{0.471\ \text{m}^2} = 73.46\ \text{kW/m}^2$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry length is roughly

$$L_h \approx L_t \approx 10D = 10 \times 0.03 = 0.3\ \text{m}$$

$$V_m = \frac{\dot{V}}{A_c} = \frac{0.010\ \text{m}^3/\text{min}}{7.069 \times 10^{-4}\ \text{m}^2} = 14.15\ \text{m/min} = 0.236\ \text{m/s}$$
$$\text{Re} = \frac{V_m D}{\nu} = \frac{(0.236\ \text{m/s})(0.03\ \text{m})}{0.658 \times 10^{-6}\ \text{m}^2/\text{s}} = 10{,}760$$



$$V_m = \frac{\dot{V}}{A_c} = \frac{0.010 \text{ m}^3/\text{min}}{7.069 \times 10^{-4} \text{ m}^2} = 14.15 \text{ m/min} = 0.236 \text{ m/s}$$

$$\text{Re} = \frac{V_m D}{\nu} = \frac{(0.236 \text{ m/s})(0.03 \text{ m})}{0.658 \times 10^{-6} \text{ m}^2/\text{s}} = 10{,}760$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry length is roughly

$$L_h \approx L_t \approx 10D = 10 \times 0.03 = 0.3 \text{ m}$$

which is much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe and determine the Nusselt number from

$$\text{Nu} = \frac{hD}{k} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.4} = 0.023(10{,}760)^{0.8} (4.34)^{0.4} = 69.5$$

Then,

$$h = \frac{k}{D} \text{Nu} = \frac{0.631 \text{ W/m} \cdot {}^\circ\text{C}}{0.03 \text{ m}} (69.5) = 1462 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

and the surface temperature of the pipe at the exit becomes

$$T_s = T_m + \frac{q_s}{h} = 65^\circ\text{C} + \frac{73{,}460 \text{ W/m}^2}{1462 \text{ W/m}^2 \cdot {}^\circ\text{C}} = 115^\circ\text{C}$$

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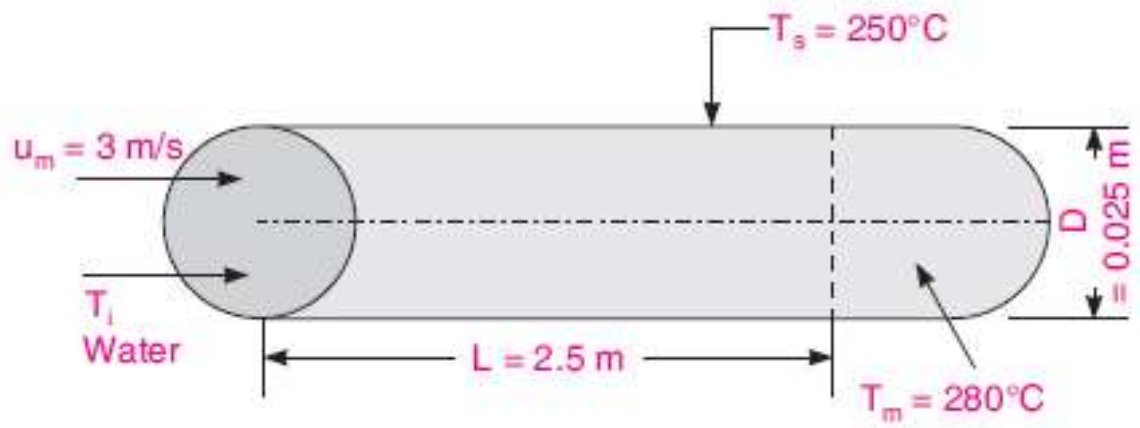
Heat transfer correlations for liquids and gases in incompressible fluid flow through tubes

Name of equation	*Formula*[a]	*Conditions for use*
Dittus-Boelter	$Nu_D = 0.023\ Re_D^{0.8}\ Pr^n$ $n \begin{cases} = 0.4 \text{ for heating} \\ = 0.3 \text{ for cooling} \end{cases}$	$0.7 < Pr < 160$ $6000 < Re_D < 10^7$
Sieder-Tate	$Nu_D = 0.027\ Re_D^{0.8}\ Pr^{0.3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$	$6000 < Re_D < 10^7$ $0.7 < Pr < 10^4$
Petukhov-Popov	$Nu_D = \frac{(f/8) Re_D Pr}{K_1 + K_2 (f/8)^{1/2} (Pr^{2/3} - 1)}$ where $f = (1.82 \log_{10} Re_D - 1.64)^{-2}$ $K_1 = 1 + 3.4f$ $K_2 = 11.7 + \frac{1.8}{Pr^{1/3}}$	$0.5 < Pr < 2000$ $10^4 < Re_D < 5 \times 10^6$
Sleicher-Rouse	$Nu_D = 5 + 0.015\ Re_D^a\ Pr_s^b$ where $a = 0.88 - \frac{0.24}{4 + Pr_s}$ $b = 1/3 + 0.5e^{-0.6 Pr_s}$	$0.1 < Pr < 10^5$ $10^4 < Re_D < 10^6$

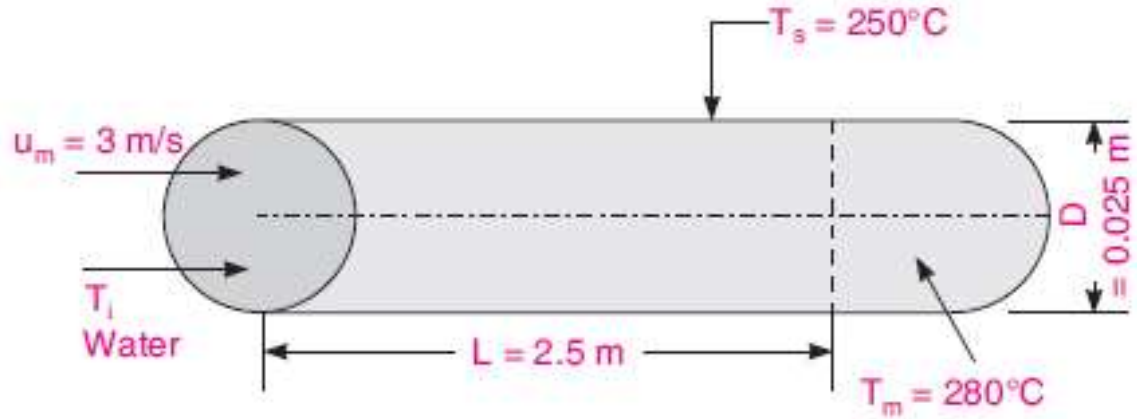
"All properties are evaluated at the bulk fluid temperature T_m except properties with subscript, they are evaluated at surface temperature T_s."

L-25

Steam is generated on the surface of tubes (surrounded by water) with pressurized water flowing inside the tubes of a heat exchanger. At a particular section, the velocity of the water in the tubes is 3 m/s. The inside diameter of the tubes is 25 mm and the tube surfaces are at 250°C. Find the convective heat transfer coefficient by different correlations at a section where the bulk temperature of the pressurized water is 280°C. This section is 2.5 m from the entrance of the water to the tube.



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Analysis : The properties of water at T_m = 280°C from Table A-7 ;

$\rho = 756$ kg/m^3, $C_p = 5.24$ kJ/kg.K,

$\mu = 97 \times 10^{-6}$ kg/ms, $k_f = 0.580$ W/m.K,

Pr = 0.87, $\mu_s = 106 \times 10^{-6}$ kg/ms (at 250°C).

Reynolds number

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{756 \times 3 \times 0.025}{97 \times 10^{-6}} = 584536$$

Re > 2300, thus the flow is turbulent.

(*i*) Using Colburn eqn. (9.77)

$$Nu_D = 0.023 Re_D^{0.8} Pr^{1/3}$$

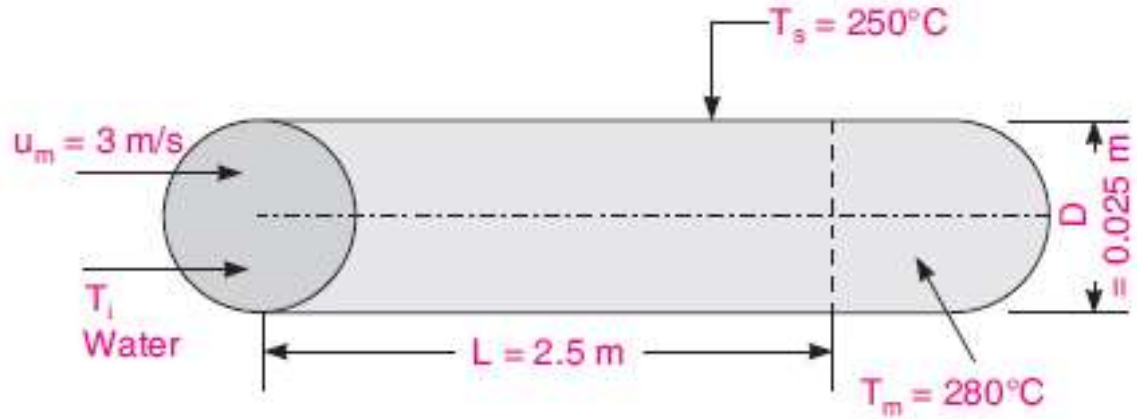
$$= 0.023 \times (584536)^{0.8} \times (0.87)^{1/3} = 901$$

Average heat transfer coefficient

$$h = \frac{Nu_D k_f}{D} = \frac{901 \times 0.580}{0.025}$$

$$= \mathbf{20917\ W/m^2.K.}\ \textbf{Ans.}$$

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(*ii*) Using Dittus-Boelter eqn. (9.78), with $n = 0.3$ for cooling ;

$$Nu_D = 0.023\ Re_D^{0.8}\ Pr^{0.3} = 906$$

and h = **21015 W/m².K.** **Ans.**

(*iii*) Using Sieder-Tate eqn. (9.79)

$$Nu_D = 0.027\ Re_D^{0.8}\ Pr^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$= 0.027 \times (584536)^{0.8} \times (0.87)^{1/3} \times \left(\frac{97 \times 10^{-6}}{106 \times 10^{-6}}\right)^{0.14} = 1045$$

and h = **24252 W/m².K.** **Ans.**

) Using Gnielinsky eqn. (9.81) for $0 < Pr < 1.5$

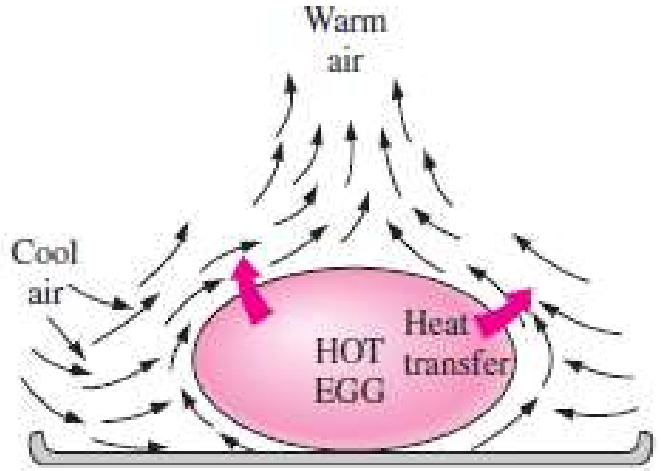
$$Nu_D = 0.0214\ (Re_D^{0.8} - 100)\ Pr^{0.4} \left[1 + \left(\frac{D}{L}\right)^{2/3}\right]$$

$$= 0.0214 \times [(584536)^{0.8} - 100) \times (0.87)]^{0.4} \times \left[1 + \left(\frac{0.025}{2.5}\right)^{2/3}\right] = 867.6$$

h = **20128 W/m².K.** **Ans.**

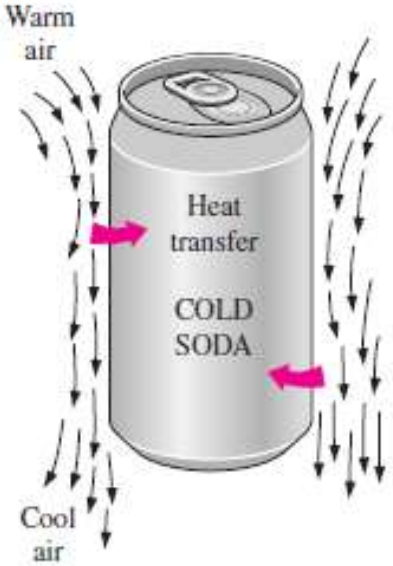
PHYSICAL MECHANISM OF NATURAL CONVECTION

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heated air rises

continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a natural convection current

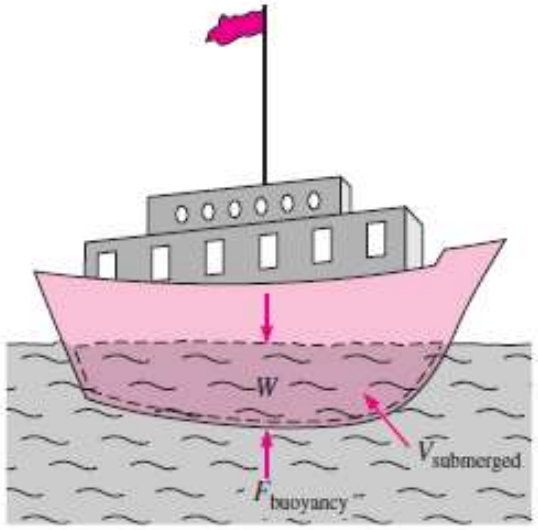


direction of fluid motion is reversed

Buoyancy force and natural convection

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• *The magnitude of the buoyancy force is equal to the weight of the fluid displaced by the body.*



$$F_{buoyancy} = \rho_{fluid}\, g V_{body}$$

$$\begin{aligned} F_{net} &= W - F_{buoyancy} \\ &= \rho_{body}\, g V_{body} - \rho_{fluid}\, g V_{body} \\ &= (\rho_{body} - \rho_{fluid})\, g V_{body} \end{aligned}$$

•*Thus, a body immersed in a fluid will experience a "weight loss" in an amount equal to the weight of the fluid it displaces. This is known as Archimedes' principle.*

•*without buoyancy, heat transfer between a hot (or cold) surface and the fluid surrounding it would be by conduction instead of by natural convection.*

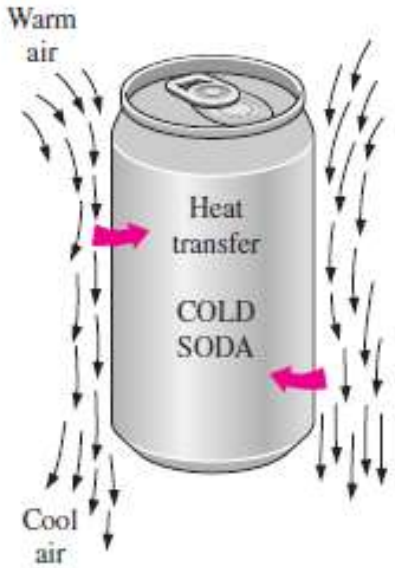
Buoyancy force and natural convection

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- *In heat transfer studies, the primary variable is temperature, and it is desirable to express the net buoyancy force in terms of temperature differences.*

- *This requires expressing the density difference in terms of a temperature difference, which requires a knowledge of a property that represents the variation of the density of a fluid with temperature at constant pressure.*

- *The property that provides that information is the volume expansion coefficient (β)*



$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \quad \text{(at constant } P\text{)}$$

$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \quad \text{(at constant } P\text{)}$$

$$\beta_{\text{ideal gas}} = \frac{1}{T}$$

T is the absolute temperature.

Buoyancy force and natural convection

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$\beta_{\text{ideal gas}} = \frac{1}{T}$ *T is the absolute temperature.*

•A large value of β for a fluid means a large change in density with temperature, and that the product β ΔT represents the fraction of volume change of a fluid that corresponds to a temperature change ΔT *at constant pressure* .

•The buoyancy force is proportional to the density difference, which is proportional to the temperature difference at constant pressure. Therefore, the larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the larger the buoyancy force and the stronger the natural convection currents, and thus the higher the heat transfer rate.

•The magnitude of the natural convection heat transfer between a surface and a fluid is directly related to the flow rate of the fluid.

•In natural convection, no blowers are used, and therefore the flow rate cannot be controlled externally. The flow rate in this case is established by the dynamic balance of buoyancy and friction.

Buoyancy force and natural convection

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•The buoyancy force is caused by the density difference between the heated (or cooled) fluid adjacent to the surface and the fluid surrounding it, and is proportional to this density difference and the volume occupied by the warmer fluid.

•whenever two bodies in contact (solid–solid, solid–fluid, or fluid–fluid) move relative to each other, a friction force develops at the contact surface in the direction opposite to that of the motion

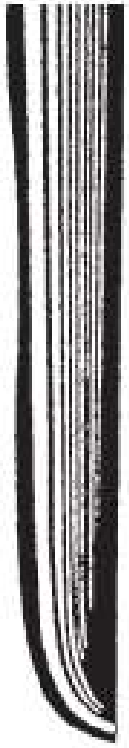
•This opposing force slows down the fluid and thus reduces the flow rate of the fluid. Under steady conditions, the air flow rate driven by buoyancy is established at the point where these two effects balance each other.

• The friction force increases as more and more solid surfaces are introduced, seriously disrupting the fluid flow and heat transfer. For that reason, heat sinks with closely spaced fins are not suitable for natural convection cooling.

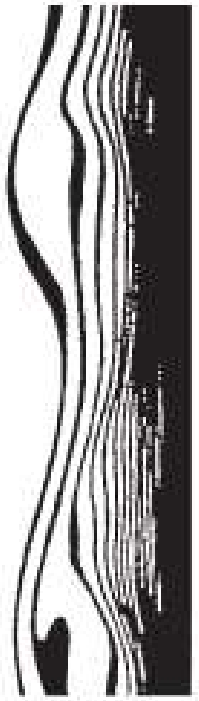
Natural convection flow patterns

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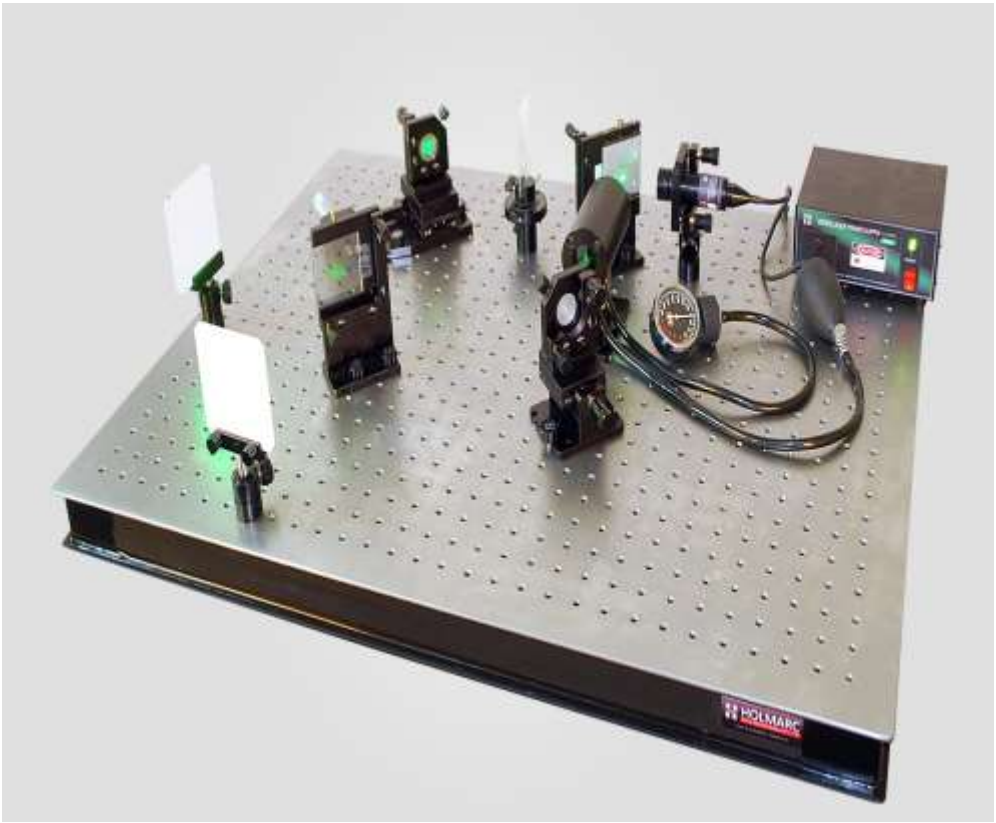
•Mach–Zehnder interferometer, which gives a plot of isotherms in the fluid in the vicinity of a surface.



(*a*) Laminar flow

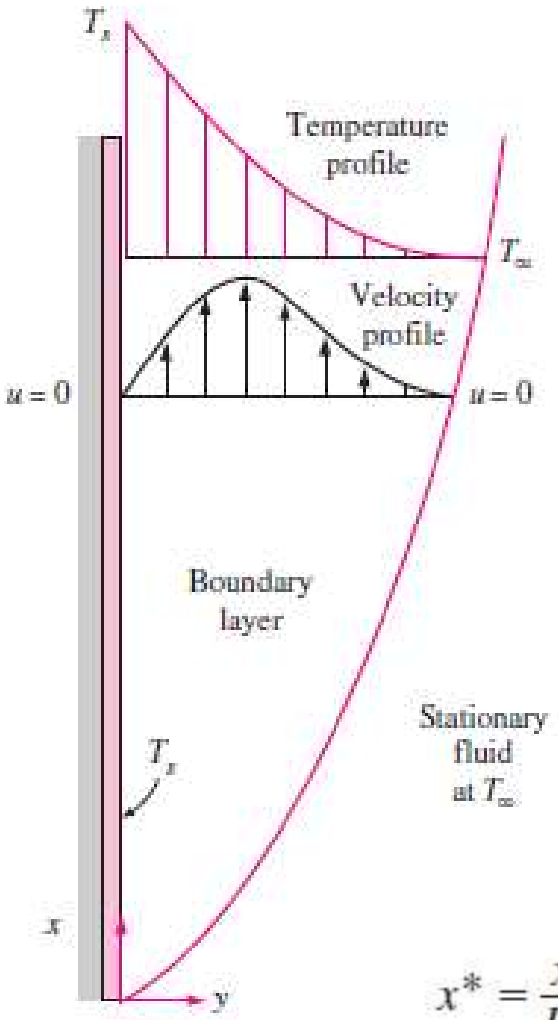


(*b*) Turbulent flow



THE GRASHOF NUMBER

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$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

This is the equation that governs the fluid motion in the boundary layer due to the effect of buoyancy.

The governing equations of natural convection and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by suitable constant quantities:

all lengths by a characteristic length Lc ,

all velocities by an arbitrary reference velocity (which, from the definition of Reynolds number, is taken to be Re = L /Lc),
and temperature by a suitable temperature difference (which is taken to be Ts - T) as

$$x^* = \frac{x}{L_c} \qquad y^* = \frac{y}{L_c} \qquad u^* = \frac{u}{\nu} \qquad v^* = \frac{v}{\nu} \qquad \text{and} \qquad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

THE GRASHOF NUMBER

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$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

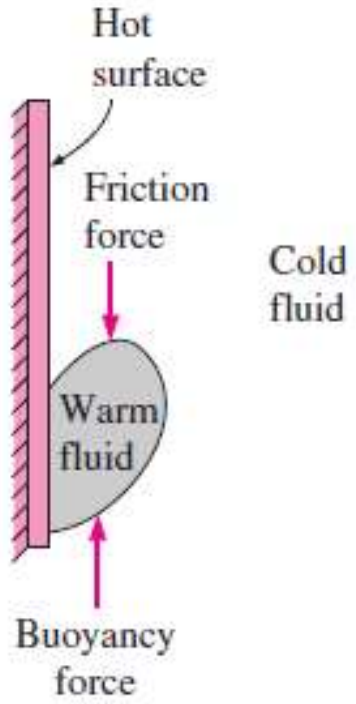
$$x^* = \frac{x}{L_c} \qquad y^* = \frac{y}{L_c} \qquad u^* = \frac{u}{V} \qquad v^* = \frac{v}{V} \qquad \text{and} \qquad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right]\frac{T^*}{\mathrm{Re}_L^2} + \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

The Grashof number Gr is a measure of the relative magnitudes of the *buoyancy force and the opposing viscous force acting on the fluid.*

g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, m^2/s



THE GRASHOF NUMBER

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The flow regime in natural convection is governed by the dimensionless *Grashof number,* which represents the ratio of the *buoyancy force to the viscous force acting* on the fluid.

The role played by the Reynolds number in forced convection is played by the Grashof number in natural convection.

As such, the Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection.

For vertical plates, for example, the critical Grashof number is observed to be about 10^9. Therefore, the flow regime on a vertical plate becomes turbulent at Grashof numbers greater than 10^9.

When a surface is subjected to external flow, the problem involves both natural and forced convection. The relative importance of each mode of heat transfer is determined by the value of the coefficient GrL/ReL^2

Natural convection effects are negligible if GrL/ReL^2 <<1, free convection dominates and the forced convection effects are negligible if GrL/ReL^ 2 >>1, and both effects are significant and must be considered if GrL /ReL ^2 = 1.

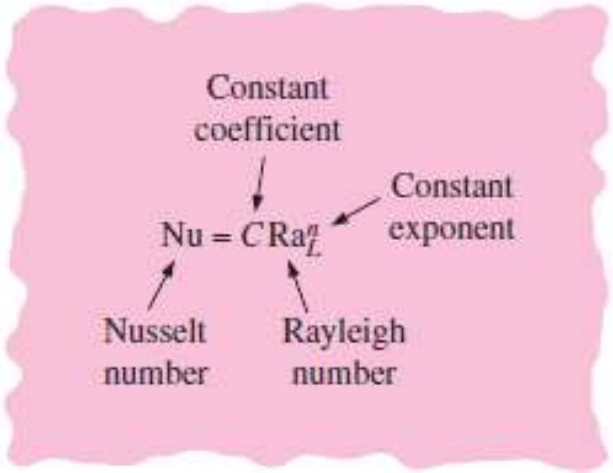
NATURAL CONVECTION OVER SURFACES

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Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid involved.

The simple empirical correlations for the average *Nusselt number Nu in natural* convection are of the form

$$\mathrm{Nu} = \frac{hL_c}{k} = C(\mathrm{Gr}_L\,\mathrm{Pr})^n = C\,\mathrm{Ra}_L^n \qquad \mathrm{Ra}_L = \mathrm{Gr}_L\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\mathrm{Pr}$$



The values of the constants C and n depend on the geometry of the surface and the flow regime, which is characterized by the range of the Rayleigh number.

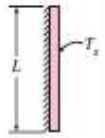
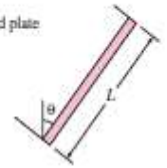
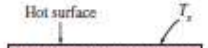
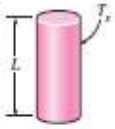
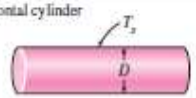
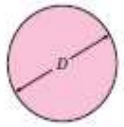
n= ¼ laminar

N=1/3 Turbulent

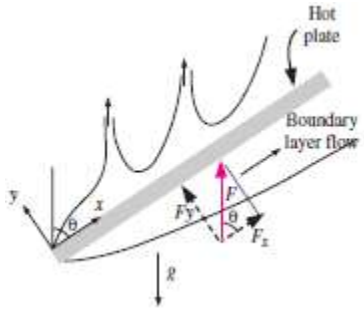
C= <1

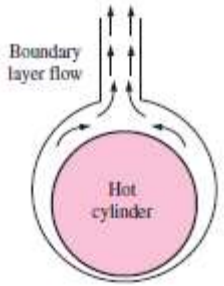
All fluid properties are to be evaluated at the film temperature *Tf = (Ts + Ta) / 2.*

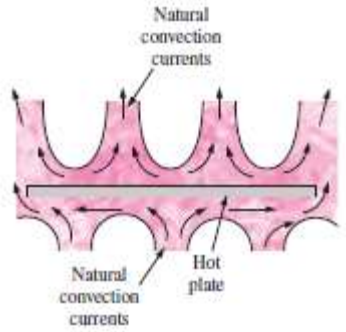
Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ (9-19) $Nu = 0.1Ra_L^{1/3}$ (9-20) $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $Ra < 10^9$
Horiontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$Nu = 0.54Ra_L^{1/4}$ (9-22) $Nu = 0.15Ra_L^{1/3}$ (9-23) $Nu = 0.27Ra_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere 	D	$Ra_D \leq 10^{11}$ ($Pr \geq 0.7$)	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (9-26)

Vertical Plates (*Ts= constant)*
Vertical Plates (*qs =constant)*
Vertical Cylinders
Inclined Plates
Horizontal Plates
Horizontal Cylinders and Spheres

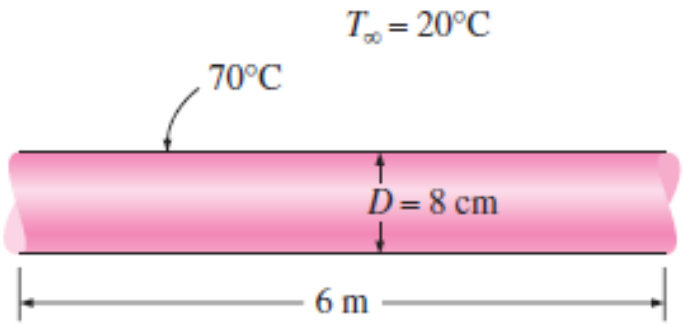






A 6-m-long section of an 8-cm-diameter horizontal hot water pipe passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.

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1 Steady operating conditions exist.
2 Air is an ideal gas.
3 The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (70 + 20)/2 = 45°C$ and 1 atm are (Table A–15)

$$k = 0.02699 \text{ W/m} \cdot {}^\circ\text{C} \qquad \text{Pr} = 0.7241$$
$$\nu = 1.749 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = \frac{1}{318 \text{ K}}$$

Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.08$ m. Then the Rayleigh number becomes

$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81 \text{ m/s}^2)[1/(318 \text{ K})](70 - 20 \text{ K})(0.08 \text{ m})^3}{(1.749 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.7241) = 1.869 \times 10^6$$

The natural convection Nusselt number in this case can be determined from Eq. 9-25 to be

$$\text{Nu} = \left\{0.6 + \frac{0.387 \text{ Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1869 \times 10^6)^{1/6}}{[1 + (0.559/0.7241)^{9/16}]^{8/27}}\right\}^2$$
$$= 17.40$$

- $\nu = 17.45 * 10^{-6}$
- Pr = 0.6835
- K = 0.02791

- $\beta = 1 / 318$
- $Gr = [\beta.g.(Ts - Ta)D^3] / \nu^2$

- $$Gr = \frac{\left(\frac{1}{318}\right)9.81\times(70-20)\times0.08^3}{(17.45 * 10^{-6})^2} = 2593528.3$$

$$= 2.5 \times 10^6$$

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- Gr.Pr $= 2.5 \times 10^6 \times 0.6835$

  $= 1.77 \times 10^6$

$$Nu_x = 0.508\, Pr^{0.5} (0.952 + Pr)^{-0.25} \cdot Gr_x^{0.25}$$

$$Nu = 0.508 (\quad)(\quad)(2.5 \times 10^6)^{0.25}$$

$$Nu = 14.76$$

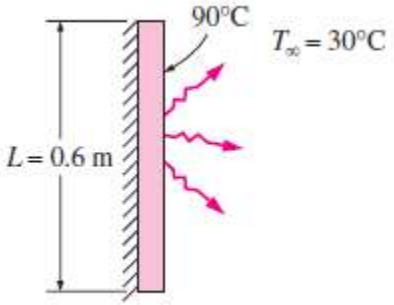
$$Nu = h.D / K$$

$$h = Nu.(k/D) = 14.76 \times (0.02791/0.08) = 5.14\ W/m^2.K$$

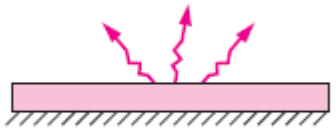
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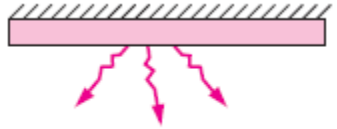
- $Q = h.As.(Ts - Ta)$
- $Q = 5.14 \times 3.14 \times 0.08 \times 6\ (70\text{-}20)$
- $Q = 387\ W$

Consider a 0.6-m × 0.6-m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated, as shown in Figure 9–15. Determine the rate of heat transfer from the plate by natural convection if the plate is (*a) vertical, (b) horizontal with hot surface facing* up, and (*c) horizontal with hot surface facing down.*



(*a*) Vertical

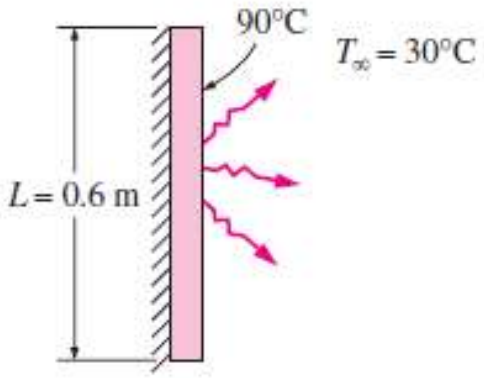

(*b*) Hot surface facing up


(*c*) Hot surface facing down

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (90 + 30)/2 = 60°C$ and 1 atm are (Table A-15)

$$k = 0.02808 \text{ W/m} \cdot {}^\circ\text{C} \qquad \text{Pr} = 0.7202$$

$$\nu = 1.896 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = \frac{1}{333 \text{ K}}$$



(*a*) Vertical

Analysis (*a*) *Vertical.* The characteristic length in this case is the height of the plate, which is $L = 0.6$ m. The Rayleigh number is

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr}$$

$$= \frac{(9.81\ \mathrm{m/s^2})[1/(333\ \mathrm{K})](90 - 30\ \mathrm{K})(0.6\ \mathrm{m})^3}{(1.896 \times 10^{-5}\ \mathrm{m^2/s})^2}(0.722) = 7.656 \times 10^8$$

Then the natural convection Nusselt number can be determined from Eq. 9-21 to be

$$\mathrm{Nu} = \left\{0.825 + \frac{0.387\ \mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$= \left\{0.825 + \frac{0.387(7.656 \times 10^8)^{1/6}}{1 + (0.492/0.7202)^{9/16}]^{8/27}}\right\}^2 = 113.4$$

Note that the simpler relation Eq. 9-19 would give $\mathrm{Nu} = 0.59\ \mathrm{Ra}_L^{1/4} = 98.14$, which is 13 percent lower. Then,

$$h = \frac{k}{L}\mathrm{Nu} = \frac{0.02808\ \mathrm{W/m \cdot °C}}{0.6\ \mathrm{m}}(113.4) = 5.306\ \mathrm{W/m^2 \cdot °C}$$

$$A_s = L^2 = (0.6\ \mathrm{m})^2 = 0.36\ \mathrm{m^2}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.306\ \mathrm{W/m^2 \cdot °C})(0.36\ \mathrm{m^2})(90 - 30)°\mathrm{C} = \mathbf{115\ W}$$

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- $Gr = [\beta.g.(Ts - Ta)L^3] / \nu^2$
- $Gr = \frac{\left(\frac{1}{333}\right)9.81\times(90-30)\times0.6^3}{(18.97 * 10^{-6})^2}$ $= 1.06 \times 10^9$

$Gr.Pr = 1.06 \times 10^9 \times 0.696 = 7.37 \times 10^8$

$$Nu_x = 0.508\ Pr^{0.5}\ (0.952 + Pr)^{-0.25}\ .\ Gr_x^{0.25}$$

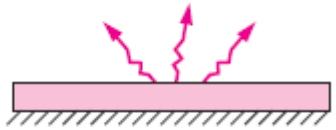
- $Nu = 0.508\ (0.696)^{0.5}\ (0.952+0.696)^{-0.25}(1.06 \times 10^9)^{0.25}$

  $= 67.49$

$h = Nu\ (k/L) \quad = 67.49\ (0.02896/0.6)$

$= 3.25\ W/m^2.K$

$Q = h.As.(Ts - Ta) = 3.25\ (0.6 \times 0.6)\ (90\text{-}30) = 70.2\ W$



(*b*) Hot surface facing up

(*b*) *Horizontal with hot surface facing up.* The characteristic length and the Rayleigh number in this case are

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6\text{ m}}{4} = 0.15\text{ m}$$

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr}$$

$$= \frac{(9.81\text{ m/s}^2)[1/(333\text{ K})](90 - 30\text{ K})(0.15\text{ m})^3}{(1.896 \times 10^{-5}\text{ m}^2/\text{s})^2}(0.7202) = 1.196 \times 10^7$$

The natural convection Nusselt number can be determined from Eq. 9-22 to be

$$\text{Nu} = 0.54\,\text{Ra}_L^{1/4} = 0.54(1.196 \times 10^7)^{1/4} = 31.76$$

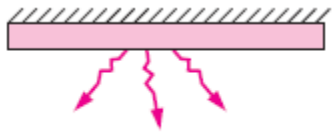
Then,

$$h = \frac{k}{L_c}\text{Nu} = \frac{0.0280\text{ W/m} \cdot {}^\circ\text{C}}{0.15\text{ m}}(31.76) = 5.946\text{ W/m}^2 \cdot {}^\circ\text{C}$$

$$A_s = L^2 = (0.6\text{ m})^2 = 0.36\text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.946\text{ W/m}^2 \cdot {}^\circ\text{C})(0.36\text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{128\text{ W}}$$



(c) Hot surface facing down

(c) *Horizontal with hot surface facing down*. The characteristic length, the heat transfer surface area, and the Rayleigh number in this case are the same as those determined in (*b*). But the natural convection Nusselt number is to be determined from Eq. 9-24,

$$\mathrm{Nu} = 0.27\,\mathrm{Ra}_L^{1/4} = 0.27(1.196 \times 10^7)^{1/4} = 15.86$$

Then,

$$h = \frac{k}{L_c}\mathrm{Nu} = \frac{0.02808\ \mathrm{W/m \cdot {}^\circ C}}{0.15\ \mathrm{m}}(15.86) = 2.973\ \mathrm{W/m^2 \cdot {}^\circ C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (2.973\ \mathrm{W/m^2 \cdot {}^\circ C})(0.36\ \mathrm{m^2})(90 - 30)^\circ\mathrm{C} = 64.2\ \mathrm{W}$$

Note that the natural convection heat transfer is the lowest in the case of the hot surface facing down. This is not surprising, since the hot air is “trapped” under the plate in this case and cannot get away from the plate easily. As a result, the cooler air in the vicinity of the plate will have difficulty reaching the plate, which results in a reduced rate of heat transfer.

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Unit 4

Introduction

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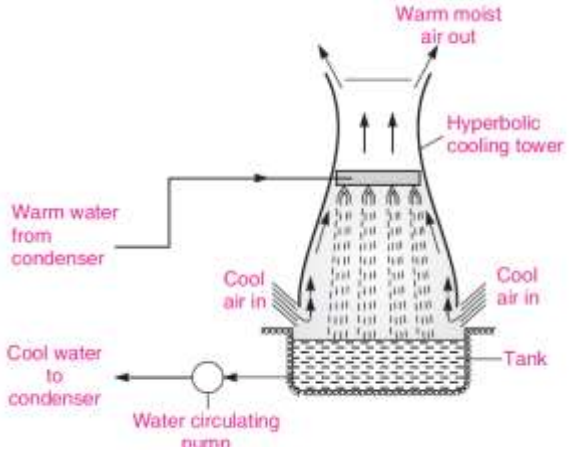
- A device used for exchange of heat between the two fluids that are at different temperatures, is called the *heat exchanger.*
- *Applications*
  - In a car as radiator, where hot water from the engine is cooled by atmospheric air.
  - In a refrigerator, the hot refrigerant from the compressor is cooled by convection into atmosphere by passing it through finned tubes.
  - In a steam condenser, the latent heat of condensation is removed by circulating water through the tubes.
  - In space heating and air-conditioning, waste heat recovery and chemical processing.
  - Intercoolers and Preheaters
  - Regenerators

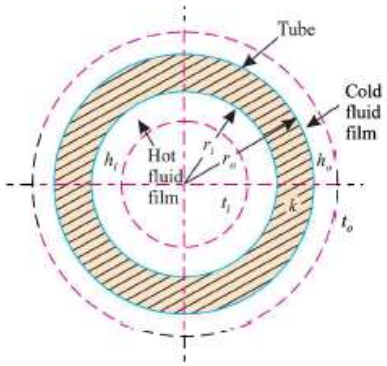
Classification

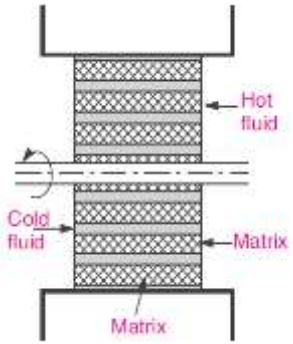
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1. According to Heat Transfer Process

- **Direct contact type heat exchanger**
  - In this type of heat exchanger, the two immiscible fluids at different temperatures are come in direct contact. (Mixed with each other)
  - one fluid is sprayed through the other.
  - Cooling towers, jet condensers, desuperheaters, open feed water heaters and scrubbers
- **Transfer type heat exchangers or recuperator**
  - The cold and hot fluids flow simultaneously through the device and the heat is transferred through the wall separating them.
  - Most commonly used heat exchangers
- **Regenerators or storage type heat exchangers**
  - the heat energy is transferred directly from one fluid to another across a solid surface which is alternatively in contact with both fluids.



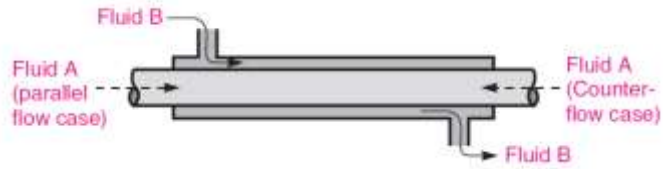




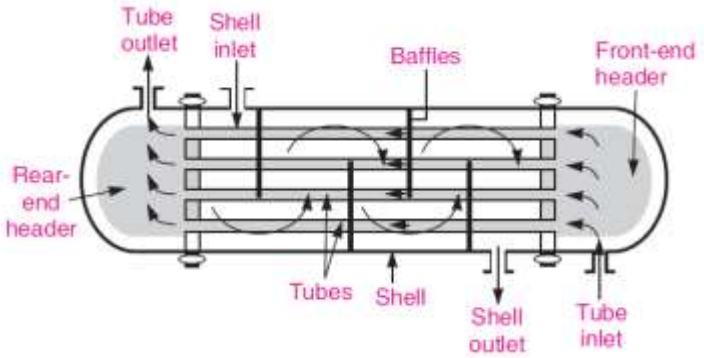
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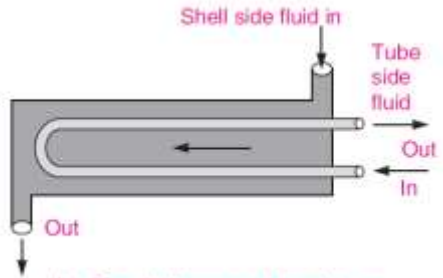
2. According to Constructional Features

- **Tubular heat exchangers**
  - Also called as ‘tube in tube’ or ‘concentric tube’ or ‘double pipe’ heat exchanger
  - These are widely used in many sizes and different flow arrangements and type.

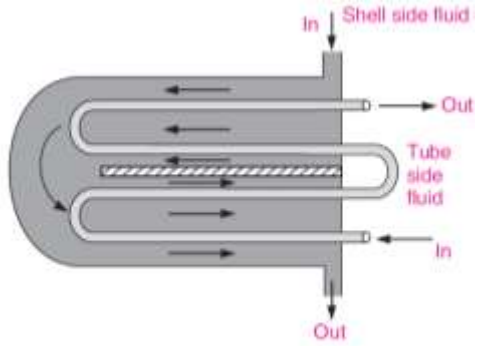


- **Shell and tube type heat exchangers**
  - also called as *surface condensers*
  - It consists of a shell and a large number of parallel tubes
  - The heat transfer takes place as one fluid flows through the tubes and other fluid flows outside the tubes through the shell
  - The baffles are commonly used on the shell to create turbulence and to keep the uniform spacing between the tubes.
  - Large surface area in small volume.





(a) One shell pass and two tube pass heat exchanger



(b) Two shell pass and two tube pass heat exchanger

- **Finned tube type**
  - an enhanced heat transfer rate is required, the extended surfaces (fins) are used on one side of the heat exchanger.
  - Used in gas turbines, automobiles, aeroplanes, heat pumps, refrigeration, electronics, etc.
  - The radiator of an automobile is an example of such heat exchanger.
- **Compact heat exchanger**
  - The heat transfer surface area per unit volume is very large.
  - A heat exchanger with an area density greater than 700 m^2/m^3 is called compact heat exchanger.
  - Plate -fin, tube-fin, and rotary regenerators are examples of compact heat exchangers



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3. According to Flow Arrangement

— Parallel flow

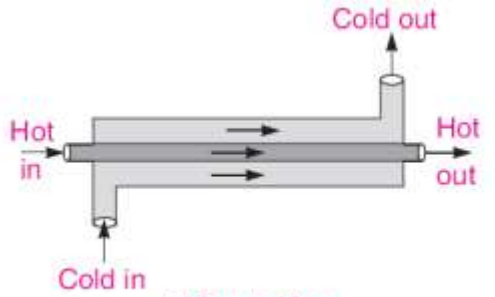
- The hot and cold fluids enter at same end of the heat exchanger, flow through in same direction and leave at other end.

— Counter flow

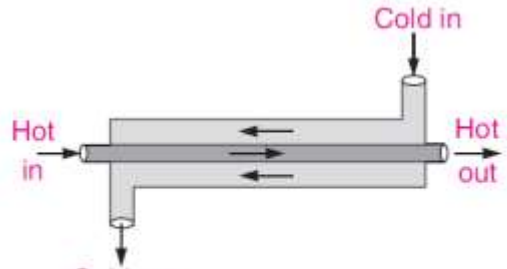
- The hot and cold fluids enter at the opposite ends of heat exchanger, flow through in opposite direction and leave at opposite ends.

— Cross flow

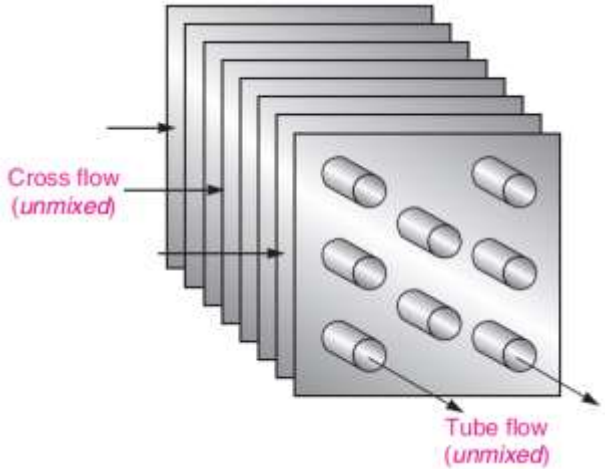
- The two fluids flow at right angle to each other.



(a) Parallel flow



(b) Counter flow heat exchanger



(c) Cross Flow Heat Exchanger

Overall heat transfer coefficien

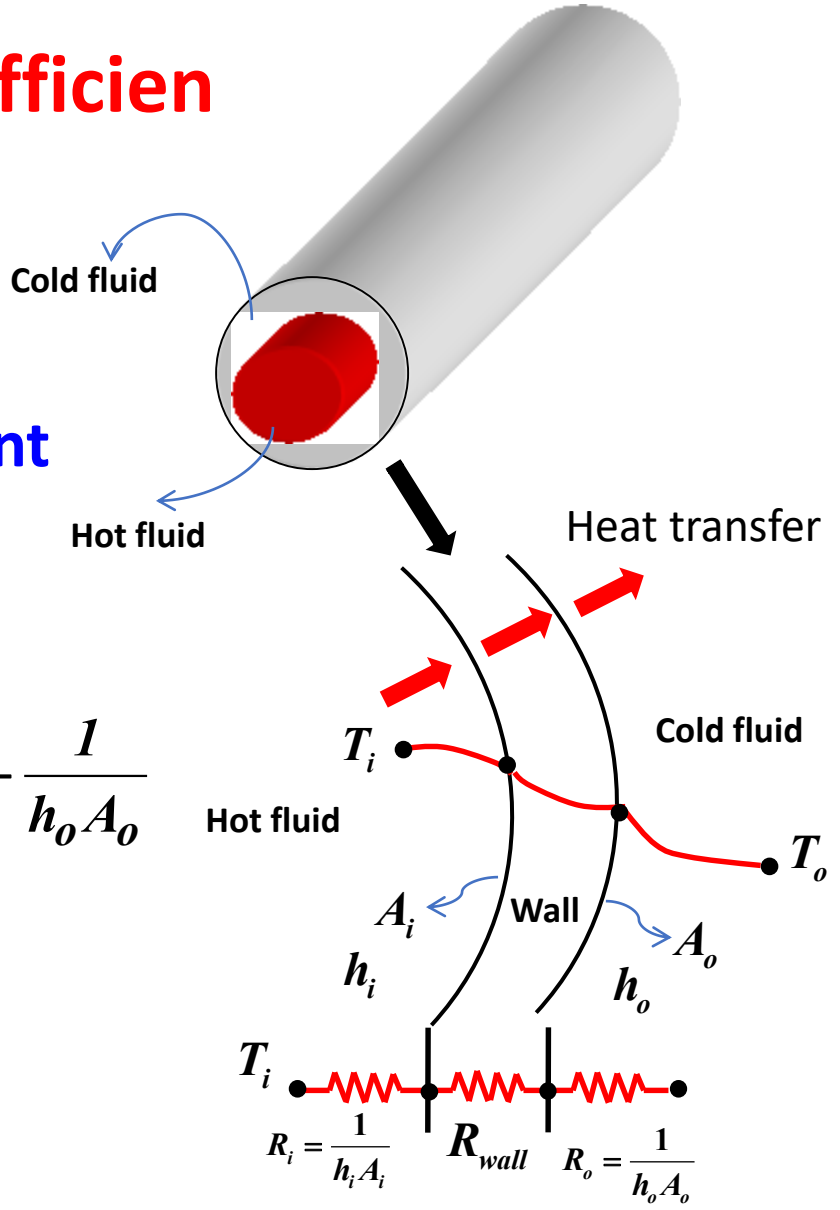
A_i – Inside area of the tube

A_o – Outside area of the tube

h_i – inside heat transfer coefficient

h_o – Outside heat transfer coefficient

k – thermal conductivity of tube



$$R_{Total} = R_i + R_{wall} + R_o = \frac{1}{h_i A_i} + \frac{\ln\left(r_o / r_i\right)}{2\pi k L} + \frac{1}{h_o A_o}$$

$$\frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{\ln\left(r_o / r_i\right)}{2\pi k L} + \frac{1}{h_o A_o}$$

U is meaningless unless area is specified

If $R_{wall} = 0$ and $A_i = A_o$ then $$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

U is dominated by smaller convection co efficient

- If one side fluid is gas and other side it is liquid, then to improve h.t.c. on gas side fin is introduced

Fin- Gas side –UA_s ↑

Tube is finned $$\frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{\eta_i h_i A_i} + \frac{\ln\left(r_o / r_i\right)}{2\pi k L} + \frac{1}{\eta_o h_o A_o}$$

η = Fin efficiency

Overall Heat Transfer Coefficient

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$$R_{wall} = \frac{\ln(d_o/d_i)}{2\pi L k}$$

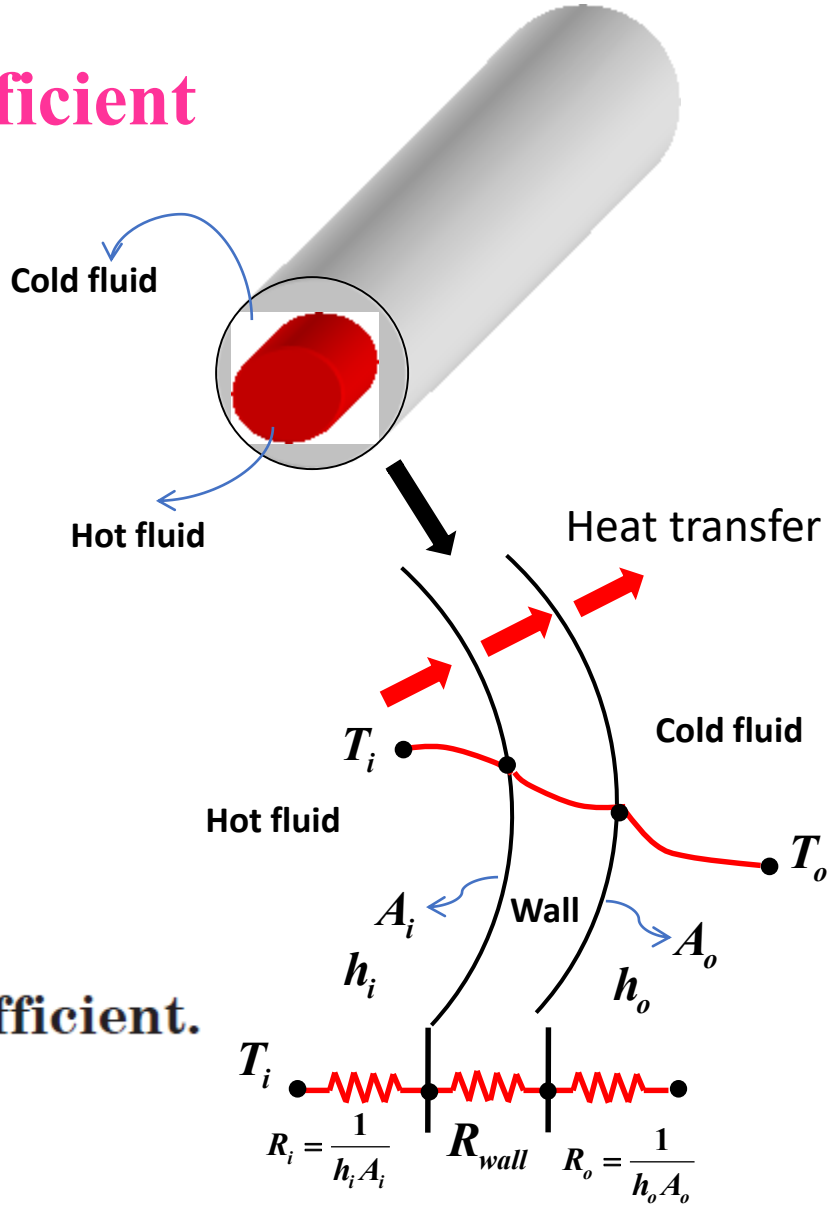
$$\Sigma R_{th} = R_{total} = R_{conv,\,i} + R_{wall} + R_{conv,\,o}$$

$$\Sigma R_{th} = \frac{1}{h_i A_i} + \frac{\ln(d_o/d_i)}{2\pi L k} + \frac{1}{h_o A_o}$$

$$Q = \frac{\Delta T}{R_{th}} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

where U is **overall heat transfer coefficient.**

$$\Sigma R_{th} = \frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$



The overall heat transfer coefficient based on outside tube surface can be expressed as :

$$U_o = \frac{1}{\Sigma R_{th} A_o}$$

$$= \frac{1}{\frac{A_o}{A_i h_i} + \frac{A_o}{2\pi L k} \ln\left(\frac{d_o}{d_i}\right) + \frac{1}{h_o}}$$

$$= \frac{1}{\frac{d_o}{d_i h_i} + \frac{d_o}{2k} \ln\left(\frac{d_o}{d_i}\right) + \frac{1}{h_o}}$$

Similarly, overall heat transfer coefficient based on inside tube surface can be expressed as :

$$U_i = \frac{1}{\Sigma R_{th} A_i}$$

$$= \frac{1}{\frac{1}{h_i} + \frac{A_i}{2\pi L k} \ln\left(\frac{d_o}{d_i}\right) + \frac{A_i}{A_o h_o}}$$

$$= \frac{1}{\frac{1}{h_i} + \frac{d_i}{2k} \ln\left(\frac{d_o}{d_i}\right) + \frac{d_i}{d_o h_o}}$$

Heat Exchanger Analysis

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Assumptions:

1. The mass flow rate of each fluid remains constant.
2. The fluid properties like temperature, velocity etc. at inlet and outlet also remain unchanged.
3. The changes in kinetic energy, potential energy, specific heat and conduction along the length are negligible.
4. The outer surface of outer tube of heat exchanger is well insulated and it does not allow any heat transfer to the surroundings, therefore, heat transfer occurs between hot and cold fluids only.
5. No fluid is undergoing phase change within heat exchanger.

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- Applying SFEE to heat exchanger

The rate of heat transfer from the hot fluid becomes equal to the rate of heat transfer to the cold fluid.

The rate of heat transfer by hot fluid

$$Q = m_h.C_{ph}(T_{hi} - T_{ho}) \quad \text{---------- (1)} \qquad \text{Watts}$$

The rate of heat transfer to cold fluid

$$Q = m_c.C_{pc}(T_{ci} - T_{co}) \quad \text{---------- (2)} \qquad \text{Watts}$$

where m = mass flow rate, kg/s

Cp = specific heat of the fluid, J/kg.K

T = temperature, °C

Suffix h = for hot fluid,

c = cold fluid,

i = for inlet condition,

o = for outlet condition.

Logarithmic Mean Temperature Difference (LMTD) Method

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- The total heat transfer rate between the hot and cold fluids can also be calculated by using overall heat transfer coefficient and surface area as :

$$Q = UA\Delta T_{lm}$$

where
U = overall heat transfer coefficient,
A = surface area for heat exchange,
ΔT_{lm} = log mean temperature difference.

- Logarithmic mean temperature difference is given by

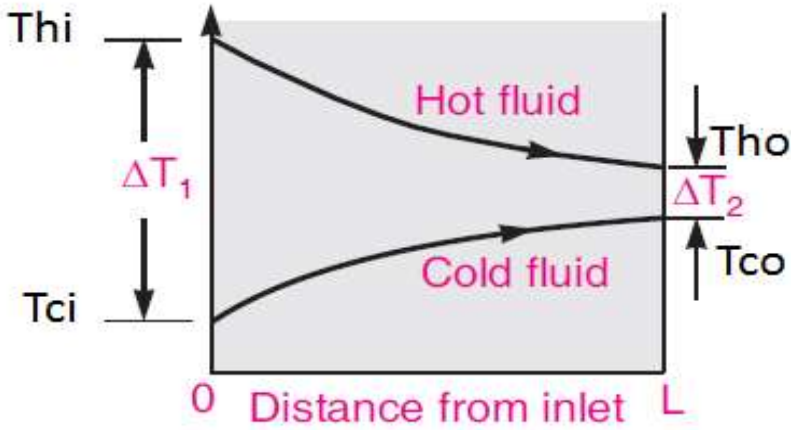
$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left[\frac{\Delta T_1}{\Delta T_2}\right]} = \frac{\Delta T_2 - \Delta T_1}{\ln\left[\frac{\Delta T_2}{\Delta T_1}\right]}$$

- **Where,**

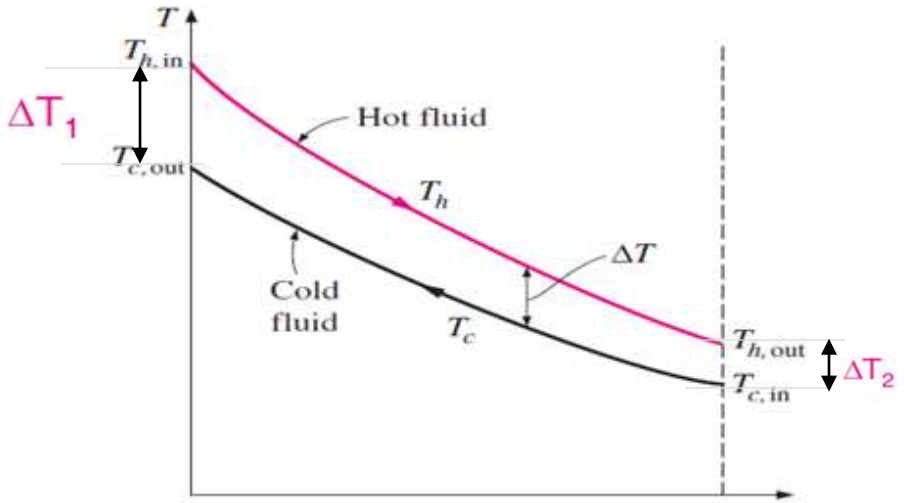
ΔT1 and ΔT2 can be evaluated for different heat exchangers (HE) from temperature distribution diagram.

Temperature distribution diagram

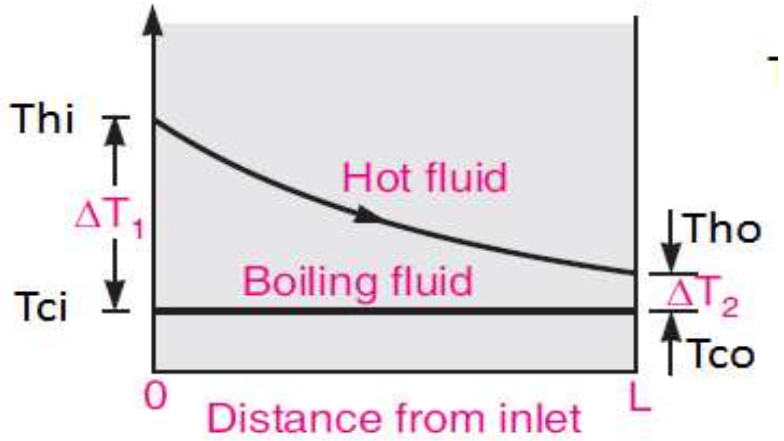
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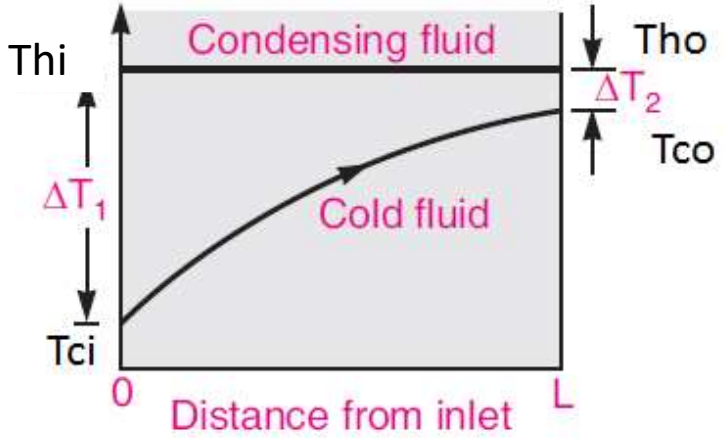
(a) Parallel flow heat exchanger



(b) Counter flow heat exchanger



(c) Temperature distribution for an evaporator



(d) Temperature distribution for a condenser

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- For multipass and cross flow heat exchangers, the temperature distribution and flow conditions are complicated.
- the appropriate mean temperature difference can be obtained by introducing a correction factor F with LMTD.

$$\Delta T_{lm} = F\ \Delta T_{lm,\ \text{counter flow}}$$

- The correction factor F = 1, for counter flow heat exchangers and less than unity for cross flow and multipass arrangement for shell and tube type heat exchangers.

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Example 1: *A heat exchanger is required to cool 55,000 kg/h of alcohol from 66°C to 40°C using 40,000 kg/h of water entering at 5°C. Calculate (i) exit temperature of water, (ii) heat transfer rate, (iii) surface area required for (a) parallel flow type, (b) counter flow type of heat exchanger.*

Take overall heat transfer coefficient

$$U = 580\ W/m^2.K$$

$$C_{p\ (alcohol)} = 3760\ J/kg.K$$

$$C_{p\ (water)} = 4180\ J/kg.K.$$

Solution

Given: A double pipe heat exchanger

Alcohol, $\dot{m}_h = 55000$ kg/h

Water, $\dot{m}_c = 40000$ kg/h

$T_{h,\,i} = 66°C$

$C_{p,\,c} = 4180$ J/kg.K

$T_{h,\,o} = 40°C$

$T_{c,\,i} = 5°C$

$C_{p,\,h} = 3760$ J/kg.K

$U = 580$ W/m^2.K.

To find :

(*i*) Exit temperature $T_{c,\,o}$ of water,

(*ii*) Heat transfer rate Q,

(*iii*) Surface area of heat exchanger if it is (*a*) parallel type, (*b*) counter flow type.

Analysis : (*i*) Making energy balance on the heat exchanger

$$\dot{m}_h C_{p,\,h} (T_{h,\,i} - T_{h,\,o}) = \dot{m}_c C_{p,\,c} (T_{c,\,o} - T_{c,\,i})$$

$$55000 \times 3760 \times (66 - 40) = 40{,}000 \times 4180 \times (T_{c,\,o} - 5)$$

or $$T_{c,\,o} = \mathbf{37.15°C.}$$ **Ans.**

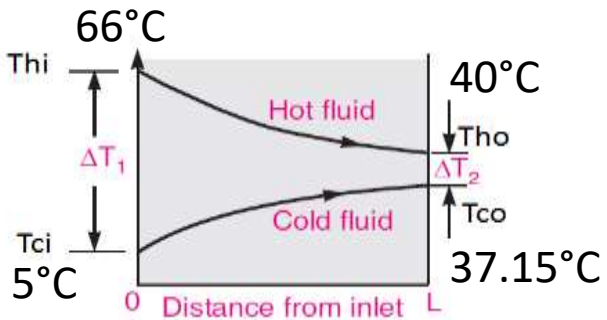
(*ii*) Heat transfer rate

$$Q = \dot{m}_h C_{p,\,h} (T_{h,\,i} - T_{h,\,o})$$

$$= 55000 \times 3760 \times (66 - 40)$$

$$= 537.68 \times 10^7 \text{ J/h}$$

$$= \mathbf{1493.55 \text{ kW}}.$$ **Ans.**



(*iii*) Surface area of heat exchanger,

(*a*) Parallel flow type

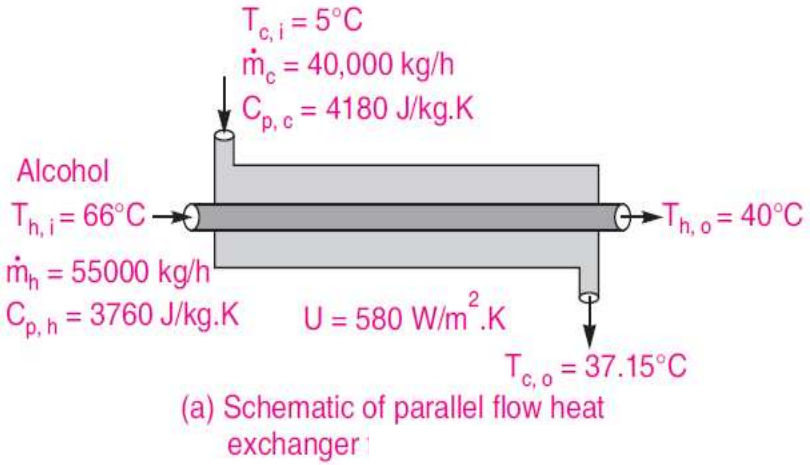
$$\Delta T_1 = T_{h,\,i} - T_{c,\,i} = 66 - 5 = 61°C$$

$$\Delta T_2 = T_{h,\,o} - T_{c,\,o} = 40 - 37.15 = 2.85°C$$

Log mean temperature difference

$$\Delta T_{lm,\ \text{parallel}} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

$$= \frac{61 - 2.85}{\ln\left(\frac{61}{2.85}\right)} = 18.98°C$$



(a) Schematic of parallel flow heat exchanger

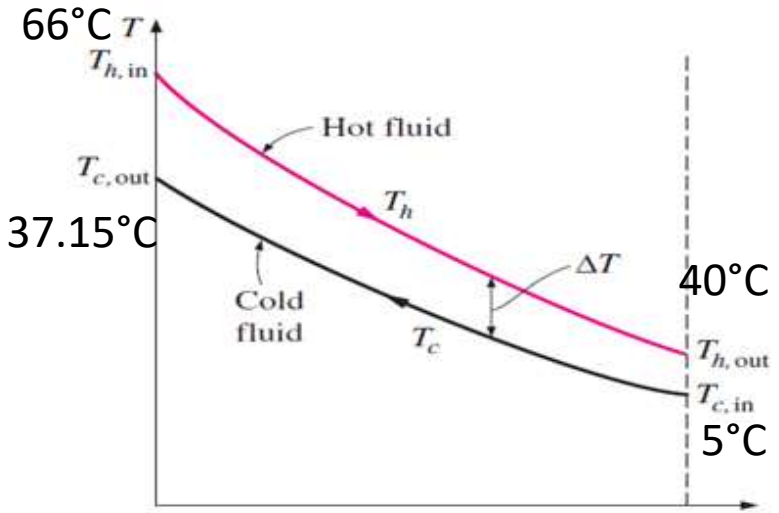
$$Q = UA\Delta T_{lm}$$

Heat transfer area,

$$A = \frac{Q}{U\,\Delta T_{lm,\,\text{parallel}}}$$

$$= \frac{1493.55 \times 10^3}{580 \times 18.98}$$

$$= \mathbf{135.66\ m^2.}\quad \textbf{Ans.}$$



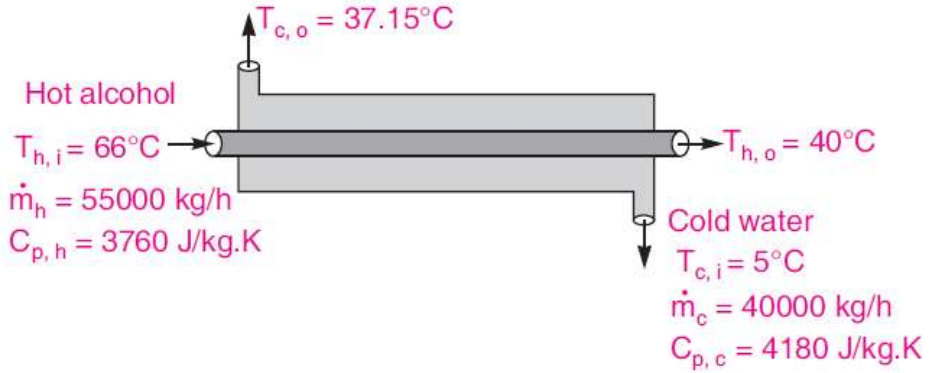
(*b*) Counter flow type

$$\Delta T_1 = T_{h,\,i} - T_{c,\,o} = 66 - 37.15 = 28.85°C$$

$$\Delta T_2 = T_{h,\,o} - T_{c,\,i} = 40 - 5 = 35°C$$

$$\Delta T_{lm,\,\text{counter}} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

$$= \frac{28.85 - 35}{\ln\left(\frac{28.85}{35}\right)} = 31.82°C$$



(b) Schematic of counter flow heat exchanger

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Surface area,

$$A = \frac{Q}{U\ \Delta T_{lm,\ \text{counter}}}$$

$$= \frac{1493.55 \times 10^3}{580 \times 31.82}$$

$$= \mathbf{80.91\ m^2.}\quad \textbf{Ans.}$$

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Example 2: *A simple heat exchanger consisting of two concentric flow passages is used for heating 1110 kg/h of oil (sp. heat = 2.1 kJ / kg.K) from a temperature of 27°C to 49°C. The oil flows through the inner pipe made of copper (O.D. = 2.86 cm, I.D. = 2.54 cm) and the surface heat transfer coefficient on the oil side is 635 W/m^2.K. The oil is heated by hot water supplied at the rate of 390 kg/h and at an inlet temperature of 93°C. The water side heat transfer coefficient is 1270 W/m^2.K. Take the thermal conductivity of copper to be 350 W/m.K and the fouling factors on the oil and water sides to be 0.0001 and 0.0004 m^2.K/W. What is the length of the heat exchanger for : (i) parallel flow, and (ii) counter flow ?*

Solution

Given : Double heat exchanger

$\dot{m}_c$ = 1110 kg/h,

$\dot{m}_h$ = 390 kg/h,

$C_{p,c}$ = 2.1 kJ/kg. K = 2100 J/kg.K,

$T_{h,i}$ = 93°C,

$T_{c,i}$ = 27°C,

$T_{c,o}$ = 49°C,

d_i = 2.54 cm = 0.0254 m,

d_o = 2.86 cm = 0.0286 m,

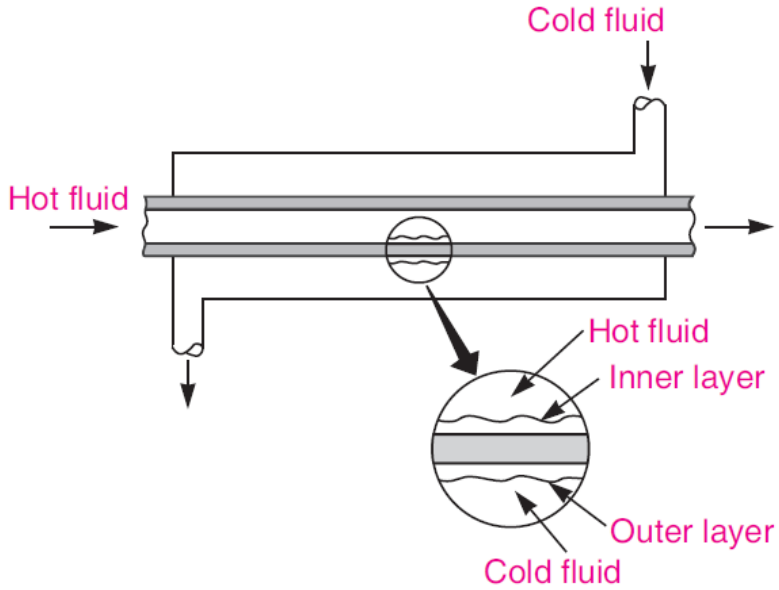
h_i = 635 W/m^2.K,

h_o = 1270 W/m^2.K,

k = 350 W/m.K,

$R_{f,i}$ = 0.0001 m^2.K/W,

$R_{f,o}$ = 0.0004 m^2.K/W.



To find : Length of heat exchanger for

(*i*) Parallel flow arrangement,

(*ii*) Counter flow arrangement.

Analysis : The exit temperature of water : The energy balance between the two fluids ;

$$\dot{m}_h C_{p,h}(T_{h,i} - T_{h,o}) = \dot{m}_c C_{p,c}(T_{c,o} - T_{c,i})$$

$$390 \times 4180 \times (93 - T_{h,o}) = 1110 \times 2100 \times (49 - 27)$$

or $$T_{h,o} = 93 - 31.45 = 61.55°C$$

The heat transfer rate,

$$Q = \dot{m}_c C_{p,c}(T_{c,o} - T_{c,i})$$

$$Q = (1100/3600)(kg/s) \times (2100\ J/kg.K) \times (49 - 27)(K)$$

$$= 14245\ W$$

The overall heat transfer coefficient, U_o

$$U_o = \frac{1}{\frac{d_o}{d_i h_i} + \frac{d_o}{d_i} R_{f,i} + \frac{d_o \ln(r_o/r_i)}{2k} + R_{f,o} + \frac{1}{h_o}}$$

$$= \frac{1}{\frac{0.0286}{0.0254} \times \left\{\frac{1}{635} + 0.0001\right\} + \frac{0.0286}{2 \times 350} \times \ln\left(\frac{0.0286}{0.0254}\right) + 0.0004 + \frac{1}{1270}}$$

$$= 325\ W/m^2.K$$

Uo = 1 / (Ro.Ao)

Uo = 1 / [Ao(R1+R2+R3+R4+R5)

R1 = 1/hi.Ai

Ao/Ai = πDoL / πDiL = Do/Di

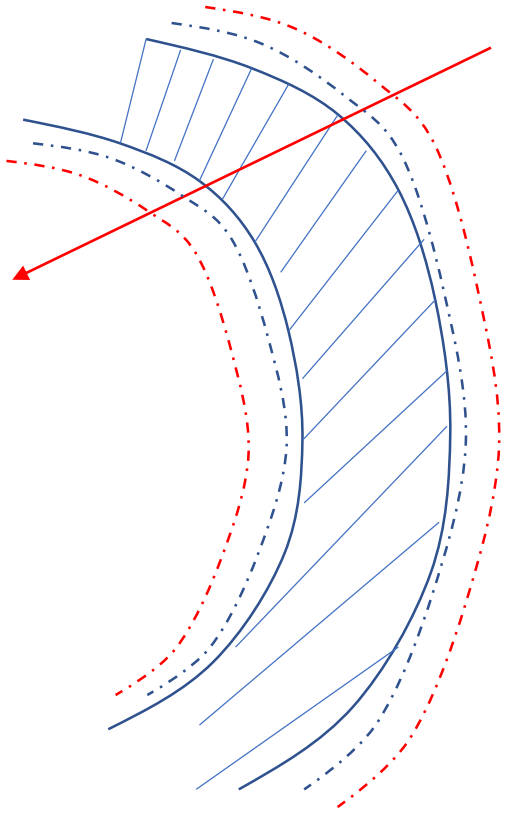
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$U_o = 1 / (R_o.A_o)$

$U_o = 1 / [A_o(R_1+R_2+R_3+R_4+R_5)$

$R_1 = 1/h_i.A_i$

$A_o/A_i = \pi D_o L / \pi D_i L = D_o/D_i$



Oil

Hot water

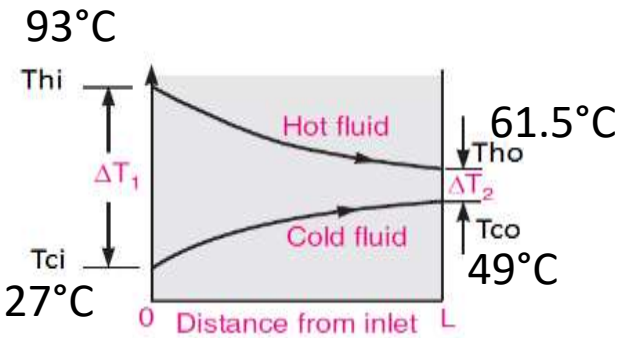
$$\text{LMTD, } \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left[\frac{\Delta T_1}{\Delta T_2}\right]}$$

(i) For parallel flow arrangement :

$$\Delta T_1 = (T_{h,i} - T_{c,i}) = 93 - 27 = 66°C$$

and $$\Delta T_2 = (T_{h,o} - T_{c,o}) = 61.55 - 49 = 12.55$$

$$\Delta T_{lm} = \frac{66 - 12.55}{\ln\left(\frac{66}{12.55}\right)} = 32.2°C$$



The length of heat exchange can be calculated as

$$Q = U_o A_o \Delta T_{lm} = U_o(\pi d_o L)\, \Delta T_{lm}$$

or $$14245 = 325 \times (\pi \times 0.0286\ L) \times 32.2$$

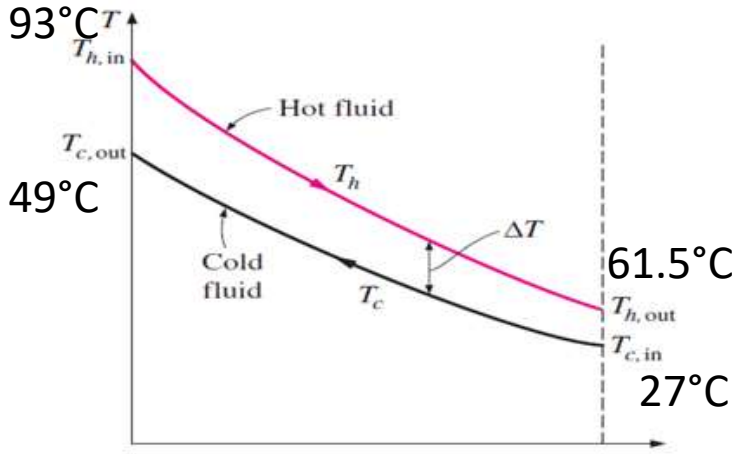
or $$L = \mathbf{15.15\ m.}$$ **Ans.**

(ii) For counter flow arrangement :

$$\Delta T_1 = T_{h,o} - T_{c,i} = 61.55 - 27 = 34.55°C$$

$$\Delta T_2 = T_{h,i} - T_{c,o} = 93 - 49 = 44°C$$

$$\text{LMTD}\Delta T_{lm} = \frac{34.55 - 44}{\ln\left[\frac{34.55}{44}\right]} = 39.07°C$$



The length of heat exchanger

$$Q = U_o A_o \Delta T_{lm} = U_o(\pi d_o L)\, \Delta T_{lm}$$

or $$14245 = 325 \times (\pi \times 0.0286\ L) \times 39.07$$

or $$L = \mathbf{12.48\ m.}$$ **Ans.**

The counter flow heat exchanger will require less space than parallel flow due to more effective temperature difference.

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Example 3: *Steam enters a counter flow heat exchanger, dry saturated at 10 bar and leaves at 350°C. The mass flow rate of the steam is 720 kg/min. The hot gas enters the exchanger at 650°C with mass flow rate of 1320 kg/min. If the tubes are 30 mm in diameter and 3 m long, determine the number of tubes required. Neglect the resistance offered by metallic tubes. Use following data:*

For steam T_{sat} = 180°C (at 10 bar),

$C_{p,\,s}$ = 2.71 kJ/kg.K,

h_i = 600 W/m^2.K

For gas $C_{p,\,g}$ = 1 kJ/kg.K,

h_o = 250 W/m^2.K.

Solution

Given : A counter flow heat exchanger ;

For steam ; $\dot{m}_c$ = 720 kg/min = 12 kg/s

$T_{c,\,i}$ = 180°C

$T_{c,\,o}$ = 350°C

$C_{p,\,c}$ = 2.71 kJ/kg.K,

h_i = 600 W/m^2.K

For gas; $\dot{m}_h$ = 1320 kg/min = 22 kg/s

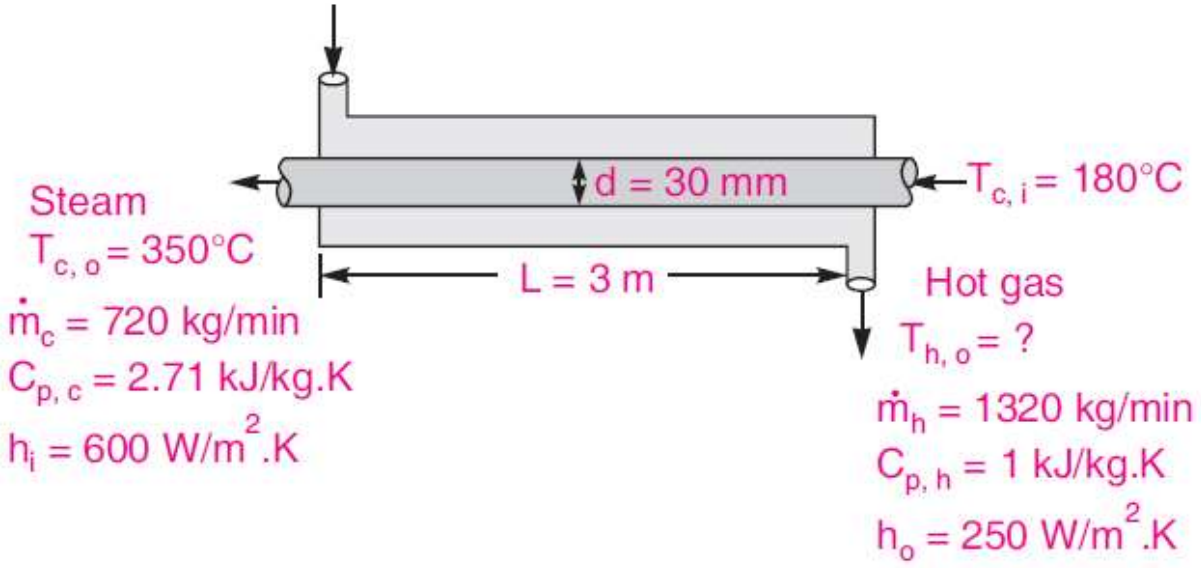
$C_{p,\,h}$ = 1 kJ/kg.K

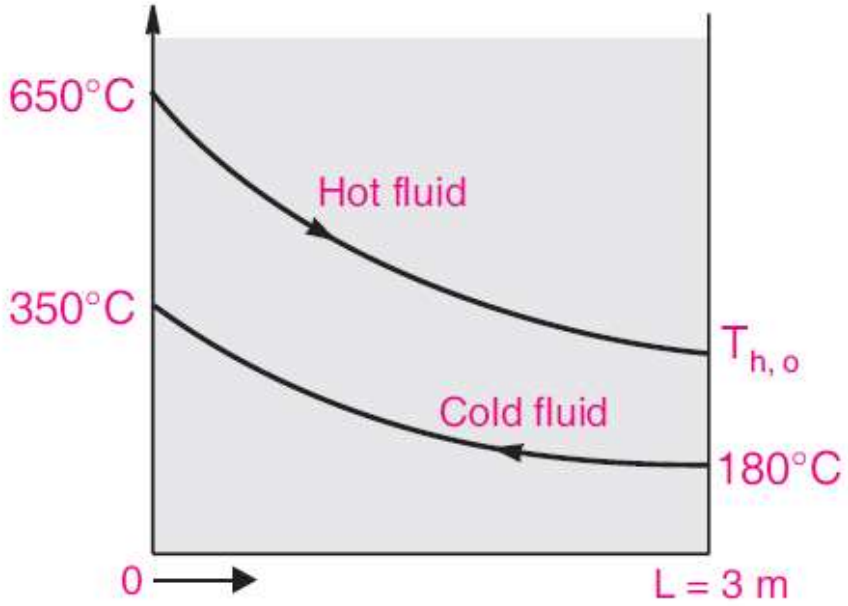
$T_{h,\,i}$ = 650°C,

h_o = 250 W/m^2.K

d = 30 mm = 0.03 m,

L = 3 m.


Steam
$T_{c,o} = 350°C$
$\dot{m}_c = 720$ kg/min
$C_{p,c} = 2.71$ kJ/kg.K
$h_i = 600$ W/m^2.K
d = 30 mm
L = 3 m
$T_{c,i} = 180°C$
Hot gas
$T_{h,o} = ?$
$\dot{m}_h = 1320$ kg/min
$C_{p,h} = 1$ kJ/kg.K
$h_o = 250$ W/m^2.K


650°C
Hot fluid
350°C
$T_{h,o}$
Cold fluid
180°C
0
L = 3 m

To find : Number of tubes required.

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Analysis : The exit temperature of hot gas can be determined by energy balance on the heat exchanger.

Heat lost rate by gas = Heat gain rate by steam

$$\dot{m}_h \, C_{p,\,h} \, (T_{h,\,i} - T_{h,\,o}) = \dot{m}_c C_{p,\,c} (T_{c,\,o} - T_{c,\,i})$$

$$22 \times 1 \times (650 - T_{h,\,o}) = 12 \times 2.71 \times (350 - 180)$$

$$T_{h,\,o} = 398.7°C$$

Heat transfer rate :

$$Q = \dot{m}_h C_{p,\,h} (T_{h,\,i} - T_{h,\,o})$$

$$= 22 \times 1 \times (650 - 398.7)$$

$$= 5528.6 \text{ kW}$$

Log mean temperature difference (LMTD)

$$\Delta T_1 = T_{h,\,i} - T_{c,\,o} = 650 - 350$$

$$= 300°C$$

$$\Delta T_2 = T_{h,\,o} - T_{c,\,i} = 398.7 - 180$$

$$= 218.7°C$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

$$= \frac{300 - 218.7}{\ln\left(\frac{300}{218.7}\right)} = 257.21°C$$

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Overall heat transfer coefficient U ; without tube wall resistance

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{600} + \frac{1}{250}}$$

$$= 176.47 \text{ W/m}^2\text{.K}$$

Further, the heat transfer rate in terms of U and ΔT_{lm} is expressed as

$$Q = UA\Delta T_{lm}$$

or

$$A = \frac{Q}{U\Delta T_{lm}}$$

$$= \frac{5528.6 \times 10^3}{176.47 \times 257.21} = 121.8 \text{ m}^2$$

Further, $A = \pi d L N$

$$N = \frac{121.8}{\pi \times 0.03 \times 3} = \textbf{431 tubes.}$$ Ans.

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Example 4: *A heat exchanger contains 4500 tubes, each having a diameter of 2.54 cm, through which 11.3 kg/s of air to be cooled from 538°C to 148°C. Water passes in counterflow over the tubes, rising in temperature from 38°C to 85°C. Determine the tube length required for this duty, if the water side resistance to heat flow is negligible.*

Solution

Given : A counter flow heat exchanger

$T_{h,i}$ = 538°C,

$T_{c,i}$ = 38°C

$T_{h,o}$ = 148°C,

$T_{c,o}$ = 85°C

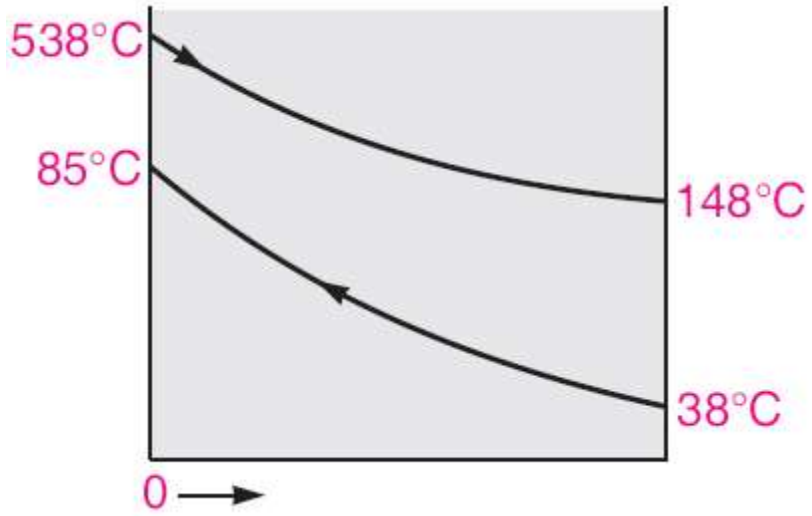
$C_{p,h}$ = 1.0082 kJ/kg.K,

$\dot{m}_h$ = 11.3 kg/s,

d = 2.54 cm = 0.0254 m

N = 4500,

$h_o = 0$



To find : Length of the heat exchanger.

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From data book at mean film temp. Tf = (538+148)/2 = 343°C

The properties of air are taken

Pr = 0.696

$\rho = 1.009\ kg/m^3$, $k = 3.003 \times 10^{-5}\ kW/m.K$,

$\mu = 2.075 \times 10^{-5}\ kg/ms$ $C_p = 1.0082\ kJ/kg.K$.

Analysis : The heat transfer rate through the heat exchanger can be calculated as :

$$Q = \dot{m}_h C_{p,h}(T_{h,i} - T_{h,o})$$
$$= (11.3\ kg/s) \times (1.0082\ kJ/kg.K)$$
$$\times (538 - 148)(K) = 4443.13\ kW$$

The calculation of heat transfer coefficient on air side :

The mass flow rate of air per tube :

$$\dot{m}_{tube} = \frac{11.3}{4500} = 2.51 \times 10^{-3}\ kg/s$$

$$Re = \frac{4\dot{m}_{tube}}{\pi d \mu}$$

$$= \frac{4 \times (2.51 \times 10^{-3}\ kg/s)}{\pi \times (0.0254\ m) \times (2.075 \times 10^{-5}\ kg/ms)}$$

$$= 6066$$

Pr = 0.6966

m = ρAV

$$V = \frac{m}{\rho A} = \frac{4.m}{\rho.\pi.d^2}$$

$$Re = \frac{\rho V d}{\mu}$$

$$Re = \frac{\rho.d}{\mu}\left(\frac{4.m}{\rho.\pi.d^2}\right)$$

$$Re = \frac{4.m}{\pi.d.\mu}$$

$$\text{Nu} = \frac{h_i\, d}{k_f} = 0.023 \times (\text{Re})^{0.8} \times (\text{Pr})^{0.4}$$

or $$h_i = \frac{0.03003}{0.0254} \times 0.023 \times (6066)^{0.8} \times (0.6966)^{0.4}$$

$$= 25.92 \text{ W/m}^2\text{.K} = 0.02592 \text{ kW/m}^2\text{.K}$$

The overall heat transfer coefficient; if water side and tube material resistances are neglected.

$$U = h_i = 0.02592 \text{ kW/m}^2\text{.K}$$

The log mean temperature difference ;

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{(538 - 85) - (148 - 38)}{\ln\left[\frac{(538 - 85)}{(148 - 38)}\right]}$$

$$= 242.4°\text{C}$$

The heat transfer area ;

$$A = \frac{Q}{U\, \Delta T_{lm}}$$

$$A = \frac{4443.13}{0.02592 \times 242.4} = 707.16 \text{ m}^2$$

The length of each pipe ;

$$A = \pi d L\, pN$$

(where p is number of tube passes)

$$L = \frac{707.16}{\pi \times 0.0254 \times 1 \times 4500} = \textbf{1.97 m. Ans.}$$

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Example 5: *Assume the condenser of a large power plant to be a shell and tube type heat exchanger consisting of a single shell and 30,000 tubes, each executing two passes. The tubes are of thin wall constructed with 25 mm in diameter, and steam condenses on their outer surface with an associated convection coefficient of 11,000 W/m^2.K. The heat transfer rate that must be effected by exchanger is 2000 MW and this is accomplished by cooling water through the tubes at the rate of 30,000 kg/s. The water enters at 20°C, while the steam condenses at 50°C. What is the temperature of cooling water coming out the condenser ? What is the required tube length per pass ?*

Solution

Given : A shell and tube type condenser of a power plant ;

No. of shell = 1,

No. of tubes, N = 30,000

No. of passes, p = 2,

d = 25 mm, Q = 2000 MW

Cold fluid	Hot fluid
$\dot{m}_c$ = 30,000 kg/s,	h_o = 11,000 W/m^2.K
$T_{c,i}$ = 20°C,	T_h = 50°C.

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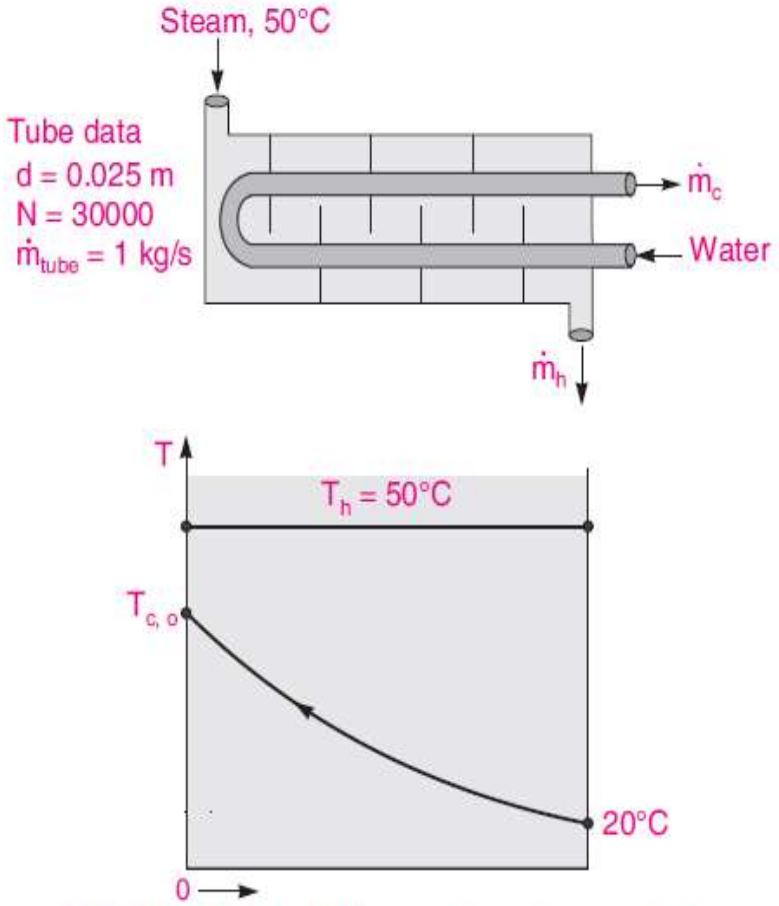


Fig. 14.24. One shell pass two tube pass heat exchanger

To find :

(*i*) Outlet temperature of the cooling water.

(*ii*) Tube length per pass.

Properties : Properties of water at $Tf = \frac{T_s + T_\infty}{2} = \frac{50 + 20}{2} = 35°C = 308K$

C_p = 4180 J/kg.K,

$\mu = 855 \times 10^{-6}$ Ns/m^2,

k_f = 0.613 W/m.K,

Pr = 5.83.

Analysis : (*i*) The heat transfer rate through heat exchanger

$$Q = \dot{m}_c C_{p,c} (T_{c,o} - T_{c,i})$$

$$2000 \times 10^6 \text{ W} = (30{,}000 \text{ kg/s}) \times (4180 \text{ J/kg.K}) \times (T_{c,o} - 20)(\text{K or °C})$$

or $T_{c,o} = \mathbf{36.0°C.}$ **Ans.**

(*ii*) The LMTD for counter flow arrangement

$$\Delta T_1 = T_{h,i} - T_{c,o} = 50 - 36 = 14°C$$

$$\Delta T_2 = T_{h,o} - T_{c,i} = 50 - 20 = 30°C$$

$$\Delta T_{lm,\,\text{counter}} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{14 - 30}{\ln\left(\frac{14}{30}\right)} = 21°C$$

The correction factor F can be obtained from Data book

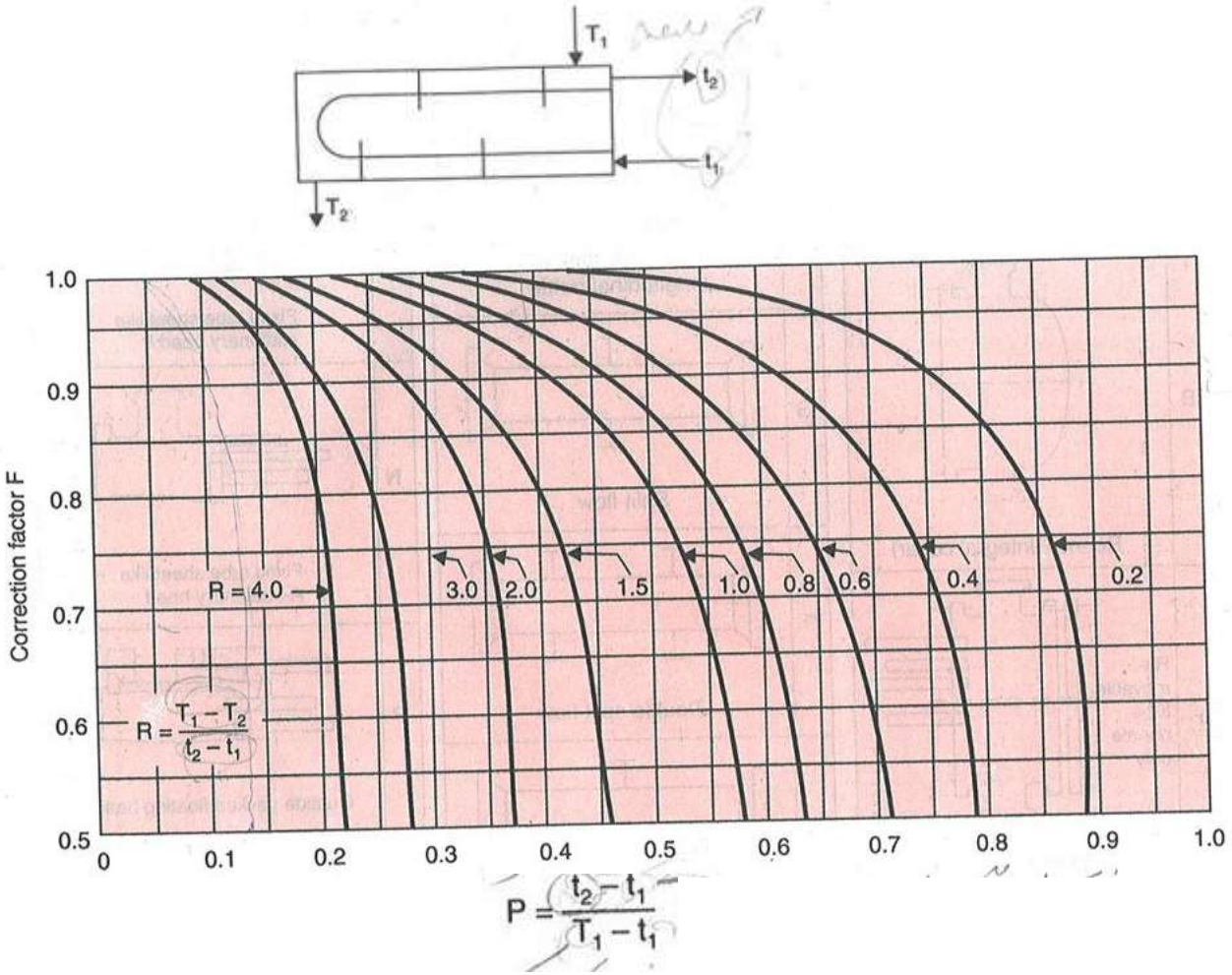
$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{36 - 20}{50 - 20} = 0.466$$

and $$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{50 - 50}{36 - 20} = 0$$

Hence the correction factor F = 1, and the corrected LMTD

$$\Delta T_{lm} = F\,\Delta T_{lm,\,\text{counter}} = 1 \times 21 = 21°C$$

CORRECTION FACTOR PLOT FOR EXCHANGER WITH ONE SHELL PASS AND TWO, FOUR OR MULTIPLE TUBE-PASSES



This is sixth edition data book, page number varies according to data book.

Since the heat transfer coefficient on waterside is not given, thus its calculation is required to determine overall heat transfer coefficient.

The mass flow rate per tube,

$$\dot{m}_{tube} = \frac{30{,}000\ (kg/s)}{30{,}000\ tubes} = 1\ kg/s\ per\ tube$$

Reynolds number

$$Re = \frac{4\dot{m}_{tube}}{\pi\, d\, \mu} = \frac{4 \times 1}{\pi \times 0.025 \times 855 \times 10^{-6}}$$

$$= 59{,}567$$

Using relation,

$$Nu = 0.023\ Re^{0.8}\ Pr^{0.4}$$

or
$$h_i = 0.023 \times \frac{k_f}{d} \times Re^{0.8}\ Pr^{0.4}$$

$$= \frac{0.023 \times 0.613}{0.025} \times (59{,}567)^{0.8} \times (5.83)^{0.4}$$

$$= 7552\ W/m^2.K$$

The overall heat transfer coefficient ;

$$U_o = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{7552} + \frac{1}{11000}}$$

$$= 4478\ W/m^2.K$$

The length of heat exchanger ;

$$Q = U_o\, A_o\, \Delta T_{lm}$$

or
$$A_o = \frac{2000 \times 10^6}{4478 \times 21} = 21{,}268\ m^2$$

The length of each pipe ;

$$A_o = \pi\, d\, L\, p\, N$$

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$$L = \frac{21{,}268}{\pi \times 0.025 \times 2 \times 30{,}000}$$

$$= \mathbf{4.51\ m.} \quad \textbf{Ans.}$$

The length of each tube per pass is 4.51 m.

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Unit 5

Radiation - Basics

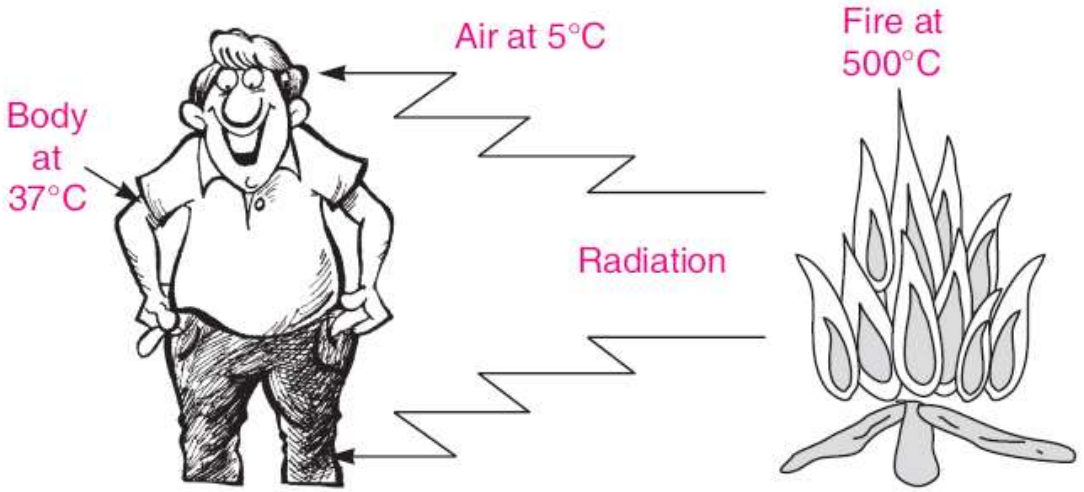
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- Radiation refers to the heat energy emitted by the bodies because of their temperatures.
- All bodies at a temperature above absolute zero temperature emit energy by a process of electromagnetic radiation.
- The intensity of such radiation depends upon the temperature and nature of the surface.
- The energy transfer by radiation does not require any medium between hot and cold surfaces.
- Energy transfer by radiation is fastest (at the speed of light).

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- Heat transfer through an evacuated space can occur only by radiation. For example, the energy of the sun reaches the earth by radiation.

- Radiation heat transfer can occur between two bodies separated by a medium colder than both bodies.



Theories of Radiation

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Maxwell's Theory

- *James Clerk Maxwell* postulated that accelerated charges or changing electric currents give rise to electric and magnetic fields.
- These rapidly moving fields are called **electromagnetic waves** or **electromagnetic radiation.**
- According to *Maxwell electromagnetic theory,* the energy is transferred from a hot body to cold body in the form of *electromagnetic waves.*
- The *electromagnetic waves* possess the energy emitted by a body.
- The energy is due to change in electronic configuration of atoms or molecules.

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- These electromagnetic waves travel with the speed of light in vacuum.

- The electromagnetic waves are characterized by their frequency *f* and wavelength λ, in a medium as

$$\lambda = \frac{C}{f}$$

Where,

C is the speed of propagation of wave in the medium.

$C = \frac{co}{n}$

c_o is velocity of light in vacuum = 2.998×10^8 m/s.

n is index of refraction of the medium.

(The refractive index is unity for air and most gases, about 1.5 for glass, and about 1.33 for water)

Max Planck's Theory

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- According to *Max Planck's concept*, the propagation of thermal radiation takes place in form of discrete packets of energy called as photons or quanta.

- Each photon having an energy of

$$E = h.f = \frac{h.C}{\lambda}$$

Where,

h is Planck's constant = 6.6256×10^{-34} J-s

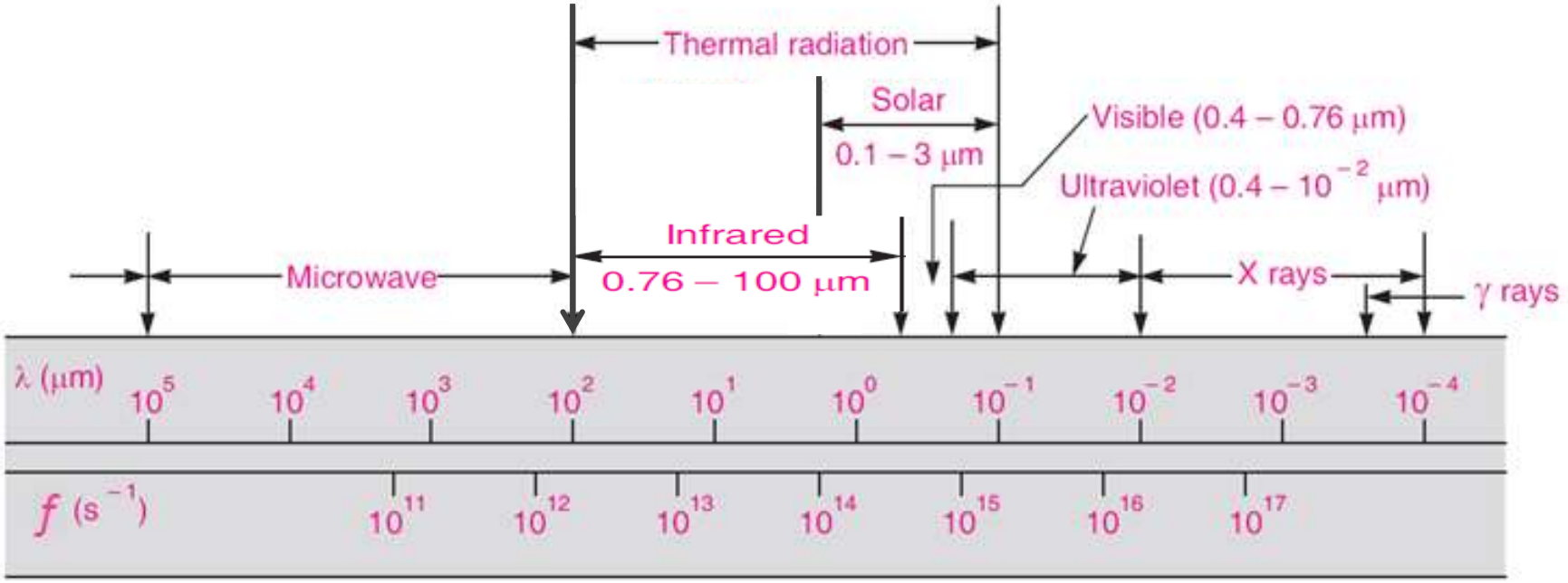
f is frequency of photons.

- Hence energy of the photons is inversely proportional to its wavelength.
- Hence the shorter wavelength radiation possesses the larger photon energy.

Spectrum of Electromagnetic Radiation

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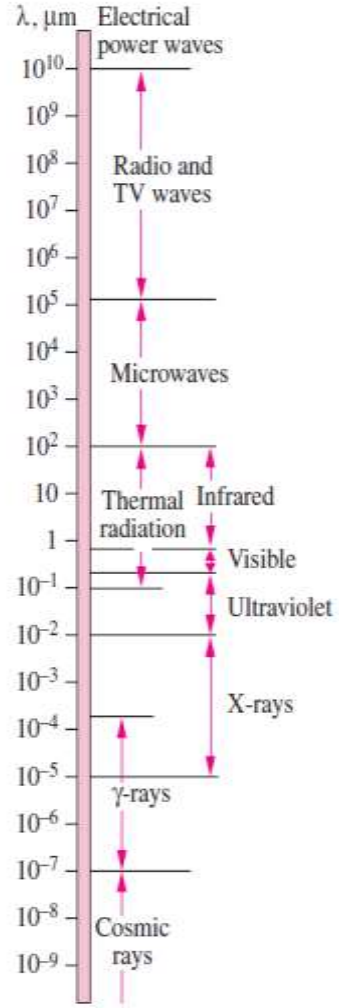
- The radiation energy in form of electromagnetic waves, is emitted at all wave length from $\lambda = 0$ to $\lambda = \infty$.
- Waves of different wavelength shows different behavior.



- The visible spectrum (light) having wavelength range for different colors is as shown in below table.
- The colour of a surface depends on its ability to reflect the radiation in certain wavelength.

Colour	*Wavelength band*
Violet	0.40–0.44 μm
Blue	0.44–0.49 μm
Green	0.49–0.54 μm
Yellow	0.54–0.60 μm
Orange	0.60–0.63 μm
Red	0.63–0.76 μm

- Different types of electromagnetic radiation are produced through various mechanisms.
  - *gamma rays* are produced by nuclear reactions,
  - *X-rays* by the bombardment of metals with high-energy electrons,
  - *microwaves* by special types of electron tubes such as klystrons and magnetrons, and
  - *radio waves* by the excitation of some crystals or by the flow of alternating current through electric conductors.



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- The type of electromagnetic radiation responsible for heat transfer is the **thermal radiation**

- These radiations are emitted as a result of energy transitions of molecules, atoms, and electrons of a substance.

- The rate of thermal radiation emission increases with increasing temperature

- The **ultraviolet** radiation is low wavelength ($0.1 \leq \lambda \leq 0.4$ μm) radiation in the thermal radiation range.

- These rays are harmful, since they can kill micro-organisms and cause serious damage to human, and other living beings.

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- the ozone (O_3) layer in the atmosphere absorbs most of the ultraviolet solar radiation.

- Ultraviolet rays remaining in the sunlight is still sufficient to cause sun burn, skin cancer etc.

Black Body Radiation

- A *black body* is defined as a body which is a perfect emitter and absorber of radiation.

- Properties of a black body are
  - It absorbs all incident radiation from all directions at all wavelengths.
  - At a specified temperature and wavelength, nobody can emit energy more than a black body.
  - The radiations emitted by a Blackbody are independent of direction.
  - A black body neither reflects nor transmits any amount of incident radiation.

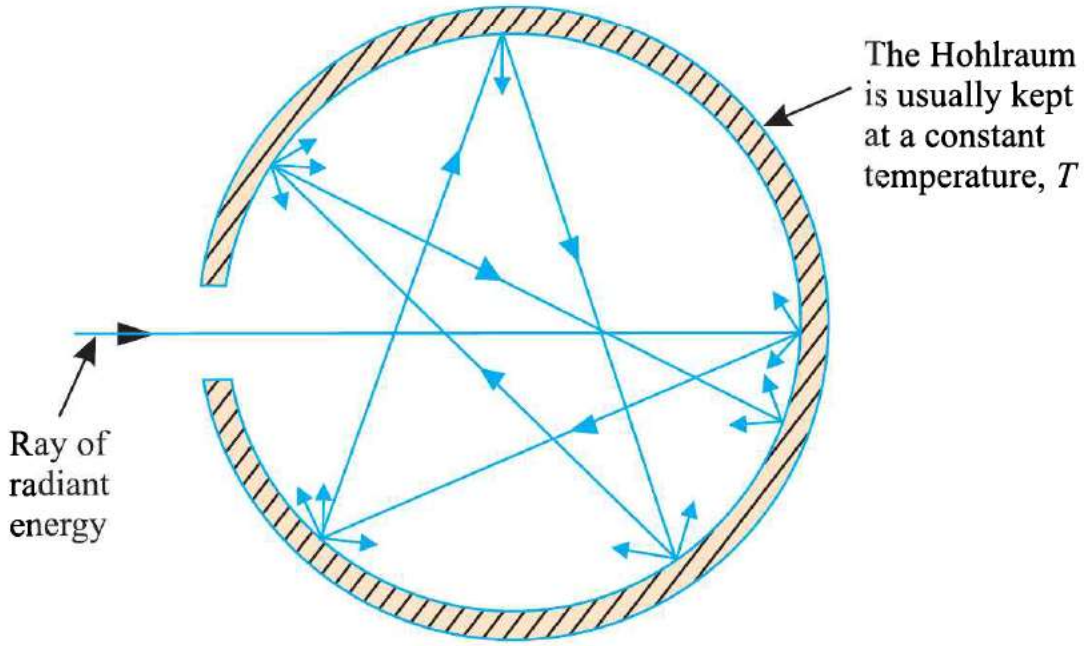
- In actual practice, no surface has all the properties mentioned above for a blackbody.

- The ice and snow appear quite bright to human eye but acts as a blackbody since it absorbs all incident thermal radiation at all wavelength from $\lambda = 0$ to $\lambda = \infty$.

- Hence blackbodies may or may not appear black in colour to human eyes.

• Concept of black body

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- The radiation energy emitted by a blackbody per unit time and per unit surface area was determined experimentally by Joseph Stefan in 1879.

$$E_b = \sigma T^4 \qquad (W/m^2)$$

Where,

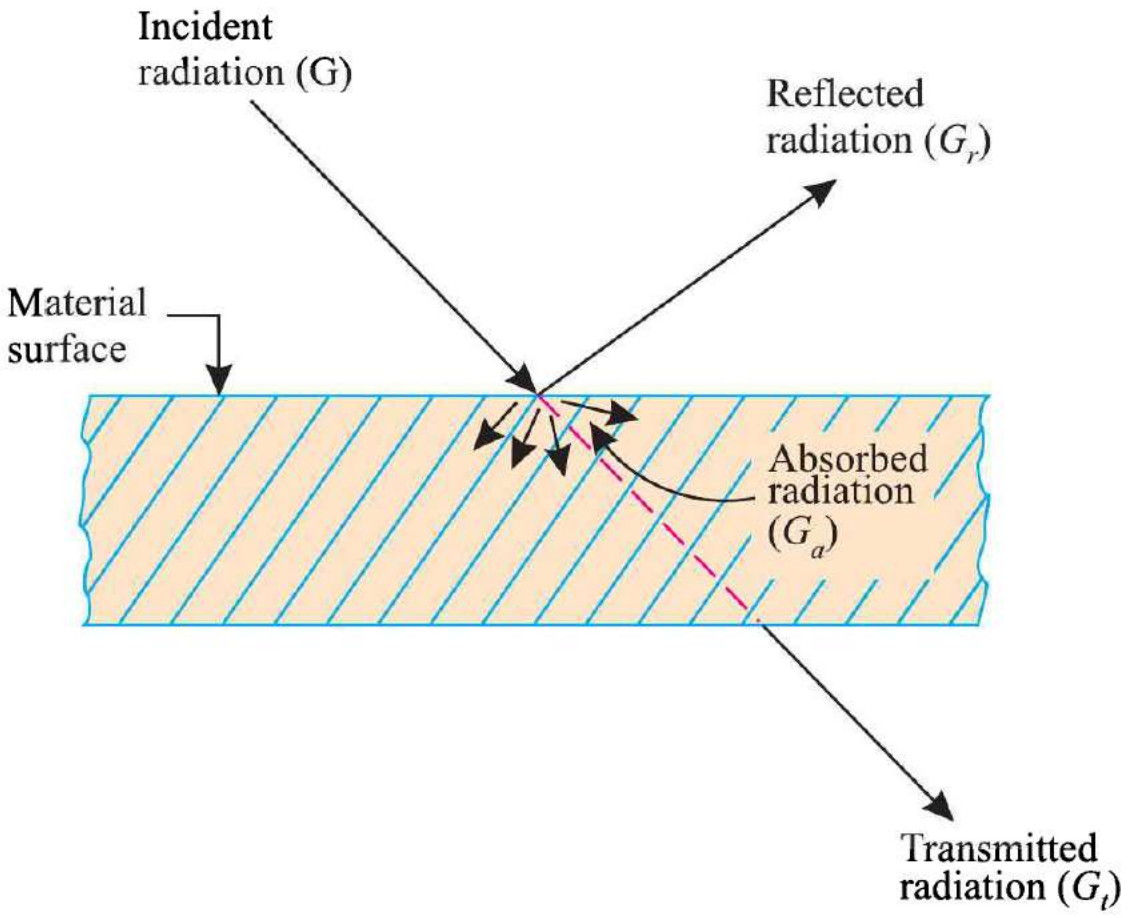
σ = 5.67 × 10^{-8} $W/m^2.K^4$ is Stefan-Boltzmann constant
T is the absolute temperature of the surface in **K**.

- This relation was theoretically verified in 1884 by L. Boltzmann and the equation is known as the **Stefan–Boltzmann law**

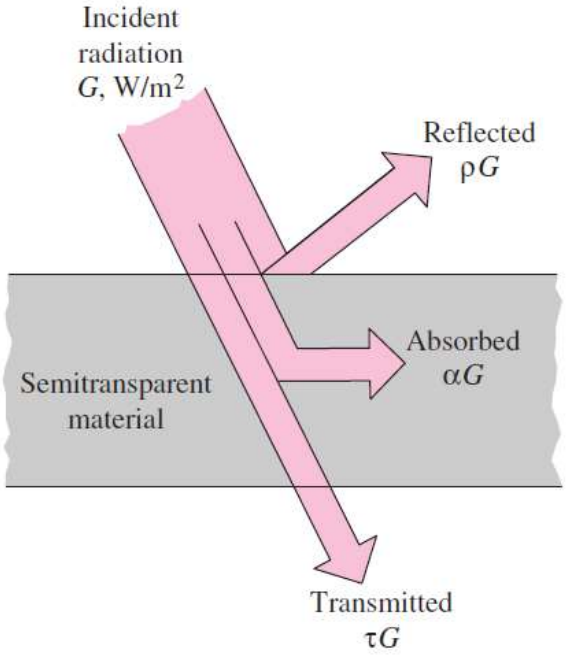
- E_b is called as **blackbody emissive power.**

Absorptivity, Reflectivity and Transmissivity

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By the conservation of energy principle,

$$G_a + G_r + G_t = G$$

Dividing both sides by G, we get

$$\frac{G_a}{G} + \frac{G_r}{G} + \frac{G_t}{G} = \frac{G}{G}$$

$$\alpha + \rho + \tau = 1$$

where

α = *absorptivity* (or fraction of incident radiation absorbed),

ρ = *reflectivity* (or fraction of incident radiation reflected), and

τ = *transmittivity* (or fraction of incident radiation transmitted).

Black body: For perfectly absorbing body

$$\alpha = 1,\ \rho = 0,\ \tau = 0.$$

Opaque body: When no incident radiation is transmitted through the body, it is called an '*opaque body*'.

For the opaque body $\tau = 0$, and eqn. reduces to

$$\alpha + \rho = 1$$

Solids generally do not transmit unless the material is of very thin section. Metals absorb radiation within a fraction of a micrometre, and insulators within a fraction of a millimetre. Glasses and liquids are, therefore, generally considered as opaque.

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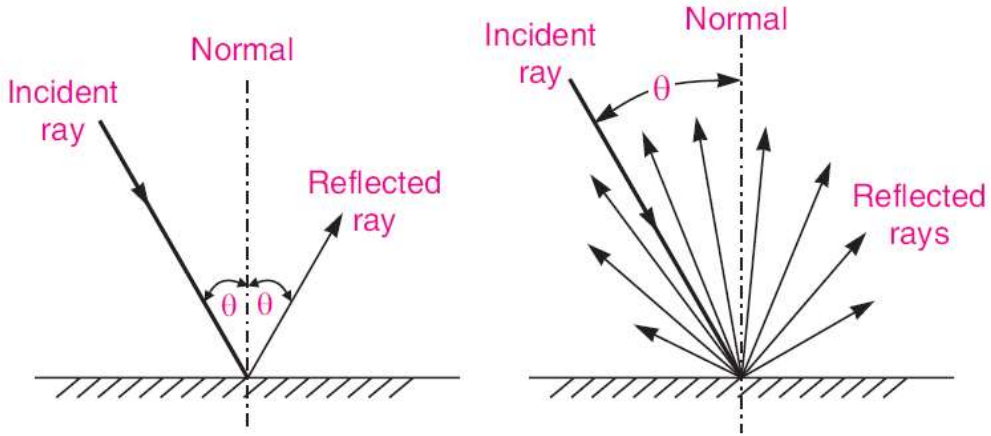
White body: If all the incident radiation falling on the body are reflected, it is called a '*white body*'.

For a white body, $\rho = 1$, $\alpha = 0$ and $\tau = 0$.

Gray body: If the radiative properties, α, ρ, τ of a body are assumed to be uniform over the entire wavelength spectrum, then such a body is called *gray body*. A *gray body* is also defined as one *whose absorptivity of a surface does not vary with temperature and wavelength of the incident radiation* [$\alpha = (\alpha)_\lambda$ = constant.].

Reflection

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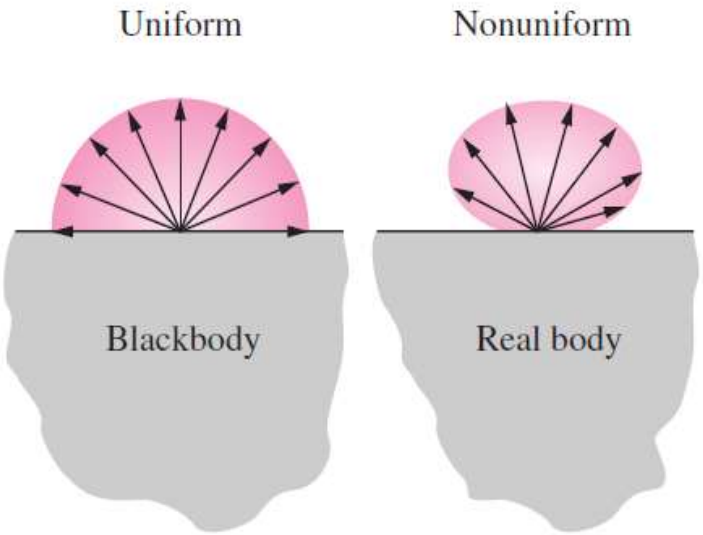
(a) Specular or mirror-like reflection of incoming ray.

(b) Diffuse reflection

- If the surface is perfectly smooth and the angle θ of incident and reflected rays is equal, then the reflection is called the **specular** (or *mirror like*) **reflection**
- If the surface has some roughness, the incident radiation is scattered in all directions after reflection, such reflection is called the **diffuse reflection**

Emission

- A blackbody emits radiation energy uniformly in all directions per unit area normal to direction of emission.
- A blackbody is a *diffuse* emitter. The term *diffuse* means “independent of direction.”



A blackbody is said to be a *diffuse* emitter since it emits radiation energy uniformly in all directions.

Spectral And Total Emissive Power

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- At a given temperature, the total amount of heat energy emitted by a surface in all direction over entire wavelength per unit area, per unit time is called the *emissive power.*

- The emissive power depends on characteristics and temperature of the surface.

$$E = f(\varepsilon, \lambda, T)$$

where ε = emissivity, a surface characteristics,

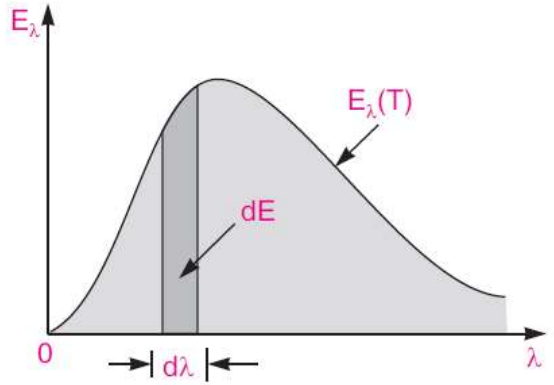
λ = wavelength of radiation, μm,

T = absolute temperature of the surface, K.

- The **spectral** or **monochromatic emissive power (E_λ)** is defined as amount of radiation energy emitted by a surface at an absolute temperature T, per unit time, per unit surface area and per unit wavelength $d\lambda$

The term *spectral* indicates dependence on wavelength.

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The radiation is emitted over the wavelength ranges from $\lambda = 0$ to $\lambda = \infty$. The emissive power of surface over the wavelength ranges from λ to $\lambda + d\lambda$ is given by

$$dE = E_\lambda d\lambda$$

where dE = emissive power in given wave band between λ to $\lambda + d\lambda$, (W/m^2)

E_λ = monochromatic emissive power, $(W/m^2.\mu m)$.

The total emissive power of a surface over entire spectrum of wavelength is

$$E = \int_0^\infty E_\lambda d\lambda$$

Emission from real surface

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- From Stefan Boltzmann law, blackbody emissive power is given by eq.

$$E_b = \sigma T^4 \qquad W/m^2$$

$$E_b = \sigma A T^4 \qquad W$$

- At any specified temperature, black body emits maximum amount of radiations.

$$E = \varepsilon \sigma A T^4 \qquad (W)$$

- However other bodies (real surfaces) emits radiations for a specified temperature can be given by eq.

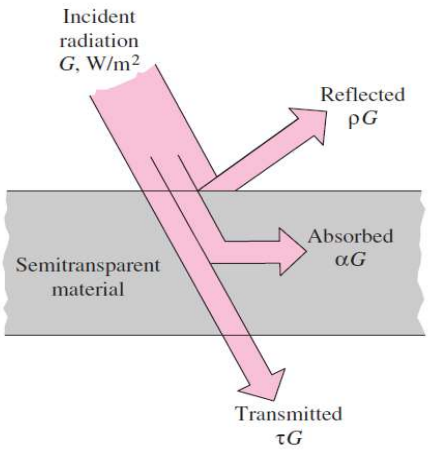
where, ε = Emissivity of the material.

***Emissivity* (ε).** It is defined as the *ability of the surface of a body to radiate heat*. It is also defined as the *ratio of the emissive power of any body to the emissive power of a black body of equal temperature* (*i.e.*, $\varepsilon = \frac{E}{E_b}$). Its values varies for different substances ranging from 0 to 1. For a black body $\varepsilon = 1$, for a white body surface $\varepsilon = 0$ and for gray bodies it lies between 0 and 1. It may vary with temperature or wavelength.

Monochromatic Irradiation

- The absorptivity, transmissivity and reflectivity are average properties of a surface for all directions and all wavelengths.

- However, the incident radiations of specific wavelength or direction is called as *monochromatic irradiation or spectral irradiation*



Hence monochromatic irradiation (G_λ) are the radiant heat flux incident on a surface per unit wavelength from all directions.

$$G_\lambda = \frac{dG}{d\lambda}$$

- Hence total hemispherical irradiation evaluated by

$$G = \int_0^\infty G_\lambda \, d\lambda$$

Black body Spectral Emissive Power

The spectral or monochromatic emissive power for a black surface is highest at every wavelength at any given temperature and it is given by *Max Planck's distribution law*, based on quantum theory, as :

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 \{\exp\,[C_2/(\lambda T)] - 1\}}$$

where C_1 and C_2 are constant ;

$C_1 = 2\pi h c_0^2 = 3.742 \times 10^8$ W.μm^4/m^2, and

$C_2 = h c_0 / k_B = 1.438 \times 10^4$ μm.K ;

$k_B = 1.3805 \times 10^{-23}$, *Boltzmann constant.*

T = absolute temperature, K

λ = wavelength, μm ;

$E_{b\lambda}(T)$ = spectral black body emissive power at absolute temperature T, W/m^2.μm.

This relation is valid for black body spectral emission in a vacuum or a gas. For other mediums, the constant C1 should be replaced by C1/n^2

Wien's Displacement Law

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- Fig. shows spectral blackbody emissive power distribution over a certain range of wavelength.
- For a given temperature, there is a definite peak value of λ, at a particular wavelength.
- The relationship between the wavelength λ_{max} and absolute temperature T at which $E_{b\lambda}$ reaches a maximum value is given by the **Wien's displacement law.**
- It can be derived from *Planck's distribution law.*
- differentiating E*b*λ with respect to λ and equating it to zero

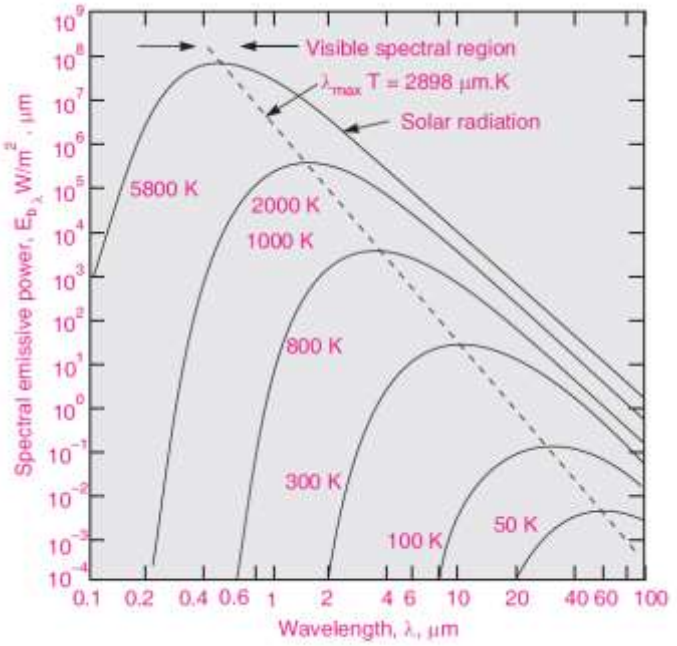


Fig. Spectral black body emissive power

$$\frac{dE_{b\lambda}(T)}{d\lambda} = \frac{d}{d\lambda}\left[\frac{C_1\lambda^{-5}}{\exp\left[C_2/(\lambda T)\right]-1}\right] = 0$$

$$\lambda_{max} T = 2897.6 \ \mu m.K.$$

Substituting λ_{max} as $\frac{2897.6}{T}$ in eqn $E_{b\lambda}(T) = \frac{C_1}{\lambda^5 \{\exp [C_2/(\lambda T)] - 1\}}$

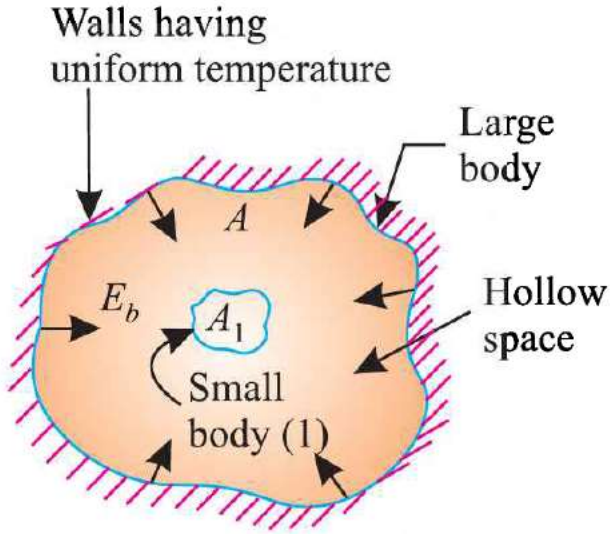
We get

$E_{b\lambda\ max} = 12.87 \times 10^{-10}\ T^5$ $W/m^2.\mu m$

Kirchoff's Law

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The law states that *at any temperature the ratio of total emissive power E to the total absorptivity α is a constant for all substances which are in thermal equilibrium with their environment.*



Let us consider a large radiating body of surface area A which encloses a small body (1) of surface area A_1 (as shown in Fig. Let the energy fall on the unit surface of the body at the rate E_b.

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Of this energy, generally, a fraction α, will be absorbed by the small body. Thus, this energy absorbed by the small body (1) is $\alpha_1 A_1 E_b$, in which α_1 is the absorptivity of the body. When thermal equilibrium is attained, the *energy absorbed* by the body (1) must be equal to the *energy emitted*, say, E_1 per unit surface. Thus, at equilibrium, we may write

$$A_1 E_1 = \alpha_1 A_1 E_b$$

Now we remove body (1) and replace it by body (2) having absorptivity α_2. The radiative energy impinging on the surface of this body is again E_b. In this case, we may write

$$A_2 E_2 = \alpha_2 A_2 E_b$$

By considering generality of bodies, we obtain

$$E_b = \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E}{\alpha}$$

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Also, as per definition of emissivity ε, we have

$$\varepsilon = \frac{E}{E_b}$$

or,
$$E_b = \frac{E}{\varepsilon}$$

By comparing eqns. (11.11) and (11.12), we obtain

$$\varepsilon = \alpha$$

Thus, Kirchhoff's law also states that *the emissivity of a body is equal to its absorptivity when the body remains in thermal equilibrium with its surroundings.*

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- Example 1: A hot water radiator of overall dimensions 2 × 1 × 0.2 m is used to heat the room at 18°C. The surface temperature of radiator is 60°C and its surface is black. The actual surface of the radiator is 2.5 times the area of its envelope for convection for which

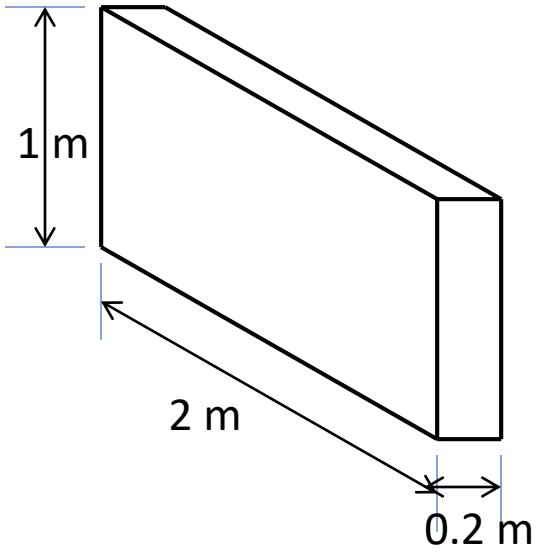
  the convection coefficient is given by $h_c = 1.3(\Delta T)^{1/3}$ W/m^{2}K.

  Calculate the rate of heat loss from the radiator by convection and radiation.

Solution:

The area of the radiator ;

$A_{rad} = 2 \times \{2 \times 1 + 2 \times 0.2 + 1 \times 0.2\} = 5.2$ m^2



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Radiation heat transfer rate ;

$$Q_{rad} = A_{rad}\,\sigma\,(T_s^4 - T_\infty^4)$$
$$= 5.2 \times 5.67 \times 10^{-8} \times (333^4 - 291^4)$$
$$= 1511.2 \text{ W}$$

The convection heat transfer coefficient,

$$h_c = 1.3(\Delta T)^{1/3}$$
$$= 1.3 \times (60 - 18)^{1/3}$$
$$= 4.51 \text{ W/m}^2\text{.K}$$

The convection heat transfer area,

$$A_s = 2.5 \times A_{rad} = 2.5 \times 5.2 = 13 \text{ m}^2$$

The convection heat transfer rate ;

$$Q_{conv} = h_c\, A_s (T_s - T_\infty)$$
$$= 4.51 \times 13 \times (60 - 18) = 2462.24 \text{ W}$$

The total heat transfer rate from the radiator,

$$Q = Q_{rad} + Q_{conv}$$
$$= 1511.2 + 2462.24$$
$$= \mathbf{3973.67\ W.}\quad \textbf{Ans.}$$

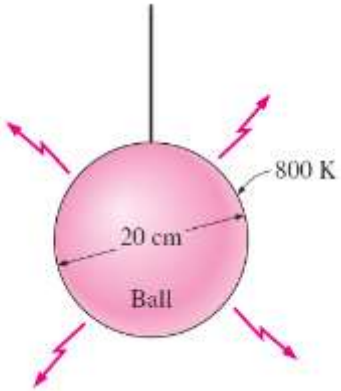
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Example 2: Consider a 20-cm-diameter spherical ball at 800 K suspended in air as shown in Fig. Assuming the ball closely approximates a blackbody, determine:

(i) the total blackbody emissive power,

(ii) the total amount of radiation emitted by the ball in 5 min, and

(iii) the spectral blackbody emissive power at a wavelength of 3μm.



- Solution

(i)The total blackbody emissive power is determined from the Stefan–Boltzmann law

$$E_b = \sigma T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(800\ \text{K})^4 = 23.2 \times 10^3\ \text{W/m}^2$$

$$= 23.2\ \text{kW/m}^2$$

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(ii) The total amount of radiation energy emitted from the entire ball in 5 min is determined by multiplying the blackbody emissive power obtained above by the total surface area of the ball and the given time interval:

$$A_s = \pi D^2 = \pi(0.2\ \text{m})^2 = 0.1257\ \text{m}^2$$

$$\Delta t = (5\ \text{min})\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 300\ \text{s}$$

$$Q_{\text{rad}} = E_b A_s\, \Delta t = (23.2\ \text{kW/m}^2)(0.1257\ \text{m}^2)(300\ \text{s})\left(\frac{1\ \text{kJ}}{1000\ \text{W} \cdot \text{s}}\right)$$

$$= \mathbf{876\ kJ}$$

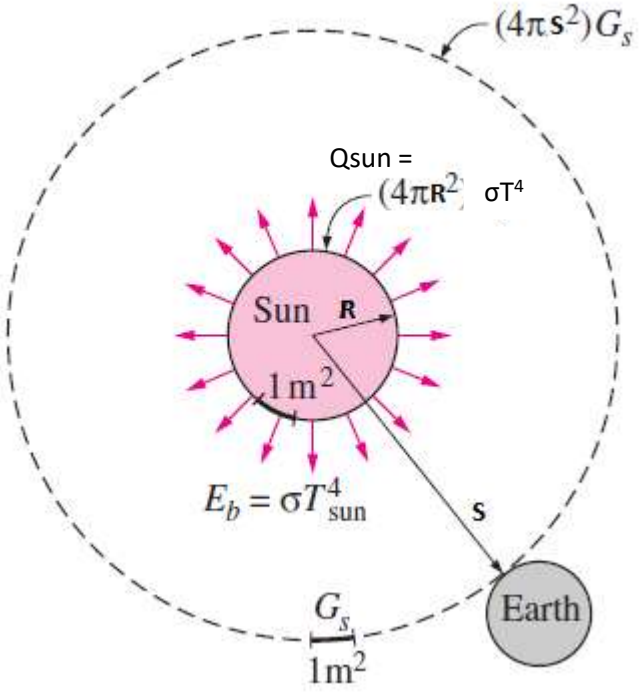
(iii) The spectral blackbody emissive power at a wavelength of 3 μm is determined from Planck's distribution law

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]} = \frac{3.743 \times 10^8\ \text{W} \cdot \mu\text{m}^4/\text{m}^2}{(3\ \mu\text{m})^5\left[\exp\left(\frac{1.4387 \times 10^4\ \mu\text{m} \cdot \text{K}}{(3\ \mu\text{m})(800\ \text{K})}\right) - 1\right]}$$

$$= \mathbf{3848\ W/m^2 \cdot \mu m}$$

Example 3: The average solar radiation flux on the earth's atmosphere is 1353 W/m^2 and it is known as solar constant. Calculate the temperature of sun (a black body), 1.392 × 10^6 km in diameter, when it has mean distance of 1.496 × 10^{11} km from the earth's atmosphere

Solution



$$(4\pi S^2)G_s = (4\pi R^2)\sigma T^4$$

- The left-hand side of this equation represents the total solar energy passing through a spherical surface whose radius is the mean earth–sun distance.
- the right-hand side represents the total energy that leaves the sun's outer surface. The conservation of energy principle requires that these two quantities be equal to each other, since the solar energy experiences no attenuation (or enhancement) on its way through the vacuum.

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Analysis : The energy radiated by sun (black body)

$$Q_{sun} = A_{sun}\ \sigma\ T_{sun}^4 = \pi\,(D_{sun})^2\ \sigma\ T_{sun}^4$$

$$= \pi \times (1.392 \times 10^9\ m)^2 \times 5.67 \times 10^{-8}\ T_{sun}^4$$

$$= 3.45 \times 10^{11}\ T_{sun}^4 \qquad ...(1)$$

The sun is considered as source at a distance $s = 1.496 \times 10^{11}$ m from earth's surface.

Mean area, which receives solar radiation

$$A = 4\pi s^2 = 4\pi \times (1.496 \times 10^{11})^2$$

$$= 2.812 \times 10^{23}\ m^2$$

The solar flux incidence on the earth is :

$$G_s = \quad q = \frac{Q_{sun}}{A} \quad \text{or} \quad 1353 = \frac{3.45 \times 10^{11}\ T_{sun}^4}{2.812 \times 10^{23}}$$

or $$T_{sun}^4 = 1.102 \times 10^{15}$$

or $$T_{sun} = \mathbf{5762.2\ K.}\quad \textbf{Ans.}$$

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- *Problem 4: Calculate the following quantities for an industrial furnace (black body) emitting radiation at 2650°C.*

  (*i*) *Spectral emissive power at* $\lambda = 1.2$ *μm,*

  (*ii*) *Wavelength at which the emissive power is maximum,*

  (*iii*) *Maximum spectral emissive power,*

  (*iv*) *Total emissive power,*

  (*v*) *Total emissive power of the furnace, if it is treated as gray and diffuse body with an emissivity of 0.9.*

Given :

An industrial furnace as black body radiating at T = 2650°C = 2923 K.

Solution:

(*i*) The spectral emissive power at $\lambda = 1.2$ μm :

The I $E_{b\lambda} = \dfrac{C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]}$ qn.

$C_1 = 3.742 \times 10^8$ W.μm^4/m^2

$C_2 = 1.438 \times 10^4$ μm.K.

$$E_{b\lambda} = \frac{3.742 \times 10^8}{(1.2)^5 \times \left[\exp\left(\frac{1.438 \times 10^4}{1.2 \times 2923}\right) - 1\right]}$$

$$= \mathbf{2.53 \times 10^6\ W/m^2.\mu m.} \quad \textbf{Ans.}$$

(*ii*) The wavelength at which the emissive power is maximum :

Using Wien's displacement law, eqn.(12.20)

$$\lambda_{max}\, T = 2897.6\ \mu m.K$$

$$\lambda_{max} = \frac{2897.6}{2923} = \mathbf{0.9913\ \mu m.} \quad \textbf{Ans.}$$

(*iii*) Maximum spectral emissive power, by eqn.(12.21)

$$E_{b\lambda\,max} = 12.87 \times 10^{-10}\, T^5\ W/m^2.\mu m$$

$$= 12.87 \times 10^{-10} \times (2923)^5$$

$$= \mathbf{2.746 \times 10^8\ W/m^2.\mu m.} \quad \textbf{Ans.}$$

(*iv*) Total emissive power

$$E_b = \int_0^\infty E_{b\lambda}\, d\lambda = \sigma\, T^4$$

$$= 5.67 \times 10^{-8} \times (2923)^4$$

$$= \mathbf{4.139 \times 10^6\ W/m^2.} \quad \textbf{Ans.}$$

(*v*) Total emissive power with $\varepsilon = 0.9$

$$E = \varepsilon E_b = \varepsilon\, \sigma\, T^4$$

$$= 0.9 \times 4.139 \times 10^6$$

$$= \mathbf{3.725 \times 10^6\ W/m^2.} \quad \textbf{Ans.}$$

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Example 5: Calculate the equilibrium temperature for a plate, exposed to a solar flux of 700 W/m^2 and convection environment at 25°C, with convection coefficient of 10 W/m^2.K. If the plate is coated with

(*i*) *White paint :* $\alpha_{sun} = 0.12$; $\alpha_{plate} = 0.9$.

(*ii*) *Flat black paint :* $\alpha_{sun} = 0.96$; $\alpha_{plate} = 0.95$.

Given: A plate exposed to solar flux and convection environment ;

Solar flux = 700 W/m^2, T∞ = 25°C = 298 K, h = 10 W/m^2.K,

(i) $\alpha_{sun} = 0.12$; $\varepsilon_{plate} = 0.9$.

(*ii*) $\alpha_{sun} = 0.96$; $\varepsilon_{plate} = 0.95$.

Solution:

Solar flux on the plate = Convection flux + Radiation flux

$$\alpha_{sun}\frac{Q}{A} = h(T - T_\infty) + \varepsilon\sigma(T^4 - T_\infty{}^4)$$

(i) When the plate is coated with white paint

$$\alpha_{sun} = 0.12 \; ; \; \varepsilon_{plate} = 0.9.$$

$$0.12 \times 700 = 10 \times (T - 298) + 0.9 \times 5.67 \times 10^{-8} \times (T^4 - 298^4)$$

$$84 = 10\,T - 2980 + 5.103 \times 10^{-8} T^4 - 402.43$$

$$T^4 + 195.963 \times 10^6\,T - 67.929 \times 10^9 = 0$$

T = 303.4 K

(ii) When the plate is coated with black paint

$$\alpha_{sun} = 0.96 \; ; \; \varepsilon_{plate} = 0.95$$

$$0.96 \times 700 = 10 \times (T - 298) + 0.95 \times 5.67 \times 10^{-8} \times (T^4 - 298^4)$$

T = 337.65 K

Example 6:

Determine :

(a) the wavelength at which the spectral emissive power of a tungsten filament at 1400 K is maximum,

(b) the spectral emissive power at that wavelength, and

(c) the spectral emissive power at 5 μm.

Given: $T_s = 1400$ K

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Analysis : (*a*) The wavelength corresponds to maximum emissive power.

$$\lambda_{max} T = 2897.6 \ \mu m.K$$

or $$\lambda_{max} = \frac{2897.6}{1400} = \mathbf{2.07\ \mu m.}$$ **Ans.**

(*b*) Spectral emissive power at $\lambda = \lambda_{max}$ can be obtained from eqn. (12.21) ;

$$E_{b\lambda,\,max} = 12.87 \times 10^{-10}\, T^5$$
$$= 12.87 \times 10^{-10} \times (1400)^5$$
$$= \mathbf{69.23 \times 10^3\ W/m^2.\mu m.}$$ **Ans.**

(*c*) Monochromatic emissive power at 5 μm.

$$E_{b\lambda} = \frac{C_1}{\lambda^5 \left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]}$$

$$= \frac{3.743 \times 10^8\ W \cdot \mu m^4/m^2}{(5\mu m)^5 \left[\exp\left(\frac{1.4387 \times 10^4\ \mu m \cdot K}{(5\mu m)(1400\ K)}\right) - 1\right]}$$

$$= \mathbf{1.76 \times 10^5\ W/m^2.\mu m.}$$ **Ans.**

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Example 11.4. *Assuming the sun (diameter = 1.4 × 10^9 m) as a black body having a surface temperature of 5750 K and at a mean distance of 15 × 10^{10} m from the earth (diameter = 12.8 × 10^6 m), estimate the following:*

(i) The total energy emitted by the sun,

(ii) The emission received per m^2 just outside the atmosphere of the earth,

(iii) The total energy received by the earth if no radiation is blocked by the atmosphere of the earth, and

(iv) The energy received by a 1.6 m × 1.6 m solar collector whose normal is inclined at 50° to the sun. The energy loss through the atmosphere is 42 percent and diffuse radiation is 22 percent of direct radiation.

Solution: Radius of the sun, $r_s = \frac{1.4 \times 10^9}{2} = 0.7 \times 10^9$ m

Mean distance of the sun from the earth,

$$R = 15 \times 10^{10} \text{ m}$$

Radius of the earth $r_e = \frac{12.8 \times 10^6}{2} = 6.4 \times 10^6$ m

Surface temperature of the sun, $T = 5750\ K$

(*i*) The total energy emitted by the sun, E_b:

$$E_b = \sigma A T^4 = 5.67 \times 10^{-8} \times 4\pi r_s^2 \times (5750)^4$$

$$= 5.67 \times 4\pi \times (0.7 \times 10^9)^2 \times \left(\frac{5750}{100}\right)^4$$

$$= \mathbf{3.816 \times 10^{26}\ W\ (Ans.)}$$

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(ii) The emission received per m^2:

The sun may be regarded as a point source at a distance of 15×10^{10} m from the earth. The *mean area* just outside the earth's atmosphere over which the radiation will fall is

$$= 4 \pi R^2 = 4 \pi \times (15 \times 10^{10})^2 \text{ m}^2$$

$\therefore$ The emission received outside the earth's atmosphere

$$= \frac{3.816 \times 10^{26}}{4\pi \times (15 \times 10^{10})^2} = \textbf{1349.6 W/m}^2 \textbf{ (Ans.)}$$

(iii) The total energy received by the earth:

Assuming the earth a spherical body, the energy received by it will be proportional to the perpendicular projected area, *i.e.*, that of a circle ($= \pi r_e^2$).

$\therefore$ Energy received by the earth $= 1349.6 \times \pi \times (6.4 \times 10^6)^2$

$$= \textbf{1.736} \times \textbf{10}^{\textbf{17}} \textbf{ W (Ans.)}$$

(iv) The energy received by the solar collector:

The direct energy reaching the earth $= (1 - 0.42) \times 1349.6 = 782.77$ W/m^2

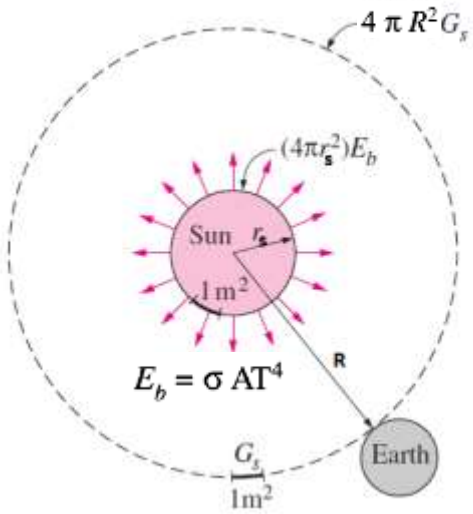
The diffuse radiation $= 0.22 \times 782.77 = 172.21$ W/m^2

$\therefore$ Total radiation reaching the collector $= 782.77 + 172.21 = 955$ W/m^2

The projected area $= A \cos \theta = 1.6 \times 1.6 \times \cos 40° = 1.961$ m^2

$\therefore$ Energy received by the solar collector

$$= 955 \times 1.961 = \textbf{1872.7 W (Ans.)}$$

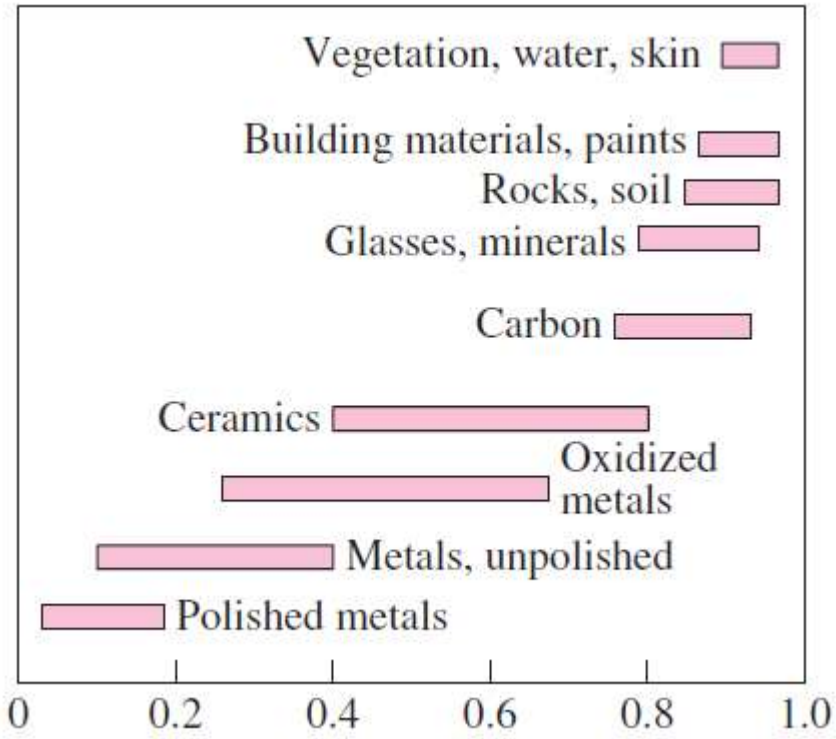


Hemispherical and Total Emissivity

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- The emissivity of a surface which is averaged over all directions is called the *hemispherical emissivity*
- The emissivity averaged over all wavelengths is called the *total emissivity.*
- Hence the *total hemispherical emissivity* ε of a surface is defined as ratio of the radiation heat flux emitted over all wavelengths into a hemispherical space (all directions) to that which would have been emitted by a blackbody at same temperature.

- The emissivity of the metallic surfaces is very small having the values as low as 0.02 for highly polished gold and silver.
- The presence of oxide layers may improve the emissivity of metallic surfaces.
- The non conductors have the large value of emissivity, generally exceeding 0.6.
- The emissivity of conducting materials increase with increase in temperature, but emissivity of non-conducting materials may either increase or decrease with increasing temperature.



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Summary

- **Maxwell's Theory**

$$\lambda = \frac{C}{f}$$

- **Max Planck's Theory**

$$E = h.f = \frac{h.C}{\lambda}$$

- **Blackbody emissive power**

$$E_b = \sigma T^4 \qquad \text{(W/m}^2\text{)}$$

$$E_b = \sigma A T^4 \qquad \text{(W)}$$

- **Emissive power of real surface**

$$E = \varepsilon \sigma A T^4 \qquad \text{(W)}$$

- **spectral or monochromatic emissive power**

$$dE = E_\lambda d\lambda$$

- **Total Emissive Power over entire spectrum of wavelength**

$$E = \int_0^\infty E_\lambda d\lambda$$

- **Monochromatic Irradiation**

$$G_\lambda = \frac{dG}{d\lambda}$$

  - **Total hemispherical irradiation**

$$G = \int_0^\infty G_\lambda \, d\lambda$$

- **Black body Spectral Emissive Power**

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 \{\exp\,[C_2/(\lambda T)] - 1\}}$$

$$E_{b\lambda\ max} = 12.87 \times 10^{-10}\ T^5 \quad W/m^2.\mu m$$

- **Wien's Displacement Law**

$$\lambda_{max} T = 2897.6\ \mu m.K.$$

- **Sum of absorptivity, reflectivity and transmissivity**

$$\alpha + \rho + \tau = 1$$

Terms	*Definition*
Absorption	The process of converting the radiation intercepted by the matter to internal thermal energy.
Absorptivity, α	Fraction of the incident radiation absorbed by the surface.
Black body	An ideal body which absorbs all incident radiation and emits maximum energy.
Black body radiation function	Fraction of radiation energy emitted by a black body at temperature T in wavelength band $\lambda = 0$ to λ.
Diffuse	A surface, whose properties are independent of directions.
Directional	The property pertains to a particular direction, denoted by θ.
Emission	The process of radiation production by the surface at a finite temperature.
Emissive power, E	The rate of radiant energy emitted by a surface in all direction per unit area of the surface. It is measured in W/m^2.
Emissivity, ε	Ratio of the emissive power of a surface to the emissive power of the blackbody at the same temperature.
Gray surface	A surface for which the spectral absorptivity and emissivity are independent of the wavelength over the spectral region of the surface irradiation and emission.
Green house effect	The warming up process due to remission between earth's surface and atmosphere.
Hemispherical	The quantity pertains to all directions above the surface.
Irradiation	The rate at which the radiation is incident on a surface from all direction per unit area of the surface, $G(W/m^2)$.
Kirchhoff 's law	Relation between emission and absorption properties of a surface at thermal equilibrium.
Planck's law	It is associated with spectral distribution of emission from a black body.
Radiation intensity	The rate of radiation energy propagation in a particular direction, per unit area normal to the direction, per unit solid angle about the direction, I $(W/m^2.sr)$.
Radiosity	Rate at which energy leaving the surface due to emission and reflection in all directions per unit area per unit time, J (W/m^2).
Reflection	The process of redirection of radiation energy incident on a surface.
Reflectivity, ρ	The fraction of incident radiation energy reflected by the surface.
Semitransparent	It is a medium in which radiation absorption is the volumetric process.
Solar energy	It is energy coming out the sun.
Solar constant	Rate at which the solar radiation flux is received on a surface normal to sun's rays just outside the earth's atmosphere, $G_s(W/m^2)$.

Terms	*Definition*
Solid angle, ω	Ratio of area of spherical surface enclosed by a conical surface with vertex of the cone at the centre of sphere to square of radius of sphere. It is measured in straradian (sr).
Spectral	It refers to a single wavelength (monochromatic) radiation. The quantity is denoted by subscript λ.
Spectral distribution	It refers to properties variation with wavelength.
Specular	It refers to the surface for which the angle of reflected radiation is equal to the angle of incident radiation.
Stefan Boltzmann law	The emissive power of the blackbody is directly proportional to fourth power of the absolute temperature ; $E_b = \sigma T^4$, where $\sigma = 5.67 \times 10^{-8}$ W/m^2.K^4, is Stefan Boltzmann constant, and T is an absolute temperature in K.
Thermal radiation	It is the electromagnetic energy emitted by a matter at a finite temperature in the spectral region from approximately from 0.1 to 100 μm.
Total solar radiation	Sum of direct and diffuse solar radiation.
Transmission	It is process of the thermal radiation passing through the matter.
Transmissivity	It is the fraction of radiation energy transmitted by the matter.
Wien's displacement law	Relation between wavelength λ_{max} and absolute temperature T at which $E_{b\lambda}$ reaches a maximum ; $\lambda_{max} T = 2897.6$ μm.K.