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Operation Research and Management

Course Code: 23BTMEL213

B. Tech - IV Semester

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Teaching and Evaluation Scheme

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Course Title: **Operation Research and Management**										
Semester	IV	Teaching Scheme				Evaluation Scheme				
						Theory			Practical	
Term	Even	Th	Tu	Pr	Credits	TAE	CAE	ESE	INT	EXT
Course Category	EEMC 2	2 Hrs	-	-	02	10	15	50	-	-
Course Code	23BTMEL213									
Teaching Mode	Offline	2 Hrs.			Total	75			-	
Duration of ESE	2.5 Hrs.					75				

Course Objectives and Outcomes

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Course Objectives	1. To study the importance of Operations Research.
	2. To study the optimization techniques for linear programming, transportation, assignment problems
	3. To learn various inventory and project management techniques.
Course Outcomes	Upon successful completion of this course, student shall be able to:
	CO 1. Define real world problem into a mathematical form and generate solutions.
	CO 2. Evaluate real world Assignment and Transportation Models for feasible solution.
	CO 3. Analyze the various inventory control and simulation techniques
	CO 4. Apply Game theory to real-life problems.
	CO 5. Understand the concept and knowledge of project management to achieve the project goals..

Mapping of Course Outcomes with Program Outcomes and Program Specific Outcomes:

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Following mapping is only for reference

Course Outcomes	Program Outcomes and Program Specific Outcomes													
	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO 10	PO 11	PSO 1	PSO 2	PSO 3
CO1	3	3	3	2	2	-	-	-	-	-	3	3	2	2
CO2	3	3	3	2	2	-	-	-	-	-	3	3	2	2
CO3	3	3	3	2	2	-	-	-	-	-	3	3	2	2
CO4	2	2	3	2	2	-	-	-	-	-	3	3	2	2
CO5	3	3	3	2	2	-	-	-	-	-	3	3	2	2

Syllabus

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Unit	Contents
Unit 1	**Linear Programming:** Introduction and methodology of Operation Research, Linear Programming- Definition, mathematical formulation, standard form, solution space, solution – feasible, basic feasible, optimal, infeasible, multiple, optimal, Redundancy, Degeneracy. Graphical and simplex method.
Unit 2	**Transportation Problems:** Formulation of transportation model, Basic feasible solution using different methods Optimality Methods, Unbalanced transportation problem, Variants in Transportation Problems, Applications of Transportation problems.
Unit 3	**Assignment Problems** Formulation of the Assignment problem, unbalanced assignment problem, typical assignment & travelling salesman problem.

Syllabus

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Unit	Contents
Unit 4	**Inventory Control &Replacement Model** Inventory control models including both deterministic and stochastic models, Replacement model.
Unit 5	**Project Planning & Management:** Introduction to Project Management, Project planning tools, CPM & PERT, Gantt chart, Probability of completion of project, crashing of project, Resource levelling, Cost analysis of project, Allocation & updating of Network

Study Material

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Text Books	1.	Operations Research ,D.S. Hira& P.K Gupta, ,S.Chand Co. 2016
	2.	Operations Research Problem and Solution', J.K. Sharma, Macmillan publication, 2013.
	3.	Operations Research, Theory and Applications, Sixth Edition, J K Sharma, Trinity Press, Laxmi Publications Pvt. Ltd. 2016
E-Books	1.	Handbook of Operations Management and Science, Financial Engineering, John Bhinge, Elsevier, 2007.
Reference Books	1.	Introduction to Operations Research, Fredrick S Hiller, 10th Edition, McGraw Hill Publication 2017
	2.	Introduction to Optimization, J C Pant, 7th reprint edition, Jain brother, Delhi 2015.

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Unit 1

L01

Introduction

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- Operation Research is a professional discipline which deals with application of scientific methods for making decision specially to allocate scare resources.

- It uses tools form different discipline such as mathematics, economics, psychology, statistics, etc.

- It represents study of optimal resources allocation

- It is often considered as sub field of mathematics

L01

Definition of OR

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- Operations Research is the art of winning wars without actually fighting. - Aurther Clarke.
- Operations Research is the art of giving bad answers to problems where otherwise worse answers are given. - T.L. Satty.
- Operations Research is Research into Operations. - J. Steinhardt
- Operations Research is defined as Scientific method for providing executive departments a quantitative basis for decisions regarding the operations under their control. - P.M. Morse and G.E. Kimball.
- Operations Research is the application of scientific methods, techniques and tools to operation of a system with optimum solution to the problem. - Churchman, Ackoff and Arnoff.

L01

Definition of OR

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- Operations Research is the application of the theories of Probability, Statistics, Queuing, Games, Linear Programming etc., to the problems of War, Government and Industry.
- Operations Research is the use of Scientific Methods to provide criteria or decisions regarding man-machine systems involving repetitive operations.
- Operations Research is applied decision theory. It uses any scientific, mathematical or logical means to attempt to cope with problems that confront the executive, when he tries to achieve a thorough going rationally in dealing with his decision problem. D.W. Miller and M.K. Starr.

LO2

Methodology of OR

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- Formulating Problem
- Constructing model to represent the system under
- Deriving a solution from the model
- Testing a model and solution derived from it.
- Establishing controls over the solution
- Implementation of solution to work

L02

Linear Programming

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- A model, which is used for optimum allocation of scarce or limited resources to competing products or activities under such assumptions as certainty, linearity, fixed technology, and constant profit per unit, is linear programming.

- Requirement of Linear Programming
  - There must be a well defined objective function
  - There must be constraints on the amount or extent of attainment of the objective functions.
  - There must be alternative courses of action
  - Decision variables should be interrelated and non negative
  - Resources must be in limited supply.

L02

Assumption in Linear Programming

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- Proportionality
- Additivity
- Continuity
- Certainty
- Finite Choice

Formulation of Linear Programming Problem

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- *A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below.*

Machine	*Time per unit (minutes)*			*Machine capacity (minutes day)*
	Product 1	*Product 2*	*Product 3*	
M1	*2*	*3*	*2*	*440*
M2	*4*		*3*	*470*
M3	*2*	*5*		*430*

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1, 2 and 3 is 7 4, 7 3 and 7 6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical (L.P.) model that will maximize the daily profit.

Solution

- Step 1: In the given situation key decision is to decide the number of units of products 1, 2 and 3 to be produced daily.
- Step 2: Let the number of units of products, 1, 2 and 3 manufactured daily be x_1, x_2 and x_3.
- Step 3 : x_1, x_2 and $x_3 \geq 0$, since negative production has no meaning and is not feasible.
- Step 4: Mention the *objective* quantitatively and express it as a linear function of variables. In the present situation, objective is to maximize the profit.
- *i. e.,* maximize $Z = 4x_1 + 3x_2 + 6x_3$

Continue

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- Step 5: Express these constraints also as linear equations/inequalities in terms of variables. Here, constraints are on the machine capacities and can be mathematically expressed as

$$2x_1 + 3x_2 + 2x_3 \leq 440$$

$$4x_1 + 0x_2 + 3x_3 \leq 470$$

$$2x_1 + 5x_2 + 0x_3 \leq 430$$

Therefore, the complete mathematical (L.P) model for the problem can be written as

Maximize $Z = 4x_1 + 3x_2 + 6x_3$

Subjected to constraints

$$2x_1 + 3x_2 + 2x_3 \leq 440$$

$$4x_1 + 0x_2 + 3x_3 \leq 470$$

$$2x_1 + 5x_2 + 0x_3 \leq 430$$

Graphical Method

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- Represent the given problem in mathematical form
- Draw the x_1 and x_2 axes. The non negativity $x_1 \geq 0$ and $x_2 \geq 0$ imply that the solution lies in the first quadrant only.
- Plot each of the constraint on the graph.
- Identify the feasible region that satisfies all the constraints simultaneously
- Use iso – profit function line approach.

Example

A company manufactures two products, *X* and *Y* by using three machines *A*, *B*, and *C*. Machine *A* has 4 hours of capacity available during the coming week. Similarly, the available capacity of machines *B* and *C* during the coming week is 24 hours and 35 hours respectively. One unit of product *X* requires one hour of Machine *A*, 3 hours of machine *B* and 10 hours of machine *C*. Similarly one unit of product *Y* requires 1 hour, 8 hour and 7 hours of machine *A*, *B* and *C* respectively. When one unit of *X* is sold in the market, it yields a profit of Rs. 5/- per product and that of *Y* is Rs. 7/- per unit. Solve the problem by using graphical method to find the optimal product mix.

Solution

Let the company manufactures x units of X and y units of Y, and then the L.P. model is:

Maximize $Z = 5x + 7y$

Subjected to $1x + 1y \leq 4$

$3x + 8y \leq 24$

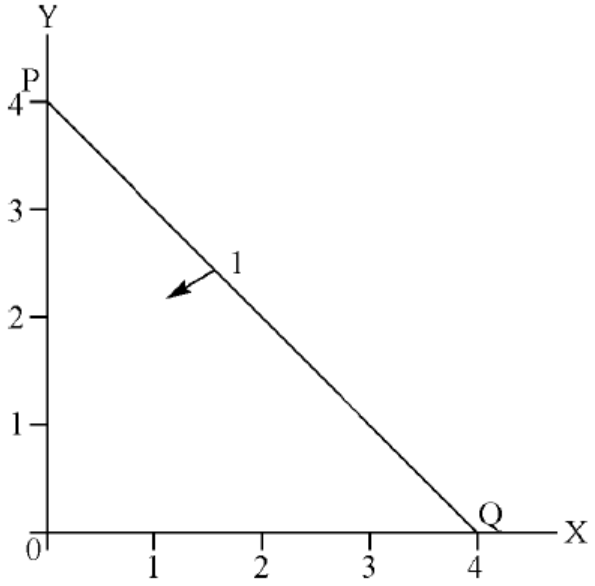
$10x + 7y \leq 35$ and

Both x and y are ≥ 0.

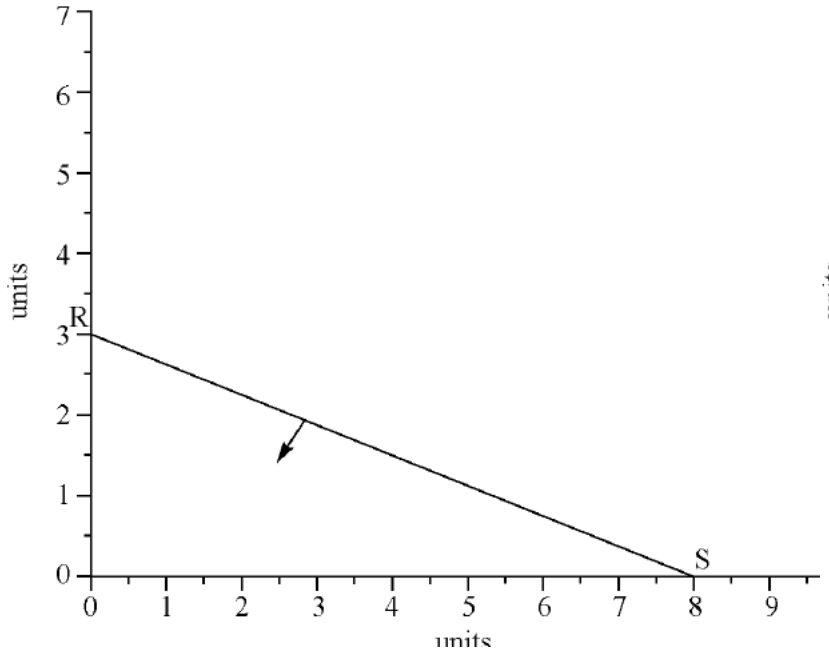
Let us take machine A. and find the boundary conditions. If $x = 0$, machine A can manufacture $4/1 = 4$ units of y. Similarly, if $y = 0$, machine A can manufacture $4/1 = 4$ units of x.

- Machine B When $x = 0$, $y = 24/8 = 3$ and when $y = 0$ $x = 24/3 = 8$
- Machine C When $x = 0$, $y = 35/10 = 3.5$ and when $y = 0$, $x = 35 / 7 = 5$.

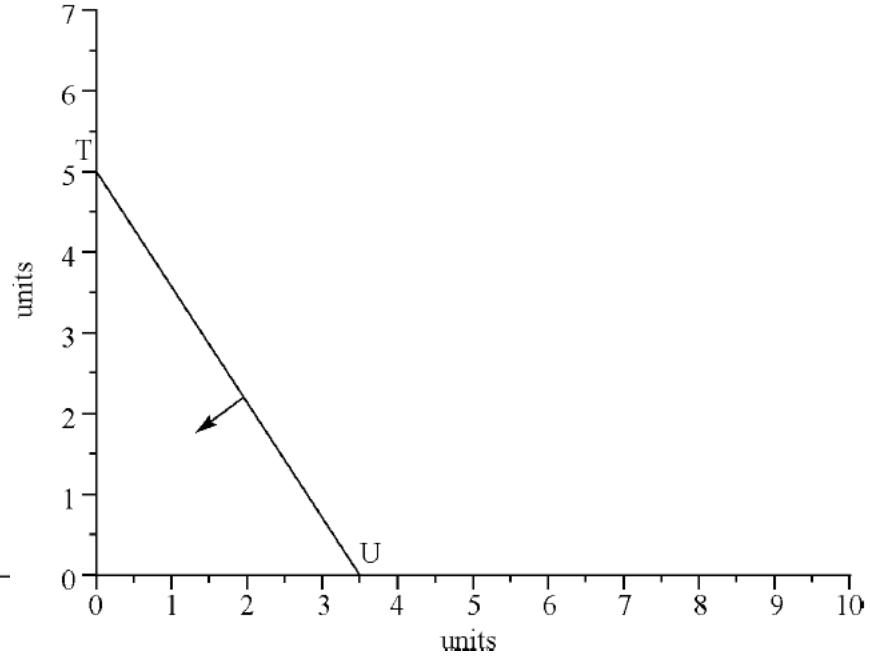
Solution



line PQ which represents
$1x + 1y = 4$.



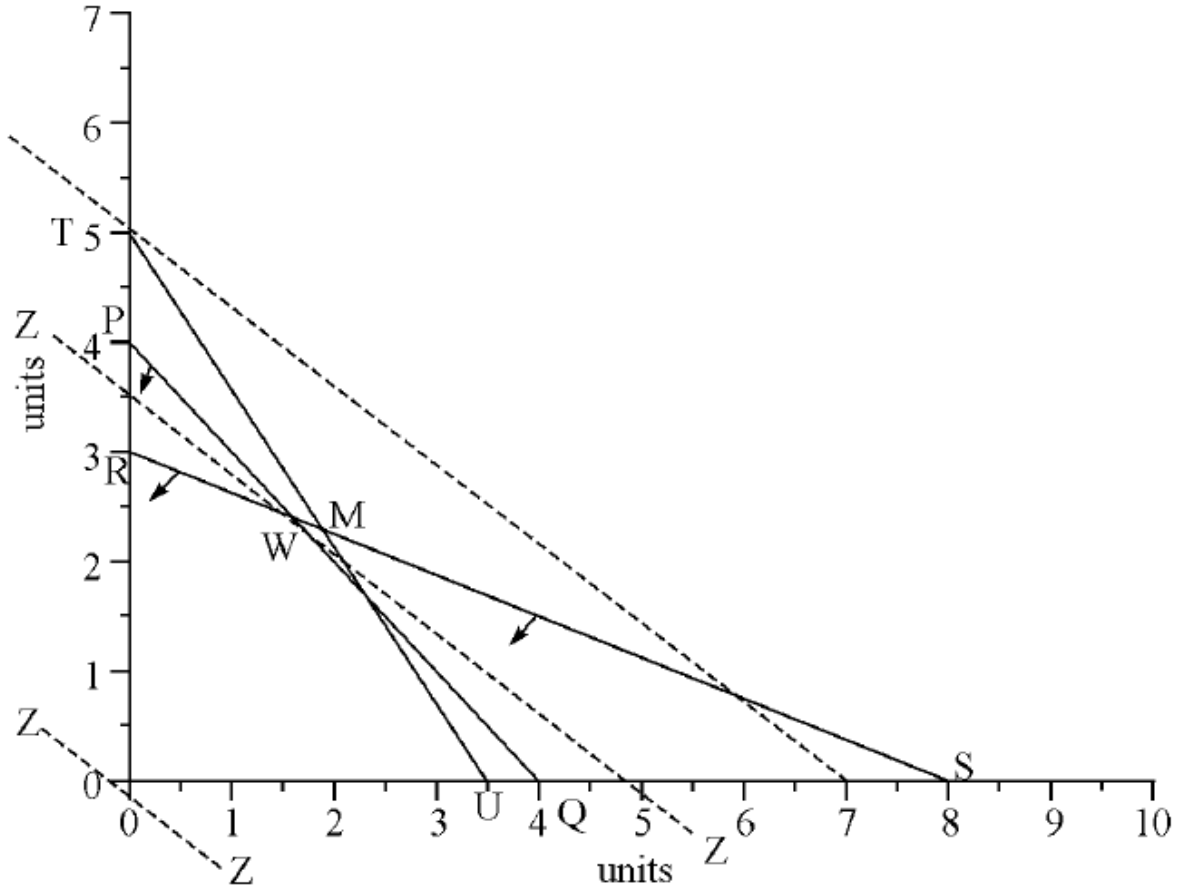
line RS which represents
$3x + 8y = 24$



line TU which represents
$10x + 7y = 35$

Solution

Figure shows the feasible area for all the three machines combined. This is the fact because a products *X* and *Y* are complete when they are processed on machine *A*, *B*, and *C*. The area covered by all the three lines *PQ*. *RS*, and *TU* form a closed polygon *ROUVW*. This polygon is the feasible areafor the three machines.



Method 1.

Here we find the co-ordinates of corners of the closed polygon ROUVW and substitute the values in the objective function.

R = (0, 3.5), O = (0,0), U = (3.5,0), V = (2.5, 1.5) and W = (1.6,2.4). Substituting these values in objective function:

Z(0,3.5) = 5 × 0 + 7 × 3.5 = Rs. 24.50, at point R

Z (0,0) = 5 × 0 + 7 × 0 = Rs. 00.00, at point O

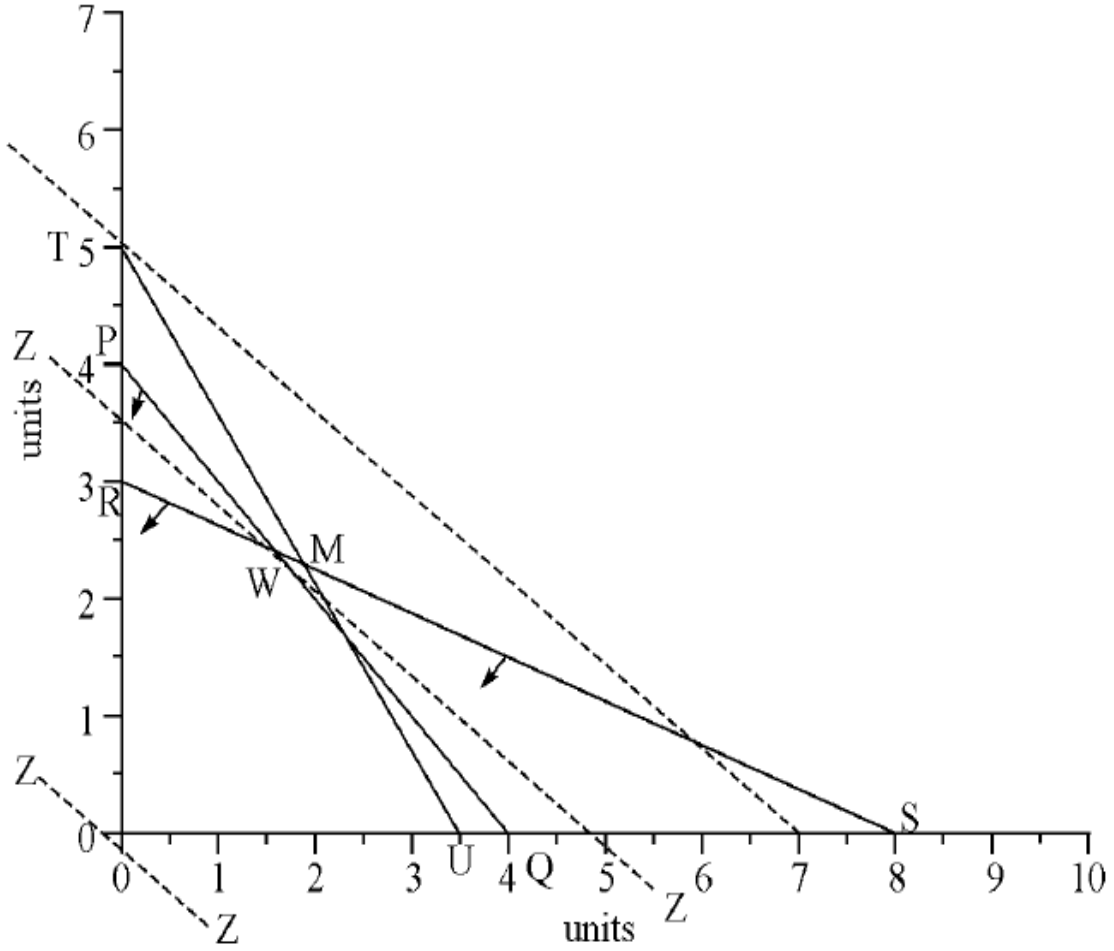
Z(3.5,0) = 5 × 3.5 + 7 × 0 = Rs. 17.5 at point U

Z (2.5, 1.5) = 5 × 2.5 + 7 × 1.5 = Rs. 23.00 at point V

Z (1.6, 2.4) = 5 × 1.6 + 7 × 2.4 = Rs. 24.80 at point W

Solution

- **Method 2. Isoprofit Line Method:** Isoprofit line, a line on the graph drawn as per the objective function, assuming certain profit.
- For example in the given problem, the objective function is Maximise $Z = 5x + 7y$. If we assume a profit of Rs. 35/-, to get Rs. 35, the company has to manufacture either 7 units of X or 5 units of Y.
- we draw line ZZ (preferably dotted line) for $5x + 7y = 35$
- Then draw parallel line to this line ZZ at origin
- The line at origin indicates zero rupees profit
- While moving it touches corners of the polygon showing certain higher profit.
- Finally, it touches the farthermost corner covering all the area of the closed polygon the line ZZ passing through point W covers the entire area of the polygon, hence it is the point that yields highest profit. Now point W has co-ordinates (1.6, 2.4).
- Now **Optimal profit Z = 5 × 1.6 + 7 × 2.4 = Rs. 24.80.**

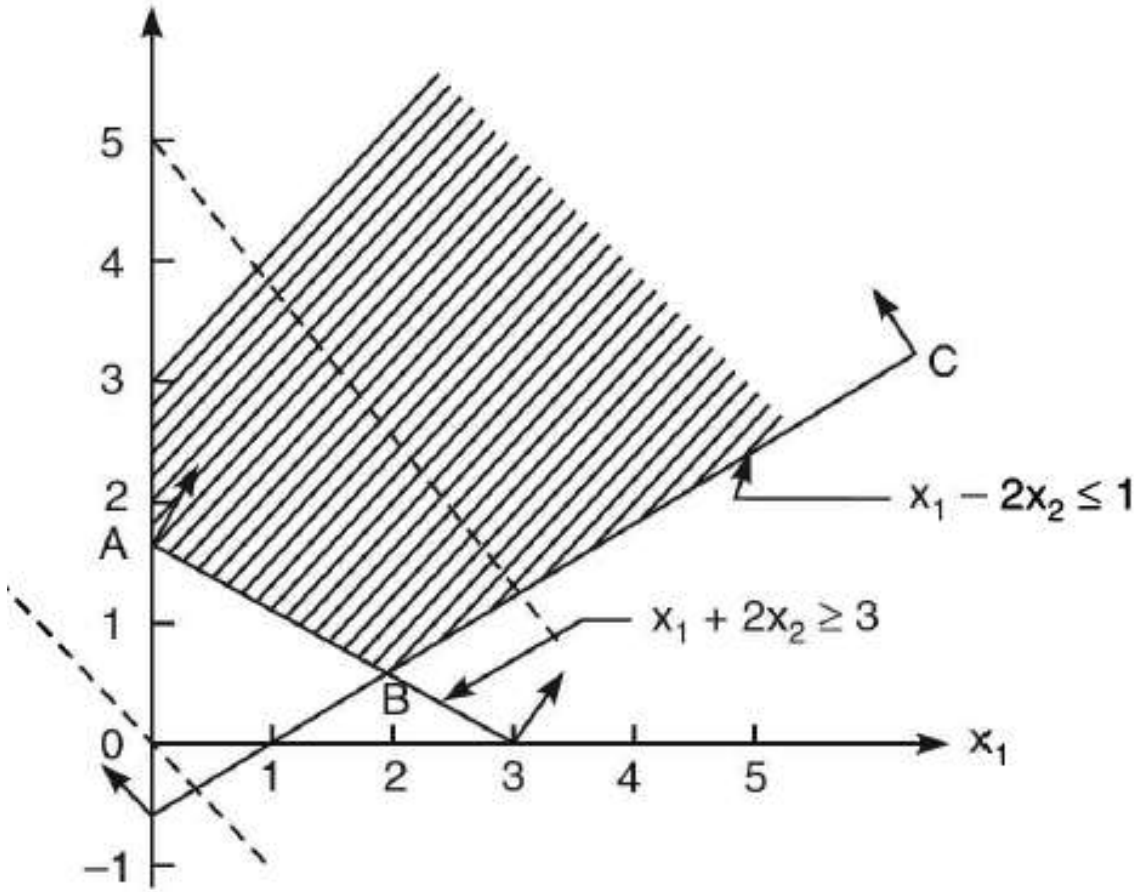


Unbounded Solution example

- Maximize $Z = 5x_1 + 4x_2$
- Subjected to $x_1 - 2x_2 + \leq 1$

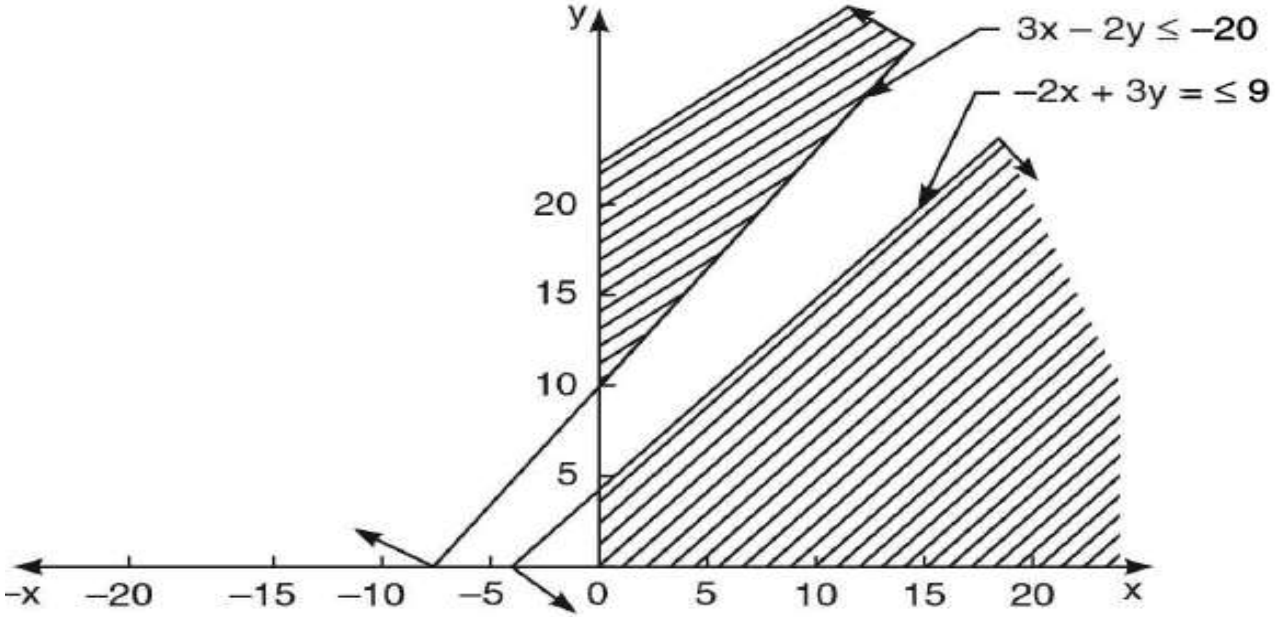
  $x_1 + 2x_2 \geq 3$

  $x_1, x_2 \geq 0.$
- The solution space satisfying the constraints $x_1 - 2x_2 \leq 1$, $x_1 + 2x_2 \geq 3$ and non negativity is plotted.
- Plot line $5x_1 + 4x_2 = 0$
- As it is not possible to get farthest corner for the shaded convex region, maximum Z cannot be found as it occurs at infinity. Therefore problem has unbounded solution



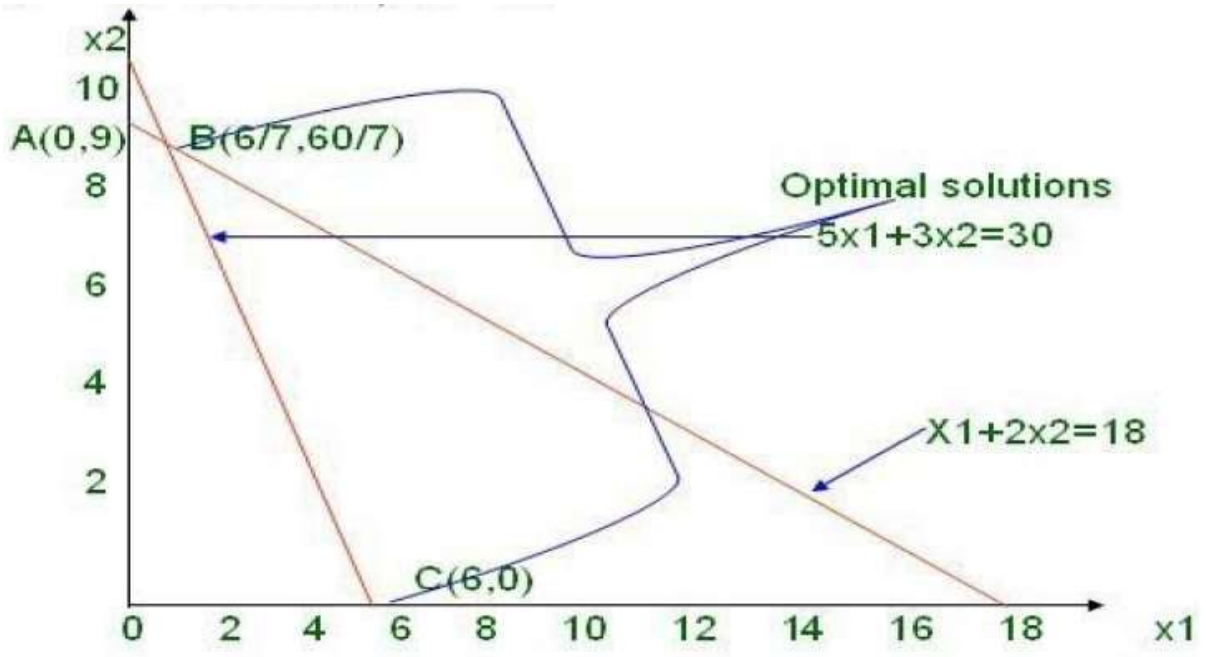
Infeasible solution example

- Maximize Z = 3x + 2y
- Subjected to -2x+3y ≤ 9 , 3x-2y≤ -20 x, y ≥ 0.
- Plot the graph satisfying constraints.
- Two shaded region do not overlap with the result that there is no point (x,y) in common to both the shaded region.
- The feasible solution to the problem does not exist i.e. infeasible solution



Multiple Solution example

- Max $Z = 10x_1 + 6x_2$
- Subjected to $5x_1 + 3X_2 \leq 30$
- $x_1+2x_2 \leq 18$, $x_1, x_2 \geq 0$
- Plot the graph satisfying constraints.
- Here we find the co-ordinates of corners of the closed polygon OABC and substitute the values in the objective function.
- O = (0,0), A = (0,9), B = (6/7, 60/7) and C = (6,0) Substituting these
- values in objective function:
- Z(0,0) = 0
- Z (0,9) = 54
- Z(6/7, 60/7) = 60
- Z (6,0) = 60
- **Z max = 60**



Standard form of LP

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Let us consider the general linear programming problem

$$\text{maximize} \qquad Z = \sum_{j=1}^{n} c_j x_j ,$$

$$\text{subject to} \qquad \sum_{j=1}^{n} a_{ij} x_j \le b_i, \ (b_i \ge 0), \qquad i = 1, 2, 3, \ldots, m,$$

$$x_j \ge 0, \qquad j = 1, 2, 3, \ldots, n.$$

This is expressed in the standard form as

$$\text{maximize } Z = \sum_{j=1}^{n} c_j x_j ,$$

$$\text{subject to} \qquad \sum_{j=1}^{n} a_{ij} x_j + s_i = b_i, \qquad i = 1, 2, 3, \ldots, m,$$

$$x_j \ge 0, \qquad j = 1, 2, 3, \ldots, n,$$

$$s_i \ge 0, \qquad i = 1, 2, 3, \ldots, m.$$

Simplex Method

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$$\begin{aligned}
&\text{maximize} \quad Z = 3x_1 + 4x_2, && \text{(Objective function)}\\
&\text{subject to} \quad x_1 + x_2 \le 450, \\
&\qquad\qquad\ \ 2x_1 + x_2 \le 600, && \text{(Machine } M_1 \text{ time constraint)}\\
&\qquad\qquad\ \ && \text{(Machine } M_2 \text{ time constraint)}\\
&\text{where} \quad x_1, x_2 \ge 0.
\end{aligned}$$

Accordingly, the problem in standard form, can be written as

$$\text{maximize } Z = 3x_1 + 4x_2 + 0s_1 + 0s_2,$$

$$\begin{aligned}
&\text{subject to} \quad x_1 + x_2 + s_1 = 450,\\
&\qquad\qquad\ \ 2x_1 + x_2 + s_2 = 600,\\
&\text{where} \quad x_1, x_2, s_1, s_2 \ge 0.
\end{aligned}$$

Step 2 : Find initial basic feasible solution

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Contribution unit cj		3	4	0	0	
	Basic Variable	Body Matrix		Identity Matrix		
c_B		X1	X2	S1	S2	b
0	S1	1	1	1	0	450
0	S2	2	1	0	1	600

- First row indicates the coefficient of cj in the objective function
- c_B column represents coefficient of current basic variable in objective function
- body matrix represents coefficient of aij in the constraints
- Identity matrix represents coefficient of slack variables in constraints
- b column is quantity column or solution values.

Step 3: perform optimality test

	c_j	3	4	0	0		
c_B	Basis	x_1	x_2	s_1	s_2	b	θ
0	s_1	1	(1)	1	0	450	450 ← (*key row*)
0	s_2	2	1	0	1	600	600
	Z_j	0	0	0	0	0	(Profit lost/unit)
(N.E.R.)	c_j-Z_j	3	4	0	0		(Net profit/unit)
			↑				
			K				

Step 4: Iterate towards optimal solution

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	c_j	3	4	0	0		
c_B	Basis	x_1	x_2	s_1	s_2	b	
4	x_2	1	1	1	0	450	
0	s_2	1	0	– 1	1	150	
	Z_j	4	4	4	0	1,800	*2nd feasible solution*
	$c_j – Z_j$	– 1	0	– 4	0		*(Optimal solution)*

Step 5: Perform optimality test for second feasible solution

Compute $Z_j \left(= \sum c_B \, a_{ij} \right)$ elements for variable columns. They are 4, 4, 4, 0. Value of Z_j under b-column represents the value of objective function as per the second solution. Note that the profit has increased from 0 to ₹ 1,800 after the iteration. Next compute c_j-Z_j row. Elements in this row are $3 - 4 = -1$, $4 - 4 = 0$, $0 - 4 = -4$ and $0 - 0 = 0$. Since all elements are either zero or negative, the second feasible solution is optimal. The computational procedure comes to an end.

Values of the basic variables are seen in the b-column (quantity column). Values of other non-basic variables are zero each.

Hence the optimal solution is

$$x_1 = 0,$$
$$x_2 = 450,$$
$$Z_{max} = ₹\ 1{,}800.$$

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Unit 2

Transportation Model

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Assumption of transportation model

- Total quantity of the item available at different sources is equal to the total requirement
- Item can be transported conveniently from all sources to destination
- The unit transportation cost of the item from all sources to destination is certainly precisely known
- The transportation cost on a given routes is directly proportional to the number of units shipped on that route
- The objective is to minimize the total transportation cost for the organization as a whole and not for the individual supply and destination centres

Definition of transportation model

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Suppose that there are m sources and n destinations. Let a_i be the number of supply units available at source $i(i = 1, 2, 3, \ldots, m)$ and let b_j be the number of demand units required at destination $j(j = 1, 2, 3, \ldots, n)$. Let c_{ij} represent the unit transportation cost for transportating the units from source i to destination j. The objective is to determine the number of units to be transported from source i to destination j so that the total transportation cost is minimum.

minimize $$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij},$$

subject to $$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, 2, 3, \ldots, m,$$

and $$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, 3, \ldots, n,$$

where $$x_{ij} \geq 0.$$

Problem

- Find the optimal solution for the given transportation problem

		Distribution Centre				
		1	2	3	4	Supply
Plants	1	2	3	11	7	6
	2	1	0	6	1	1
	3	5	8	15	9	10
Requirement		7	5	3	2	

Solution

- Calculate total supply and requirement. If both are equal it is balanced problem. If not then dummy supply or requirement is added to make is balanced one.

		Distribution Centre				
		1	2	3	4	Supply
Plants	1	2	3	11	7	6
	2	1	0	6	1	1
	3	5	8	15	9	10
Requirement		7	5	3	2	17

Total Supply =6+1+7 = 17

Total requirement = 7+5+3+2 = 17

North – West Corner Rule

- Start in the northwest corner
- If D1<S1, then set x11 equal to D1 & proceed horizontally.
- If D1=S1, then set x11 equal to D1 & proceed diagonally.
- If D1>S1, then set x11 equal to S1 & proceed vertically.
- Continue in this manner, step by step away from north – west to corner until, finally a value is reached in south – east corner

Distribution Centre

		1	2	3	4	Supply	
	1	2(6)	3	11	7	6	6/0
Plants	2	1(1)	0	6	1	1	1/0
	3	5	8(5)	15(3)	9(2)	10	10/5/2/0
Requirement		7/1/0	5/0	3/0	2/0	17	

- Zmin = (2x6+1x1+8x5+15x3+9x2)x100 = 11600

Row minima method

- This method consists in allocating as much as possible in the lowest cost cell of the 1st row so that either the capacity of the 1st plant is exhausted or the requirement at the jth distribution center is satisfied or both
- Three cases arises:
- If the capacity of the 1st plant is completely exhausted, cross off the 1st row & proceed to the 2nd row.
- If the requirement of the jth distribution center is satisfied, cross off the jth column & reconsider the 1st row with the remaining capacity.
- • If the capacity of the 1st plant as well as the requirement at jth distribution center are completely satisfied, make a 0 allocation in the 2nd lowest cost cell of the 1st row. Cross of the row as well as the jth column & move down to the 2nd row.

		Distribution Centre					
		1	2	3	4	Supply	
Plants	1	2 (6)	3	11	7	6	6/0
	2	1	0 (1)	6	1	1	1/0
	3	5 (1)	8(4)	15 (3)	9(2)	10	10/9/5/3/0
Requirement		7/1/0	5/4/0	3/0	2/0	17	

- Zmin = (2x6+0x1+5x1+8x4+15x3+9x2)x100 = 11200

Column Minima Method

- This method consists in allocating as much as possible in the lowest cost cell of the 1st column so that either the capacity of the 1st plant is exhausted or the requirement at the jth distribution center is satisfied or both
- Three cases arises:
- If the capacity of the 1st distribution is completely exhausted, cross off the 1st column & proceed to the 2nd column.
- If the requirement of the ith plant is satisfied, cross off the ith row & reconsider the 1st column with the remaining capacity.
- If the capacity of the 1st distribution center as well as the capacity at ith plant are completely satisfied, Cross of the column as well as the ith row & move right to the 2nd Column.

		Distribution Centre					
		1	2	3	4	Supply	
Plants	1	2 (6)	3	11	7	6	6/0
	2	1 (1)	0	6	1	1	1/0
	3	5	8(5)	15 (3)	9(2)	10	10/5/3/0
Requirement		7/6/0	5/0	3/0	2/0	17	

$$Z_{min} = (2x6+1x1+8x5+15x3+9x2)x100 = 11600$$

Least Cost Method

- This method consists of allocating as much as possible in the lowest cost cell/cells& then further allocation is done in the cell with the 2nd lowest cost.
- In case of tie amongst the cost, select the cell where allocation of more number of units can be made.

		Distribution Centre					
		1	2	3	4	Supply	
Plants	1	2 (6)	3	11	7	6	6/0
	2	1	0(1)	6	1	1	1/0
	3	5 (1)	8(4)	15 (3)	9(2)	10	10/9/5/3/0
Requirement		7/1/0	5/4/0	3/0	2/0	17	

- $Z_{min} = (2x6+0x1+5x1+8x4+15x3+9x2)x100 = 11200$

Vogel Approximation Method

- Compute the difference between two lowest cost for each row and column.
- Select row or column with highest opportunity cost.
- Make allocation to least cost cell in selected row or column.

Distribution Centre

		1	2	3	4	Supply			
	1	2 (1)	3 (5)	11	7	6/1/0	[1]	[1]	[5] ←
Plants	2	1	0	6	1 (1)	1/0	[1]		
	3	5 (6)	8	15 (3)	9(1)	10/4/3/0	[3]	[3]	[4]
Requirement		7/6/0	5/0	3/0	2/1/0	17			
		[1]	[3]	[5]	↑ [6]				
		[3]	↑ [5]	[4]	[2]				
		[3]		[4]	[2]				

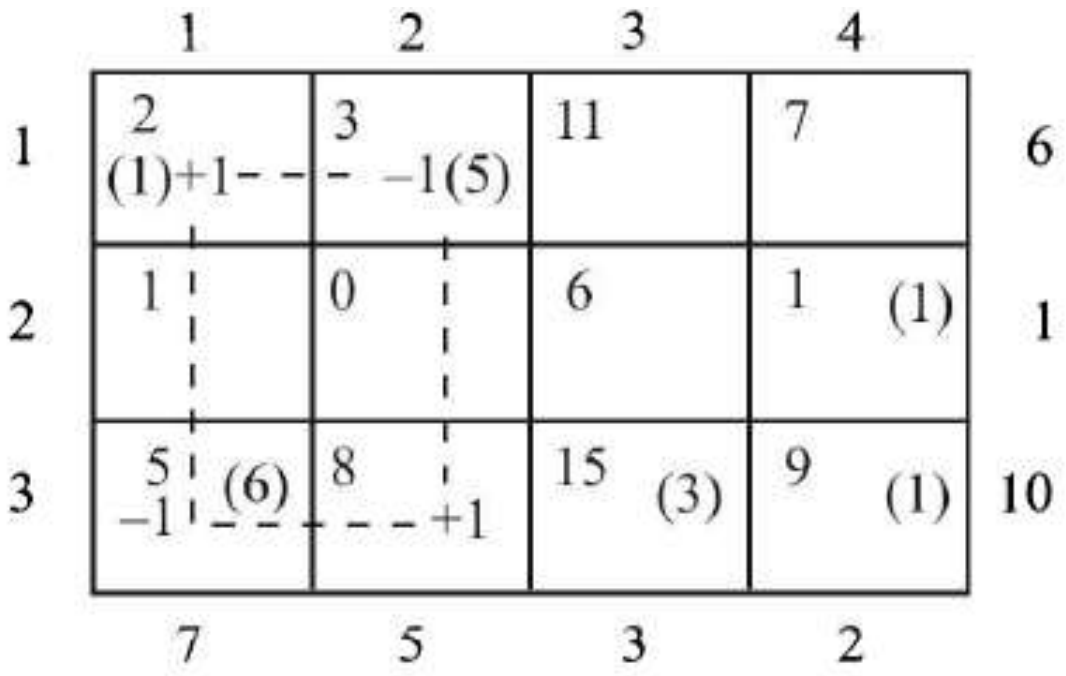
- Zmin = (2x1+3x5+1x1+5x6+15x3+9x1)x100 = 10200

Optimality test : Stepping Stone Method

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Optimality Test

- Number of allocation = m+n-1. where m and n are number of rows and column.
- Allocation should be in independent position.



Cell (3,2)= (8x1-5x1+2x1-5x1)x100= 0

Cell (1,3) = -1, Cell (1,4) = 1, Cell(2,1)=4

Cell (2,2) =2, Cell (2,3) = -1, Cell (3,2) = 2

- If the cell evaluation is negative, solution is not optimal and can be improved.
- If the cell evaluation is positive or zero solution is optimal

Modified distribution Method or u-v Method

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- Step 1: Set up a cost matrix containing the unit costs associated with the cells for which allocation have been made.

2	3		
			1
5		15	9

- Step 2: Introduce dual variable corresponding to supply and demand constraints.
- Let u_i (i=1,2,…m)
- v_j (1,2,….n)
- $u_i + v_j = c_{ij}$ for all occupied cell

$$u_1 + v_1 = 2,$$
$$u_1 + v_2 = 3,$$
$$u_2 + v_4 = 1,$$
$$u_3 + v_1 = 5,$$
$$u_3 + v_3 = 15,$$
$$u_3 + v_4 = 9.$$

v_j / u_i	0	1	10	4
2	2	3		
–3				1
5	5		15	9

Modified distribution Method or u-v Method

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- Step 3:Fill the vacant cells with the sum of ui and vj
- Step 4:Substract the cell values of the matrix of step 3 from the original cost matrix

v_j / u_i	0	1	10	4
2	·	·	12	6
–3	–3	–2	7	·
5	·	6	·	·

·	·	11–12	7–6
1+3	0+2	6–7	·
·	8–6	·	·

·	·	–1	1
4	2	–1	·
·	2	·	·

Cell evaluation matrix

Modified distribution Method or u-v Method

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- Step 5 : Signs of the values in the cell evaluation matrix indicates whether optimal solution has been obtained or not
  - A negative value in an unoccupied cell indicates that a better solution can be obtained by allocating units to this cell.
  - A positive value in an unoccupied cell indicates that a poorer solution will result by allocating units to the cell.
  - A zero value in an unoccupied cell indicates that another solution of the same total value can be obtained by allocating units to this cell.
  - Two cell evaluation is negative so better solution can be obtained by allocating to these cell.

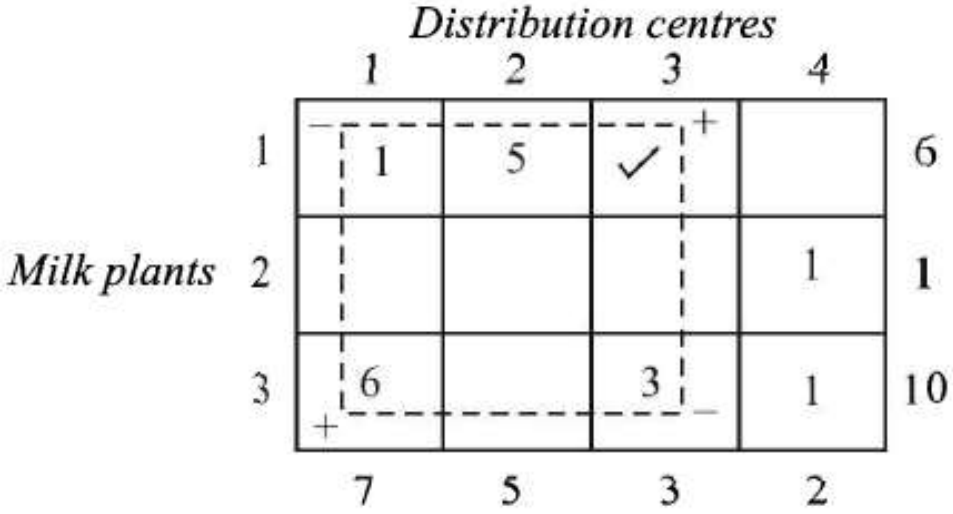
Modified distribution Method or u-v Method

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Step 6: Iterate towards optimal solution

Step 6.1 Identify the cell with most negative cell evaluation. In case of tie in cell evaluation, select the cell where maximum allocation can be made

Step 6.2 Write down again the initial basic feasible solution that is to improved. Check mark the identified cell.



Modified distribution Method or u-v Method

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Step 6.3 : Trace a closed path in the matrix

- It begins and terminates in the identified cell
- It consists of a series of alternate horizontal and vertical lines only
- It can be traced clockwise or anticlockwise
- All other corners of the path lie in the allocated cell only.
- The path may skip over any number of allocated or vacant cells.
- There will always be one and only one closed path, which may be traced.

Modified distribution Method or u-v Method

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- Step 6.4 : Mark the identified cell as positive and each occupied cell at the corner of the path –ve, +ve and – ve so on.
- Step 6.5 : Make a new allocation in the identified cell by entering the smallest allocation on the path that has been assigned a – ve sign. Add and subtract this new allocation from the cell at the corner of the path, maintaining the row and column requirement.

1 – 1	5	+1		6
		·	1	1
6 + 1		3 – 1	1	10
7	5	3	2	

→

	5	1		6
			1	1
7		2	1	10
7	5	3	2	

2nd feasible solution

Zmin = (3x5+11x1+1x1+7x5+15x2+9x1)x100 = 10100

Modified distribution Method or u-v Method

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- Check for optimality

v_j / u_i	0	2	10	4
1		3	11	
–3				1
5	5		15	9

v_j / u_i	0	2	10	4
1	1	·	·	5
–3	–3	–1	7	·
5	·	7	·	·

Thus, $u_1 + v_2 = 3$, $u_3 + v_1 = 5$,
$u_1 + v_3 = 11$, $u_3 + v_3 = 15$,
$u_2 + v_4 = 1$, and $u_3 + v_4 = 9$.

Let $v_1 = 0$.

Then $u_3 = 5$, $u_2 = -3$,
$v_3 = 10$, $u_1 = 1$,
$v_4 = 4$, $v_2 = 2$.

Modified distribution Method or u-v Method

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- Check for optimality

2 – 1	·	·	7 – 5
1 + 3	0 + 1	6 – 7	·
·	8 – 7	·	·

→

1	·	·	2
4	1	–1	·
·	1	·	·

	5	1	
		+ ✓	– 1
7		2 –	1 +

	5	1	
		+ 1	1 – 1
7		2 – 1	1 + 1

→

	5	1	
		1	
7		1	2

Third feasible solution

Zmin = (5x3+1x11+1x6+1x15+2x9+7x5)x100 = 10000

Modified distribution Method or u-v Method

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- Check for optimality

u_i \ v_j	0	2	10	4
1		3	11	
–4			6	
5	5		15	9

u_i \ v_j	0	2	10	4
1	1	·	·	5
–4	–4	–2		0
5	·	7	·	·

$u_1 + v_2 = 3$, $u_3 + v_1 = 5$,

$u_1 + v_3 = 11$, $u_3 + v_3 = 15$,

$u_2 + v_3 = 6$, and $u_3 + v_4 = 9$.

Let $v_1 = 0$.

Then, $u_3 = 5$, $u_2 = -4$,

$v_3 = 10$, $u_1 = 1$,

$v_4 = 4$, and $v_2 = 2$.

Modified distribution Method or u-v Method

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- Check for optimality

2 – 1	·	·	7 – 5
1 + 4	0 + 2	·	1 – 0
·	8 – 7	·	·

→

1	·	·	2
5	2	·	1
·	1	·	·

Cell evaluation matrix

All cell values are positive, third feasible solution is the optimal solution

Distribution centres

Plants	1	2	3	4	*Supply*
1	2	3 (5)	11 (1)	7	6
2	1	0	6 (1)	1	1
3	5 (7)	8	15 (1)	9 (2)	10
Requirement	7	5	3	2	

Zmin = (5x3+1x11+1x6+1x15+2x9+7x5)x100 = 10000

Unbalanced transportation problem

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- Consider the following transportation problem

Factories	Stores 1	2	3	4	Production capacity
A	4	6	8	13	50
B	13	11	10	8	70
C	14	4	10	13	30
D	9	11	13	8	50
Demand	25	35	105	20	

- Determine the extent of deliveries from each factories to each of the store so that the transportation cost is minimum

Solution

- Here total production capacity = 200 units
- Total demand = 185 units
- Surplus =15 units
- Demand and supply not balanced, add dummy store with a demand of 15 units

		Stores					
		1	2	3	4	*d*	*Capacity*
	A	4	6	8	13	0	50
Factories	B	13	11	10	8	0	70 Total capacity = 200
	C	14	4	10	13	0	30 Total demand = 200
	D	9	11	13	8	0	50
Demand		25	35	105	20	15	

Solution

- Basic feasible solution using Vogel's approximation method.

		Stores 1	2	3	4	*d*	*Capacity*
Factories	A	4 (25)	6 (5)	8 (20)	13	0	50/25/20/0 [4] [2] [2] [2] [5] ←
	B	13	11	10 (70)	8	0	70/0 [8] [2] [2] [2] [2] [2]
	C	14	4 (30)	10	13	0	30/0 [4] [6] ←
	D	9	11	13 (15)	8 (20)	0 (15)	50/35/15/0 [8] [1] [1] [3] [5] [5] ←
Demand		*25/0*	*35/5/0*	*105/85/15/0*	*20/0*	*15/0*	
		[5]	[2]	[2]	[0]	[0]	
		[5]	[2]	[2]	[0]		
		[5] ↑	[5]	[2]	[0]		
			[5] ↑	[2]	[0]		
				[2]	[0]		
				[3]	[0]		

Solution

- Perform Optimality Test

u_i \ v_j	0	2	4	−1	−9
4	4	6	8		
6			10		
2		4			
9			13	8	0

→

u_i \ v_j	0	2	4	−1	−9
4	·	·	·	3	−5
6	6	8	·	5	−3
2	2	·	6	1	−7
9	9	11	·	·	·

·	·	·	10	5
7	3	·	3	3
12	·	4	12	7
0	0	·	·	·

- Since cell values are non negative, first feasible solution is optimal.

- Z= (4x25+6x5+8x20+10x70+4x30+13x15+8x20+0x15) = 1465

Maximization Problem

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- As maximization of a function is equivalent to minimization of negative of that function, the given problem can be converted to minimization problem by multiplying the profit matrix by -1.

- It may be converted into minimization problem, by subtracting all the profits from the highest profit in the matrix and thus can be solved by usual method.

- While finding the initial basic feasible solution, allocation are to be made to in the highest profit cell, instead of lowest cost cell. Solution will be optimal when all cell evaluations are non positive.

Numerical

A company has 3 factories manufacturing the same product and 5 sale agencies in different parts of the country. Production costs differ from factory to factory and the sales prices from agency to agency. The shipping cost per unit product from each factory to each agency is known. Given the following data, find the production and distribution schedules most profitable to the company.

Factory i	Production cost/unit (Rs.)	Max. capacity (No. of units)
1	18	140
2	20	190
3	16	115

	1	2	2	6	10	5
Factory i	2	10	8	9	4	7
	3	5	6	4	3	8
Ajency j		1	2	3	4	5
Demand		74	94	69	39	119
Sales price (Rs.)		35	37	36	39	34

} Shipping cost (Rs.)

Solution

- Prepare the transportation table
- Profit/unit = sales price/unit – production cost/unit – transportation cost/ unit.

		Sales agencies					*Production capacity*
		1	2	3	4	5	
Factories	1	35–18–2 = 15	37–18–2 = 17	36–18–6 = 12	39–18–10 = 11	34–18–5 = 11	140
	2	35–20–10 = 5	37–20–8 = 9	36–20–9 = 7	39–20–4 = 15	34–20–7 = 7	190
	3	35–16–5 = 14	37–16–6 = 15	36–16–4 = 16	39–16–3 = 20	34–16–8 = 10	115
Demand		74	94	69	39	119	445 / 395

- Total production capacity = 445 units
- Total demand = 395 units
- Surplus capacity = 50 units

Solution

- Balanced Maximization Problem

		Sale agencies						
		1	2	3	4	5	Dm	*Capacity*
Factories	1	15	17	12	11	11	0	140
	2	5	9	7	15	7	0	190
	3	14	15	16	20	10	0	115
Demand		74	94	69	39	119	50	445/445

Balanced maximization problem

Solution

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- Convert maximization problem to minimization problem

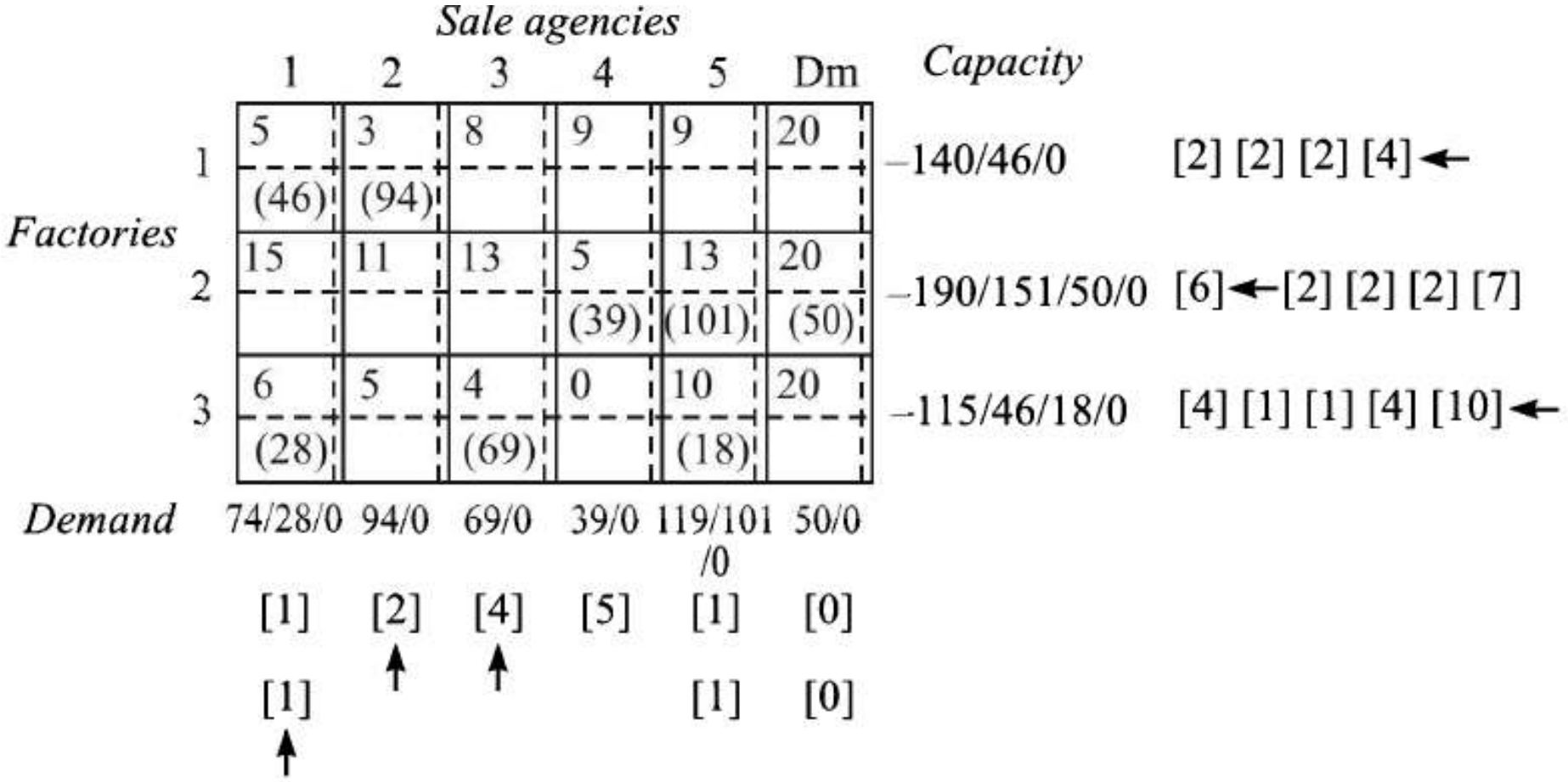
		Sale agencies						
		1	2	3	4	5	Dm	*Capacity*
Factories	1	–15	–17	–12	–11	–11	0	140
	2	–5	–9	–7	–15	–7	0	190
	3	–14	–15	–16	–20	–10	0	115
Demand		74	94	69	39	119	50	

		Sale agencies						
		1	2	3	4	5	Dm	*Capacity*
Factories	1	5	3	8	9	9	20	140
	2	15	11	13	5	13	20	190
	3	6	5	4	0	10	20	115
Demand		74	94	69	39	119	50	445 / 445

Relative loss matrix

Solution

- Find initial basic feasible solution by Vogel approximation method



Solution

- Perform Optimality Test

	0	−2	−2	−4	4	11
5	5	3				
9				5	13	20
6	6		4		10	

→

	0	−2	−2	−4	4	11
5	·	·	3	1	9	16
9	9	7	7	·	·	·
6	·	4	·	2	·	17

·	·	5	8	0	4
6	4	6	·	·	·
·	1	·	−2 ✓	·	3

- Perform Optimality Test

46	94				
			39	101	50
28		69	✓	18	

→

46	94				
			21	119	50
28		69	18		

·	·	5	10	2	6
4	2	4	·	·	·
·	1	·	·	2	5

Zmax = (15x46+17x94+15x21+7x119+0x50+14x28+16x69+20x18)= 5292

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Unit 3

Assignment model

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- Mathematical representation of assignment model

Let x_{ij} denote the assignment of facility i to job j such that

$$x_{ij} = \begin{cases} 0, \text{ if the } i\text{th facilitiy is not assigned to } j\text{th job,} \\ 1, \text{ if the } i\text{th facility is assigned to } j\text{th job.} \end{cases}$$

Then, the model is given by

$$\text{minimize } Z = \sum_{j=1}^{n} \sum_{i=1}^{n} c_{ij} x_{ij} \left\{ = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \right\},$$

$$\text{subject to constraints } \sum_{j=1}^{n} x_{ij} = 1,\ i = 1, 2, 3, \ldots, n, \text{ (one job is assigned to the } i\text{th facility)}$$

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1, 2, 3, \ldots, n, \text{ (one facility is assigned the } j\text{th job)}$$

$$\text{and} \qquad x_{ij} = 0 \text{ or } 1 \text{ (or } x_{ij} = x_{ij}^2 \text{).}$$

Hungarian Method for solution of Assignment Problem

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Step 1: Prepare a square matrix

1. If matrix is nxn, matrix is square
2. If mxn (m!=n) dummy column or row is added to make it square

Step 2: Reduce the matrix

1. Subtract the smallest element of each row from all the elements of row. So there will be at least one zero in each row.
2. Examine if there is at least one zero in each column. If not, subtract the minimum element of the column(s) not containing zero from all elements of that column(s).

Hungarian Method for solution of Assignment Problem

Step 3 : Check whether an optimal assignment can be made in reduced matrix or not

a) Examine rows successively until a row with exactly one unmarked zero is obtained. Make an assignment tom m this single zero. Cross all other zero in the same column. Proceed in this way until all rows have been examined.

b) Now examine columns successively until a column with exactly one unmarked zero found. Make assignment and cross other zero in same row.

c) In case there is no row or column containing single unmarked zero , mark any unmarked zero arbitrarily and cross all other zeros in its row and column. Proceed in this manner till there is no unmarked zero left.

d) Repeat sub steps a and b till one of the following two things occurs

1. There is assignment in each row and column. In this case current feasible solution is the optimal solution.
2. There is some row and /or column without assignment. Follow Step 4.

Hungarian Method for solution of Assignment Problem

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- Step 4: Find the minimum number of lines crossing all zeros.
  a) Mark the row that do not have assignment
  b) Marks the column (not already marked) that have zeros in the marked column.
  c) Mark the row (not already marked) that have assignment in marked column.
  d) Repeat sub steps b and c till no more rows or column can be marked.
  e) Draw straight line through all unmarked rows and marked columns. If the number of lines equal to the order matrix, then it is an optimal solution.
- Step 5: Iterate towards the optimal solution
  - Examine the uncovered elements. Select thee smallest elements and substract it from all uncovered elements. Add this smallest element to every element that lies at the intersection of two lines. Leave the remaining elements of the matrix as such.
- Step 6: Repeat steps 3 to 5 successively until optimal solution is reached.

Numerical

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A machine tool company decides to make four subassemblies through four contractors. Each contractor is to receive only one subassembly. The cost of each subassembly is determined by the bids submitted by each contractor and is shown in table 4.7 in hundreds of rupees.

TABLE 4.7

Subassemblies \ Contractors	1	2	3	4
1	15	13	14	17
2	11	12	15	13
3	13	12	10	11
4	15	17	14	16

(i) *Formulate the mathematical model for the problem.*

(ii) *Show that the assignment model is a special case of the transportation model.*

(iii) *Assign the different subassemblies to contractors so as to minimize the total cost.*

Solution

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Objective is to minimize the total cost involved

i.e., minimize

$$Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} = \sum_{i=1}^{4}\sum_{j=1}^{4} = \begin{matrix} 15x_{11} + 13x_{12} + 14x_{13} + 17x_{14} + 11x_{21} + 12x_{22} + 15x_{23} + 13x_{24} \\ +13x_{31} + 12x_{32} + 10x_{33} + 11x_{34} + 15x_{41} + 17x_{42} + 14x_{43} + 16x_{44}. \end{matrix}$$

Constraints: (*a*) Constraints on subassemblies are

$$x_{11} + x_{12} + x_{13} + x_{14} = 1,$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1,$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1,$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1.$$

(*b*) Constraints on contractors are

$$x_{11} + x_{21} + x_{31} + x_{41} = 1,$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1,$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1,$$
$$x_{14} + x_{24} + x_{34} + x_{44} = 1.$$

Solution

Subassemblies \ Contractors	1	2	3	4
1	2	0	1	4
2	0	1	4	2
3	3	2	0	1
4	1	3	0	2

Subassemblies \ Contractors	1	2	3	4
1	2	[0]	1	3
2	[0]	1	4	1
3	3	2	~~0~~	[0]
4	1	3	[0]	1

	1	2	3	4
1	2	0	1	3
2	0	1	4	1
3	3	2	0	0
4	1	3	0	1

Initial basic feasible solution

Zmin = (13x1+11x1+11x1+14x1)x100 = 4900

Numerical

Four different jobs can be done on four different machines. The set-up and take-down time costs are assumed to be prohibitively high for changeovers. The matrix below gives the cost in rupees of producing job i on machine j.

Jobs \ Machines	M_1	M_2	M_3	M_4
J_1	5	7	11	6
J_2	8	5	9	6
J_3	4	7	10	7
J_4	10	4	8	3

How should the jobs be assigned to the various machines so that the total cost is minimized?

Solution

	M_1	M_2	M_3	M_4
J_1	0	2	6	1
J_2	3	0	4	1
J_3	0	3	6	3
J_4	7	1	5	0

→

	M_1	M_2	M_3	M_4
J_1	0	2	2	1
J_2	3	0	0	1
J_3	0	3	2	3
J_4	7	1	1	0

First feasible solution

	M_1	M_2	M_3	M_4
J_1	[0]	2	2	1
J_2	3	[0]	~~0~~	1
J_3	~~0~~	3	2	3
J_4	7	1	1	[0]

	M_1	M_2	M_3	M_4	
J_1	[0]	2	2	1	✓
J_2	~~3~~	[0]	~~0~~	~~1~~	
J_3	~~0~~	3	2	3	✓
J_4	~~7~~	~~1~~	~~1~~	[0]	
	✓				

Solution

- Iterate towards optimality

	M_1	M_2	M_3	M_4
J_1	0	1	1	0
J_2	4	0	0	1
J_3	0	2	1	2
J_4	8	1	1	0

Second feasible solution

	M_1	M_2	M_3	M_4	
J_1	~~0~~	1	1	~~0~~	✓
J_2	~~4~~	[0]	~~0~~	~~1~~	
J_3	[0]	2	1	2	✓
J_4	8	1	1	[0]	✓
	✓			✓	

	M_1	M_2	M_3	M_4
J_1	[0]	~~0~~	~~0~~	~~0~~
J_2	5	[0]	~~0~~	2
J_3	~~0~~	1	[0]	2
J_4	8	~~0~~	~~0~~	[0]

Third feasible solution

Zmin = (5+5+10+3)= 23

Variation in Assignment model

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- Non square Matrix

A company has one surplus truck in each of the cities A, B, C, D and E and one deficit truck in each of the cities 1, 2, 3, 4, 5 and 6. The distance between the cities in kilometres is shown in the matrix below (Table 4.37). Find the assignment of trucks from cities in surplus to cities in deficit so that the total distance covered by vehicles is minimum.

TABLE 4.37

	1	2	3	4	5	6
A	12	10	15	22	18	8
B	10	18	25	15	16	12
C	11	10	3	8	5	9
D	6	14	10	13	13	12
E	8	12	11	7	13	10

Solution

- Prepare a square matrix

		Cities in deficit					
		1	2	3	4	5	6
Cities with surplus	A	12	10	15	22	18	8
	B	10	18	25	15	16	12
	C	11	10	3	8	5	9
	D	6	14	10	13	13	12
	E	8	12	11	7	13	10
	Dummy	0	0	0	0	0	0

- Reduce the matrix

	1	2	3	4	5	6
A	4	2	7	14	10	0
B	0	8	15	5	6	2
C	8	7	0	5	2	6
D	0	8	4	7	7	6
E	1	5	4	0	6	3
Dummy	0	0	0	0	0	0

Solution

- Check for optimal allotment

	1	2	3	4	5	6	
A	4	~~2~~	~~7~~	~~14~~	~~10~~	[0]	
B	[0]	8	15	5	6	2	✓
C	~~8~~	~~7~~	[0]	~~5~~	~~2~~	~~6~~	
D	~~8~~	8	4	7	7	6	✓
E	~~1~~	~~5~~	~~4~~	[0]	~~6~~	~~3~~	
Dummy	~~8~~	[0]	~~8~~	~~8~~	~~8~~	~~8~~	

- Iterate towards optimal solution

	1	2	3	4	5	6
A	6	2	7	14	10	0
B	0	6	13	3	4	0
C	10	7	0	5	2	6
D	0	6	2	5	5	4
E	3	5	4	0	6	3
Dummy	2	0	0	0	0	0

Second feasible solution

Solution

- Check for optimal allotment

	1	2	3	4	5	6	
A	6	2	7	14	10	[0]	✓
B	[0]	6	13	3	4	~~0~~	✓
C	~~10~~	~~7~~	[0]	~~5~~	~~2~~	~~6~~	
D	~~0~~	6	2	5	5	4	✓
E	~~3~~	~~5~~	~~4~~	[0]	~~6~~	~~3~~	
Dummy	~~2~~	[0]	~~0~~	~~0~~	~~0~~	~~0~~	
	✓					✓	

- Iterate towards optimal solution

	1	2	3	4	5	6
A	6	0	5	12	8	0
B	0	4	11	1	2	0
C	12	7	0	5	2	8
D	0	4	0	3	3	4
E	5	5	4	0	6	5
Dummy	4	0	0	0	0	2

Third feasible solution

Solution

	1	2	3	4	5	6
A	6	[0]	5	12	8	~~0~~
B	~~0~~	4	11	1	2	[0]
C	12	7	[0]	5	2	8
D	[0]	4	~~0~~	3	3	4
E	5	5	4	[0]	6	5
Dummy	4	~~0~~	~~0~~	~~0~~	[0]	2

Since there is no row and no column without assignment, the third feasible solution is the optimal solution. The optimal assignment pattern is

city A should supply the vehicle to city 2,
city B should supply the vehicle to city 6,
city C should supply the vehicle to city 3,
city D should supply the vehicle to city 1,
city E should supply the vehicle to city 4, and
minimum distance travelled = (10 + 12 + 3 + 6 + 7) km = 38 km.
No truck is supplied to city 5.

Maximization problem

A company has a team of four salesmen and there are four districts where the company wants to start its business. After taking into account the capabilities of salesmen and the natrue of districts, the company estimates that the profit per day in rupees for each salesman in each district is as below.

Salesman	District 1	2	3	4
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

Find the assignment of salesmen to various districts which will yield maximum profit.

Solution

- Convert maximization problem to minimization by subtracting all thee elements of the matrix from the largest elements.

	1	2	3	4
A	0	6	2	5
B	2	5	1	1
C	1	1	3	4
D	3	4	2	1

Opportunity loss matrix (minimization problem)

- Reduce the matrix

	1	2	3	4
A	0	6	2	5
B	1	4	0	0
C	0	0	2	3
D	2	3	1	0

Matrix after substep I (initial feasible solution)

Solution

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- Check if optimal assignment can be made.

	1	2	3	4
A	[0]	6	2	5
B	1	4	[0]	~~0~~
C	~~0~~	[0]	2	3
D	2	3	1	[0]

salesman A should be assigned district 1,
salesman B should be assigned district 3,
salesman C should be assigned district 2,
salesman D should be assigned district 4.
Maximum profit per day = ₹ (16 + 15 + 15 + 15) = ₹ 61.

Restriction on Assignment

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Four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. C_{ij}, the assignment cost of machine i to place j in rupees is shown below.

	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	–	5	4
M_3	–	6	9	6	2
M_4	9	3	7	2	3

Find the optimal assignment schedule.

Solution

- Prepare a square matrix

	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	∞	5	4
M_3	∞	6	9	6	2
M_4	9	3	7	2	3
Dummy machine	0	0	0	0	0

- Prepare a square matrix

	A	B	C	D	E
M_1	0	2	6	1	2
M_2	3	0	∞	1	0
M_3	∞	4	7	4	0
M_4	7	1	5	0	1
Dummy	0	0	0	0	0

Matrix after substep 1

Solution

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- Check for optimality

	A	B	C	D	E
M_1	[0]	2	6	1	2
M_2	3	[0]	∞	1	~~0~~
M_3	∞	4	7	4	[0]
M_4	7	1	5	[0]	1
Dummy	~~0~~	~~0~~	[0]	~~0~~	~~0~~

Machine M_1 to place A,
machine M_2 to place B,
machine M_3 to place E,
machine M_4 to place D,
and place C will remain vacant.
Total assignment cost = ₹ (4 + 4 + 2 + 2) = ₹ 12.

Travelling Salesman Problem

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- There are a number of cities a salesman must visit. The distance (or time and cost) between every pair of cities is known. He starts from his home city, passes through each city once and only once and return to his home city. The problem is to find the routes shortest in distance (or time or cost).

If the salesman is to visit only two cities there is, of course, no choice. If the number of cities is three (A, B and C), of which the starting base is A, there are two possible routes $A \rightarrow B \rightarrow C$ and $A \rightarrow C \rightarrow B$. For four cities, there are 6 possible routes:

$A \rightarrow B \rightarrow C \rightarrow D$, $A \rightarrow B \rightarrow D \rightarrow C$, $A \rightarrow C \rightarrow B \rightarrow D$, $A \rightarrow C \rightarrow D \rightarrow B$, $A \rightarrow D \rightarrow B \rightarrow C$ and $A \rightarrow D \rightarrow C \rightarrow B$.

Numerical

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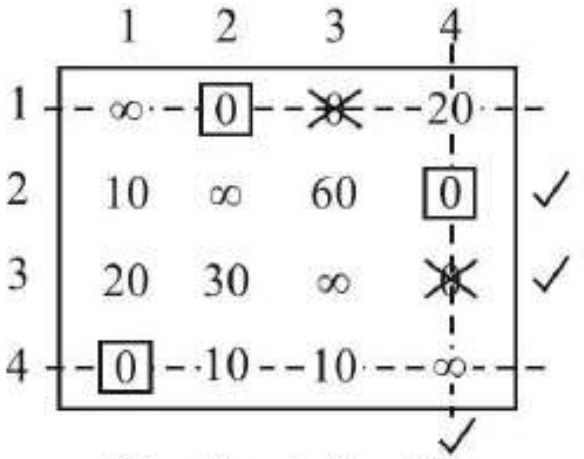
A salesman wants to visit cities 1, 2, 3 and 4. He does not want to visit any city twice before completing the tour of all the cities and wishes to return to his home city, the starting station. Cost of going from one city to another in rupees is given in table 4.202. Find the least cost route.

		To city			
		1	2	3	4
From city	1	0	30	80	50
	2	40	0	140	30
	3	40	50	0	20
	4	70	80	130	0

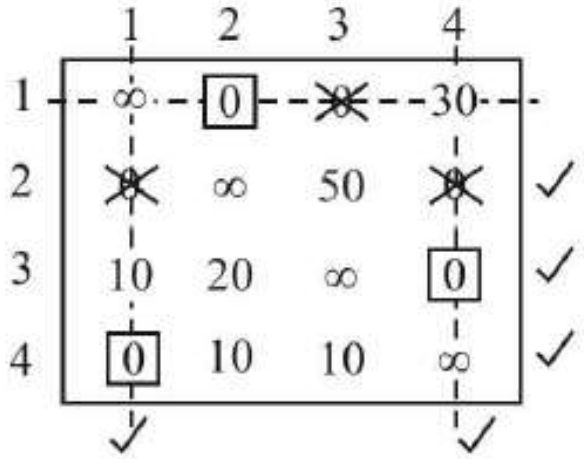
Solution

	1	2	3	4
1	∞	0	0	20
2	10	∞	60	0
3	20	30	∞	0
4	0	10	10	∞

Reduced matrix with zero in every row and column



First basic feasible solution



Second basic feasible solution

	1	2	3	4
1	∞	∞	0	40
2	0	∞	40	∞
3	10	10	∞	0
4	∞	0	∞	∞

Optimal solution

$$\text{city } 1 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

Numerical

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A salesman wants to visit cities A, B, C, D and E. He does not want to visit any city twice before completing his tour of all the cities and wishes to return to the point of starting journey.

	A	B	C	D	E
A	0	2	5	7	1
B	6	0	3	8	2
C	8	7	0	4	7
D	12	4	6	0	5
E	1	3	2	8	0

Solution

	A	B	C	D	E
A	∞	1	3	6	0
B	4	∞	0	6	0
C	4	3	∞	0	3
D	8	0	1	∞	1
E	0	2	0	7	∞

	A	B	C	D	E
A	∞	1	3	6	[0]
B	4	∞	[0]	6	~~0~~
C	4	3	∞	[0]	3
D	8	[0]	1	∞	1
E	[0]	2	~~0~~	7	∞

$$A \rightarrow E,\ E \rightarrow A,\ B \rightarrow C,\ C \rightarrow D,\ D \rightarrow B$$

Solution

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- Case 1

	A	B	C	D	E
A	∞	[1]	3	6	~~∞~~
B	4	∞	[0]	6	~~∞~~
C	4	3	∞	[0]	3
D	8	~~∞~~	1	∞	[1]
E	[0]	2	~~∞~~	7	∞

$A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow E, E \rightarrow A$

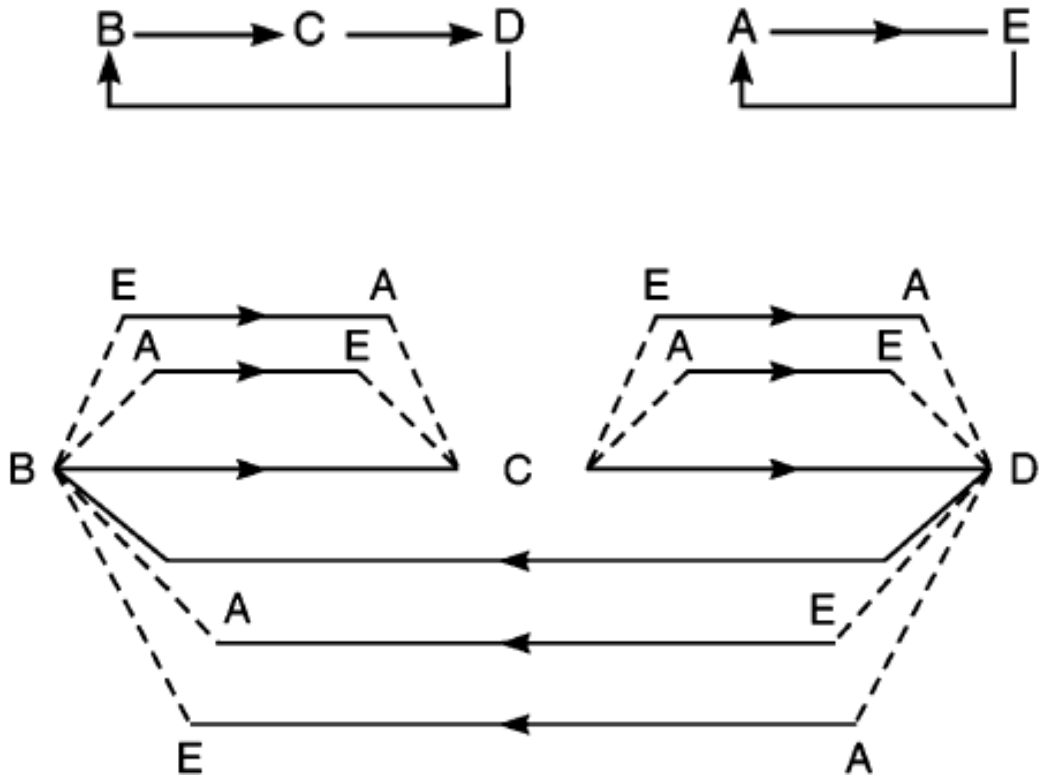
- Case 2

	A	B	C	D	E
A	∞	[1]	3	6	~~∞~~
B	4	∞	~~∞~~	6	[0]
C	4	3	∞	[0]	3
D	8	~~∞~~	[1]	∞	1
E	[0]	2	~~∞~~	7	∞

$A \rightarrow B \rightarrow E \rightarrow A, C \rightarrow D \rightarrow C,$

Solution

- Method 2



BA and EC : 4 + 0 = 4,
BE and AC : 0 + 3 = 3,
CA and ED : 4 + 7 = 11,
CE and AD : 3 + 6 = 9,
DE and AB : 1 + 1 = 2,
DA and EB : 8 + 2 = 10.

$B \rightarrow C \rightarrow D \rightarrow E \rightarrow A \rightarrow B$

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Unit 4

Inventory Control

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- Inventory consists of usable but idle resources.
- Necessity of inventory
  - Smooth and efficient running of enterprise
  - Provides service at very short notice.
  - In absence, need to pay high price.
  - Reduces product cost
  - Acts as buffer stock
  - Helps in maintaining economy

Classification of inventory

- Direct Inventory
  - Product inventory
  - Work in process inventory
  - Finished good inventory
  - MRO inventory
  - Miscellaneous inventory
- Indirect inventory
  - Transit or pipeline inventory
  - Buffer inventory
  - Decoupling inventory
  - Seasonal inventory
  - Lot size inventory
  - Anticipation inventory

Inventory Cost

- Purchase cost
- Inventory carrying cost / stock holding cost
  - Cost of money
  - Cost of storage
  - Depreciation cost
  - Pilferage cost
  - Obsolesce cost
  - Handling cost
  - Record keeping cost
  - Taxes and Insurance
- Procurement cost
- Shortage cost

Inventory Control Problem

- When to order ?
- How much to order ?
- How much safety stock should be kept ?

Inventory Model with Deterministic Demand

- A Stockist has to order to supply goods to customer at a uniform rate R per unit time.
- Demand is fixed and known.
- No shortage allowed , C_2 is infinity
- Time t is fixed
- Ordering cost per order is C_3.

Numerical

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A stockist has to supply 12,000 units of a product per year to his customer. The demand is fixed and known and the shortage cost is assumed is to be infinite. The inventory holding cost is ₹ 0.20 per unit per month and the ordering cost per order is ₹ 350. Determine

(i) The optimum lot size q_0

(ii) optimum scheduling period t_0,

(iii) minimum total variable yearly cost.

Supply rate, $R = \frac{12,000}{12} = 1,000$ units/month,

$C_1 =$ ₹ 0.20 per unit per month, $C_3 =$ ₹ 350 per order.

(*i*) $q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 350 \times 1,000}{0.20}} = 1,870$ units/order.

(*ii*) $t_0 = \sqrt{\frac{2C_3}{C_1R}} = \sqrt{\frac{2 \times 350}{0.20 \times 1,000}} = 1.87$ months $= 8.1$ weeks between orders.

(*iii*) $C_0 = \sqrt{2C_1C_3R} = \sqrt{2 \times .20 \times 12 \times 350 \times (1,000 \times 12)} =$ ₹ 4,490 per year.

Numerical

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A particular item has a demand of 9,000 units/year. The cost of one procurement is ₹100 and the holding cost per unit is ₹ 2.40 per year. The replacement is instantaneous and no shortages are allowed. Determine

(i) the economic lot size,
(ii) the number of orders per year,
(iii) the time between orders,
(iv) the total cost per year if the cost of one unit is ₹ 1.

Solution

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$R = 9{,}000$ units/year,

$C_3 = ₹\ 100$/procurement, $C_1 = ₹\ 2.40$/unit/year.

(*i*) $q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 100 \times 9{,}000}{2.40}} = 866$ units/procurement.

(*ii*) $n_0 = \frac{1}{t_0} = \sqrt{\frac{C_1R}{2C_3}} = \sqrt{\frac{2.40 \times 9{,}000}{2 \times 100}} = \sqrt{108} = 10.4$ orders/year.

(*iii*) $t_0 = \frac{1}{n_0} = \frac{1}{10.4} = 0.0962$ years $= 1.15$ months between procurement.

(*iv*) $C_0 = 9{,}000 \times 1 + \sqrt{2C_1C_3R}$

$= 9{,}000 + \sqrt{2 \times 2.40 \times 100 \times 9{,}000}$

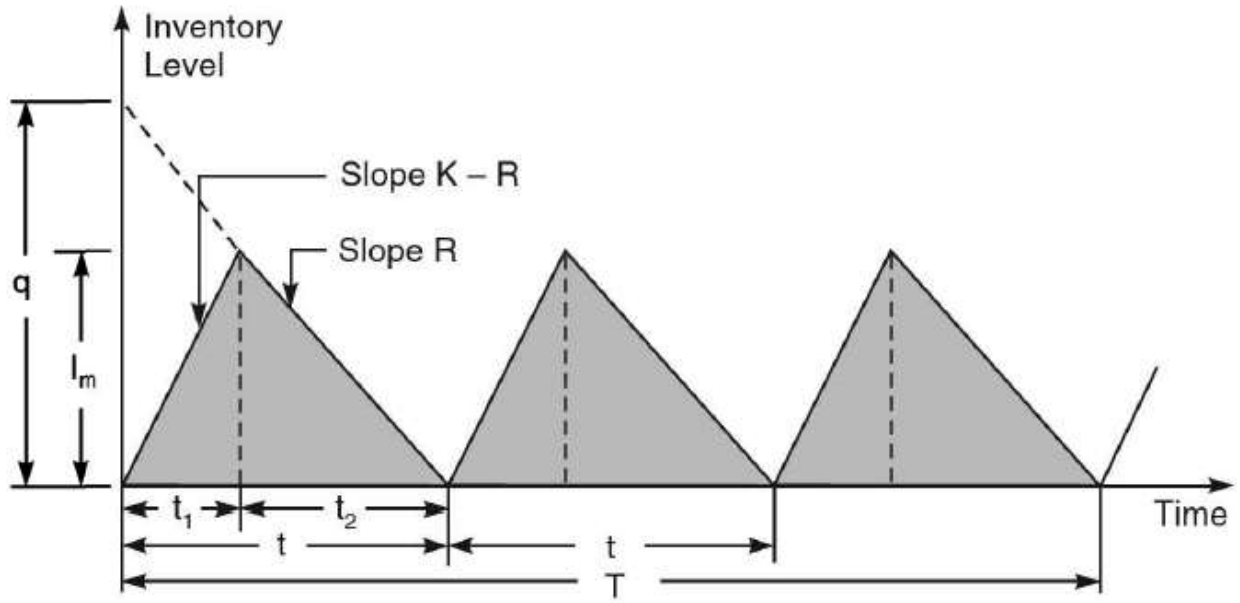
$= 9{,}000 + 2{,}080 = ₹\ 11{,}080$/year.

Inventory Model with Deterministic Demand

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- Demand rate uniform, Production rate finite

R = number of items required per unit time,
K = number of items produced per unit time,
C_1 = cost of holding per item per unit time,
C_3 = cost of setting up a production run,
q = number of items produced per run, $q = Rt$,
t = interval between runs.



Numerical

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A company has a demand of 12,000 units/year for an item and it can produce 2,000 such items per month. The cost of one setup is ₹ 400 and the holding cost/unit/month is ₹ 0.15. Find the optimum lot size and the total cost per year, assuming the cost of 1 unit as ₹ 4. Also find the maximum inventory, manufacturing time and total time.

$R = 12{,}000$ units/year,

$K = 2{,}000 \times 12 = 24{,}000$ units/year,

$C_3 = ₹\ 400$/setup,

$C_1 = ₹\ 0.15 \times 12 = ₹\ 1.80$/unit/year.

$$q_0 = \sqrt{\frac{K}{K-R}} \cdot \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{24{,}000}{24{,}000-12{,}000}} \cdot \sqrt{\frac{2 \times 400 \times 12{,}000}{1.80}}$$

$$= 3{,}264 \text{ units/setup.}$$

$$C_0 = 12{,}000 \times 4 + \sqrt{2C_1C_3R} \cdot \sqrt{\frac{K-R}{K}}$$

$$= 48{,}000 + \sqrt{2 \times 1.80 \times 400 \times 12{,}000 \times \frac{12{,}000}{24{,}000}}$$

$$= 48{,}000 + 2{,}940 = ₹\ 50{,}940/\text{ year.}$$

$$I_{m_0} = \frac{K-R}{K} \cdot q_0 = \frac{24{,}000-12{,}000}{24{,}000} \times 3{,}264 = 1{,}632 \text{ units.}$$

$$t_1 = \frac{I_{m0}}{K-R} = \frac{1{,}632}{12{,}000} = 0.136 \text{ years.}$$

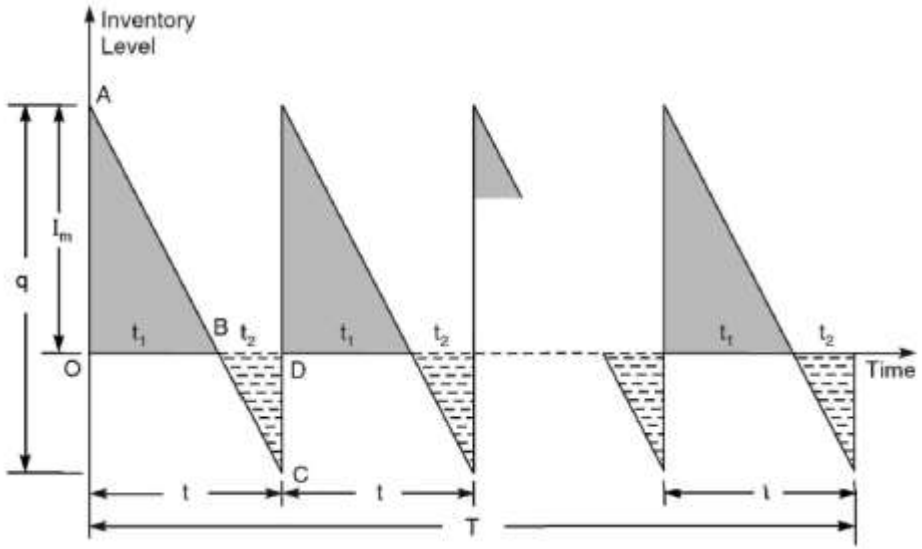
$$t_0 = \frac{q_0}{R} = \frac{3{,}264}{12{,}000} = 0.272 \text{ years.}$$

Inventory Model with Deterministic Demand

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- Demand Rate Uniform, Replenishment Rate Infinite, Shortage Allowed

R = number of items required per unit time *i.e.*, demand rate,
C_1 = cost of holding the item per unit time,
C_2 = shortage cost per item per unit time,
C_3 = ordering cost/order,
q = number of items ordered in one order,
$q = Rt$,
t = interval between orders,
I_m = number of items that form inventory at the beginning of time interval t.

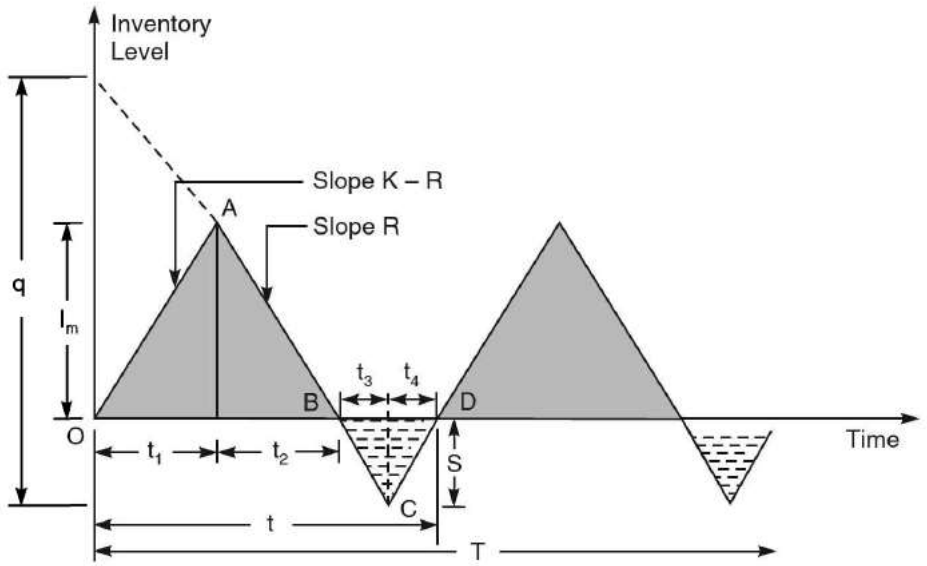


Inventory Model with Deterministic Demand

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- Demand Rate Uniform, Production Rate Finite, Shortage Allowed

R = number of items required per unit time *i.e.*, demand rate,
C_1 = cost of holding the item per unit time,
C_2 = shortage cost per item per unit time,
C_3 = ordering cost/order,
q = number of items ordered in one order,
$q = Rt$,
t = interval between orders,
I_m = number of items that form inventory at the beginning of time interval



Inventory Model with Deterministic Demand

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A particular item has a demand of 9,000 units/years. The cost of one procurement is Rs 100 and holding cost per unit is Rs. 2.40 per year. The replacement is instantaneous and shortage cost is Rs. 5 per unit per year. Determine

i. The economic lot size

ii. The number of order per year

iii. The time between orders

iv. The total cost per year if the cost of one unit is Rs. 1

$$q_0 = \sqrt{\frac{C_1 + C_2}{C_2}} \cdot \sqrt{\frac{2C_3 R}{C_1}} = \sqrt{\frac{2.40 + 5}{5}} \cdot \sqrt{\frac{2 \times 100 \times 9,000}{2.40}}$$

$$= \sqrt{11,10,000} = 1,053 \text{ units/procurement.}$$

$$n_0 = \frac{9,000}{1,053} = 8.55.$$

$$C_0(I_m, q) = 9,000 \times 1 + \sqrt{\frac{C_2}{C_1 + C_2}} \sqrt{2C_1C_3R}$$

$$t_0 = \frac{1}{n_0} = \frac{1}{8.55} = 0.117 \text{ year} = 1.4 \text{ months.}$$

Replacement Model

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- All equipment's gets worn with time and usage and its functions with decreasing efficiency
- Items which do not detoriate with time but suddenly fails
- Items get obsolete due to new discoveries and better design
- Staff in an organisation that gradually decreases due to death, retirement and other reasons.

Types of Failure

- Gradual Failure
  - Gradual failure is progressive in nature it results in
  - Increase running cost
  - Decreased productivity
  - Decreased resale or scrap value
- Sudden Failure
  - Progressive Failure
  - Retrogressive Failure
  - Random Failure

Placement of item that deteriorate

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- Replacement of item whose maintenance and repair costs increased with time, ignoring change in the value of money during the period.

- The maintenance cost and resale value per year of a machine whose purchase price is Rs. 7000 is given below.

Year	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Maintenance cost in ₹	*900*	*1,200*	*1,600*	*2,100*	*2,800*	*3,700*	*4,700*	*5,900*
Resale value in ₹	*4,000*	*2,000*	*1,200*	*600*	*500*	*400*	*400*	*400*

- When should the machine be replaced ?

Solution

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(1) *Years of service*	(2) *Resale value* (₹)	(3) *Purchase price–resale value (C – S)* (₹)	(4) *Annual maintenance cost* $f(t)$ (₹)	(5) *Summation of maintenance cost* $\sum_{t=0}^{n} f(t)$ (₹)	(6) *Total cost* $\left[C - S + \sum_{t=0}^{n} f(t)\right]$ (₹)	(7) *Average annual cost* $\frac{1}{n}\left[C - S + \sum_{t=0}^{n} f(t)\right]$ (₹)
1	4,000	3,000	900	900	3,900	3,900
2	2,000	5,000	1,200	2,100	7,100	3,550
3	1,200	5,800	1,600	3,700	9,500	3,166.67
4	600	6,400	2,100	5,800	12,200	3,050
5	500	6,500	2,800	8,600	15,100	**3,020**
6	400	6,600	3,700	12,300	18,900	3,150
7	400	6,600	4,700	17,000	23,600	3,371.43
8	400	6,600	5,900	22,900	29,500	3,687.50

We observe from the table that average annual cost is minimum (₹ 3,020) in the 5th year. Hence the machine should be replaced at the end of 5 years of service.

Placement of item that deteriorate

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- Replacement of item whose maintenance and repair costs increased with time, value of money also changes with time.

A machine costs ₹ 500. Operation and maintenance costs are zero for the first year and increase by ₹ 100 every year. If money is worth 5% every year, determine the best age at which the machine should be replaced. The resale value of the machine is negligibly small. What is the weighted average cost of owning and operating the machine?

Discount rate,

$$v = \frac{1}{1+r} = \frac{1}{1+0.05} = 0.9524.$$

Solution

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(1) Years of service (r)	(2) Maintenance cost (R_r) (₹)	(3) Discount factor (v^{r-1})	(4) Discounted cost ($R_r v^{r-1}$) (₹)	(5) Discounted total cost $C+\sum_{r=1}^{n} R_r v^{r-1}$ (₹)	(6) Cumulative discount factor $\sum_{r=1}^{n} v^{r-1}$	(7) Weighted average annual cost $\frac{C+\sum_{r=1}^{n} R_r v^{r-1}}{\sum_{r=1}^{n} v^{r-1}}$ (₹)
1	0	1.0000	0.00	500.00	1.0000	500.00
2	100	0.9524	95.24	595.24	1.9524	304.88
3	200	0.9070	181.40	776.64	2.8594	**217.61 Replace**
4	300	0.8638	259.14	1,035.78	3.7232	278.20
5	400	0.8227	329.08	1,364.86	4.5459	300.25

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Unit 5

Project Planning and Management

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- Project
  - Combination of interrelated activities which must be executed in a certain order before entire task is completed.

- Salient Characteristics of a project
  - Project must have beginning and end
  - It is not permanent entity
  - It is schedule to be completed by a target date.
  - Objective are clear and output or end product is define
  - It involves heavy investment
  - Execution of project activities and hence completion of the project is always subjected to some uncertainties and risk.

Basic requirement of project

- It should be completed without any delay.
- It should use as small man power and other resources as possible
- It should involve as small investment as possible.

Project Management

- Project Planning
- Project Scheduling
- Project Controlling

Project Planning

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Project planning involves the following steps:

1. Setting the objectives of the project and the assumptions to be made.
2. Developing the W.B.S. (*Work Breakdown Structure*). Depending upon the objectives of the management, the extent of control desired and the availability of computational aids, the project is broken down into clearly definable activities.
3. Determination of time estimates of these activities.
4. Estimation of resources–financial, managerial and operational.
5. Study of alternative ways of attaining the goals.
6. Establishment of inter-dependence relationship between the activities *i.e.*, the sequence of performing the activities.

Project Scheduling

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1. Start and finish times of each activity and the earliest and latest times at which events can occur.
2. Critical activities that require special attention.
3. Allocating resources—men, machines, materials, etc. to each activity.
4. Slacks and floats for non-critical activities.
5. Various constraints due to limitation of resources.

Project Controlling

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1. Setting standards and targets with regard to time and cost of the project.
2. Reviewing the progress by comparing the work accomplished to the work scheduled at different stages of time and finding the deviations.
3. Evaluating the effect of deviations on the project plan.
4. Updating the project schedule.
5. The corrective measures to rectify the deviations from the plan should be suggested. This requires decision-making with regard to scheduling of resources, scheduling of jobs, crashing of projects, etc.

Network Techniques

- Program Evaluation and Review Technique (PERT)
  - It is preferred for the project that are non – repetitive.
  - There is no significant past experience to guide
- Critical Path Method (CPM)
  - It consists of number of well recognised jobs , tasks or activities
  - It is generally used for simple, repetitive types of projects
  - It is deterministic model than a probabilistic model.

Network

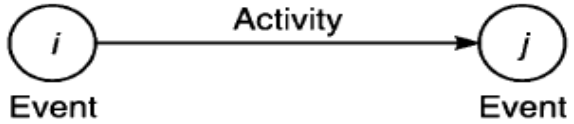
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Activity

It is physically identifiable part of a project which requires time and resources for its execution. An activity is represented by an arrow, the tail of which represents the start and the head, the finish of the activity. The length, shape and direction of the arrow has no relation to the size of the activity.

Event

The beginning and end points of an activity are called events or nodes. Event is a point in time and does not consume any resources. It is represented by a circle. The head event, called the jth event, has always a number higher than the tail event, called the ith event *i.e.*, $j > i$. For example



Path

An unbroken chain of activity arrows connecting the initial event to some other event is called a path.

Network

It is the graphical representation of logically and sequentially connected arrows and nodes representing activities and events of a project. Networks are also called *arrow diagrams*.

Dummy

An activity which only determines the dependency of one activity on the other, but does not consume any time is called a dummy activity. Dummies are usually represented by dotted line arrows.

Network Construction

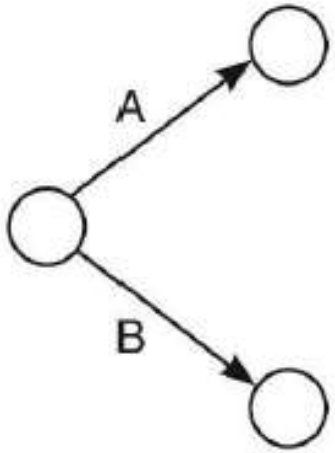
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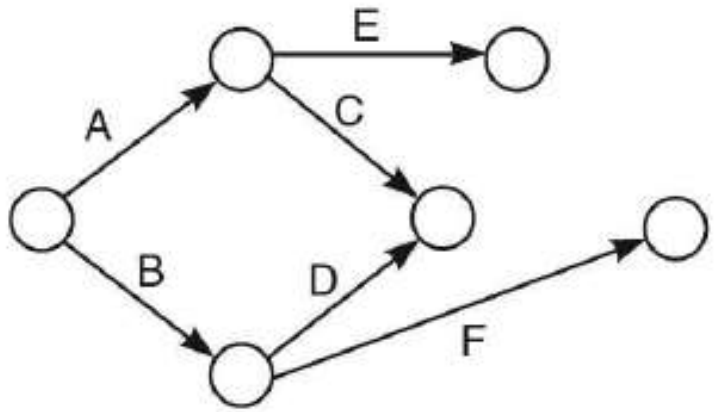
Draw a network for the simple project of erection of steel works for a shed. The various activities of the project are as under:

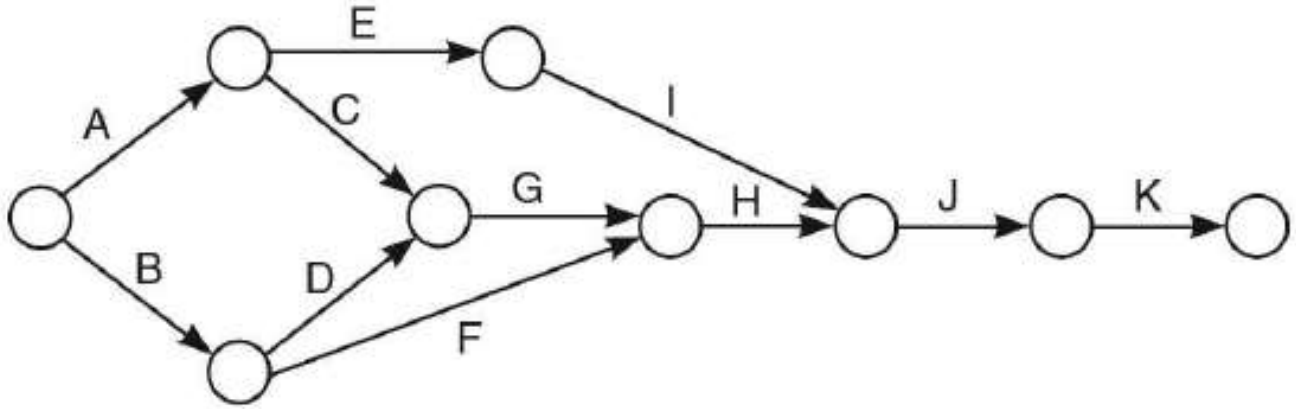
Activity	*Description*	*Preceded by*
A	*Erect site workshop*	—
B	*Fence site*	—
C	*Bend reinforcement*	*A*
D	*Dig foundation*	*B*
E	*Fabricate steel work*	*A*
F	*Install concrete pillars*	*B*
G	*Place reinforcement*	*C, D*
H	*Concrete foundation*	*G, F*
I	*Erect steel work*	*E*
J	*Paint steel work*	*H, I*
K	*Give finishing touch*	*J*

Solution

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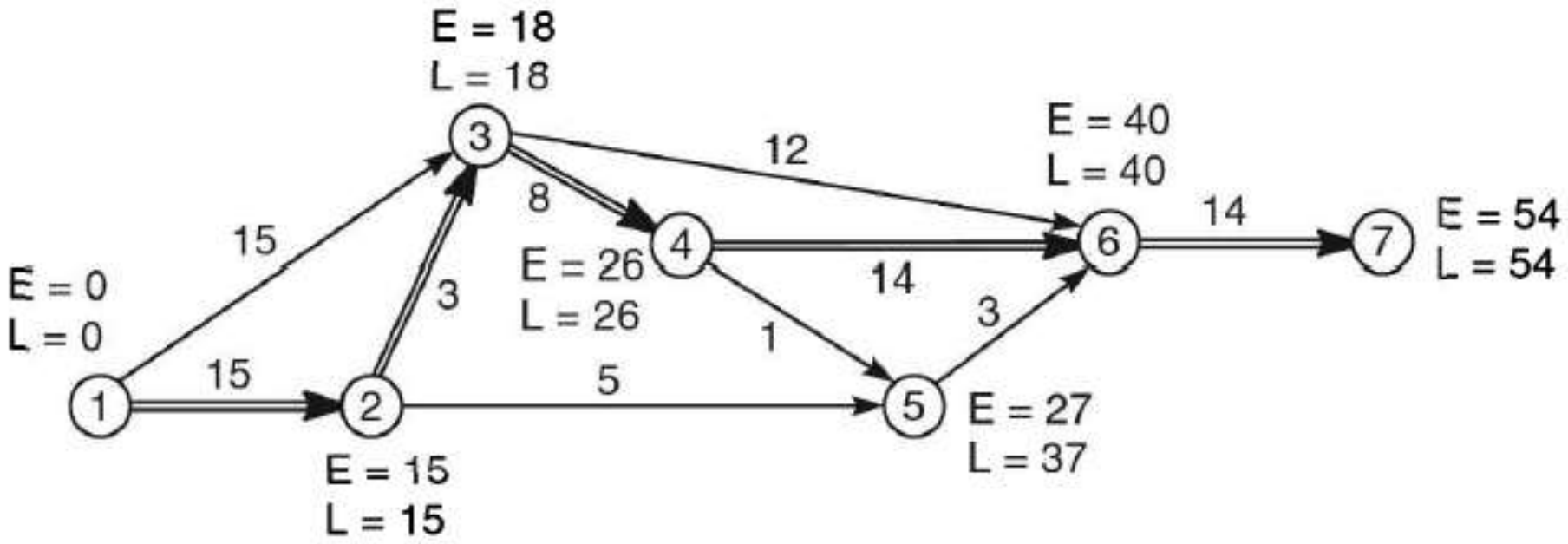




Critical Path Method

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Activity	:	1-2	1-3	2-3	2-5	3-4	3-6	4-5	4-6	5-6	6-7
Duration (weeks)	:	15	15	3	5	8	12	1	14	3	14



Solution

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Activity (i – j)	Duration (D)	Start time		Finish time		Total Float
		Earliest	Latest	Earliest	Latest	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1-2	15	0	0	15	15	0
1-3	15	0	3	15	18	3
2-3	3	15	15	18	18	0
2-5	5	15	32	20	37	17
3-4	8	18	18	26	26	0
3-6	12	18	28	30	40	10
4-5	1	26	36	27	37	10
4-6	14	26	26	40	40	0
5-6	3	27	37	30	40	10
6-7	14	40	40	54	54	0

Project Evaluation and Review Technique

(*i*) *The Optimistic Time Estimate (t_o or a)* : The shortest possible time required for the completion of an activity, if all goes extremely well. No provisions are made for delays or setbacks while estimating this time.

(*ii*) *The Pessimistic Time Estimate (t_p or b)* : The maximum possible time the activity will take if everything goes bad. However, major catastrophes such as earthquakes, floods, storms and labour troubles are not taken into account while estimating this time.

(*iii*) *The Most Likely Time Estimate (t_m or m)* : The time an activity will take if executed under normal conditions. It is the modal value. It is assumed that the situation is normal with usual setbacks or lapses.

Variance $$V = \sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$$

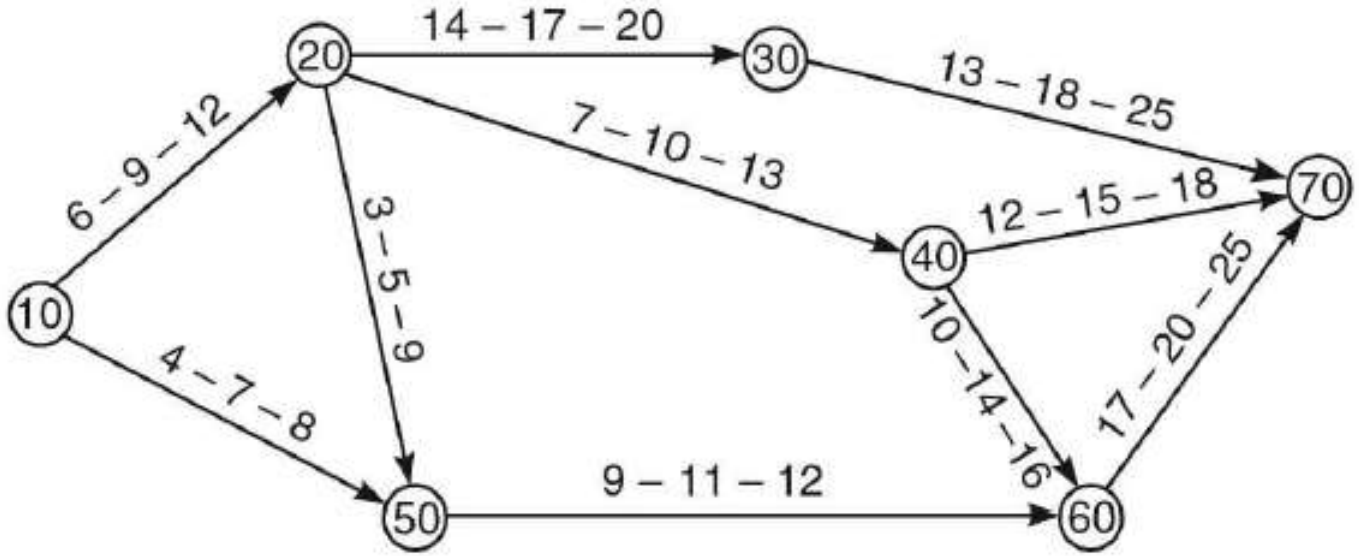
or $$\sigma = \frac{t_p - t_o}{6},$$

and $$\mu = \frac{t_0 + 4t_m + t_p}{6}.$$

$$t_e = \mu = \frac{t_0 + 4t_m + t_p}{6}.$$

Numerical

Consider the network shown Fig. 14.31. For each activity, the three time estimates t_0, t_m and t_p are given along the arrows in the $t_0 - t_m - t_p$ order. Determine variance and expected time for each activity.



Solution

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Activity i-j		t_o	t_m	t_p	$\sigma^2 = V = \left(\frac{t_p - t_0}{6}\right)^2$	$t_e = \frac{t_0 + 4t_m + t_p}{6}$
Predecessor event i	*Successor event j*					
10	20	6	9	12	1.00	9.0
10	50	4	7	8	0.44	6.7
20	30	14	17	20	1.00	17.0
20	40	7	10	13	1.00	10.0
20	50	3	5	9	1.00	5.33
30	70	13	18	25	4.00	18.33
40	60	10	14	16	1.00	13.67
40	70	12	15	18	1.00	15.00
50	60	9	11	12	0.25	10.83
60	70	17	20	25	1.78	20.33

Cost Analysis and Crashing the network

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- Project Cost
  - Direct Cost
  - Indirect Cost
- Crashing the network
- Time – Cost Trade off Procedure

Numerical

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The following table gives data on normal time and cost and crash time and cost for a project:

Activity	Normal		Crash	
	Time (days)	Cost (₹)	Time (Days)	Cost (₹)
1-2	6	60	4	100
1-3	4	60	2	200
2-4	5	50	3	150
2-5	3	45	1	65
3-4	6	90	4	200
4-6	8	80	4	300
5-6	4	40	2	100
6-7	3	45	2	80
		470		

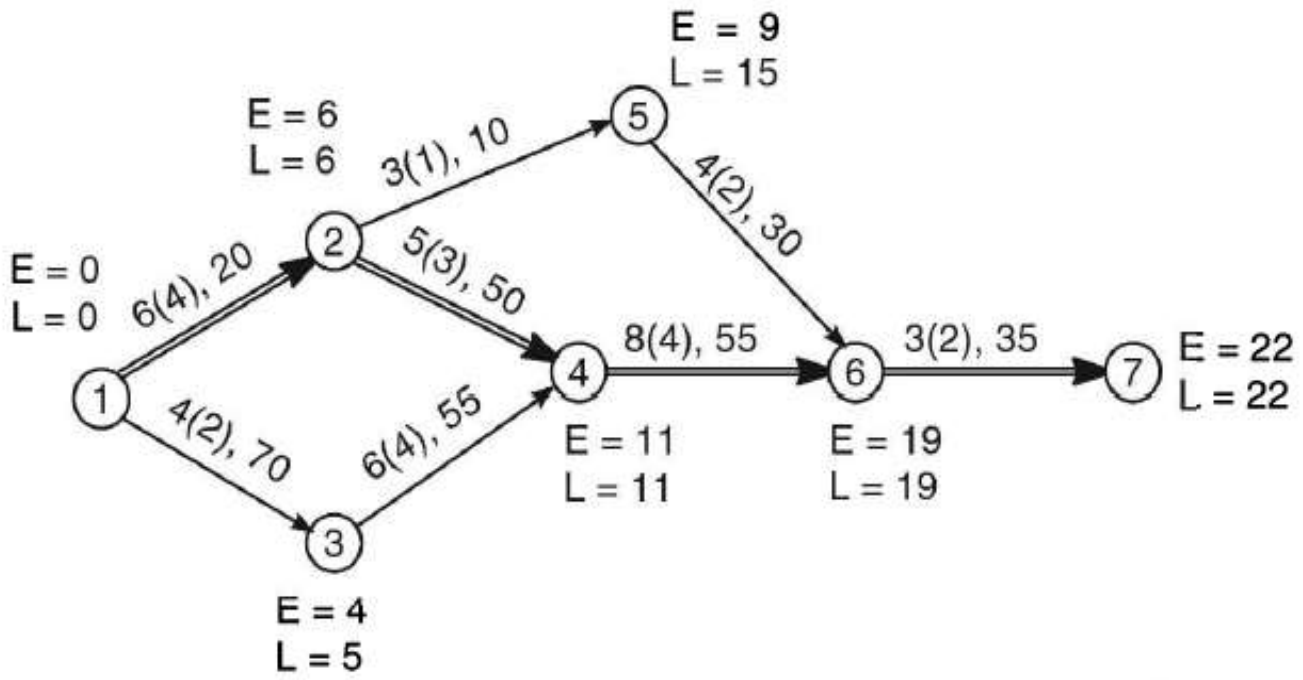
The indirect cost per day is ₹ 10.

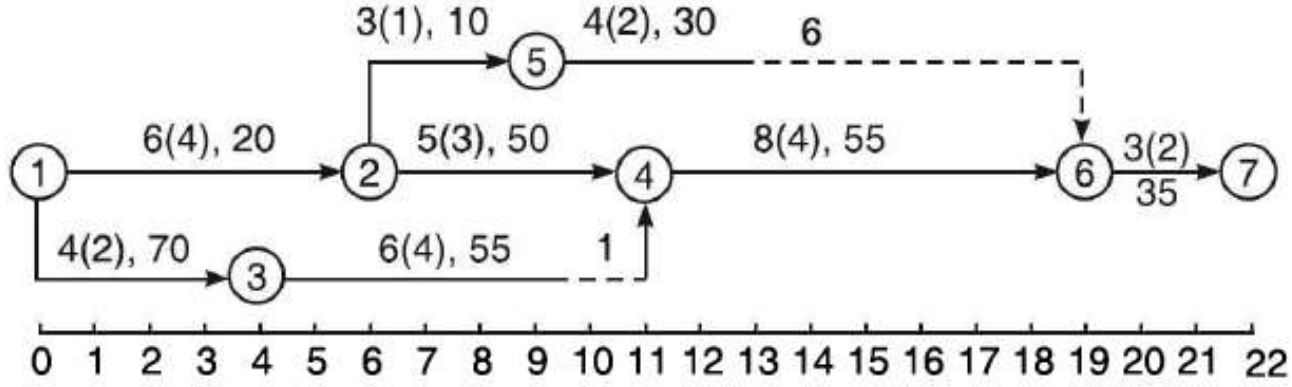
(i) Draw the network for the project.

(ii) Find the critical path.

(iii) Determine minimum total time and corresponding cost.

Solution


E = 9
L = 15
E = 6
L = 6
3(1), 10
5
4(2), 30
E = 0
L = 0
6(4), 20
2
5(3), 50
8(4), 55
3(2), 35
7
E = 22
L = 22
1
4(2), 70
6(4), 55
4
6
E = 11
L = 11
E = 19
L = 19
3
E = 4
L = 5


3(1), 10
4(2), 30
6
5
6(4), 20
5(3), 50
8(4), 55
3(2)
35
1
2
4
6
7
4(2), 70
6(4), 55
1
3
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

Solution

Crash activity 1-2 by 1 day.

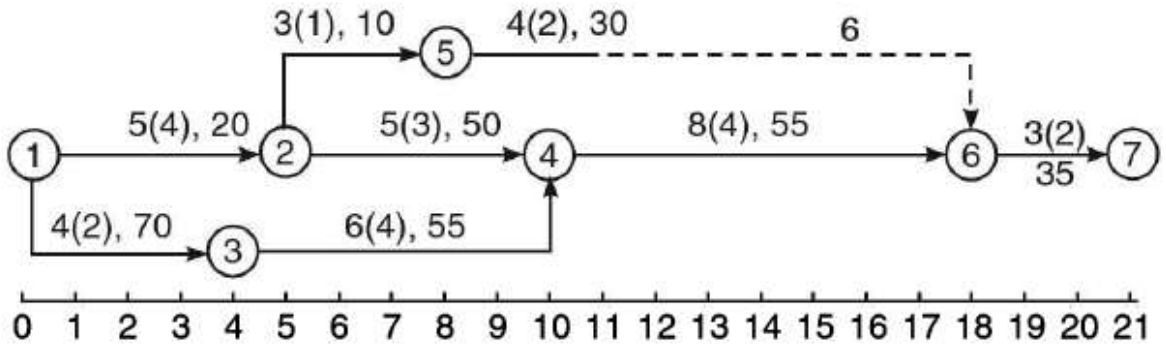
The various alternative activities and their crash costs are given below.

(a) Activity	*Cost (₹)*	*(b) Activity*	*Cost (₹)*	*(c) Activity*	*Cost (₹)*	*(d) Activity*	Cost *(₹)*
1-2	20	2-4	50	4-6	55	6-7	35
1-3/3-4	Nil	2-5/5-6	Nil	2-5/5-6	Nil		
		1-3/3-4	Nil				
	20		50		55		35

Since activity 1-2 has the lowest associated crash cost of ₹ 20 per day, it is crashed by one day.

∴ Crash cost = ₹ 20,

project duration = 21 days.



Solution

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Crash activity 6-7 by 1 day.

The various alternative activities and their crash costs are given below.

(a) Activity	*Cost (₹)*	*(b) Activity*	*Cost (₹)*	*(c) Activity*	*Cost (₹)*	*(d) Activity*	*Cost (₹)*
1-2	20	2-4	50	4-6	55	6-7	35
1-3/3-4	55	2-5/5-6	Nil	2-5/5-6	Nil		
		1-3/3-4	55				
	75		105		55		35

Since activity 6-7 has the lowest associated crash cost of ₹ 35 per day, it is crashed by one day.

∴ Crash cost = ₹ (20 + 35) ₹ 55,

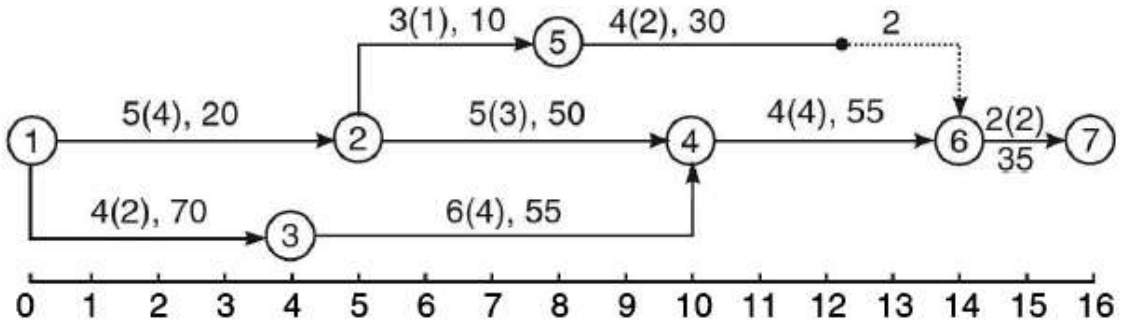
project duration = 20 days.

Crash activity 4-6 by 4 days.

As evident from the above table, next activity 4-6 has the lowest associated crash cost of ₹ 55 per day and as seen from Fig. 14.61 it can be crashed by 4 days.

∴ Crash cost = ₹ (55 + 4 × 55) = ₹ 275,

project duration = 16 days.



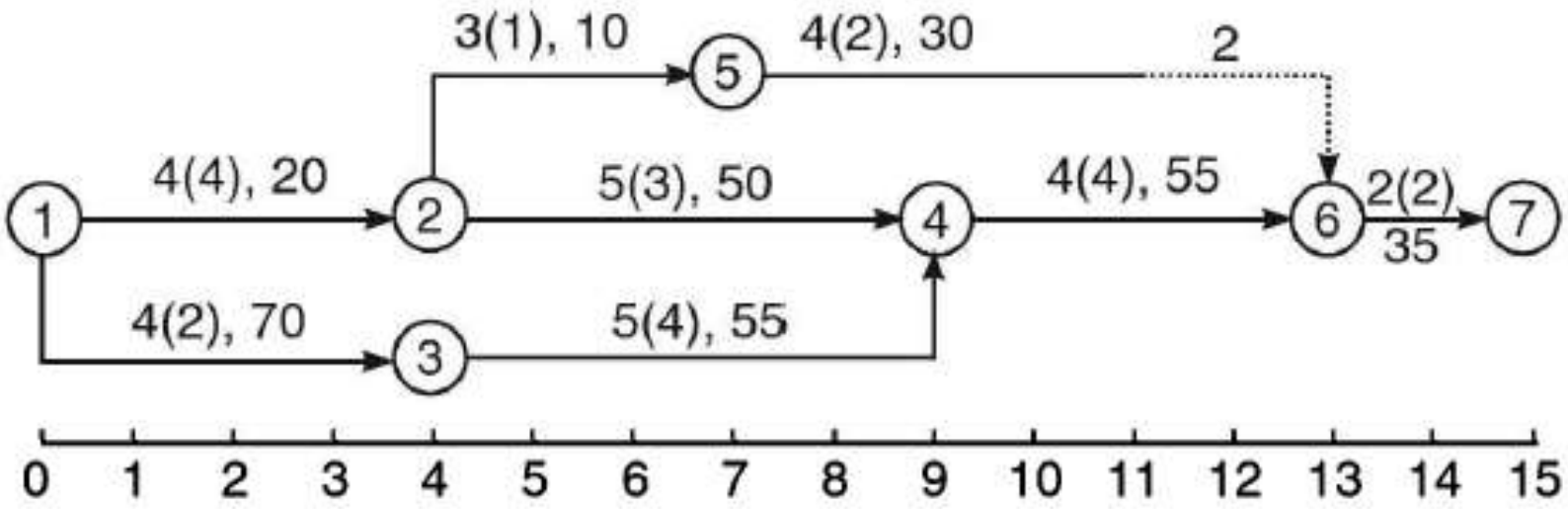
Solution

Crash activity 1-2 by 1 day.

Next, activity 1-2 is crashed by 1 day at a cost of ₹ 75 per day.

∴ Crash cost = ₹ (275 + 75) = ₹ 350,

project duration = 15 days.



Solution

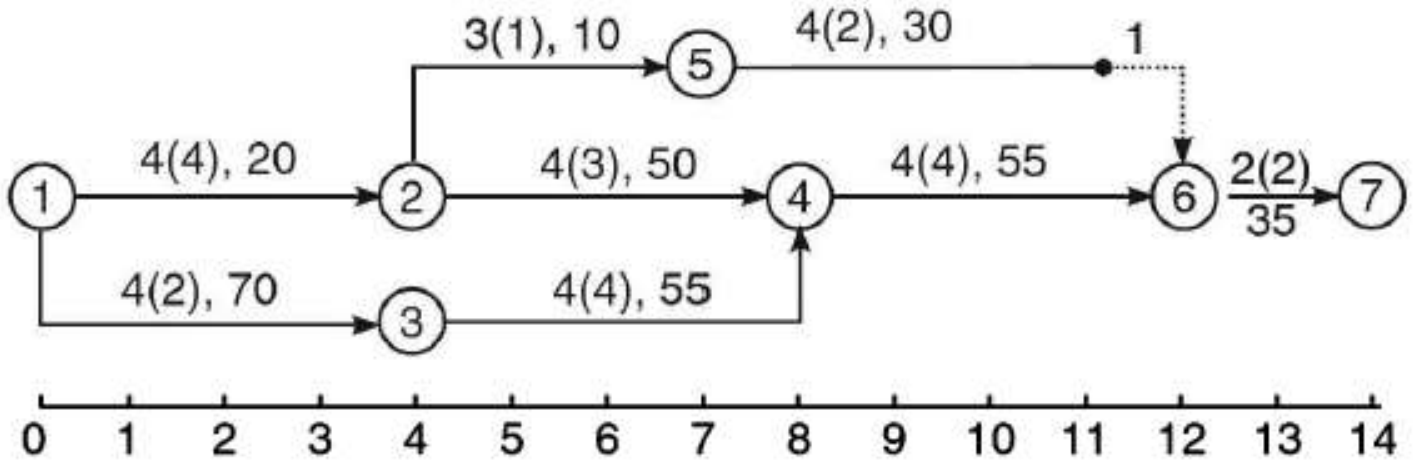
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Crash activity 2-4 by 1 day.

Now activity 2-4 is crashed by 1 day at a cost of ₹ 105 per day.

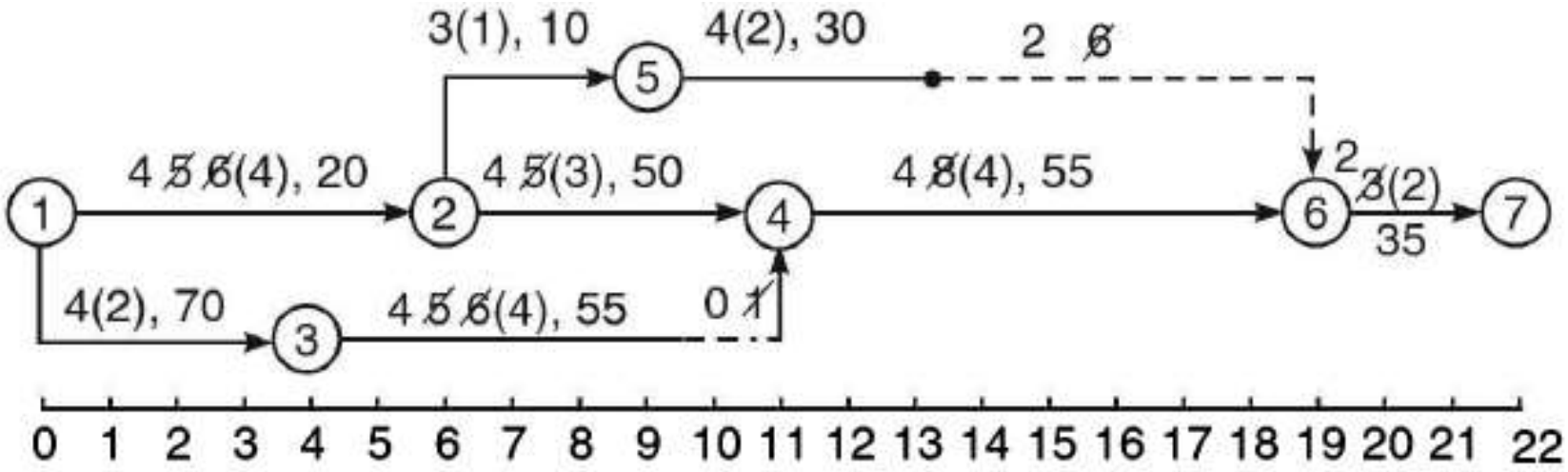
∴ Crash cost = ₹ (350 + 105) = ₹ 455,

project duration = 14 days.



Solution

- Complete crashing of the network from 22 days to 14 days can be represented in a single diagram.



Minimum total time = 14 days,

corresponding cost = ₹ (470 + 14 × 10 + 455) = ₹ 1,065.

Resource Levelling

- Variable Resource Levelling
- Fixed Resource Levelling
- Combined Resource Levelling

Updating of Network

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After 15 days of working the following progress is noted for the network of an erection job:

(a) Activities 1-2, 1-3 and 1-4 completed as per original schedule.

(b) Activity 2-4 is in progress and will be completed in 3 more days.

(c) Activity 3-6 is in progress and will need 18 days more for completion.

(d) Activity 6-7 appears to present some problem and its new estimated time of completion is 12 days.

(e) Activity 6-8 can be completed in 5 days instead of originally planned for 7 days.

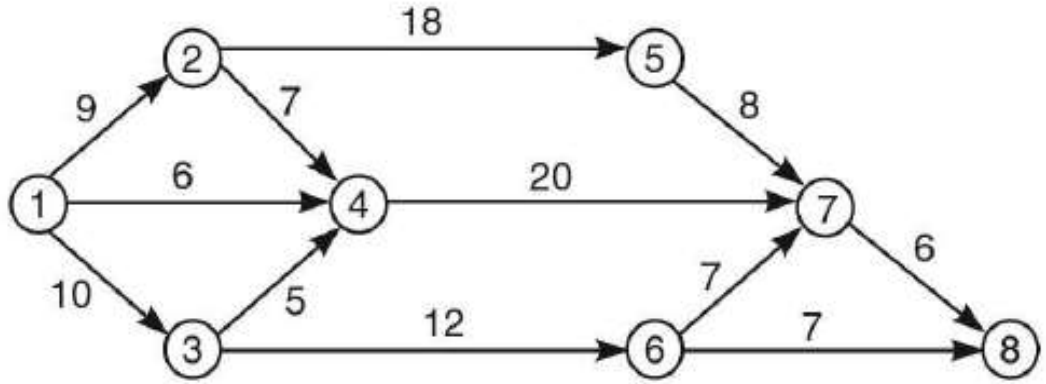


Fig. 14.101

Update the above network after 15 days of start of work. Find the critical path and project duration of the updated network.

Solution

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This problem will be solved by method 1. Activities 1-2, 1-3, 1-4 have already been completed and nothing needs to be done about them. Activities 2-4 and 3-6 are in progress; their revised durations will be calculated.

Revised duration of activity 2-4 = (15 + 3) – 9 = 9 days, and revised duration of activity 3-6 = (15 + 18) – 10 = 23 days.

Activities 6-7 and 6-8 have not started on 16th day; they are assigned the new estimated times of 12 days and 5 days respectively in the updated network. The durations of the remaining activities are kept unchanged in the updated network (Fig. 14.102).

